Margin Adaptive Resource Allocation in Downlink OFDMA system with Outdated Channel State Information

Ayaz Ahmad and Mohamad Assaad
Ecole Supérieure d'électricité (SUPELEC), 91192 Gif-sur-Yvette, France
Email: {ayaz.ahmad, mohamad.assaad}@supelec.fr

Abstract—Most of existing works on resource allocation in TDMA and OFDMA systems assume the availability of perfect channel state information (CSI) at the transmitter. The availability of only a noisy estimate of the channel and the feedback delay are the problems which are faced more often. In this paper, we study the effect of feedback delay on resource allocation in a downlink OFDMA system. We approach the problem by using convex optimization framework and get the optimal solution with zero optimality gap. By considering the probability distribution function of the current CSI conditioned on outdated CSI in the resource allocation framework, we propose an algorithm that minimizes the total transmit power of the system subject to strict constraints on users’ conditional expected capacities. Results show that the system performance is sensitive to feedback delay and provide guidelines for system dimensioning and design.

I. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) is a very popular transmission scheme for many wireless communication systems. It is used in IEEE 802.16 WirelessMAN, both for uplink and downlink air interfaces. Recently it is used for downlink air interface in 3GPP Long Term Evolution of the third generation cellular systems, and is also the candidate access technique for the IEEE 802.22 Wireless Regional Area Networks. OFDMA is based on OFDM technique; therefore it inherits the immunity to inter symbol interference (ISI) in frequency selective fading channel and offers good flexibility and performance for a reasonable complexity [1]. The users of a same cell are multiplexed in frequency, each user’s data being transmitted on a subset of the sub-carriers of an orthogonal frequency division multiplexing (OFDM) symbol. In order to achieve the challenging spectral efficiency and user throughput targets, adaptive resource allocation and link adaptation are very essential [2]-[4].

In OFDMA systems, resource allocation techniques take advantage of the frequency and time diversities of the system in order to optimize the use of the available resources. It exploits available channel state information (CSI) at the transmitter side for accomplishing power allocations and sharing the sub-carriers among users. The resource allocation has two main objectives: First maximize the efficiency of the system (i.e. either by maximizing throughput or minimizing total power) by allocating the resources to the most appropriate users and second achieve the Quality of Service (QoS) constraints of each user. These two objectives are conflicting and there is a risk in achieving one at the expense of the other. Therefore, a trade-off between these objectives should be achieved.

The optimal resource allocation in an OFDMA system has been addressed in a number of publications [5]-[6]. In [5] the authors show that the optimal OFDMA resource allocation problem is computationally feasible and that an ergodic capacity maximization problem can be solved with a complexity of \( O(KN) \) for \( K \) users and \( N \) subcarriers. To solve the problems of weighted sum rate maximization and weighted sum power minimization, the Lagrange dual decomposition method is used [6]. But in major portion of the previous research the availability of perfect channel state information (CSI) is assumed which is unfortunately an unrealistic approach because in most wireless communication systems the transmitter has only imperfect CSI at its disposal. The two major causes of imperfection in CSI are the channel estimation error and the channel feedback delay. Thus, this problem of unavailability of perfect CSI has necessitated the design of such optimal resource allocation techniques that ensures communication with the required QoS in spite of the presence of channel estimation error and feedback delay. The objective of this paper is to provide an efficient resource allocation strategy for the case where only outdated (imperfect) CSI is available.

The effect of imperfect CSI is well considered for single user OFDM [7]-[9]. In [7] it is shown that the effect of outdated channel information on the performance of an adaptive OFDM system can be overcome by taking advantage of the channel prediction. The effect of channel estimation error as well as that of outdated channel state information on the performance of adaptive OFDM for the variable bit rate case is studied in [8]. In [9] an optimal power loading algorithm for rate maximization in OFDM is derived which is based on average and outage capacity criteria and it is investigated that the outage rate of the system may be highly reduced due to the CSI error. In [10] the authors have discussed the effect of imperfect channel information on the performance of multicarrier-OFDM and Digital Multi-Tone (DMT) systems, and have derived expressions for the signal-to-interference ratio (SIR) and the probability of error in terms of signal to noise ratio (SNR) and the mean square error in channel estimation. However, very less work is done for multiuser OFDM (OFDMA) case. In [11] the authors have considered a downlink OFDMA wireless communication system where the imperfect CSI at the transmitter is due to channel estimation error. In addition, the resource allocation problem in [11] has been modeled as an ergodic weighted-sum capacity maximization with total power constraint. No assurance of users’ QoS requirements is provided in [11] (i.e. no constraint on minimum bit rate to achieve per user).

In this paper, we consider a downlink wireless OFDMA system, and investigate the impact of feedback delay on the performance of the system and derive an optimal iterative algorithm which ensures the required QoS in spite of the presence of feedback delay. More specifically, our work is aimed at minimizing the total transmit power of the system subject to strict data rate constraints.
We solve the problem under the assumption of outdated CSI, and propose an algorithm which allocates the subcarriers to the users in such a manner that the individual data rate requirements of all the users are satisfied while transmitting the minimum possible power (margin adaptive). We formulate the problem by using convex optimization framework, and show that the optimal power allocation is multilevel water-filling. The proposed algorithm has polynomial complexity and solves the problem with zero optimality gap.

The rest of this paper is organized as follows: in sections II and III the system model and problem formulation are described. The optimization framework and proposed resource allocation algorithm are provided in section IV and V. Results are presented in section VI and section VII concludes the paper.

II. SYSTEM MODEL

We consider a downlink OFDMA system with N subcarriers and K active users around the base station. N and K are chosen such that N > K. The channel is time varying but it is assumed to stay invariant over an entire OFDM symbol. The signal received by user k at time t, is given as

\[ y_k(t) = H_k(t)s_k(t) + w_k(t) \]  

where \( N \times 1 \) vectors \( s_k(t) \) and \( y_k(t) \) are the transmitted and received signals respectively; \( w_k(t) \) is assumed to be a zero mean, circular symmetric, complex Gaussian (ZMCSG) noise vector. We consider that the channels \( h_k(t) \) are the complex valued wireless channel fading random processes. We assume that the channels \( h_k(t) \sim C\mathcal{N}(0, \sigma^2_{h}) \) have Gaussian distribution and are i.i.d over different users.

Each user terminal estimates the CSI and sends it to the base station through a feedback channel. Due to the feedback delay, the transmitter has only outdated CSI at its disposal and makes decisions on the basis of CSI estimated at time \( t - \tau \) for the transmission at time \( t \), where \( \tau \) is the feedback delay. We consider a stationary ergodic Gauss-Markov block fading process, where the channel variation between times \( t \) and \( t - \tau \) is modeled as

\[ h_k(t) = \rho h_k(t - \tau) + e_k(t) \]  

where \( \rho \) is the time-autocorrelation and \( e_k(t) = [e_{k,1}(t), ..., e_{k,N}(t)]^T \) is the channel error vector defined as \( e_k(t) = \sqrt{1 - \rho^2}u_k(t) \). The entries of \( u_k(t) \) are i.i.d complex Gaussian random variables with zero mean and unit variance i.e., \( u_{k,n}(t_1) \) is independent of \( u_{k,n}(t_2) \) for \( t_1 \neq t_2 \). Furthermore, the entries of \( u_k(t) \) are also i.i.d over different users. The variance of channel error vector is \( \sigma^2_e = 1 - \rho^2 \) and it is uncorrelated with \( h_k(t - \tau) \). The correlation coefficient \( \rho \) is defined as

\[ \rho = E[h_{k,n}(t)h_{k,n}(t - \tau)] \]  

We use Clarke’s isotropic scattering model [12] for which \( \rho \) can be related to the user velocity \( v \) by \( \rho = J_0(2\pi f_v \frac{v}{c}) \), where \( J_0 \) is the zeroth order Bessel function of the first kind, \( f_v \) is the carrier frequency, and \( c \) is the speed of light.

Based on the channel model given by (2), the marginal distribution of fading channel for user \( k \) on subcarrier \( n \) conditioned on the outdated CSI can be modeled as a non-zero mean complex Gaussian random variable [13] [14] with mean \( h_{k,n}(t - \tau) \) i.e.,

\[ h_{k,n}(t)|h_{k,n}(t) \sim C\mathcal{N}(h_{k,n}(t), \sigma^2_z) \]

where \( h_{k,n}(t) \) is the actual channel gain and \( h_{k,n}(t - \tau) \) is the outdated channel gain respectively, and \( \sigma^2_z \) is the channel error variance. Let \( g_k,n \) be the channel to noise ratio (CNR) given as

\[ g_k,n = \frac{|h_{k,n}(t)|^2}{\sigma^2_w + \sigma^2_z} \]

then the distribution of \( g_k,n \) conditioned on \( \hat{g}_{k,n} = \frac{|h_{k,n}(t - \tau)|^2}{\sigma^2_e} \) can be modeled as a non-central Chi-squared distribution with two degrees of freedom. The probability distribution function of \( g_k,n \) conditioned on \( \hat{g}_{k,n} \) is given as [13, Section 2.1-4]

\[ f_{g_k,n}(g_k,n|\hat{g}_{k,n}) = \frac{\sigma^2_w}{\sigma^2_e} e^{-\frac{\sigma^2_w}{\sigma^2_e}(g_k,n + \rho^2 g_k,n)} I_0(2\frac{\sigma^2_w}{\sigma^2_e}\sqrt{g_k,n + \rho^2 g_k,n}) \]  

where \( I_0 \) is the zeroth order modified Bessel function of the first kind.

III. PROBLEM FORMULATION

The problem can be formulated as follows: having outdated CSIs, how the base station should allocate power and subcarriers in order to minimize the total transmit power of the system subject to constraints on users’ throughput. Note that in OFDMA technique, a subcarrier cannot be allocated to more than one users at a time. Let \( R = [R_1, ..., R_K]^T \) be the data rate requirements of the users; \( P = [P_1, ..., P_K]^T \) be the transmit powers vector where \( P_k = [p_{k,1}, ..., p_{k,N}]^T \), and \( I = [I_1, ..., I_K]^T \) be the vector of the sets of subcarriers assigned to the users. The data rate for user \( k \) on subcarrier \( n \) is given by the Shannon capacity

\[ C_{k,n} = \log(1 + p_{k,n}g_{k,n}) \text{ nats/s/Hz} \]

The base station does not have knowledge of the current CSI of the users, however the distribution of this CSI conditioned on outdated CSI is described in the previous section. The base station can then use this distribution function in the resource allocation in order to improve the users’ QoS. Hence, the rate constraint of user \( k \) will be written in terms of conditional average capacity as follows:

\[ E_{g_k,n|\hat{g}_{k,n}} \left\{ \sum_{n \in I_k} \log(1 + p_{k,n}g_{k,n}|\hat{g}_{k,n}) \right\} \geq R_k \]  

where \( p_{k,n} \) is the allocated power; \( g_{k,n} \) is the actual CNR; and \( \hat{g}_{k,n} \) is CNR obtained from the available outdated CSI for user \( k \) on subcarrier \( n \). Notation \( E_{g_k,n|\hat{g}_{k,n}} \) means Expectation with respect to \( g_{k,n} \) conditioned on \( \hat{g}_{k,n} \). Since we assume the availability of only outdated CSI, therefore \( p_{k,n} \) is a function of \( \hat{g}_{k,n} \) and \( \rho \). To ensure that a subcarrier is always assigned to a single user at a time, \( I_l \cap I_m = \emptyset \) for \( l \neq m \). The overall optimization problem with these two constraints can now be stated as

\[ \min_{P} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \]  

subject to,

\[ E_{g_k,n|\hat{g}_{k,n}} \left\{ \sum_{n \in I_k} \log(1 + p_{k,n}g_{k,n}|\hat{g}_{k,n}) \right\} \geq R_k \]

\[ p_{k,n} \geq 0 \]

\[ I_l \cap I_m = \emptyset, \forall l \neq m \]
Due to the presence of exclusive subcarrier assignment constraint, the optimization problem (7) is a combinatorial problem. Since each subcarrier can only be assigned to one user, therefore, there are $K^N$ possible subcarrier assignments. So, finding the optimal solution of (7) needs a search over all power and subcarrier allocations and thus, it is a mixed integer programming problem with exponential complexity.

IV. Minimization of Total Transmit Power via Convex Optimization

A. Convex optimization and KKT conditions

We approach the optimization problem (7) using convex optimization to make it more tractable. We convert it into convex optimization problem by taking advantage of time sharing technique [3]. The idea is that we introduce a time sharing factor $\delta_{k,n}$ for the $k$th user on the $n$th subcarrier, such that $\delta_{k,n} \in [0, 1]$ and $\sum_{n=1}^{N} \delta_{k,n} \leq 1$. The time sharing factor $\delta_{k,n}$ indicates the portion of time for which the subcarrier $n$ is allocated to user $k$ during each OFDM frame transmission. The capacity of user $k$ on subcarrier $n$ after the introduction of $\delta_{k,n}$ will be given as

$$C_{k,n} = \delta_{k,n} \log \left( 1 + \frac{q_{k,n} g_{k,n}}{\delta_{k,n}} \right)$$  \hspace{1cm} (8)

Further, we change the objective variable from $p_{k,n}$ to $q_{k,n} = \delta_{k,n} p_{k,n}$, where $q_{k,n}$ is the actual power allocated to user $k$ on subcarrier $n$. After making this variable change, the capacity in (8) will have the following expression

$$C_{k,n} = \delta_{k,n} \log \left( 1 + \frac{q_{k,n}}{\delta_{k,n}} g_{k,n} \right)$$  \hspace{1cm} (9)

which is a concave function for $\delta_{k,n} \geq 0$ and $q_{k,n} \geq 0$. The optimization problem (7) can now be written as

$$\min_{p} \sum_{k=1}^{K} \sum_{n=1}^{N} q_{k,n}$$ \hspace{1cm} (10)

subject to,

$$-E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \sum_{n=1}^{N} \delta_{k,n} \log \left( 1 + \frac{q_{k,n} g_{k,n}}{\delta_{k,n}} \right) \right\} + R_k \leq 0$$ \hspace{1cm} (10a)

$$\sum_{k=1}^{K} \delta_{k,n} - 1 \leq 0$$ \hspace{1cm} (10b)

$$-q_{k,n} \leq 0$$ \hspace{1cm} (10c)

$$-\delta_{k,n} \leq 0$$ \hspace{1cm} (10d)

Thus, (10) is a convex optimization problem in standard form. As the objective function and the inequality constraints are continuously differentiable convex functions, therefore, Karush–Kuhn–Tucker (KKT) conditions are sufficient for global optimality. So, we solve the optimization problem (10) by using KKT conditions. The inequality constraints (10a)-(10d) satisfy the KKT primal feasibility condition and states that $q_{k,n}$ and $\delta_{k,n}$ are primal feasible. Similarly, let $\lambda_{k}$, $\alpha_{k,n}$, $\mu_{n}$ and $\nu_{k,n}$ be the Lagrange multiplier associated with inequality constraints (10a)-(10d) respectively. All these multipliers are $\geq 0$ so that the KKT dual feasibility condition is satisfied. The Lagrangian for (10) can be written as

$$L(q, \delta, \lambda, \alpha, \mu, \nu) = \min_{p} \sum_{k=1}^{K} \sum_{n=1}^{N} q_{k,n}$$

$$- \sum_{k=1}^{K} \lambda_{k} E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \sum_{n=1}^{N} \delta_{k,n} \log \left( 1 + \frac{q_{k,n} g_{k,n}}{\delta_{k,n}} \right) \right\} - R_k$$

$$+ \sum_{n=1}^{N} \mu_{n} \left( \sum_{k=1}^{K} \delta_{k,n} - 1 \right) - \sum_{k=1}^{K} \sum_{n=1}^{N} \alpha_{k,n} q_{k,n}$$

$$- \sum_{k=1}^{K} \sum_{n=1}^{N} \nu_{k,n} \delta_{k,n}$$ \hspace{1cm} (11)

The KKT stationarity conditions for optimal power and subcarrier allocation can be found by differentiating (11) with respect to $q_{k,n}$, and $\delta_{k,n}$ respectively and then setting them to zero. The two KKT stationarity conditions are given as

$$\lambda_{k} E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \frac{q_{k,n} g_{k,n}}{1 + p_{k,n} g_{k,n}} \delta_{k,n} \right\} = 1 - \alpha_{k,n}$$ \hspace{1cm} (12)

$$\lambda_{k} E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \log \left( 1 + \frac{p_{k,n} g_{k,n}}{1 + p_{k,n} g_{k,n}} \delta_{k,n} \right) \right\} - \lambda_{k} E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \frac{p_{k,n} g_{k,n}}{1 + p_{k,n} g_{k,n}} \delta_{k,n} \right\} = \mu_{n} - \nu_{k,n}$$ \hspace{1cm} (13)

B. Simplification of the KKT equations

The solution of (12) and (13) is based on the evaluation of the expectations with respect to the CNR $g_{k,n}$ given $\hat{g}_{k,n}$. The use of pdf (4) makes the solution of (12) and (13) very complicated, therefore in this section, we derive a closed form to $E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \frac{q_{k,n} g_{k,n}}{1 + p_{k,n} g_{k,n}} \delta_{k,n} \right\}$ and $E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \log \left( 1 + \frac{p_{k,n} g_{k,n}}{1 + p_{k,n} g_{k,n}} \delta_{k,n} \right) \right\}$ in order to simplify and reduce the complexity of the solution. To do this, we start by approximating the pdf (4) by gamma distribution [11, 14]

$$f_{g_{k,n}|\hat{g}_{k,n}}(g_{k,n} | \hat{g}_{k,n}) = \frac{q_{k,n} g_{k,n}^{a_{k,n} - 1} e^{-\frac{g_{k,n}}{\hat{g}_{k,n}}}}{\hat{g}_{k,n}^{a_{k,n}}} \Gamma(a_{k,n})$$ \hspace{1cm} (14)

where $\Gamma(z)$ is Gamma function and equation (14) is the pdf of gamma distribution with shape parameter $a_{k,n} = \frac{q^2}{\hat{E}^2} \rho^2 (\rho + 1)^2$ and scale parameter, $\theta_{k,n} = \frac{\rho^2 \hat{g}_{k,n} + \frac{q^2}{\hat{E}^2}}{\rho^2}$. Now, the conditional mean $E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \frac{g_{k,n}}{1 + p_{k,n} g_{k,n}} \right\}$ can be found easily by using the pdf in (14) and is given as

$$E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \frac{g_{k,n}}{1 + p_{k,n} g_{k,n}} \right\} = \frac{a_{k,n}}{p_{k,n} \theta_{k,n}} \left( 1 + \frac{1}{p_{k,n} \theta_{k,n}} \right) e^{-\frac{a_{k,n}}{p_{k,n} \theta_{k,n}}} \Gamma \left( -a_{k,n}, -\frac{1}{p_{k,n} \theta_{k,n}} \right)$$ \hspace{1cm} (15)

where $\Gamma(a,z)$ is the incomplete Gamma function [16, Section 8.350]. However, the solution of $E_{g_{k,n}|\hat{g}_{k,n}} \left\{ \log \left( 1 + \frac{p_{k,n} g_{k,n}}{1 + p_{k,n} g_{k,n}} \right) \right\}$
$p_{k,n}g_{k,n} | \hat{g}_{k,n}$ is not that simple and to the best of our knowledge is not available in the state of the art. In order to get a closed form expression of $E_{g_{k,n} | \hat{g}_{k,n}} \{ \log (1 + p_{k,n}g_{k,n} | \hat{g}_{k,n}) \}$ we proceed as follows: Let $\eta_{k,n} = p_{k,n}g_{k,n}$. Using the pdf of $\log (1 + p_{k,n}g_{k,n} | \hat{g}_{k,n})$ from (14), we get

$$E_{g_{k,n} | \hat{g}_{k,n}} \{ \log (1 + p_{k,n}g_{k,n} | \hat{g}_{k,n}) \} = \left\{ \begin{array}{ll} \frac{1}{\Gamma(a_{k,n})} \int_{0}^{\infty} \log (1 + \eta_{k,n}) \eta_{k,n}^{a_{k,n}-1} e^{-\eta_{k,n}} d\eta_{k,n} \\
\end{array} \right. \}$$

(16)

There is no closed form solution to the integral in (16), but we solve it by first making the use of the lower bound for the logarithmic term $\log (1 + \eta_{k,n})$ as follows [17].

$$\log (1 + \eta_{k,n}) \leq \beta_{k,n} + \gamma_{k,n} \log (\eta_{k,n})$$

(17)

where

$$\beta_{k,n} = \log (1 + z_{0}) - \frac{z_{0}}{1 + z_{0}} \log (z_{0})$$

(18)

$$\gamma_{k,n} = \frac{z_{0}}{1 + z_{0}}$$

(19)

The bound in (17) is tight with equality for a chosen value of $z_{0}$ when the values of $\beta_{k,n}$ and $\gamma_{k,n}$ are calculated as specified above [17]. The determination method for $z_{0}$ and calculating the values of $\alpha_{k,n}$ and $\beta_{k,n}$ from it will be discussed later in this subsection. By plugging (17) into (16) we get

$$E_{g_{k,n} | \hat{g}_{k,n}} \{ \log (1 + p_{k,n}g_{k,n} | \hat{g}_{k,n}) \} = \left\{ \begin{array}{ll} \left( \frac{1}{p_{k,n} \theta_{k,n}} \right)^{a_{k,n}} \frac{1}{\Gamma(a_{k,n})} \int_{0}^{\infty} \eta_{k,n} \left( e^{-\eta_{k,n}} - \frac{\eta_{k,n}^{a_{k,n}-1} e^{-\eta_{k,n}}}{\Gamma(a_{k,n})} \right) d\eta_{k,n} \\
\end{array} \right. \}$$

(20)

where the first integral in (20) reduces to $(p_{k,n} \theta_{k,n})^{a_{k,n}} \Gamma(a_{k,n})$, and the second one also has a close form solution [16, Section 4.358]. After some mathematical manipulations, we arrive at

$$E_{g_{k,n} | \hat{g}_{k,n}} \{ \log (1 + p_{k,n}g_{k,n} | \hat{g}_{k,n}) \} = \beta_{k,n} + \gamma_{k,n} \left\{ \log (p_{k,n} \theta_{k,n}) + \psi (a_{k,n}) \right\}$$

(21)

where $\psi(z)$ is the Euler’s psi function [16, Section 8.360]. Plugging (15) and (21) into (12) and (13), the final KKT equations become:

$$\frac{a_{k,n}}{p_{k,n} \theta_{k,n}} \left( \frac{1}{p_{k,n} \theta_{k,n}} \right)^{a_{k,n}} e^{-\frac{1}{p_{k,n} \theta_{k,n}}} \Gamma \left( -a_{k,n}, \frac{1}{p_{k,n} \theta_{k,n}} \right) = \frac{1 - \alpha_{k,n}}{\lambda_{k}}$$

(22)

$$\lambda_{k} \left\{ \beta_{k,n} + \gamma_{k,n} \left\{ \log (p_{k,n} \theta_{k,n}) + \psi (a_{k,n}) \right\} \right\} = \alpha_{k,n} e^{-\frac{1}{p_{k,n} \theta_{k,n}}} \Gamma \left( -a_{k,n}, \frac{1}{p_{k,n} \theta_{k,n}} \right) = \mu_{n} - \nu_{k,n}$$

(23)

The solution of above equations gives the optimal power and subcarrier allocation with zero duality gap [15, Section 5.5.3]. The power allocation is found by solving (22) which is the well known multilevel water-filling solution. Similarly, (23) gives a criterion for exclusive subcarrier allocation. A subcarrier is only allocated to the user for which the left hand side of (23) is maximum.

1) Determination of $z_{0}$, $\beta_{k,n}$ and $\gamma_{k,n}$: The values of $\beta_{k,n}$ and $\gamma_{k,n}$ in (17) are calculated numerically by using an iterative algorithm. By selecting the initial values of $\beta_{k,n} = 0$ and $\gamma_{k,n} = 1$ (an high SNR approximation), the value of $p_{k,n}$ is calculated from (22). Then, this value of $p_{k,n}$ is used to find $z_{0} = p_{k,n}g_{k,n}$ and update $\alpha_{k,n}$ and $\beta_{k,n}$ respectively by numerically evaluating the expectation of right hand side of (18) and (19) with respect to $g_{k,n} | \hat{g}_{k,n}$. The updated $\alpha_{k,n}$ and $\beta_{k,n}$ are used to update $p_{k,n}$ and this iterative procedure is continued till convergence.

V. Optimal Subcarrier and Power Allocation Algorithm

In this section, we present an optimal resource allocation algorithm in order to solve the KKT equations (22) and (23). The iterative algorithm can be implemented by going through the following steps:

1) Start with first user and allocate the powers to the subcarriers according to equation (22) by adjusting the water-level in such a manner that the user's average data rate constraint is fulfilled. At this moment as user 1 is the only user in the system, so all the subcarriers will be allocated to it.

2) Repeat the same procedure for the other users and allocate the powers by adjusting their water-levels such that their data rate constraints are satisfied. Allocate a subcarrier only to the user for which the right hand side of equation (23) is maximum for that subcarrier.

3) When the $K$th user is done with power and subcarrier allocation, some or all the other $K-1$ users must have lost some of their subcarriers and their data rate constraints are not satisfied any more.

4) Restart with first user and go on increasing its power level till its data rate constraint is satisfied.

5) Repeat steps 2) to 4) till the average data rate constraints of all the users are satisfied.

This algorithm is of the type multi-level waterfilling and thus always converges after some iterations. It minimizes the total transmit power by searching the best user for each subcarrier. In each iteration $KN$ power allocation values are calculated from (22). Thus, the computational load in each iteration is linear in $KN$ and the overall algorithm has a polynomial complexity.

VI. Simulation Results

In this section, we present the simulation results for the proposed algorithm. We simulate the algorithm for $K = 10$ users and $N = 20$ OFDM subcarriers. The bandwidth of each subcarrier is 375 KHz. A frequency selective Rayleigh fading channel is simulated where the channel gain has a small-scale Rayleigh fading component and a large-scale path loss component with path loss exponent of 3.7. The impact of outdated CSI is considered in the simulation. In fact, we simulate a time varying dynamic environment where speed dependent Doppler spectrum is included in every tap of the power delay profile of the channel (using Clarke’s isotropic scattering model [12]). The power spectral density of noise $N_{0}$, is -174 dBm/Hz. Without loss of generality, we assume the same data rate requirement for all the users since the scope is to study the impact of outdated CSI.
Fig. 1 shows the variation of time correlation coefficient of the channel with feedback delay (using Clarke’s model). It is clear from the figure that the value of correlation coefficient decreases significantly for small increase in feedback delay. Thus, according to (2) the channel gain will decrease and consequently the CNR will decrease, and more transmit power will be required.

In Fig. 2, we plot the normalized minimum required total transmit power for different user minimum rate constraint and various values of feedback delay. The mobile speed is assumed to be equal to 30 km/h. It shows that the optimal power allocation is very sensitive to feedback delay. Fig. 2 shows that the impact of feedback delay on the total power is independent of the system load. For example, for minimum user bit rate equal to 4 Mbps the loss in power is 35% when the feedback delay increases from 2.5 to 3.5 ms. Approximately the same loss is occurred for minimum user bit rate equal to 5 Mbps. One can also notice that the total transmit power vs minimum data rate grows consistently regardless of the feedback delay. Therefore, to closely observe the effect of feedback delay on power requirement we plot the total transmit power verses feedback delay for a fixed value of user minimum rate constraint (3 Mbps) in Fig. 3. From the figure we observe that 45% more power is required when the feedback delay increases from 3 to 5 ms. The results obtained in this paper can provide guidelines for system dimensioning and design.

VII. CONCLUSION

This paper considered the problem of margin adaptive resource allocation in a downlink OFDMA system, under the assumption of only outdated CSI at the transmitter. This exponential complexity combinatorial problem is converted into a convex optimization problem by using the time sharing technique. By using the probability distribution function of the current CSI conditioned on outdated CSI in the optimization framework, an optimal resource allocation algorithm with polynomial complexity is derived which solves the problem with zero optimality gap. It is shown through simulations that the optimal power allocation is very sensitive to feedback delay and selection of an appropriate feedback delay in practice should partly consider this impact.

REFERENCES


