Reliability Analysis of Lifeline Networks Using Binary Decision Diagram

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Abstract

Disaster-induced damage and disruption to lifeline systems cause a variety of impacts, ranging from direct effects, such as physical damage and service supply interruption, to indirect impacts, such as forced relocation of community residents, threats to ongoing social and economic activity, and delays in the disaster recovery process. Recently, lifeline networks have been stressed by significant natural disasters such as earthquakes and human errors such as terrorist attacks leading to malfunction. Past events have evidenced the vulnerability of these growing networks threatening the business continuity and economy of several communities. Hence, characterization of the performance and reliability analysis of these networks is the key component to propose effective actions as a global civil defense program. In this paper, we extend the most recent method for computing two-terminal reliability based on Binary Decision Diagram (BDD), to reliability analysis of lifeline networks. The network reliability is defined as the probability that the nodes in the network can communicate to each other, taking into account the possible failures of network links. The effectiveness of this approach is demonstrated by performing experiment on a lifeline network.

Keywords: Lifeline Systems, Network Reliability, Binary Decision Diagram

1. Introduction

Lifeline systems, such as water distribution, gas, transportation, communication and power transmission systems are important to modern society. Unfortunately, lifeline systems are vulnerable to natural and manmade disasters. They have performed poorly during many previous events and their damage resulted in numerous losses and casualties. Therefore, the lifeline performance simulation under the threat environment, which can be measured by network reliability, is one of the important tasks in a global civil defense program. The lifeline network reliability has been an important research topic in research for past decades. Network reliability can be referred as the probability of success between two nodes connected by several links. Knowing the reliability of each link, one can determine the so called two-terminal reliability between nodes. The network reliability has been approached and resolved with different methods in the literature. Two kinds of computations exist for network reliability: exact, and

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approximate. The exact methods provide an exact reliability; in contrast, the simulation methods provide an approximate result. In literature, two classes of exact methods are often used to compute the network reliability. The first class deals with the enumeration of all minimal paths or cuts. A path is defined as a set of network components so that if these components are all reliable, the system is operational. A path is minimal if it has no proper subpaths. Conversely, a cut is set of network components such that if these components fail, the system is failed. If the probability of failure of nodes and links is known, the reliability expression can be calculated using different techniques, like inclusion-exclusion or sum of disjoint products (SDP) methods because this enumeration provides non-disjoint events. Numerous works related to this kind of analysis have been done in the literature, (Ahmad 1988, Locks 1987, and Hariri 1987). However, in minpaths and mincuts methods, as the number of links become large, the number of minimal paths and cuts will be large and SDPs or inclusion-exclusion expression for minpaths and mincuts will be increased too large to be stored and be processed.

In the second class, the algorithms are based on reducing process in graph topology. The first process consists of reducing the graph by replacing some special substructures by smaller ones. The replaced substructures can be elementary, such as two adjacent links (series-parallel reductions), or more complex as a subgraph (polygon-to-chain, Choi & Jun 1995), and delta-to-star reductions, (Gadani 1981). Thee reductions allow us to compute the reliability of series-parallel networks in linear time; the reductions are recursively applied until resulting in a single link. Nevertheless, for general networks, such substructure can not be found. In this case, the factoring process is applied. The idea is to choose a component, and decompose the problem into two sub-problems: the first assumes the component has failed, and the second assumes it is functioning. Satarayana & Chang (1983), and Wood (1985), have shown that the factoring algorithms with reductions are more efficient at solving this problem than the classic path or cut enumeration methods. In factoring algorithm the number of factored reliability graphs will increase exponentially with the number of links increases thereby computational time becomes prohibitive for large networks.

The increased complexity of the lifeline networks requires new analytical methods to evaluate their behavior. In the last decades, Binary Decision Diagrams (BDD) has provided an extraordinary efficient method to represent and manipulate Boolean functions. Akers (1978) proposed BDD as a powerful representation of truth tables with an easy implementation. Since his work, there has been a tremendous amount of literature in which BDD have been utilized for different applications. Bryant (1986) applied the BDD concept to Boolean function manipulation. Rauzy (1993), Coudert & Madre (1994) and Sinnamon & Andrews (1997) applied BDD to reliability analysis of fault trees. In this study, we apply the BDD to network reliability analysis of lifeline networks to achieve the exact terminal-pair reliability.

2. Binary Decision Diagram

A BDD is a directed acyclic graph (DAG) based on Shannon’s decomposition. If \( f \) is a Boolean function on variables \( x_1, x_2, \ldots, x_n \), the Shannon’s decomposition is defined as follows:

\[
f = x_1 f_{x_1 0} + \overline{x}_1 f_{x_1 1}
\]
\[ f = x.f_{x=1} + \overline{x}.f_{x=0} \]  

(1)

where \( x \) = decision variable and, \( f \) is denoted as \( f_{x=1} \) when \( x \) is true \((x = 1)\), and \( f_{x=0} \) when \( x \) is false \((x = 0)\). By choosing a total order over the variables, and applying recursively the Shannon’s decomposition, the truth table of any Boolean function can be graphically represented as a binary decision tree. Fig. 1a shows a binary tree of Boolean function \((a \land c) \lor (b \land c)\). Sink nodes are labeled either with 0 or with 1 representing the two corresponding constant expressions. Each non-sink node \( u \) is labeled with Boolean variable \( x = \text{var}(u) \) and has two out-links called 0-link \((\text{low}(u))\) and 1-link \((\text{high}(u))\). The node linked by 1-link and 0-link represent the Boolean expressions when \( x = 1 \) and \( x = 0 \) in Eq.1, i.e., \( f_{x=1} \) and \( f_{x=0} \), respectively.

Indeed, representing a Boolean expression as a binary decision tree is space consuming. Binary decision tree can be turned into more compact data structures by means of the following three transformation rules.

i. Eliminate all but one terminal node with a given label, and redirect all arcs into the eliminated vertices remaining one.

ii. If non-sink nodes \( u \) , and \( v \) have \( \text{var}(u) = \text{var}(v) \), \( \text{low}(u) = \text{low}(v) \), and \( \text{high}(u) = \text{high}(v) \), then eliminate one of two nodes, and redirect all incoming arcs to the other node. This pruning operation is called elimination of redundant tests.

iii. If non-sink node \( v \) has \( \text{low}(v) = \text{high}(v) \), then eliminate \( v \) , and redirect all incoming arcs to \( \text{low}(v) \). This operation is called merging subdags.

The transformation rules must be applied repeatedly because each transformation can expose new possibilities for further ones. Applying the transformation rules to the binary decision tree in Fig. 1a, the related BDD to the Boolean function of \( f = (a \land c) \lor (b \land c) \), is illustrated in Fig. 1b.

Figure 1. a) Binary decision tree, and b) binary decision diagram of \( f = (a \land c) \lor (b \land c) \).
3. Reliability Evaluation by BDD

In order to apply BDD to network reliability analysis, we consider the network model as a stochastic graph $G(V, E)$, where $V$ represents the nodes set and $E = \{e_1, e_2, \ldots, e_n\}$ is the links set. Each link $e_i$ is subject to failure with known failure probability of $q_i$, $q_i \in [0, 1]$, in which it is assumed that all failure events are statistically independent. Nodes are considered reliable and the probability that the link $e_i$ functions can be denoted as $p_i = 1 - q_i$. Let $x_i$ stands for state of the link $e_i$, i.e., $x_i = 0$ when link $e_i$ fails, and $x_i = 1$ when it functions. Considering the Boolean function $f$ on the variables $x_i$, the associated probability of $f$ is defined as:

$$P_r(f) = p_i \cdot p_r(f_{x_i=1}) + q_i \cdot p_r(f_{x_i=0})$$

(2)

where $p_r(f)$ is the probability of connectivity between two terminal nodes which can be referred as the two-terminal reliability. To show the probability evaluation process in BDD, consider the bridge network in Fig. 2. The network reliability can be obtained at each node of the BDD with the evaluation process presented in Fig. 3. The source to terminal ($s-t$) path function for the example network can also be constructed as shown in Fig. 3.

To the best of our knowledge, two algorithms have been proposed in the literature to derive directly the reliability function in the form of a BDD, without deriving the corresponding logical expression. The algorithm presented by Sekine & Imai (1995), revised by Hardy et al (2005), is based on a suitable factorization algorithm, consisting in pivoting along a complete sequence of links and deriving the right subgraph when the link functions and the left subgraph, when the link fails. Both papers indicate a lattice of $12 \times 12$ (144 nodes and 264 links) as an upper limit to computational power of the method. Similarly, Kuo et al (1999) presented an algorithm that the success path function of a given graph is constructed based on BDD by traversing the graph with link expansion diagram and the graph reliability is obtained by directly evaluating on this BDD recursively. Performing several case studies, they showed that their algorithm has an efficient performance even for a $2 \times 100$ lattice network. The second algorithm presented by Zang et al (2000) generates the BDD directly, without explicitly forming Boolean expression, via a recursive depth first search on the graph.

![Figure 2. A bridge network.](image-url)
Figure 3. Probability evaluation of the BDD for bridge Network.

In practice, a reduced ordered binary decision diagram (ROBDD) is utilized which is an ordered BDD (OBDD) where each node represents a distinct Boolean expression. Bryant (1986) proved that the ROBDD is a canonical form for a logic function, that is, two functions are equivalent if, and only if, the ROBDD’s for each function are isomorphic. Thus, to generate a ROBDD, the ordering of the variables has to be made first and this order of variables will not be changed during the generation. Thus, the two-terminal network reliability using ROBDD can be evaluated in three steps.

i. Ordering the network links using an efficient variable ordering heuristic. Several heuristic methods have been developed in the literature; however, a heuristic is good in the sense that it yields a compact OBDD (e.g., Bouissou 1996, and Kuo et al 1999).

ii. Generating ROBDD from the stochastic graph of the network using the algorithm described by Zang et al (2000).

iii. Evaluating the $s-t$ network reliability from ROBDD using the aforementioned method.

4. Case Study

To examine the efficiency of the proposed BDD method, it was applied to seismic reliability analysis of the Kobe city water supply system in Japan. Javanbarg & Takada (2007), and Javanbarg (2008), developed a probabilistic seismic reliability analysis for water supply systems with the aid of Monte Carlo Simulation (MCS) and flow analysis. In their study, a part of Kobe city water distribution network was examined as the case study and the results of the model were compared to the simulation results of the actual observed damage data during the 1995 Kobe Earthquake. In Fig. 4, buried pipelines layer are overlaid with observed seismic intensity of the Kobe Earthquake. Pipeline damage locations with different failure modes are also depicted in Fig. 4. Considering the actual damage locations and types of failure mode, they assigned correspondent
The amount of leakage to each link. Hydraulic analysis of seismically damaged network was deterministically performed and results of nodal reliability were found in terms of pressure availability at each demand node which we will present later in this paper. Table 1 presents the pipe material type as well as probability of failure corresponding to each link. Considering failure probability of each link, Javanbarg and Takada (2007) also performed a MCS to probabilistically model the damage entire the network. Hydraulic analysis of the seismically damaged network was also probabilistically performed by MCS. Nodal pressure availability was then computed as the reliability at each node.

Figure 4. Pipeline damage in the 1995 Kobe Earthquake (Javanbarg & Takada, 2007).

<table>
<thead>
<tr>
<th>Link No.</th>
<th>Pipe material</th>
<th>Probability of failure (q_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CIP</td>
<td>0.717</td>
</tr>
<tr>
<td>2</td>
<td>DIP</td>
<td>0.219</td>
</tr>
<tr>
<td>3</td>
<td>DIP (seismic joint)</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>DIP</td>
<td>0.537</td>
</tr>
<tr>
<td>5</td>
<td>CIP</td>
<td>0.549</td>
</tr>
<tr>
<td>6</td>
<td>DIP</td>
<td>0.046</td>
</tr>
<tr>
<td>7</td>
<td>DIP</td>
<td>0.023</td>
</tr>
<tr>
<td>8</td>
<td>DIP</td>
<td>0.062</td>
</tr>
<tr>
<td>9</td>
<td>CIP</td>
<td>0.422</td>
</tr>
<tr>
<td>10</td>
<td>CIP</td>
<td>0.329</td>
</tr>
<tr>
<td>11</td>
<td>CIP</td>
<td>0.620</td>
</tr>
<tr>
<td>12</td>
<td>DIP</td>
<td>0.080</td>
</tr>
<tr>
<td>13</td>
<td>CIP</td>
<td>0.238</td>
</tr>
<tr>
<td>14</td>
<td>CIP</td>
<td>0.222</td>
</tr>
<tr>
<td>15</td>
<td>DIP</td>
<td>0.064</td>
</tr>
<tr>
<td>16</td>
<td>DIP</td>
<td>0.158</td>
</tr>
<tr>
<td>17</td>
<td>DIP</td>
<td>0.162</td>
</tr>
<tr>
<td>18</td>
<td>CIP</td>
<td>0.354</td>
</tr>
<tr>
<td>19</td>
<td>CIP</td>
<td>0.242</td>
</tr>
<tr>
<td>20</td>
<td>DIP (seismic joint)</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 1. Probability of failure of buried pipelines as the network links (Javanbarg 2007).
Later, utilizing the same case study, Javanbarg et al. (2009) developed a minimal paths enumeration reliability analysis for lifeline networks. Accordingly, the same network model has been used for BDD reliability analysis which the schematic graph of the network is presented in Fig. 5. The results of nodal reliability using BDD are compared with the results of both deterministic and probabilistic models (Javanbarg & Takada 2007) as well as result of minimal paths enumeration (Javanbarg et al, 2009) as presented in Fig. 6. As seen, BDD could estimate the two-terminal reliability efficiently.

Figure 5. Network model by BDD reliability analysis of the case study from Javanbarg & Takada (2007).and Javanbarg et al (2009)

Figure 6. Comparison between the BDD reliability results and results of the past studies

5. Conclusions

The application of BDD to reliability analysis of lifeline networks has been presented. BDD is able to evaluate the exact two-terminal reliability for a real-life network which is defined as probability that the nodes in the network can communicate to each other, taking into account the possible failures of network links. The effectiveness of this approach has been demonstrated by performing experiment on a lifeline network. More efficient algorithm may be implemented to cope with large-scale
networks. In the analysis, the connectivity measure has been considered as the reliability measure. However, an emerging challenge is the implementation of the reliability analysis methods which are able to evaluate the capacity measure of the networks. The study of performance and dependability properties among different interacting networks, like interaction of the lifeline systems, is also an emerging field of study.

References


