Adaptive fuzzy control for strict-feedback nonlinear time-delay systems without backstepping scheme

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Abstract—In this paper, an adaptive fuzzy tracking control is presented for a class of SISO nonlinear strict-feedback systems with unknown time delays. The proposed algorithm does not use the backstepping scheme rather it converts the strict-feedback time-delayed system to the normal form. The Mamdani-type fuzzy system is employed to approximate online the unknown lumped nonlinear function. The developed controller guarantees uniform ultimate boundedness of all signals in the closed-loop system. The designed control law is independent of the time delays and has a simple form with only one adaptive parameter vector, which is required to be updated online. As a result, the proposed control algorithm is considerably simpler than the previous ones based on backstepping. Simulation results are presented to verify the effectiveness of the proposed approach.

I. INTRODUCTION

Systems with delays frequently appear in engineering applications. Typical examples of time-delay systems are electrical networks, chemical processes, rolling mill systems, teleportation systems, underwater vehicles and so on. The existence of time delays may degrade the control performance and make the stabilization problem more difficult. So far, the stability analysis and robust control for these dynamic time-delay systems have attracted a number of researchers over the past years; see, for example, [1], [2], and the references therein. Stability analysis and synthesis of time-delay systems are important issues addressed by many authors and for which surveys can be found in several articles [3]-[7].

Recently, by combining Lyapunov–Krasovskii functional and backstepping technique, an adaptive neural tracking control scheme was proposed in [8] for a class of strict-feedback nonlinear time-delay systems with unknown virtual control coefficients. The suggested controller guarantees the uniform ultimate boundedness of the adaptive closed-loop system, while the output tracking is achieved. Further improvements were given in [9, 10]. The robust stabilization methods were presented via the approximation capability of neural network [11, 12]. In [13], the adaptive $H_\infty$ control was addressed via backstepping and neural networks technique. The observer-based adaptive neural controller was designed for a special class of nonlinear time-delay systems [14]. However, these adaptive neural control methods [8]-[14] require a large number of neural weights to be adapted online simultaneously. This makes the learning time unacceptably large. In view of this disadvantage, several adaptive fuzzy control schemes have been developed in [15]-[18] for nonlinear delay-free systems. The advantage of these control schemes is that the proposed controllers require much less parameters to be updated online.

Stability analysis and control synthesis of T-S fuzzy delayed systems were proposed in [19]-[22]. An approximation-based adaptive control has also been addressed for nonlinear systems with time-delay. An attempt for using Mamdani-type fuzzy logic systems to design an adaptive fuzzy tracking controller by using the backstepping technique and Lyapunov–Krasovskii functional appeared in [23]. Most of the previous adaptive fuzzy control algorithms for strict-feedback nonlinear systems were based on the backstepping scheme, which makes the control law and stability analysis very complicated. The main advantage of the results proposed in [23] is that the developed fuzzy controllers contain less adaptation laws.

The previous adaptive fuzzy controllers that were based on the backstepping design method have some drawbacks. First, the determination of virtual control terms and their time derivatives requires tedious and complex analysis. As pointed out in [24], for the relatively simple application to dc motor control, the regression matrix in the adaptive backstepping control almost covers an entire page in [25]. The complexity exhibits an exponential increase as the order of the controlled system grows. However, this complexity must be avoided for practical implementation. Recently, in [26], an attempt for using an adaptive neural control for SISO nonlinear systems without backstepping is proposed.

Based on the above observation, a novel systematic design procedure is developed for the synthesis of a stable adaptive fuzzy controller for a class of nonlinear time-delay systems. Fuzzy logic systems are employed to approximate the unknown nonlinear function, and then, the adaptive law of adjustable parameters is obtained.

In this paper, an algorithm is proposed for adaptive fuzzy tracking control of SISO nonlinear strict-feedback systems with unknown time delays. The proposed algorithm involves two major steps: applying a change of coordinates to convert the strict-feedback time-delayed system to the normal form to avoid backstepping.
procedure and then using a Mamdani-type fuzzy system [29] to approximate the unknown nonlinear function appeared in the control law. Because the designed control law is independent of the time delays and has a simple form with only one adaptive parameter vector, it is simpler than the existing ones based on backstepping.

The paper is organized as follows. The system under investigation and its transformation to normal form is outlined in section 2. The adaptive fuzzy control design is presented in section 3. The main result is presented in section 4. Simulation results are provided in section 5, with conclusions given in section 6.

II. PROBLEM FORMULATION

Consider the SISO nonlinear time-delay dynamic systems in the following form:

\[
\begin{align*}
\dot{x}_i &= g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + h_i(\bar{x}_i(t - \tau_i)), \quad i = 1, 2, \ldots, n - 1, \\
\dot{x}_n &= g_n(x)u(t) + f_n(x) + h_n(x(t - \tau_n)),
\end{align*}
\]

where \( \bar{x}_i = [x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]^T \) is the system state vector, assumed measurable, \( u \in R \) and \( v \in R \) denote system control input and output, respectively, \( g_i(\cdot), f_i(\cdot) \) and \( h_i(\cdot) \) are unknown smooth functions and \( \tau_i \) is an unknown time delay of the state variables, \( i = 1, \ldots, n \). With regard to controllability, the following assumption must be made.

**Assumption 1:** For \( 1 \leq i \leq n \), the signs of \( g_i(\bar{x}_i) \) are known. Without loss of generality, it is assumed that \( g_i(\bar{x}_i) > 0 \).

The control objectives are to design an adaptive fuzzy tracking controller for the system (1) such that the system output tracks a desired reference signal \( y_d \) and to guarantee that all the signals in the closed-loop system are bounded. The original system (1) can be transformed into the normal form with respect to the newly defined state variables [26]

\[
\begin{align*}
z &=[z_1, z_2, \ldots, z_n]^T.
\end{align*}
\]

Let \( \Delta_i = y \) and \( \Delta_i = \dot{z}_i = g_i x_i + f_i + d_i \) and \( d_i = h_i(\bar{x}_i(t - \tau_i)) \). The time derivative of \( z_2 \) can be written as

\[
\begin{align*}
\dot{z}_2 &= a_2(\bar{x}_2) + b_2(\bar{x}_2) x_3 + l_2(\bar{x}_2(t - \tau_2)) \quad (2)
\end{align*}
\]

where \( a_2(\bar{x}_2) = \left( \frac{\partial f_1}{\partial x_1} + \frac{\partial g_1}{\partial x_1} x_2 \right) f_1 + g_1 x_2 + g_1 f_2 \), \( b_2(\bar{x}_2) = g_1 h_2 \) and

\[
\begin{align*}
l_2(\bar{x}_2(t - \tau_2)) &= \left( \frac{\partial f_1}{\partial x_1} + \frac{\partial g_1}{\partial x_1} x_2 \right) d_1 + g_1 d_2 + \frac{\partial d_1}{\partial x_1} x_1 (t - \tau_1)
\end{align*}
\]

Again, let \( \dot{z}_3 = a_3(\bar{x}) + b_3(\bar{x}_2) x_3 + l_3(\bar{x}_2(t - \tau_2)) \) and its time derivative is induced as

\[
\begin{align*}
\dot{z}_3 &= a_3(\bar{x}_3) + b_3 y_3 x_4 + l_3(\bar{x}_3(t - \tau_3)) \quad (3)
\end{align*}
\]

where \( a_3(\bar{x}_3) = \sum_{j=1}^2 \left( \frac{\partial f_j}{\partial x_j} + \frac{\partial g_j}{\partial x_j} x_2 \right) (f_j + g_j x_{j+1}) + b_2 f_3 \), \( b_3(\bar{x}_3) = b g_3 = g_1 g_2 g_3 \) and

\[
\begin{align*}
l_3(\bar{x}_3) &= \sum_{j=1}^2 \left( \frac{\partial f_j}{\partial x_j} + \frac{\partial g_j}{\partial x_j} x_2 \right) (f_j + g_j x_{j+1}) + b_2 f_3
\end{align*}
\]

In general, by induction, if we define \( a_i f_j \) and \( b_j = g_j \), the following is satisfied for \( i = 2, \ldots, n \):

\[
\begin{align*}
\dot{z}_i &= a_i(\bar{x}_i) + b_i(\bar{x}_i) x_{i+1} + l_i(\bar{x}_i(t - \tau_i))
\end{align*}
\]

where

\[
\begin{align*}
a_i(\bar{x}_i) &= \sum_{j=1}^i \left( \frac{\partial f_j}{\partial x_j} + \frac{\partial g_j}{\partial x_j} x_2 \right) (f_j + g_j x_{j+1}) + b_2 f_3 \\
b_i(\bar{x}_i) &= h_i \quad (4)
\end{align*}
\]

Therefore, with respect to state variables \( z = [z_1, z_2, \ldots, z_n]^T \), the strict-feedback time-delayed system (1) can be represented in the following normal form:

\[
\begin{align*}
\dot{z}_i &= a(x) + b(x)u + l(x_d) \\
y &= z_1
\end{align*}
\]

where \( a(x) = a_n(x), b(x) = b_n(x), l(x_d) = l_n(x_d) \) and \( x_d = [x_1(t - \tau_1), \ldots, x_n(t - \tau_n)] \) should be noted that \( a(x), b(x), l(x_d) \) are unknown functions of \( x \) and time delays. From Assumption 1, it is also noted that a constant \( \bar{b} > 0 \) exists such that \( b(x) \geq \bar{b}, \forall x \in \mathbb{R}^n \). This assumption poses a controllable condition on system (6).

III. ADAPTIVE FUZZY CONTROLLER DESIGN

A. Ideal Controller

It is assumed that the desired output trajectory and its derivatives \( \dot{y}_d = [y_d, \dot{y}_d, \ldots, y_d^{(n-1)}]^T \) are measurable and bounded where \( y_d^{(n-1)} \) denotes the \( (n-1) \)th derivative of \( y_d \) with respect to time. Define the error vector \( e \) as

\[
e = z - y_d = [e_1, e_2, \ldots, e_n]^T
\]

and the filtered tracking error \( e_

\[
e = \Lambda \int_0^t \lambda^{n-1} e_1 \quad (8)
\]

where \( \Lambda = [\Lambda^{n-1}, (n-1) \lambda^{n-2}, \ldots, (n-1) \lambda] \) and \( \lambda > 0 \) is a positive constant to be specified by the designer.

**Remark 1.** The nonlinear differential equations of the form (1) represent many physical processes such as recycled reactors, recycled storage tanks and cold rolling mills [2]. Compared with the systems in [27], the system considered in this paper is more general in the sense that the uncertainty is due to both parametric uncertainty and
unknown nonlinear functions. These unknown functions might come from inaccurate modeling or model reduction.

Assumption 2: The desired trajectory vector \( \tilde{y}_d \) is continuous, available, and satisfies \( \tilde{y}_d = [y_d^T, \dot{y}_d^{(n)}]^T \in \mathbb{R}^{n+1} \).

From (6) and (8), one obtains the following differential equation for the tracking error
\[
\dot{e}_s = a(x) + b(x)u + \sum_{i=1}^{n} \rho_i \phi_i(x_d) + v
\]  
(9)
where \( v = [\partial \mathbf{A}^T \mathbf{y} - y_d^{(n)}] \).

Though the functions \( \phi_i(x_d) \) are known, it could not be used for the construction of control law as \( x_d \) is not available due to the unknown time delays. To tackle this problem, the following Lyapunov-Krasovskii functional is introduced
\[
V_u = \frac{1}{2} \sum_{i=1}^{n} \int_{-\tau_i}^{0} \rho_i^2 (x_d(\lambda)) d\lambda
\]
The time derivative of \( V_u \) is
\[
\dot{V}_u = \frac{1}{2} \sum_{i=1}^{n} (\phi_i^2(x_d(t)) - \phi_i^2(x_d(t - \tau_i)))
\]  
(10)

Considering that (6) satisfies Assumption 1, the filtered tracking error \( e_s \) is uniformly ultimately bounded if the ideal control law is designed as
\[
u^* = -k e_s - u_{ad}(x,v)
\]
\[
u_{ad}(x,v) = \frac{a(x) + v}{b(x)}
\]
where \( k > 0 \) is a design parameter. To prove (11), consider the following Lyapunov function
\[
V_{e_s} = \frac{1}{2} e_s^2 + V_u
\]
Using (10), the time derivative of (12) along (9) is
\[
\dot{V}_{e_s} = e_s \dot{e}_s + \frac{1}{2} \sum_{i=1}^{n} (\phi_i^2(x_d(t)) - \phi_i^2(x_d(t - \tau_i)))
\]  
(13)

Using triangular inequality, we have
\[
e_s \dot{e}_s \leq e_s (a(x) + b(x)u + v) + \frac{1}{2} e_s^2 \rho^T \rho + \sum_{i=1}^{n} \frac{1}{2} \phi_i^2(x_d)
\]  
(14)

with \( \rho^T = [\rho_1 \ldots \rho_n] \).

Upon substitution of (11) and (14) in (13) one gets
\[
\dot{V}_{e_s} \leq e_s (a(x) + b(x)\dot{e}_s) - \frac{1}{2} e_s^2 \rho^T \rho + \sum_{i=1}^{n} \frac{1}{2} \phi_i^2(x_d)
\]
\[
\dot{V}_{e_s} \leq -k e_s^2 + \frac{1}{2} e_s^2 \rho^T \rho + \sum_{i=1}^{n} \frac{1}{2} \phi_i^2(x_d)
\]
\[
\dot{V}_{e_s} \leq -(\delta - 1) e_s^2 - C \phi
\]  
(15)
where \( \delta = (k\bar{b} - \frac{1}{2} \rho^T \rho) > 1 \). Therefore, for all \( e_s^2 \geq \frac{C \phi}{2} \), equation (15) becomes \( \dot{V}_{e_s} \leq -(\delta - 1)e_s^2 \) which concludes that \( e_s \) is uniformly ultimately bounded.

B. Brief Description of Fuzzy Approximator

The fuzzy system considered in this paper has a center-average defuzzifier, product inference and singleton fuzzifier [29]. This type of fuzzy logic system is given by:
\[
q(x) = \frac{\sum_{i=1}^{M} F_i^T(x_i)}{\sum_{i=1}^{M} \prod_{l=1}^{q} \mu_{G_l}^i}\]

where \( M \) is the number of IF-THEN rules in the fuzzy rule base. The IF-Then rules take the following form for \( \ell = 1, 2, \ldots, M \):
\[
R^\ell : \text{If } x_1 \text{ is } F_1^\ell \text{ and } x_2 \text{ is } F_2^\ell \text{ and } \cdots \text{ and } x_n \text{ is } F_n^\ell \text{ Then } q \text{ is } G^\ell
\]
where \( F_i^\ell \) and \( G^\ell \) are the fuzzy sets with membership functions \( \mu_{F_i}^\ell \) and \( \mu_{G_l}^l \) respectively, and \( q \) is the linguistic variables which can be considered as output of the fuzzy logic system. The parameter \( \hat{q}^\ell \) is the point at which \( \mu_{G_l}(\hat{q}^\ell) \) achieves its maximum value and we assume that \( \mu_{G_l}(\hat{q}^\ell) = 1 \). Equation (16) can be rewritten as
\[
q(x) = \psi^T \hat{q}^\ell(x)
\]  
(17)
where \( \psi = [\psi_1, \psi_2, \ldots, \psi_M]^T \) is a parameter vector, and \( \hat{q}^\ell(x) = [\hat{q}_1(x), \hat{q}_2(x), \ldots, \hat{q}_M(x)]^T \) is a regressive vector with the regressor \( \hat{q}^\ell(x) \) known as fuzzy basis function (FBF) of the form
\[
\hat{q}^\ell(x) = \sum_{i=1}^{M} \prod_{l=1}^{q} \mu_{F_l}^i(x_i)
\]

Property:

For any given real continuous function \( u_{ad}(x_m) \) on a compact set \( \Omega_{x_m} \in \mathbb{R}^{n+1} \) and an arbitrary \( e_r > 0 \), there exists a fuzzy logic system \( \hat{u}_{ad}(x_m) \) in the form of (17) and an optimal parameter vector \( \psi^* \) such that
\[
\sup_{x_m \in \Omega_{x_m}} \left| u_{ad}(x_m) - \hat{u}_{ad}(x_m) \right| < e_r
\]
where \( x_m = [x^T v]^T \).

C. Adaptive Control Law

Using fuzzy logic system of (17), the fuzzy-logic based control input is designed as follows:
\[
u = -k e_s - \psi^T \hat{q}^\ell(x_m)
\]  
(18)
where \( \hat{\psi} \) is the estimation of the optimal parameter \( \psi^* \) and can be updated as follows:
\[
\dot{\hat{\psi}} = \frac{\gamma (x_{\infty})_e - \sigma(\hat{\psi})|e_1|}{|\hat{\psi}|} \tag{19}
\]
where \( \gamma \) is a positive design parameter and the switching function \( \sigma(\hat{\psi}) \) [30] is employed to retain the learned information of FBF and to prevent the loss of information if \( e_\psi \) is chosen sufficiently large such that \( |\hat{\psi}| < \epsilon_\psi \), while guaranteeing the boundedness of \( |\hat{\psi}| \). The switching function is given by:
\[
\sigma(\hat{\psi}) = \begin{cases} 
C_\zeta, & \text{if } |\hat{\psi}| > \epsilon_\psi \\
0, & \text{otherwise}
\end{cases} \tag{20}
\]
where \( C_\zeta \) is a design parameter and \( |\hat{\psi}| > C_\zeta \). Then \( |\hat{\psi}| \leq \epsilon_\psi \).

IV. MAIN RESULTS

**Theorem:** For the adaptive system comprising (1) under Assumption 1, the controller (18) and update law (19), the filtered error \( e_\varepsilon \) is semi-globally uniformly ultimately bounded.

**Proof:** Let us consider the following Lyapunov function candidate:
\[
V = \frac{1}{2} e_\varepsilon^T \hat{\psi} + \frac{1}{2\gamma} \hat{\psi}^T \dot{\hat{\psi}} \tag{21}
\]
The time derivative of (21) is given by
\[
\dot{V} = e_\varepsilon \dot{e}_\varepsilon + \frac{1}{\gamma} \hat{\psi}^T \dot{\hat{\psi}} \tag{22}
\]
Now, using (9) and (19) in (22) to get
\[
\dot{V} = -b(x)e_\varepsilon^2 + b(x)u + v + \hat{\psi}^T (\hat{\varepsilon} + \sigma(\hat{\psi})|e_1|)/\gamma
\]
which can be written as
\[
\dot{V} = -b(x)e_\varepsilon^2 + b(x)u + v + \hat{\psi}^T (\hat{\varepsilon} + \sigma(\hat{\psi})|e_1|)/\gamma
\]
\[+\hat{\psi}^T (\hat{\varepsilon} + \sigma(\hat{\psi})|e_1|)/\gamma \tag{23}\]
where \( \hat{\psi}^T = \psi^T - \hat{\psi}^T \). Equivalently, (23) can be put in the following form
\[
\dot{V} \leq -b(x)e_\varepsilon^2 + b(x)u + v + (e_\varepsilon + C_\psi C_\zeta)
\]
\[+\hat{\psi}^T (\hat{\varepsilon} + \sigma(\hat{\psi})|e_1|)/\gamma - C_\zeta \tag{24}\]
where \( \hat{\psi}^T < C_\psi C_\zeta \). Now if \( |\hat{\psi}| > \epsilon_\psi \), then according to (20), the term \( (\sigma(\hat{\psi})|e_1|)/\gamma - C_\zeta \) will be a positive constant \( C_\phi \) and (24) becomes
\[
\dot{V} \leq -(e_\varepsilon + C_\psi C_\zeta)
\]
\[+\hat{\psi}^T (\hat{\varepsilon} + \sigma(\hat{\psi})|e_1|)/\gamma - C_\zeta \tag{25}\]
where \( \hat{\psi}^T < C_\psi \). Equation (25) implies that the filtered error \( e_\varepsilon \) is invariant to the set
\[
S = \{ e_\varepsilon \mid e_\varepsilon^2 < (e_\varepsilon + C_\psi C_\zeta)/k - C_\psi C_\zeta/bk \} \tag{26}\]
whose radius can be made arbitrarily small by increasing the controller gain \( k \) and decreasing \( C_\zeta \). This concludes the proof.

V. SIMULATION RESULTS

In this section, the effectiveness of the proposed adaptive fuzzy control algorithm is demonstrated using the following illustrative examples.

**Example 1:** In this example, we consider a second order time-delay system of the form [9]:
\[
x_1 = (1 + x_1^2)x_2 + x_1 e^{-0.5}x_1 + 2x_1^3 (t - \tau_1)
\]
\[
x_2 = (3 + \cos(x_1, x_2))u + x_1x_2^2 + 0.2x_1 (t - \tau_2) \sin x_2 (t - \tau_2)
\]
where \( \tau_1 = \tau_2 = 2 \) sec and the desired reference signal is chosen as \( y_d = 0.5(\sin t + \sin(0.5t)) \) [6]. The above system is simulated under the proposed adaptive fuzzy control given by (18) and (19) where \( x_i = [x_1, x_2, v]^T \) and \( v = \lambda e_\varepsilon - \hat{\gamma} \). In this case, seven Gaussian membership functions with centers evenly spaced between [-1.5, 1.5] for each variable \( W_i \in x_i, i = 1, 2, 3 \) are chosen as follows:
\[
\begin{align*}
\mu_{e_\varepsilon}^{\frac{-0.5}{5}} &= e_\varepsilon^{4} \\
\mu_{e_\varepsilon}^{\frac{-0.5}{5}+1} &= e_\varepsilon^{4} \\
\mu_{e_\varepsilon}^{\frac{-0.5}{5}+2} &= e_\varepsilon^{4} \\
\mu_{e_\varepsilon}^{\frac{-0.5}{5}+3} &= e_\varepsilon^{4} \\
\mu_{e_\varepsilon}^{\frac{-0.5}{5}+4} &= e_\varepsilon^{4} \\
\mu_{e_\varepsilon}^{\frac{-0.5}{5}+5} &= e_\varepsilon^{4} \\
\mu_{e_\varepsilon}^{\frac{-0.5}{5}+6} &= e_\varepsilon^{4} \\
\end{align*}
\]
The FBFs take the following form:
\[
\zeta(W_i) = \begin{bmatrix}
\prod_{i=1}^{3} \mu_{e_\varepsilon}(W_i) & \cdots & \prod_{i=1}^{3} \mu_{e_\varepsilon}(W_i)
\end{bmatrix}^T
\]
where \( D = \prod_{i=1}^{7} \prod_{i=1}^{3} \mu_{e_\varepsilon}(W_i) \).

Simulation results are depicted in Fig. 1 and Fig. 2 where the controller parameters are selected as \( k = 15; \lambda = 25; \beta = 1 \) and zero initial conditions for the model dynamics as well as for the vector \( \hat{\psi} \).

**Example 2:** In this example, a two-stage chemical reactor with delayed recycle streams is simulated with the proposed controller. The reactor model is given by [24]:

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\[
\dot{x}_1 = -0.8x_1 + x_2 + 0.5\sin(t)x_1^2(t-\tau_1),
\]
\[
\dot{x}_2 = u - 0.8x_2 + x_1(t-\tau_2) + 0.5\sin(t)x_1^3(t-\tau_2),
\]
\[y = x_1,
\]
where the delay times are \(\tau_1 = \tau_2 = 2\sec\). The desired reference signal is assumed as \(y_d = 0\). For the vector \(x_u = [x_1, x_2, v]^T\) seven Gaussian membership functions of the form (27) are used. Simulation results are shown in Fig. 3 and Fig. 4 for \(k = 10; \lambda = 1.5;\) and \(\gamma = 1\) and initial conditions of \(x(0) = [-2, 0]\) and \(\dot{\psi}(0)=0\).

Simulation results of the above examples demonstrate the effectiveness of the proposed adaptive fuzzy controller in achieving good tracking performance and smooth control signal.

### VI. CONCLUSION

Adaptive fuzzy tracking control is presented for a class of SISO strict-feedback nonlinear systems with unknown time-delays. The key aspect of the proposed method is that the state-feedback control problem of the strict-feedback system can be viewed as the output-feedback control problem of the affine system in the normal form; this results in avoiding backstepping in the controller design. It is also demonstrated that control law and stability analysis is considerably simpler than the previous backstepping based algorithms. In the developed control algorithm, Mamdani-type fuzzy systems are utilized to approximate on-line the unknown nonlinear functions. The proposed control scheme guarantees the semi-global boundedness of all the signals in the closed-loop system and good tracking performance. Moreover, the suggested adaptive fuzzy controller is simple. This makes the design scheme easier to be implemented in practical applications. Simulation examples are provided to validate the effectiveness of the proposed controller.

### REFERENCES


