EVALUATION OF MIXED CONVECTION IN INCLINED SQUARE LID-DRIVEN CAVITY FILLED WITH AL₂O₃/WATER NANO-FLUID

A. Fereidoon, S. Saedodin, M. Hemmat Esfe * and M.J. Noroozi

Department of Mechanical Engineering, Faculty of Engineering, Semnan University, Semnan, Iran
* E-Mail: m.hemmatesfe@gmail.com (Corresponding Author)

ABSTRACT: To study the mixed convection flows in a square double lid-driven cavity with various inclination angles, a numerical method based on the finite volume method is used. In this investigation Al₂O₃/water nanofluid with particle diameter of 47 nanometers in different solid volume fractions have been utilized. Flow is obtained by the top and bottom walls, which slide in opposite direction at constant speed. The study has been conducted for the Richardson number of 0.1 to 10, Reynolds number (Re) of 1 to 100, while solid volume fraction (φ) of nanoparticles altered from 0 to 0.06. The heat transfer increases with increasing solid volume fraction for a constant Re. It also increases with increasing Richardson number and Reynolds number for a particular volume fraction. Also at low Reynolds number, the solid volume fraction does not have much effect on flow and heat transfer.

Keywords: nanofluid, heat transfer, mix convection, double lid-driven cavity, numerical study

1. INTRODUCTION

Common heat transfer fluids, such as water, mineral oil, etc., have a rather low thermal conductivity. Taking into account the rising demands of modern technology, there is a need to develop new types of fluids that will be more effective in terms of heat transfer performance. To solve the current problem, a new technique, the use of nano-scale particles in the base fluid, has been innovated. Choi (1995) named these fluids nanofluids, which appear to have a high thermal conductivity and are used to meet this demand.

As noted, the use of nanoparticles in the base fluid improves heat transfer, but one of the main problems in the use of nanoparticles in the base fluid is the agglomerate coagulation due to Brownian motion of nanoparticles, which creates a barrier to fluid flow. Thus, it is important to understand how nanoparticles coagulate in agglomerate form. A new moment method for solving the coagulation equation for particles in Brownian motion was presented by Yu et al. (2008). One of their important studies was about agglomerate coagulation due to Brownian motion in the entire size regime (Yu and Lin, 2009).

In the last several years, different researches concerning the problem of heat transfer by using nanofluid have been carried out analytically, numerically, and experimentally. For instance, Masuda et al. (1993) revealed that an increase of 20% of the effective thermal conductivity of ethylene glycol was observed as a result of just 0.3% volume fraction of nanoparticles suspended in liquid. Furthermore, Lee et al. (1999) showed that oxide ceramic nanofluids in water enhanced thermal conductivity. They demonstrated that using Al₂O₃ nanoparticles having mean diameter of 13 nm at 4.3% volume fraction increase the thermal conductivity of water by 30%. Moreover, shape, size and thermal properties of the solid particles in nanofluids have an important role in the thermal conductivity (Masuda et al., 1993).

Besides, cavity filled with nanofluid has been studied by different researchers. As an example, Hwang et al. (2007) developed a numerical model to determine natural convection heat transfer in a rectangular cavity utilizing Al₂O₃ nanofluid heated from below. They utilized various models so as to obtain the effective thermal conductivity and viscosity. Based on their results, base fluid is more unstable than Al₂O₃ nanofluids in a rectangular cavity as the volume fraction of nanofluid increases, the size of nanoparticles decreases, or the average temperature of nanofluids increases. Another research in a rectangular cavity containing water–TiO₂ nanofluid has been done by Wen and Ding (2005). Their results show that for the Rayleigh number less than 10⁶, by increasing of nanoparticle concentration, the natural convection heat transfer rate decreases at a particular Rayleigh number.

Numerical simulation of natural convection in a cavity filled with nanofluid has been examined by Khanafer et al. (2003). The nanofluid in the cavity was considered to be in single phase. The impact of nanoparticle concentration on buoyancy-driven
heat transfer process was discussed. Consequently at any given Grashof number the heat transfer rate increases as the particle volume fraction is increased.

Mixed convection is a type of convection which combines both natural and forced convection. It has significant roles in many applications in industry and engineering. Cooling of electronic devices, food processing, crystal growth, lubrication technologies, drying technologies and chemical processing equipment are among its applications.

Mixed convection heat transfer is a complex phenomenon because of the interaction of buoyancy and shear forces. Combined convection flow occurs in lid-driven cavities due to both shear force generated by the movement of the lid wall of the cavity and the buoyancy force created by thermal heating of the cavity.

Mixed convection flow in a square ventilation cavity with a nanofluid at different Richardson numbers has been studied numerically by Shahi et al. (2010). They reported the impact of the presence of nanoparticles on the hydrodynamic and thermal characteristics of flow.

Recently, because of the many applications of lid-driven cavity, it has attracted a lot of attention. Mohamad and Viskanta (1995) investigated the impact of a sliding wall on the fluid flow and thermal properties in a shallow lid-driven cavity. Based on their results it was found that the highest local heat transfer rate was seen at the beginning area of the lid and decreases as the lid moves.

Zhang (2003) carried out numerical simulation of a two-dimensional square driven cavity and compared his own results with those of previous researchers. Few researches have been done about mixed convection with lid-driven cavity.

More recently, Tiwari and Das (2007) investigated numerically heat transfer enhancement in a lid-driven square cavity with nanofluids. Based on the movement of the walls in different directions, they presented three cases to study their effects on the fluid flow and heat transfer in the cavity.

The assisting and opposing mechanisms of mixed convection in a lid-driven cavity with moving vertical wall have been presented by Aydin (1999). He divided the results into three flow regimes as the forced convection (\( Ri < 0.1 \)), the mixed convection (\( 0.1 < Ri < 10 \)) and the natural convection (\( Ri > 10 \)).

In another research Muthtamilselvan et al. (2010) made a numerical investigation of the mixed convection in a lid-driven cavity with a copper-water nanofluid for different aspect ratios. The results showed the effect of both the aspect ratio and nano particle concentration on the fluid flow and heat transfer in the cavity.

Moallemi and Jang (1992) performed a numerical study of mixed convective flow in a bottom heated square lid-driven cavity. They also investigated the impact of Prandtl number on heat transfer improvement. The conclusion was that the impact of buoyancy is more appreciable for high Prandtl numbers. They proposed a correlation for the average Nusselt number with Prandtl, Reynolds, and Richardson numbers.

A numerical and experimental study on transient mixed convection in a lid-driven cavity with a top moving lid at a higher temperature than that of the bottom wall and insulated vertical sidewalls was also done by Ji et al. (2007). These boundary conditions produce a stably stratified overall configuration. They found that the temperature field shows weak oscillatory trend at the beginning in the mid and upper part of the cavity.

Also, mixed convection inside a rectangular lid-driven enclosure was investigated numerically by Prasad and Koseff (1996). The two vertical walls are insulated while the top moving wall is maintained at a high temperature and the bottom wall is at a low temperature.

Usually, the horizontal position of the cavity is considered while it is necessary to change the inclination of the cavity in real applications and engineering designs. But so far no investigation has been executed to study the heat transfer characteristics of the tilted double lid-driven cavities filled with nanofluid. According to the cavity inclination, the lid-driven shear may assist or oppose buoyancy. The results of this paper, such as streamline contours, temperature field, etc., are not the same as those of horizontal configurations.

This paper aims to present numerical analysis of mixed convective flow in a square double lid-driven cavity with a cooled top moving wall and the lower part of the left and right walls heated. The other walls are assumed to be insulated.

2. MATHEMATICAL MODELING

A schematic diagram of the square lid-driven cavity considered in the present study is shown in Fig. 1 with the boundary conditions and coordinates as indicated. The fluid in the cavity is water-based nanofluid containing Al₂O₃ nanoparticles which are assumed to be uniform in shape. Nanofluid is assumed Newtonian with the flow being laminar and incompressible.
Furthermore, base fluid and nanoparticles are assumed to be in thermal equilibrium with no slip between them. The thermo-physical properties of the nanofluid are assumed constant, except for a variation of the density which is approximated by the Boussinesq model and those of Al₂O₃ nanoparticles (47 nanometer) and water as base fluid are presented in Table 1.

Fig. 1 Schematic diagram of square lid-driven cavity.

Table 1 Thermophysical properties of base fluid and nanoparticles.

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Fluid phase (Water)</th>
<th>Solid (Al₂O₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cp(J/kg k)</td>
<td>4179</td>
<td>765</td>
</tr>
<tr>
<td>ρ (kg/m³)</td>
<td>997.1</td>
<td>3970</td>
</tr>
<tr>
<td>K (W m⁻¹ K⁻¹)</td>
<td>0.6</td>
<td>25</td>
</tr>
<tr>
<td>β×10⁻⁵ (1/K)</td>
<td>21</td>
<td>0.85</td>
</tr>
<tr>
<td>μ×10⁻⁴(kg/ms)</td>
<td>8.9</td>
<td>……</td>
</tr>
</tbody>
</table>

The governing equations for a steady, two-dimensional laminar and incompressible flow are expressed as:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{sf} \nabla^2 T \]  

The dimensionless parameters may be presented as

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{U_0}, \quad \theta = \frac{T - T_e}{\Delta T}, \quad \Delta T = T_h - T_e, \quad \frac{p}{\rho_{sf} U_0^2} \]  

Hence,

\[ \frac{Re}{\rho \mu f L} = \frac{Ra}{Pr Re^2}, \quad \frac{Ra}{Pr} = \frac{\beta \Delta T L^3}{\nu_f \alpha_f} \]  

The dimensionless forms of the above governing equations (1) to (4) become:

\[ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \]  

\[ U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\nu_f}{\nu_f} \frac{1}{Re} \nabla^2 U + \]  

\[ \frac{Ri}{Pr} \frac{\beta_{sf}}{\beta_f} \Delta \theta \sin(\gamma) \]  

\[ U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\nu_f}{\nu_f} \frac{1}{Re} \nabla^2 V + \]  

\[ \frac{Ri}{Pr} \frac{\beta_{sf}}{\beta_f} \Delta \theta \cos(\gamma) \]  

and

\[ \frac{\partial U}{\partial x} + V \frac{\partial \theta}{\partial y} = \frac{\alpha_{sf}}{\alpha_f} \nabla^2 \theta \]  

Thermal diffusivity and effective density of the nanofluid are:

\[ \alpha_{sf} = \frac{k_{sf}}{(\rho c_p)_{sf}} \]  

\[ \rho_{sf} = \rho_f \varphi + (1 - \varphi) \rho_f \]  

Heat capacity and thermal expansion coefficient of the nanofluid are therefore:

\[ (\rho c_p)_{sf} = \varphi (\rho c_p)_f + (1 - \varphi) (\rho c_p)_f \]  

\[ (\rho \beta)_{sf} = \varphi (\rho \beta)_f + (1 - \varphi) (\rho \beta)_f \]  

The effective viscosity of nanofluid was proposed by Brinkman (1952) as below:
\[ \mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{0.5}} \]  

(15)

The effective thermal conductivity of the nanofluid is calculated by the Maxwell model (1904), which is:

\[ k_{nf} = \frac{k_f + 2k_r - 2\varphi(k_f - k_r)}{k_f + 2k_r + \varphi(k_f - k_r)} \]  

(16)

The Nusselt number can be calculated as:

\[ Nu = \frac{hL}{k_f} \]  

(17)

where the heat transfer coefficient \( h \) is defined as:

\[ h = \frac{q_w}{T_h - T_c} \]  

(18)

And the thermal conductivity may be express as:

\[ k_{nf} = \frac{-q_w}{\partial T / \partial Y} \]  

(19)

By substituting Eqs. (18) and (19) in Eq. (17), the Nusselt number for the hot wall can be written as:

\[ Nu = -\frac{k_{nf}}{k_f} \left( \frac{\partial \theta}{\partial x} \right) \]  

(20)

The average Nusselt number calculated over the hot surface by Eq. (18) becomes

\[ Nu_m = \frac{1}{L} \int_0^L Nu \, dx \]  

(21)

3. NUMERICAL METHOD

Governing equations for continuity, momentum and energy equations associated with the boundary conditions in this study were calculated numerically based on a finite volume method and associated staggered grid system, using FORTRAN computer code. The SIMPLE algorithm is adopted to solve the coupled system of governing equations. The convection term is approximated by a combination of the central difference scheme and the upwind scheme (hybrid-scheme) which is conducive to a stable solution. Furthermore, a second-order central differencing scheme is utilized for the diffusion terms. The algebraic system resulting from numerical discretization was calculated using tridiagonal matrix algorithm (TDMA) applied in a line going through all volumes in the computational domain. The solution procedure is iterated until the following convergence criterion is satisfied

\[ error = \frac{\sum_{j=1}^{M} \sum_{i=1}^{N} |\lambda_{n+1}^{i,j} - \lambda_n^{i,j}|}{\sum_{j=1}^{M} \sum_{i=1}^{N} |\lambda_{n+1}^{i,j}|} < 10^{-7} \]  

(22)

Here, \( M \) and \( N \) correspond to the number of grid points in \( x \) and \( y \) directions, respectively. \( n \) is the number of iteration and \( \lambda \) denotes any scalar transport quantity. To verify grid independence, numerical procedure was carried out for nine different mesh sizes, namely; 21\times21, 31\times31, 41\times41, 51\times51, 61\times61, 71\times71, 81\times81, 91\times91 and 101\times101. Average \( Nu \) of the hot wall is obtained for each grid size as shown in Table 2. As can be observed, an 81 \( \times \) 81 uniform grid size, yielded the required accuracy and was hence applied for all simulation exercises in this work as presented in the following section.

In order to validate the proposed numerical scheme, the numerical results of Muthtamiliselvan et al. (2010) is compared with a special case of present study and shown in Table 3. The special case for validation is buoyancy driven laminar convection heat transfer in a square cavity with differentially heated side walls. The left wall was kept hot while the right wall was cooled. The top and bottom walls are insulated. This comparison revealed excellent agreement between the present results and those of Muthtamiliselvan et al. (2010).

4. RESULTS AND DISCUSSION

Fig. 2 shows variations of the streamlines inside the square cavity for different angles and different Reynolds numbers for Richardson number of value 7. For small Reynolds numbers, heat transfer is mainly in the form of conduction heat transfer. In this case, the vortex inside the cavity is symmetric and not very strong.

By increasing Reynolds number and therefore increasing shear forces due to the moving wall, the symmetry of the flow is disturbed. For angles greater than 60 degrees and Reynolds numbers more than 50, a small secondary vortex is formed at the lower right corner of the cavity due to the interaction between viscous forces, buoyancy forces and adverse pressure gradient. The greater the angle, the more powerful the vortex would be. For angle of 90 degrees and high Reynolds numbers, flow pattern is drastically altered and three vortices are formed due to interaction of the shear and the buoyancy effects. The upper vortex is caused due to the moving wall, the middle vortex is caused by the buoyancy forces and the lower vortex is caused by the presence of wall and the middle vortex.
Table 2 Average Nu of hot wall obtained for each grid size.

<table>
<thead>
<tr>
<th>Mesh size</th>
<th>Nu</th>
<th>(\varphi=0.04, \text{Re}=10, \text{Ri}=0.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 \times 31</td>
<td>3.025</td>
<td></td>
</tr>
<tr>
<td>41 \times 41</td>
<td>3.336</td>
<td></td>
</tr>
<tr>
<td>51 \times 51</td>
<td>3.567</td>
<td></td>
</tr>
<tr>
<td>61 \times 61</td>
<td>3.664</td>
<td></td>
</tr>
<tr>
<td>71 \times 71</td>
<td>3.752</td>
<td></td>
</tr>
<tr>
<td>81 \times 81</td>
<td>3.789</td>
<td></td>
</tr>
<tr>
<td>91 \times 91</td>
<td>3.791</td>
<td></td>
</tr>
<tr>
<td>101 \times 101</td>
<td>3.791</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Comparison of solutions for natural convection in square cavity.

<table>
<thead>
<tr>
<th></th>
<th>Present study</th>
<th>Muthtamilselvan et al. (2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Ra}=10^3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nu</td>
<td>1.1366</td>
<td>1.118</td>
</tr>
<tr>
<td>(N_{\text{u,max}})</td>
<td>1.5738</td>
<td>1.508</td>
</tr>
<tr>
<td>(N_{\text{u,min}})</td>
<td>0.6468</td>
<td>0.690</td>
</tr>
<tr>
<td>(\text{Ra}=10^4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nu</td>
<td>2.3256</td>
<td>2.248</td>
</tr>
<tr>
<td>(N_{\text{u,max}})</td>
<td>3.5456</td>
<td>3.545</td>
</tr>
<tr>
<td>(N_{\text{u,min}})</td>
<td>0.7172</td>
<td>0.582</td>
</tr>
<tr>
<td>(\text{Ra}=10^5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nu</td>
<td>4.4928</td>
<td>4.546</td>
</tr>
<tr>
<td>(N_{\text{u,max}})</td>
<td>7.3037</td>
<td>7.833</td>
</tr>
<tr>
<td>(N_{\text{u,min}})</td>
<td>0.9906</td>
<td>0.721</td>
</tr>
<tr>
<td>(\text{Ra}=10^6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nu</td>
<td>8.6388</td>
<td>8.975</td>
</tr>
<tr>
<td>(N_{\text{u,max}})</td>
<td>14.2521</td>
<td>18.642</td>
</tr>
<tr>
<td>(N_{\text{u,min}})</td>
<td>1.6247</td>
<td>0.959</td>
</tr>
</tbody>
</table>

With increasing Reynolds numbers, isothermal lines are inclined towards the walls, which marks forming of the thermal boundary layer and the dominance of the forced convection heat transfer. The difference in the isothermal lines corresponding to Reynolds numbers of 50 and 100 at the inclination angle of 90° is due to the presence of three vortices in the flow pattern. Therefore unlike other variables, despite domination of forced convection, isothermal lines do not show same intensity of compression in the regions close to the wall of cavity. Moreover due to opposite directions of motion of vortices in these cases, non-uniform isotherm lines are distributed in the whole enclosure.

Fig. 4 shows variations of the isothermal lines inside the square cavity with respect to Reynolds and Richardson numbers for cavity angle value of zero. For Richardson number value of 0.1, the dominant form of heat transfer is forced convection heat transfer and the flow is controlled by the moving wall. For this Richardson number, intensity of the flow increases with the increase of Reynolds number. With increasing Richardson number, strength of the buoyancy force increases, which leads to the interaction of the shear and buoyancy forces at high Reynolds numbers. And due to this interaction, a vortex is formed in the lower right corner of the cavity for Reynolds numbers 50 and 100 and high Richardson numbers. The higher the Richardson number, the stronger this vortex would be. It is noticeable that the buoyancy force originating from the hot section of the left wall is in the direction of shear force in horizontal case, while the buoyancy force originating from the hot section of the right wall is in the opposite direction of shear force and it is a major factor in creating the vortex in the right corner of cavity when the main vortex is strong at high Reynolds numbers.

Fig. 5 shows variations of the isothermal lines inside the square cavity with respect to Reynolds and Richardson numbers for cavity angle value of zero. Explanation for Figure 3 is also valid for this figure. It should be noted that in the cases corresponding to Reynolds numbers of 50 and 100 in the range of high Ri numbers, intensity of isothermal lines is reduced in the lower region of right wall because of presence of a small vortex at the lower right corner of the cavity and it causes disorder in isotherm lines in this region. However, in the regions close to the left wall, there is intense compression in the isothermal lines and thermal boundary layer.

Variation of the average Nusselt number with respect to Richardson number and volume
<table>
<thead>
<tr>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>Re</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

**Fig. 2** Variations of streamlines inside square cavity for different angles and different Reynolds numbers for Richardson number of value 7. (Solid line for $\phi=0.00$ and dashed line for $\phi=0.06$).
Fig. 3  Variations of isothermal lines inside square cavity for different angles and different Reynolds numbers for Richardson number of value 7. (Solid line for φ=0.00 and dashed line for φ=0.06).
fraction of the nanoparticles for different values of cavity angles and different values of Reynolds numbers are shown in Fig. 6. When Reynolds number is equal to unity, increasing Richardson number causes no significant change in the amount of heat transfer, however increasing the volume fraction of the nanoparticles causes an increase in the amount of heat transfer. As the dominant mechanism of heat transfer is conduction and convection heat transfer is negligible in this case, Nusselt number is approximately constant at various Richardson numbers.

By increasing Reynolds number, behavior of the Nusselt number changes with regards to Richardson number. For values of Reynolds number higher than unity, increasing Richardson number increases the amount of heat transfer which is due to dominance of convection heat transfer. Furthermore, it can be seen that for different values of Richardson number, rate of Nusselt number increase does not change with regards to volume fraction of the nanoparticles.

5. CONCLUSIONS

In the present paper, a numerical simulation of mixed convection flows in a square lid driven square cavity utilizing Al₂O₃–water nanofluid was conducted. The finite volume method was employed for the solution of the present problem. Effects of some pertinent parameters such as
Richardson number, solid concentration, and inclination angle of cavity on the flow and temperature fields, as well as the heat transfer enhancement were studied. Graphical results for various parametric conditions were presented and discussed. From this investigation, the following conclusions can be drawn:

1. At low Reynolds number and for different Richardson numbers, rate of increase of Nusselt number does not have a significant change with volume fraction of the nanoparticles.

2. Temperature patterns for the constant Reynolds and Richardson numbers do not have a considerable change with the angle of inclination of cavity.

3. With an increase in the volume fraction of the nanoparticles, the amount of heat transfer increases. This increase is larger for higher Richardson and Reynolds numbers.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_p)</td>
<td>specific heat, J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(Gr)</td>
<td>Grashof number</td>
</tr>
<tr>
<td>(g)</td>
<td>gravitational acceleration, m s(^{-2})</td>
</tr>
<tr>
<td>(h)</td>
<td>heat transfer coefficient, W m(^{-2}) K(^{-1})</td>
</tr>
<tr>
<td>(H)</td>
<td>enclosure height, m</td>
</tr>
<tr>
<td>(k)</td>
<td>thermal conductivity, W m(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(L)</td>
<td>cavity length, m</td>
</tr>
</tbody>
</table>
Fig. 6  Variation of average Nusselt number with respect to Richardson number and volume fraction of nanoparticles for different values of cavity angles and Reynolds numbers.

\[ \text{Nu} \]
Nusselt number

\[ \rho \]
pressure, N m\(^{-2}\)

\[ P \]
dimensionless pressure

\[ Pr \]
Prandtl number

\[ q \]
heat flux, W m\(^{-2}\)

\[ Ra \]
Rayleigh number

\[ Re \]
Reynolds number

\[ Ri \]
Richardson number

\[ T \]
dimensional temperature, K

\[ u, v \]
dimensional velocities components in \( x \) and \( y \) direction, m s\(^{-1}\)

\[ U, V \]
dimensionless velocities components in \( X \) and \( Y \) direction

\[ U_0 \]
lid velocity

\[ x, y \]
dimensional Cartesian coordinates, m

\[ X, Y \]
dimensionless Cartesian coordinates

Greek letters

\[ \alpha \]
thermal diffusivity, m\(^2\) s\(^{-1}\)

\[ \beta \]
thermal expansion coefficient, K\(^{-1}\)

\[ \theta \]
dimensionless temperature

\[ \mu \]
dynamic viscosity, Kg m\(^{-1}\) s\(^{-1}\)

\[ \nu \]
kineamtic viscosity, m\(^2\) s\(^{-1}\)

\[ \rho \]
density, kg m\(^{-3}\)

\[ \phi \]
volume fraction of the nanoparticles

\[ \gamma \]
cavity inclination angle

Subscripts

\[ c \]
cold

\[ f \]
fluid

\[ h \]
hot

\[ m \]
mean

\[ nf \]
nanofluid

\[ s \]
solid particles

\[ w \]
wall

REFERENCES


