Stabilization of a Buck converter: a saturated LMI approach

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Abstract: This paper deals with the problem of constrained stability and tracking of Takagi-Sugeno (T-S) fuzzy positive systems. Saturated LMIs are used to insert the constraints in the design phase while imposing positivity in closed-loop. The already available theoretical results are applied to the DC-DC buck converter which is widely used in the photovoltaic generators. Based on simulation results success of the method is shown for this application.

Keywords: T-S fuzzy systems, positive systems, stabilization, LMI, buck converter, photovoltaic generator.

1. INTRODUCTION

In this paper, we are concerned with the control of the DC-DC buck converter which is widely used in photovoltaic energy transformation. The modelization step of this system gives rise to three aspects: the system is non linear, the system is a positive one since its commonly used state variables must always be positive and the control is a duty ratio that must be constrained between 0 and 1. Hence, Takagi-Sugeno (T-S) fuzzy techniques can be used to design a control for the non linear system Takagi and Sugeno (1985). Furthermore, during the design step, asymmetrical constraints on the control, while imposing positivity in closed-loop, have to be taken into account. In consequence, the problem studied thereafter, concerns a special class of systems called constrained positive T-S models. From the history of the T-S approach, this class can be interpreted as a collection of linear models interconnected by nonlinear functions, called membership functions, which are time dependent variables. The most delicate problem is the choice of premise variables that partition the space Wang and Mendel (1992). Positive systems have been of great interest to researchers in recent years Farina (2000), Ait Rami and Tadeo (2006), Hmamed et al. (2008), Kazorek (2009) and Benzaouia (2012). The class of positive T-S fuzzy systems was considered for the first time in Benzaouia et al. (2010). The obtained results were presented using LMIs.

The buck converter has attracted the interest of many researchers Lin and Tsai (2004); Lian et al. (2006); Li and Ji (2008); El Hajjaji et al. (2009); Nachidi et al. (2011) and references therein. Almost all the available methods in the literature do not pay attention to neither the positive aspect of the system nor the asymmetrical constraints on the control of the buck converter. Based on the model given in Lian et al. (2006), the tracking of the reference is realized upon an error model using LMIs tools in Li and Ji (2008). The constraints on the control were not considered nor shown in simulation despite the main interest to obtain a duty ratio between 0 and 1. In Lin and Tsai (2004), by using backstepping and sliding mode techniques, the authors succeed to obtain an output voltage following a given reference with a variable load resistor and a variable source voltage. However, the duty ratio is shown by simulation to be restricted between 0 and 1 without taking this constraint into account in the design phase. In Nachidi et al. (2011); El Hajjaji et al. (2009), a robust output tracking approach was proposed by using LMI tool without taking into account the constraint aspect of the duty ratio. Note that El Hajjaji et al. (2009) used a different model of the buck converter. In this paper, the conditions for stabilization of T-S systems with constrained control, which are already available in the literature Benzaouia et al. (2010), are recalled. The constraints on the control while imposing positivity of the state are taken into account during the design phase. The results are obtained by using LMIs.

Furthermore, and from a practical point of view, the recalled results are successfully applied on the model of a real process consisting of a DC-DC buck converter taken from Lian et al. (2006). In fact, the simulations show that all requirements are achieved, namely, the tracking of a reference signal for the capacity voltage, the positivity of the closed-loop states and finally the limitation of the duty ratio between 0 and 1.

The rest of this paper is organized as follows: In Section 2, the model of the buck converter is presented while Section 3 deals with the description of T-S fuzzy model and fuzzy control law based on PDC structure is given for the buck converter together with conditions of stabilization for positive systems with asymmetrical constrained control. In Section 4, the application to the buck converter is given to show the need of such controllers for such systems. Some conclusions are given in Section 5.

Notation:
The objective of this work is to design a control law limited 

duction mode (CCM) in order to achieve a tracking reference 

data of the studied buck converter are given in Table 2 Li and 

Fig. 1. DC-DC Buck Converter

2. PROBLEM FORMULATION

Data of the studied buck converter are given in Table 2 Li and 

with \( i_{\text{max}} = 3A \) and \( i_{\text{min}} = 0A \). The T-S model of the DC-DC 

converter, according to Li and Ji (2008), is given by

\[
\dot{x}(t) = Ax(t) + B(x)u(t) + D, \quad 0 \leq u(t) \leq 1
\]

where

\[
x(t) = \begin{bmatrix} i(t) \\ v(t) \end{bmatrix} \in \mathbb{R}^2
\]

\[
A = \begin{bmatrix}
-\frac{R_L + \frac{R_R}{R_L + R_C}}{L} & -\frac{R}{L + LR_C} \\
\frac{C}{R} & -\frac{C}{R + CR_C}
\end{bmatrix}
\]

\[
B(x) = \begin{bmatrix}
-\frac{R_M i(t) - E - V_c}{L} \\
\frac{-V_c}{L}
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
\frac{E}{L} \\
0
\end{bmatrix}
\]

with \( i(t) \), \( v(t) \) representing the inductance current and the ca-

pacity voltage respectively, \( u(t) \) is the duty ratio of the MOS-

FET which must be within the interval [0 1]. \( R_C, R_L, R_M, E \),

and \( V_c \) are specified by figure 1. 

The objective of this work is to design a control law limited

between 0 and 1, representing the duty ratio that allows the

system to switch between two topologies in the continuous con-

duction mode (CCM) in order to achieve a tracking reference

\( v_{\text{ref}} \). Further, the system is nonlinear. The approach followed

thereafter is the use of Takagi Suggeno (T-S) techniques.

3. PRELIMINARIES

In order to built a suitable control law, we use the T-S modeliza-

tion. For this we denote \( M_1 = \text{min}(i(t)) \), \( M_m = \text{max}(i(t)) \)

and we divide the interval \([M_1, M_m] \) into \( r \) subintervals

\( \mathcal{I}_s = [M_{s-1}, M_{s+1}] \), \( s = 2, \ldots r-1 \). Consider the following fuzzy

rules:

IF \( i(t) \in F_s \) THEN

\[
\dot{x}(t) = Ax(t) + B_s u(t) + D, \quad s = 1, \ldots r
\]

\[
0 \leq u(t) \leq 1
\]

where \( B_s = \begin{bmatrix} \frac{R_M M_s - E - V_c}{L} \\
0 \end{bmatrix} \) and \( F_s \) is the fuzzy set of

the fuzzy region \( \mathcal{I}_s \), \( r \) denotes the number of rules. The global

T-S fuzzy model of the buck converter is given by

\[
\dot{x}(t) = \sum_{s=1}^r h_s(x(t))(Ax(t) + B_s u(t)) + D,
\]

where

\[
h_s(x(t)) = \frac{\mu_s(x(t))}{\sum_{s=1}^r \mu_s(x(t))}
\]

It is worth noting that according to the circuit of the Buck

converter given by Figure 1, the state space vector is positive.

Hence, one has to maintain this state positive during all the

evolution of the system. Make the following change of variab le

\( x(t) = \bar{x}(t) - w(t) \). Note that the new variable \( d(t) \) is

time varying and is not suitable for the T-S fuzzy modeling. As

\( i(t) \) belongs to \([0, i_{\text{max}}] \), one can choose the worst case \( d =

\frac{V_c}{E + V_c} \). This implies for our buck converter that \(-1 < d < 0 \).

The system becomes:

\[
\dot{x}(t) = \sum_{s=1}^r h_s(x(t))(Ax(t) + B_s w(t)),
\]

\[
-\alpha \leq w(t) \leq \alpha,
\]

\[
-\alpha \leq w(t) \leq \alpha,
\]

where \( w(t) = u(t) + d \). Note that to obtain a symmetrical saturation, one can use the following scaling:

\[
\alpha = \text{min}\{-d; 1 + d\}.
\]

Consider the following change of variables:

\[
\dot{\bar{x}}(t) = \sum_{s=1}^r h_s(x(t))(A\bar{x}(t) + B_s \bar{w}(t))
\]

\[
-1 \leq \bar{w}(t) \leq 1
\]

\[
x_0 \geq 0
\]

with \( \bar{B}_s = \alpha B_s; \bar{w}(t) = \frac{w(t)}{\alpha} \). The duty ratio is then given by

\[
u(t) = \alpha \bar{w} - d.
\]

To achieve our objective of the tracking of a given reference of 

the voltage capacity, we introduce the following idea. Let us consider the new control given by using the following PDC control

\[
\bar{w}(t) = \sum_{s=1}^r h_s(x(t))(K_s x(t) + L_s \bar{x}_{\text{ref}})
\]
where \( x_{ref} \) is the state to be followed by the closed-loop system and the gains \( L_i \) to be designed to achieve the tracking problem. Hence, the closed-loop system becomes:

\[
\dot{x}(t) = \sum_{s=1}^{r} \sum_{j=1}^{m} h_s(x(t)) h_j(x(t)) ((A + B_s K_j) x(t) + B_s L_j x_{ref}) = G(h) x(t) + B(h) L h x_{ref}
\]

(13)

where \( G(h) = A + B(h) K(h), B(h) = \sum_{s=1}^{r} h_s(x(t)) B_s \). Similar notation is used for \( K(h) \) and \( L(h) \). Taking the steady state of the closed-loop system, one can obtain:

\[
0 = G(h) x(\infty) + B(h) L(h) x_{ref}.
\]

(14)

Since the objective is that \( x(\infty) = x_{ref} \), one can choose \( L(h) = -B(h) G(h) \), with \( B(h) \) representing the pseudo inverse of matrix \( B(h) \), i.e., \( B(h)^+ = (B(h)^T B(h))^{-1} B(h)^T \).

Note that this new system (10) admits symmetrical constraints. Consequently, one needs to take into account the symmetrical constraints during the design step. It is worth noting that, to the best of our knowledge, no theoretical approach exists in the literature to solve this problem for T-S fuzzy positive systems except the work of Benzaouia et al. (2010, 2011a) where the problem is solved with LMIs.

Recall now the results on which our work will be based.

### 3.1 Controller synthesis

Consider the general T-S fuzzy system given by:

\[
\dot{x}(t) = \sum_{s=1}^{r} \sum_{j=1}^{m} h_s(x(t)) h_j(x(t)) (A x(t) + B_s u(t))
\]

\[-1 \leq u(t) \leq 1 \]

\[x_0 \geq 0\]

with \( x \in \mathbb{R}^n, u \in \mathbb{R}^m \). For the sake of brevity, \( h(x(t)) \) will be noted \( h(t) \).

The T-S fuzzy system in closed-loop with a PDC control is given by:

\[
\dot{x}(t) = \sum_{s=1}^{r} \sum_{j=1}^{m} h_s(x(t)) h_j(x(t)) (A + B_s K_j) x(t)
\]

\[x_0 \geq 0\]

Define the following sets:

\[\varepsilon(P, \rho) = \{ x \in \mathbb{R}^n, x^T P x \leq \rho, \rho > 0 \} \quad (15)\]

\[\mathcal{L}(K_j) = \{ x \in \mathbb{R}^n, -1 \leq (K_j) x \leq 1, l = 1, \ldots, m \} \quad (16)\]

where \( (K_j)_l \) stands for the \( l \)-th row of the matrix \( K_j \). Note that the set \( \varepsilon(P, \rho) \) is ellipsoidal, while the set \( \mathcal{L}(K_j) \) is polyhedral. Now, one can recall the result that will be used in the sequel for the buck converter control.

**Theorem 3.1.** Benzaouia et al. (2011a) For positive scalar \( \rho \), if there exist diagonal matrix \( P > 0 \), matrices \( K_j, j = 1, \ldots, r \) such that

\[
A^T P + PA^T + B_s K_j + K_j^T B_s^T < 0
\]

\[A + B_s K_j \enspace \text{Metzler}\]

\[\varepsilon(P, \rho) \subseteq \mathcal{L}(K_j),\]

\[j, s = 1, \ldots, r,\]

then the saturated T-S fuzzy closed-loop system (15) is asymptotically stable while respecting positivity in closed-loop \( x_0 \in \varepsilon(P, \rho) \).

The first condition in Theorem 3.1 implies the stability of the T-S fuzzy closed-loop system, while condition two ensures the positivity of the state of the closed-loop system. Further, the third condition realizes the limitation of the control within the allowed interval \([-1, 1]\).

This result which is not suitable for control synthesis can be stated in LMI form.

**Corollary 3.1.** Benzaouia et al. (2011a) For a positive scalar \( \rho \), if there exist matrix \( X = \text{diag}\{x_1, x_2, \ldots, x_n\} > 0 \) and matrices \( Y_j, j = 1, \ldots, r \) such that

\[
X A^T + A^T X + B_s Y_j + Y_j^T B_s^T < 0
\]

\[a_k x_t + b_k y_j^t \geq 0, k \neq l\]

\[\frac{1}{\rho} y_j \geq 0\]

\[j, s = 1, \ldots, r,\]

\[k, l = 1, \ldots, n,\]

then the saturated T-S fuzzy closed-loop system (15) is asymptotically stable while respecting positivity in closed-loop \( x_0 \in \varepsilon(P, \rho) \), where \( P = X^{-1}, K_j = Y_j X^{-1} \) and

\[
A = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} ; B_s = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} ; Y_j = \begin{bmatrix} y_j^1 & y_j^2 & \ldots & y_j^n \end{bmatrix} .
\]

(24)

### 4. MAIN RESULT: APPLICATION TO BUCK CONVERTER

In this section, the results of the previous section are used. However, the actual problem is a tracking one while the results of Theorem 3.1 concern only stabilization problems. This explains the transformations leading to the control law (12). To obtain a simple T-S fuzzy model of the buck converter, we choose one interval with \( r = 2 \) rules, as follows:

IF \( i(t) = M_1 = i_{min} \) THEN,

\[
B_1 = \begin{bmatrix} -R_n i_{min} + E + V_o \\ L \\ 0 \end{bmatrix}
\]

IF \( i(t) = M_2 = i_{max} \) THEN,

\[
B_2 = \begin{bmatrix} -R_n i_{max} + E + V_o \\ L \\ 0 \end{bmatrix}
\]

the membership functions are given by:

\[
h_1(t) = \frac{i_{max} - i(t)}{M_2 - M_1}
\]

\[
h_2(t) = \frac{i(t) - i_{min}}{M_2 - M_1}
\]
Remark 4.1. The direct application of the results of Corollary 3.1 by using the change of variables \( \tilde{x} = Cx \) leads to slow dynamic of the system in closed-loop. To overcome this drawback, the LMIs of Corollary 3.1 are used with \( \alpha = 1 \) realizing \(-1 \leq Kx(t) \leq 1\). The obtained gain controller is then multiplied by \( \alpha = d \) to have \(-\alpha \leq \alpha Kx(t) \leq \alpha\). The positivity and stability conditions obtained with the new gain controller are checked to remain satisfied.

Comment 4.1. To achieve the tracking goal, gains \( L_j \) were introduced. However, for the constraints the presented theoretical results do not take into account these gains. Nevertheless, as \( L_j \) are small, for the buck converter example, the constraints on the gains \( K_j \) are sufficient to compensate the variation of the tracking gains \( L_j \). This drawback is still unsolved from a theoretical point of view.

5. CONCLUSION

The conditions for stabilization of T-S fuzzy systems with constrained control while imposing positivity are presented. The constraints are taken into account during the design phase using an LMI approach. The obtained solution to Corollary 3.1 enables one to deduce the controller gains for each linear model. Similar membership functions are used to design the global controller that respects the control constraints and to achieve the tracking of the reference signal without violating the positivity of the system. The obtained results are applied to the buck DC-DC converter. Simulation results show the effectiveness of the approach. Particularly, the duty ratio respects the physical limitation between 0 and 1 which was shown by Figure 3.

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