JOINT ANGLE AND DELAY ESTIMATION OF POINT SOURCES

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ABSTRACT
In this paper, we evaluate the performance of the JADE-MUSIC (Joint Angle and Delay Estimation) algorithm in the presence of point sources. The observed snapshots are used to estimate the space-time channel matrix. The estimated channel matrix is then used to jointly estimate the Direction Of Arrival and the Time Of Arrival. The performance of the JADE-MUSIC algorithm is then evaluated in terms of bias and Mean Square Error of the DOA and TOA estimates. The obtained results are also compared to the Cramer-Rao Bound.

1. INTRODUCTION
Source localization is an issue of interest in wireless communications. Methods for mobile localization are based on Direction Of Arrival (DOA) and/or Time Of Arrival (TOA) estimation. A second type of application in wireless communications is the parametric estimation of the propagation channel, to assist equalization and directive transmission in the downlink. Estimating the propagation parameters from measurements at an antenna array also has applications in radar and sonar exploration. Parametric joint angle/delay estimation has received increased research interest lately \cite{1,2}. Many of the proposed algorithms are based on Maximum-Likelihood or multidimensional MUSIC. Unlike the traditional MUSIC and ESPRIT \cite{3,4} algorithms, a major contribution of the JADE (Joint Angle and Delay Estimation) algorithm is that it can work when the number of paths exceeds the number of antennas. In this paper, we evaluate the performance of JADE-MUSIC algorithm in the presence of point sources in terms of bias and Mean Square Error (MSE) of DOA and TOA estimates.

The paper is organized as follows. In section 2, the model is described. Section 3 provides the basic equations used by the JADE algorithm. Section 4 describes the JADE-MUSIC algorithm. In section 5, the Cramer Rao Bound (CRB) of the joint DOA and TOA estimates is derived. Simulation results are given in section 6. Section 7 draws some conclusions.

2. SYSTEM MODEL
We focus first on the case of a single user transmitting a modulated digital signal in a specular multipath environment. The received baseband signal at an M-element antenna array at time \(t\), \(x(t)\), can be written as the convolution of the transmitted digital sequence \(\{s_l\}\) with the channel \(h(t)\) \cite{5}

\[
x(t) = \sum_{l} s_l h(t - lT) + n(t), \quad (1)
\]

where \(T\) is the symbol period, \(n(t)\) is a zero mean Gaussian noise of covariance matrix \(\sigma^2 I_M\). The channel pulse response can thus be modeled as

\[
h(t) = \sum_{i=1}^{Q} a(\theta, \beta_i g(t - \tau_i) \quad (2)
\]

where \(Q\) is the number of paths, \(\beta_i\) is the \(i\)-th path attenuation, \(\tau_i\) is the \(i\)-th path delay, \(\theta_i\) is the DOA of the \(i\)-th path, \(g(t)\) is the pulse shaping function and \(a(\theta)\) is the array manifold. For an Uniform Linear Antenna (ULA), we have

\[
a(\theta) = \left[1, e^{-j \frac{2\pi df}{c} \sin \theta}, \ldots, e^{-j \frac{2\pi df}{c} (M-1) \sin \theta} \right]^T,
\]

where \(d\) is the distance between two successive sensors and \(f_c\) is the carrier frequency. Assume that this is a quasistationary model: the \(\{\theta_i, \beta_i, \tau_i\}\) vary with time as well.

Let \(LT = L_gT + \Delta \tau\) be the length of the channel impulse response where \(L_gT\) is the length of the pulse shaping function and \(\Delta \tau\) is the maximum path delay. Collect data over \(N\) symbol periods. Let \(P\) be the oversampling factor. Then (1) leads to \cite{6}

\[
X = HS + N, \quad (3)
\]

where

\[
X = \begin{bmatrix}
x(0) & \cdots & x((N-1)T) \\
\vdots & \ddots & \vdots \\
x((1-\frac{1}{P})T) & \cdots & x((N-\frac{1}{P})T)
\end{bmatrix}_{MP \times N}
\]

\[
H = \begin{bmatrix}
h(0) & \cdots & h((L-1)T) \\
\vdots & \ddots & \vdots \\
h((1-\frac{1}{P})T) & \cdots & h((L-\frac{1}{P})T)
\end{bmatrix}_{MP \times L}
\]
and $N$ is defined similarly to $X$. It will be convenient to rearrange $H$ it into

$$\tilde{H} = [h(0) \; h(T/P) \; ... \; h((L - 1/P)T)]_{M \times LP} \quad (4)$$

According to (2), $\tilde{H}$ satisfies the factorization

$$\tilde{H} = [a(\theta_1), \ldots, a(\theta_Q)]_{M \times Q} \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ 0 & \beta_Q & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_Q \end{bmatrix}_{Q \times Q} \begin{bmatrix} g^T(\tau_1) \\ \vdots \\ g^T(\tau_Q) \end{bmatrix}_{Q \times LP} \quad (5)$$

where $g(\tau_i) = [g(kT - \tau_i)]_{k=0,1/P,\ldots,L-1/P}$ is an $LP$-dimensional column vector containing the samples of $g(t - \tau_i)$, $\theta = [\theta_1, \ldots, \theta_Q]$, $\tau = [\tau_1, \ldots, \tau_Q]$, $D = \text{diag}[\beta]$ and $\beta = [\beta_1, \ldots, \beta_Q]$.

3. BASIC EQUATIONS FOR JADE

3.1. Space-time manifold

Let $h = \text{vec}(\tilde{H})$ be a column vector of length $MPL$ obtained by taking the transpose of each row of the matrix $\tilde{H}$ and stacking it below the transposes of the previous row. Applying the vec() operation to (5) and using the general relation $\text{vec}(A \; \text{diag}[b] C) = (C^T \circ A)b$ yields

$$h = (A(\theta) \circ G(\tau))\beta = U(\theta, \tau)\beta. \quad (6)$$

where $\circ$ represents the Khatri-Rao (column-wise Kronecker) product,

$$U(\theta, \tau) = A(\theta) \circ G(\tau) = [u(\theta_1, \tau_1), \ldots, u(\theta_Q, \tau_Q)], \quad (7)$$

and $u(\theta, \tau)$ is an $MPL \times 1$ vector called the space-time vector given by

$$u(\theta, \tau) = a(\theta) \otimes g(\tau), \quad (8)$$

where $\otimes$ denotes the Kronecker product.

3.2. Method outline

Since the angle/delay parameters vary very slowly, we assume that the space-time matrix $U(\theta, \tau)$ is time-invariant over the observation interval (say, $S$ slots). Thus, we have at each slot $n$,

$$\text{vec}(\tilde{H}^{(n)}) = U(\theta, \tau)\beta^{(n)}, \quad n = 1, \ldots, S. \quad (9)$$

The first step in our approach consists in estimating the channel impulse response from the user to the antenna array. This can be accomplished by using, for example, training bits. In practice (3) gives

$$X^{(n)} = H^{(n)}S^{(n)} + N^{(n)}, \quad n = 1, \ldots, S \quad (10)$$

The channel matrix can be estimated as follows

$$H^{(n)}_{\text{est}} = X^{(n)}g^{(n)}\dagger \quad (11)$$

$$= H^{(n)} + V^{(n)}_{\text{est}}, \quad n = 1, \ldots, S \quad (12)$$

where $V^{(n)}_{\text{est}} = N^{(n)}S^{(n)}\dagger$ is the channel estimation noise matrix at time slot $n$ and $\dagger$ is the matrix pseudo-inverse. According to (4) and (12), we obtain

$$\tilde{H}^{(n)}_{\text{est}} = \tilde{H}^{(n)} + \tilde{V}^{(n)}_{\text{est}}, \quad n = 1, \ldots, S \quad (13)$$

We denote by $v_k^{(n)}$ the $k$-th vector of $\tilde{V}^{(n)}_{\text{est}}$. We have

$$E\{v_k^{(n)}v_k^{(n)H}\} = \sigma^2 E\{(S^{(n)}S^{(n)H})^{-1}\}K_1. \quad \text{Assuming that the training data is perfect e.i.} \quad SS^H = NI \quad \text{where} \quad N \quad \text{is the number of training symbols, we have}$$

$$E\{v_k^{(n)}v_k^{(n)H}\} = \frac{\sigma^2}{N} \delta_{kl}I \quad (14)$$

Applying the vec operation to (13), we obtain

$$y^{(n)} = U(\theta, \tau)\beta^{(n)} + v^{(n)}, \quad n = 1, \ldots, S, \quad (15)$$

where $y^{(n)} = \text{vec}(\tilde{H}^{(n)}_{\text{est}})$ and $v^{(n)} = \text{vec}(\tilde{V}^{(n)}_{\text{est}})$. In matrix form, the above equation becomes

$$Y = [y^{(1)}, \ldots, y^{(S)}] = U(\theta, \tau)B + \tilde{V}, \quad (16)$$

where $B = [\beta^{(1)}, \ldots, \beta^{(S)}]$, and similarly for $\tilde{V}$. The joint angle/delay estimation (JADE) problem amounts for given channel estimates $\{y^{(1)}, \ldots, y^{(S)}\}$, find the angles $\theta$ and delays $\tau$ using the model (16). As an aside, note the resemblance of the JADE model to the DOA model [8].

The next and last step of the method thus consists of jointly estimating the parameters $\eta = [\theta \; \tau]^T$ that satisfy the model in (16). When we have more than one user in the same time slot, we can independently estimate the channel matrices $\tilde{H}$ using each user’s unique training signal. We can then proceed as above for each user.

4. JADE-MUSIC

Similarly to the conventional MUSIC algorithm [3], the JADE-MUSIC algorithm is based on the decomposition of the estimated correlation matrix, $\tilde{R}_Y$, into a signal subspace $\tilde{E}_s$ and a noise subspace $\tilde{E}_n$.

$$\tilde{R}_Y = \tilde{E}_s\tilde{A}_s\tilde{E}_s^H + \tilde{E}_n\tilde{E}_n^H\sigma_e^2, \quad (17)$$

where $\tilde{R}_Y = YY^H/S, \quad \sigma_e^2 = \frac{\sigma^2}{N}$ is the variance of the each entry in the estimation noise matrix. The eigenvalues have been ordered so that $\tilde{A}_s$ is a diagonal matrix containing the $Q$ largest eigenvalues of $\tilde{R}_Y$ in decreasing order.

\footnote{For a full rank matrix $A^1 = (A^H A)^{-1}A^H = A^H(A^H A)^{-1}$}
and the columns of $\hat{E}_n$ are the corresponding eigenvectors. The columns of $\hat{E}_n$ are the remaining $MPL - Q$ eigenvectors.

We know that the true space-time channel vectors $u(\eta_i)$, $i = 1 \cdots Q$ are orthogonal to the noise subspace $\hat{E}_n$. In practice, a scalar measure of the distance between the space-time manifold and the estimated noise subspace is formed, $\|\hat{E}_n^H u(\eta)\|^2 / \|u(\eta)\|^2$. Its inverse, called the JADE-MUSIC spectrum, is searched for peaks

$$P_{\text{JADE}}(\eta) = \frac{u(\eta)^H u(\eta)}{u(\eta)^H \hat{E}_n \hat{E}_n^H u(\eta)}$$

where $\eta = [\theta \tau]^T$. The spatio-temporal parameter estimates are determined by the location of the $Q$ largest peaks. For azimuth only, this spectrum is two-dimensional, as shown in Figure 1.

5. CRAMER-RAO BOUND

The Cramer-Rao Bound (CBR) provides a lower bound on the covariance matrix of any unbiased estimator. In [9], the CBR was derived for only DOA estimation only. Adapting the results of [9], we obtain [7]

$$CBR(\eta) = \frac{\sigma_n^2}{2} \left\{ \sum_{i=1}^S \text{Re} \left( B(n)^H \hat{D}_U^T \hat{P}_U \hat{D}_U B(n) \right) \right\}^{-1}$$

where $B(n) = I_2 \otimes \text{diag} [\beta(n)]$, $P_U = I - UU^T$, and $D_U = [G \circ A', G' \circ A]$. Here, the prime denotes differentiation where each column is differentiated with respect to the corresponding parameter and all matrices are evaluated at the true parameter values

$$A' = A'(\theta) = \left[ \frac{\text{d}a(\theta_1)}{\text{d}\theta_1}, \ldots, \frac{\text{d}a(\theta_Q)}{\text{d}\theta_Q} \right],$$

and similarly for $G' = G'(\tau)$. The proof of this claim is similar to the one in [9].

6. SIMULATION RESULTS

In this section, we evaluate the performance of JADE-MUSIC in terms of bias and MSE of DOA and TOA estimates. We assume a single user, 3 paths, and a 4-elements antenna. The DOAs are $[-5, 0, 10]$° relative to the array broadside and the corresponding path delays are $[0.1, 1, 0.5]\mu s$ seconds where $T = 3.7\mu s$. The path gains are generated from a complex Gaussian distribution with zero mean and variance $[1, 0.6, 0.4]$ respectively for the three rays. The modulation waveform is a raised cosine pulse with excess bandwidth $0.35$. The over sampling factor is set to $P = 2$. Data is collected over $S = 40$ time slots, and at each time slot, the channel is estimated using $N = 26$ training symbols. The experimental bias and MSE of the DOA and delay estimates is computed from 500 Monte Carlo simulations.

The Experiments are run with the JADE-MUSIC algorithm for various Signal to Noise Ratios (SNR). The SNR is defined as the ratio of the power of the strongest path to the variance of the noise: $\frac{\|b_i\|^2}{\sigma_n^2}$. In figure 1, we present a typical JADE-MUSIC spectrum for a SNR=20 dB. We remark the presence of three peaks. In figures 2 and 3, we present respectively the evolution of the bias of the DOA and the TOA estimates of the path with delay $\tau = 0.1\mu s$ and $\theta = -5^\circ$ with respect to the SNR. The figures show that the estimator is unbiased for large SNRs. In figures 4 and 5, we present the evolution of the MSE of DOA and TOA estimates with respect to the SNR. We note that for a large SNRs, the JADE-MUSIC reaches nearly the CRB. In figure 6 we present the performance of JADE-MUSIC algorithm in terms of MSE of DOA estimates with respect to the number of slots for SNR =10 dB. We notice that the JADE-MUSIC performance is far from the CRB. However, the MSE of delay estimates shown in figure 7, reaches the CRB for a large number of slots.

7. CONCLUSION

In this paper, we evaluated the performance of the JADE-MUSIC algorithm for point sources. The concept of space-time array manifold was introduced and the Cramer-Rao bound for the JADE-MUSIC estimator was provided, along with simulations showing the performance of the algorithm in terms of bias and MSE. This work is expected to be extended to joint azimuth, site and delay estimation of point sources.

8. REFERENCES


Fig. 4. MSE of DOA estimates with respect to SNR.

Fig. 5. MSE of TOA estimates with respect to SNR.

Fig. 6. MSE of DOA estimates with respect to the number of slots.

Fig. 7. MSE of TOA estimates with respect to the number of slots.