Compressive Sensing Based Multipath Exploitation for Stationary and Moving Indoor Target Localization

Michael Leigsnering, Student Member, IEEE, Fauzia Ahmad, Senior Member, IEEE, Moeness G. Amin, Fellow, IEEE, and Abdelhak M. Zoubir, Fellow, IEEE

Abstract—Compressive sensing (CS) based multipath exploitation has been successfully applied to stationary indoor scenes in through-the-wall radar imaging (TWRI). The benefits of using significantly reduced data are also desirable for moving targets. Hence, we bring CS based multipath exploitation to the non-stationary target domain and are able to treat moving and stationary targets simultaneously. In general, multipath propagation has adverse effects on the image quality. However, by using proper modeling, multipath can be used to one’s advantage. In this paper, we apply CS to both stationary and moving targets under interior wall scatterings. Assuming knowledge of the room geometry, we solve the inverse problem of joint localization and velocity estimation of the targets in an indoor multipath environment. We develop an effective method that permits reconstruction of the scene from a few measurements. We also propose a scheme to estimate and correct wall position errors that introduce distortions in the reconstruction. Effectiveness of the proposed methods is demonstrated using both simulated and experimental data.

Index Terms—Compressive sensing (CS), sparse reconstruction, multipath exploitation, through-the-wall radar imaging (TWRI), MIMO, Doppler, moving targets.

I. INTRODUCTION

Radar imaging of building interiors has gained much interest due to the rising use in civilian, security, and defense applications [1]–[8]. Through-the-wall radar imaging (TWRI) has the ability to reveal stationary and moving targets behind walls, thereby greatly improving situational awareness in urban areas for the said applications.

The TWRI objective of acquiring precise information on target location and velocity is challenged by multipath propagation due to secondary reflections at interior walls. This results in heavy multipath associated with the target. Specular multipath causes “ghosts” in the imaged scene which stem from the energy being focused at non-target locations. Also, the front wall may cause additional ghosts due to multiple reflections inside the wall. This so-called wall ringing multipath leads to a sequence of target replicas, equally spaced in range, located behind the target. Instead of treating multipath as clutter, it is prudent to utilize the energy and information contained in these additional target returns. This is usually referred to as multipath exploitation. Another issue is the large amount of data that needs to be acquired, stored and processed to obtain highly resolved images of the scene using conventional approaches, such as backprojection. This calls for an efficient, logistically viable data acquisition scheme to reduce the recording time and system cost.

The above outlined challenges have been addressed in part by prior work. The problem of efficient data collection in TWRI was first addressed in [9] and further developed in [10]–[13] through the use of compressive sensing (CS). In these works, an accurate image of a sparse scene of stationary targets was reconstructed using only a fraction of the original measurements. CS has been examined for moving targets in the context of TWRI in [14], showing similar advantages. However, none of these contributions has considered multipath propagation. Multipath exploitation in backprojection based radar imaging was first attempted in [15], wherein multipath returns were used to reveal information of hidden target areas, which were not in the line-of-sight of the radar. Following a similar idea, the work in [16] made use of the energy in the ghost targets resulting from secondary reflections at known interior walls to generate a ghost-free image with improved signal-to-clutter ratio (SCR). Multipath exploitation within the CS framework was addressed in [17]–[19] for sparsely-populated stationary indoor scenes. By using proper modeling under known wall locations, sparse reconstruction in this case yielded an image where ghosts were eliminated and their energy was added to the real targets.

In this paper, we bring CS based multipath exploitation to a general sparse indoor scene of stationary and moving targets. Assuming knowledge of the building layout, a forward linear multipath model based on ray-tracing for multistatic operation is developed, which is then used by a group sparse reconstruction approach for the underlying indoor scene. Our model is quite similar to the multi-input multi-output (MIMO) setup in [20], but cast in terms of multipath. We show that multipath propagation provides Doppler velocity diversity, which yields reconstruction benefits and improved velocity resolution capability. We also propose a method to tackle the problem of imprecisely known locations of the interior walls. We apply an image quality metric to a series of reconstruction results corresponding to different assumed wall positions, and mitigate the adverse effects of wall position uncertainties.

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M. Leigsnering and A. M. Zoubir are with the Signal Processing Group, Institute of Telecommunications, Technische Universität Darmstadt, 64283 Darmstadt, Germany (e-mail: leigsnering@spg.tu-darmstadt.de; zoubir@spg.tu-darmstadt.de).
F. Ahmad and M. G. Amin are with the Radar Imaging Laboratory, Center for Advanced Communications, Villanova University, Villanova, PA 19085 USA (e-mail: fauzia.ahmad@villanova.edu; moeness.amin@villanova.edu).
by opting for the reconstruction that optimizes the metric. We demonstrate the effectiveness of the proposed multipath exploitation approach both with and without wall position errors through simulation and experimental results.

The remainder of the paper is organized as follows. In Section II, we introduce the multipath-free model for a wideband multistatic pulsed radar. Multipath propagation is incorporated in the model in Section III. In Section IV, the compressive sensing based image reconstruction method is proposed and a method to deal with errors in the assumed wall locations is suggested. Subsequently, in Section V, we provide supporting simulation and experimental results. Finally, we conclude this paper in Section VI.

II. SIGNAL MODEL

In this section, we describe the signal model for a wideband multistatic radar system with \( M \) transmitters and \( N \) receivers. We consider a sequential sensing operation, i.e., only a single transmitter is active at a time and all \( N \) receivers are recording the returns. The model essentially follows [14].

We assume that the targets follow a translatory or linear motion with constant velocity in a two-dimensional (2D) space. Stationary targets are included as a special case of moving targets with zero velocity. Let \( K \) wideband pulses be transmitted by each transmitter with a pulse repetition interval (PRI) of \( T_r \). The pulse index \( k = 0, \ldots, K - 1 \) is referred to as slow time. The PRI is assumed to be sufficiently small and the multiplexing of the transmitters sufficiently fast, such that i) the indoor scene can be considered approximately stationary during the sequential use of the \( M \) transmitters, i.e., over an interval of length \( MT_r \), and ii) the movement of the indoor targets is approximately of constant velocity and slow enough so that the targets do not move out of a range cell during the observation interval of length \( KMT_r \). Considering a scene of \( P \) targets and using the aforementioned assumptions, we can establish that the \( p \)-th target at pulse \( k \) is located at position

\[
x_p(k) = (x_p + v_{x_p} kMT_r, y_p + v_{y_p} kMT_r),
\]

where \((x_p, y_p)\) is the target position at \( t = 0 \) and \((v_{x_p}, v_{y_p})\) is the target velocity vector.

Let the transmit signal be a modulated wideband pulse of duration \( T_p \) given by \( \Re\{s(t) \exp(j2\pi f_c t)\} \), where \( t \) is the fast time, \( s(t) \) is the pulse in the complex baseband, and \( f_c \) is the carrier frequency. With the \( m \)-th transmitter active, the emitted pulse travels through the wall to the target scene and the reflections are measured at the receive array. The baseband received signal corresponding to the \( m \)-th transmitter, \( n \)-th receiver, \( k \)-th pulse, and \( p \)-th target can be expressed as

\[
z_{mnk}^p(t) = \sigma_p s \left( t - kMT_r - mT_r - \tau_{pmn}(k) \right) \exp(-j2\pi f_c (kMT_r + mT_r + \tau_{pmn}(k))),
\]

where \( \sigma_p \) is the reflectivity of the \( p \)-th point target and \( \tau_{pmn}(k) \) is the bistatic two-way delay between the \( m \)-th transmitter, \( p \)-th target, and the \( n \)-th receiver. Assuming that the \( P \) targets do not interact with each other, the total baseband signal received by the \( n \)-th receiver, corresponding to the \( k \)-th pulse and the \( m \)-th transmitter, is the superposition of all \( P \) target returns,

\[
z_{mnk}(t) = \sum_{p=0}^{P-1} \sigma_p s \left( t - kMT_r - mT_r - \tau_{pmn}(k) \right) \times \exp(-j2\pi f_c (kMT_r + mT_r + \tau_{pmn}(k))).
\]

Note that the delays and the received signal generally depend on the slow time index \( k \). However, if the velocity of the \( p \)-th target is zero, i.e., the stationary case, the delays do not change with \( k \).

The measurements \( \{z_{mnk}(t), m = 0, \ldots, M - 1, n = 0, \ldots, N - 1, k = 0, \ldots, K - 1\} \) can be discretized and vectorized to obtain a linear model of the system in matrix-vector form. The targets are assumed to reside on a discrete spatial grid of size \( N_x \times N_y \), where a non-existing target can be represented by a zero reflectivity. The velocities are sampled at \( N_v \) points, such that we have velocity pairs \((v_{xl}, v_{yl}), l = 0, 1, \ldots, N_v - 1\). Hence, in total, we have \( N_x N_y N_v \) possible target states, which can be stacked into an \( N_x N_y N_v \times 1 \) vector \( \mathbf{v} \). The received signal \( z_{mnk}(l) \) is sampled uniformly at \( T \) time steps with sampling interval \( T_s \). The sampling interval should be chosen to attain the Nyquist rate of the wideband pulse \( s(t) \). The samples can be stacked into a \( T \times 1 \) vector \( \mathbf{z}_{mnk} \), which, using (1) and (2), can be expressed as

\[
z_{mnk} = \mathbf{\Psi}_{mnk} \mathbf{v},
\]

where \( \mathbf{\Psi}_{mnk} \) are the dictionary matrices obtained by discretizing the right hand side of (2), and are given by

\[
[\mathbf{\Psi}_{mnk}]_{l,p} = s \left( t_i - kMT_r - mT_r - \tau_{pmn}(k) \right) \times \exp(-j2\pi f_c (kMT_r + mT_r + \tau_{pmn}(k))),
\]

where \( l = 0, \ldots, T - 1 \), \( p = 0, \ldots, N_x N_y N_v - 1 \).

Stacking of the received signal vectors \( \mathbf{z}_{mnk} \) corresponding to all \( K \) pulses for all \( MN \) transmitter-receiver pairs results in a \( T MNK \times 1 \) measurement vector \( \mathbf{z} \) as

\[
\mathbf{z} = \mathbf{\Psi} \mathbf{v},
\]

where \( \mathbf{\Psi} \) is a \( T MNK \times N_x N_y N_v \) dictionary matrix given by

\[
\mathbf{\Psi} = \begin{bmatrix}
\mathbf{\Psi}_T V_{M-1} & \cdots & \mathbf{\Psi}_U V_{M-1} \cdots & \mathbf{\Psi}_M V_{M-1} \cdots
\end{bmatrix}.
\]

Note that the linear model in (6) does not take multipath propagation into account. This will be treated in detail in Section III.

Besides multipath propagation, there is clutter caused by signal reflections from the front and interior walls. These type of returns originate from stationary building features and are not associated with indoor targets. In realistic scenarios, the front wall radar returns can be very strong and may mask the behind-the-wall targets. As such, we incorporate the clutter returns in the received signal model. We consider two types of clutter, namely, wall returns and corner returns. The incident wave is reflected at any wall-air interface, for instance, at the front wall or a parallel interior wall, thereby causing wall returns. Further, any corner formed by two perpendicular walls acts as a dihedral reflector, resulting in corner returns. Since
the walls and corners are stationary, the clutter does not change from pulse to pulse, i.e., it is invariant in the slow time domain.

Owing to their flat and smooth surfaces at the frequencies typically employed for TWRI, the walls reflect EM waves in a specular manner. As such, the signal propagates along a path with equal angles of incidence and reflection at the wall surface. Considering \( N_w \) exterior and interior walls, the wall returns can be described as

\[
z_{k,m,n}(t) = \sum_{q=0}^{N_w-1} \sigma_q \cdot \text{LL}(t - kMT + mTR - \tau_{q,m,n})
\]

\[
\times \exp \left( -j2\pi/t_c \left( kMT + mTR + \tau_{q,m,n} \right) \right),
\]

where \( \sigma_q \) is the reflectivity of the \( q \)-th wall and \( \tau_{q,m,n} \) is the two-way propagation delay between the \( m \)-th transmitter, the \( q \)-th wall and the \( n \)-th receiver. The delay is independent of the slow time index \( k \) and can be determined from geometric considerations as described above. Additionally, the returns from \( N_c \) corners can be expressed as

\[
z_{\text{corner},k,m,n}(t) = \sum_{u=0}^{N_c-1} \sigma_{u,m,n} \cdot \text{LL}(t - kMT + mTR - \tau_{u,m,n})
\]

\[
\times \exp \left( -j2\pi/t_c \left( kMT + mTR + \tau_{u,m,n} \right) \right),
\]

where \( \sigma_{u,m,n} \) is the reflectivity of the \( u \)-th corner and \( \tau_{u,m,n} \) is the two-way propagation delay between the \( m \)-th transmitter, the \( u \)-th corner and the \( n \)-th receiver. Note that, similar to the wall returns, the delay is independent of the slow time index \( k \). However, the corner reflectivity depends on the transmitter and the receiver. The delay can be calculated in the same way as for point targets, while the reflectivity is given by [21]

\[
\sigma_{u,m,n} = \left( \frac{2\sqrt{\pi}}{\lambda} \right)^{1/2} 2L_u
\]

\[
\times \text{sinc} \left[ \frac{4\pi L_u}{\lambda} \left( \cos \left( \psi_{u,m}^t - \psi_{u} \right) - \cos \left( \psi_{u,m}^r - \psi_{u} \right) \right) \right]
\]

\[
\times \left\{ \begin{array}{l}
\sin \left( \frac{\psi_{u,m}^t - \psi_{u,m}^r}{2} \right),
\cos \left( \frac{\psi_{u,m}^t - \psi_{u,m}^r}{2} \right),
\end{array} \right.
\]

\[
\psi_{u,m}^t, \psi_{u,m}^r \in \left[ \psi_{u} - \frac{\pi}{4}, \psi_{u} + \frac{\pi}{4} \right],
\]

where \( L_u \) is the length of the sides of the \( u \)-th corner, \( \psi_u \) is the orientation angle of the \( u \)-th corner, \( \psi_{u,m}^t, \psi_{u,m}^r \) are the respective angles of incidence and reflection, and \( \lambda = c/f_c \) is the wavelength. Note that the angles are measured counterclockwise from the positive \( x \)-axis.

Finally, the overall signal is composed as

\[
z_{k,m,n}(t) = z_{k,m,n}(t) + z_{\text{wall},k,m,n}(t) + z_{\text{corner},k,m,n}(t),
\]

which can be vectorized in the same fashion as described above for \( z_{k,m,n}(t) \).

Due to the dominance of exterior wall clutter in TWRI, several wall mitigation techniques have been proposed in the literature. These include spatial filtering [22] and subspace projection [23], [24] methods, which have been successfully applied in conjunction with compressive sensing and sparse scene reconstruction [9], [12], [25]. Both of these methods also remove the contributions of interior parallel walls as long as they are not shadowed by the contents of the building. In the sequel, we assume that the wall clutter has been properly mitigated. Further, we ignore the corner clutter and consider only the target returns for extending the received signal model to multipath propagation. However, in Section V, in addition to reconstruction examples based on target direct and multipath returns, we provide examples including the corner clutter to illustrate its effect on the performance of the proposed multipath exploitation scheme.

### A. Conventional Image Formation

Conventional image formation for TWRI is carried out using backprojection or delay-and-sum beamforming (DSBF) [26]–[28]. Considering the spatial grid of size \( N_x \times N_y \), the complex image value \( I_q(k) \), corresponding to the \( q \)-th grid point \( (x_q, y_q) \) and slow time index \( k \), is obtained by summing delayed copies of the \( MN \) received signals corresponding to the \( k \)-th pulse, followed by applying a matched filter with impulse response \( s^*(t) \) to the result, and then sampling the filtered data [28],

\[
I_q(k) = \frac{1}{MN} \sum_{m=0}^{N_1-1} \sum_{n=0}^{N_2-1} z_{mnk}(t + \tau_{q,m,n}(k)) \ast s^*(t) \big|_{t=0},
\]

\[
q = 0, 1, \ldots, N_x N_y - 1
\]

where \( \tau_{q,m,n} \) is the focusing delay for the \( m \)-th transmitter, \( n \)-th receiver and \( q \)-th spatial grid point.

To obtain an overall image, \( I_q(k), k = 0, 1, \ldots, K - 1 \) cannot be simply combined coherently, as moving targets will be blurred and possibly mislocated. Instead, we can include the linear velocity model in the beamforming approach. We discretize the target space, as explained above, into a four-dimensional (4D) grid with two spatial and two velocity dimensions. Hence, we obtain an image value \( I_p \) at the \( p \)-th space-velocity grid point \( (x_p, y_p, v_p, \delta_p) \) by a summation over the \( K \) pulses

\[
I_p = \frac{1}{KMN} \sum_{k=0}^{K-1} \sum_{m=0}^{N_1-1} \sum_{n=0}^{N_2-1} z_{mnk}(t + \tau_{pmn}(k)) \ast s^*(t) \big|_{t=0},
\]

\[
p = 0, 1, \ldots, N_x N_y N_v - 1
\]

where \( \tau_{pmn} \) is the focusing delay for the \( (m,n) \)-th transmitter-receiver pair and the \( p \)-th space-velocity grid point for the \( k \)-th pulse. Thus, we obtain a set of 2D spatial images, each matched to a particular velocity vector. The matrix-vector equivalent of (13) can be obtained by using the adjoint of (7) as

\[
\hat{\sigma} = \Psi^H z,
\]

where \( \hat{\sigma} \) contains vectorized spatial images corresponding to all considered velocities.

Note that the DSBF resolution is limited by the point spread function or Rayleigh resolution [29]. Further, DSBF provides severely degraded image quality in the case of missing or undersampled data. As such, sparse reconstruction is used for image formation from a few measurements. This will be detailed in the following sections and performance comparison with the velocity-matched beamforming will also be provided.
III. Multipath Propagation

Multipath propagation corresponds to an indirect path, which involves reflection at one or more secondary reflectors in addition to diffuse scattering at the target of interest. In TWRI, even the direct path, i.e., the path to the target that is not subject to any secondary reflections, is influenced by the exterior building wall between the radar and the interior scene of interest. The transmitted waves are refracted twice at the front and back interfaces of the front wall. The backscattered wave is subject to the same double refraction before reaching the receiver. Assuming ray theory and homogeneous front wall, these effects can be described by Snell’s law [30].

Depending on the characteristic reflections, specular multipath can be categorized as follows [18]: 1) Interior wall multipath, which involves specular reflection at one or more interior walls on the transmit and/or receive paths to the target; 2) Floor/ceiling multipath involving specular reflections at the floor and/or ceiling; and 3) Wall ringing multipath, wherein the wave traveling to/from the target undergoes multiple reflections within the front wall. We only deal with interior wall multipath returns in this work. Floor/ceiling multipath is not considered as it is usually weak, if not present, when using antennas with a narrow elevation beamwidth. We note, however, that this type of multipath can be treated in essentially the same way as interior wall multipath. Wall ringing multipath has been treated in [18] for stationary scenes and can be similarly incorporated in this work.

The interior wall multipath returns can be further subdivided into the following classes:

- **First order multipath:** This scattering scenario involves a direct propagation to the target on transmit and one secondary reflection at an interior wall on the way back to the receiver, or vice versa. This is the dominant case of multipath.
- **Second order multipath:** The signal on the round-trip path to the target undergoes secondary reflections twice. Two cases can further be distinguished:
  - **Quasi-monostatic:** There is one specular reflection on transmit and one on receive, both occurring at the same interior wall. Note that this corresponds to scattering at the target with a very small bistatic angle, as compared to first order multipath, when using a bistatic radar with a small baseline.
  - **Bistatic:** The specular reflections take place at two different walls. Either the two reflections both take place before or after the wave reaching the target, or one occurs before and one after.
- **Higher-order multipath:** Three or more specular reflections during the round-trip path may occur as well.

In the following, we will only consider first and second order multipath returns from interior walls. The signal is attenuated at each secondary wall reflection and, as such, the higher-order multipath returns are usually weak enough to be safely neglected. Assuming that the quasi-monostatic reflection from indoor targets is stronger than bistatic scattering, we only take the quasi-monostatic second order multipath into consideration. As we are assuming perfect knowledge of the building layout, i.e., location, thickness, and permittivity of the front wall as well as the locations of the interior walls, we can accurately describe the multipath. Using a ray tracing model, we will now calculate the exact delays corresponding to each path. The derivation follows essentially from [16], [18], [30].

A. Interior Wall Multipath

Interior wall multipath can easily be described by making use of the notion of a virtual target. This can be illustrated through the example in Fig. 1, where the front wall has been ignored in the geometry for simplicity. We also do not explicitly state the dependence of the target location, propagation delay, and other associated parameters on the slow time index \( k \) in this section for simplicity of notation. The scene consists of a target located at \( x_p = (x_p, y_p) \) and one interior wall (side wall of the room) parallel to the \( y \)-axis and located at \( x = w_1 \). Now, consider propagation along the path \( P' \) from the target to a receiver via secondary reflection at the interior wall. Because of the specular nature of the interior wall reflection, a virtual target can be assumed to be present at the mirror image \( x'_p = (2w_1 - x_p, y_p) \) of the point \( x_p \). Then, the receive path \( P'' \) involving the secondary reflection is equivalent to the direct path \( P' \) between the virtual target and the receiver. Hence, the calculation of the one-way propagation delay associated with path \( P'' \) can be carried out by using the direct propagation path \( P' \). Note that the case of the path from the transmitter to the target or reflection from a different wall can be treated in a similar manner.

With the \( m \)th transmitter active, the delay corresponding to the receive path only between receiver \( n \) and target \( p \), is denoted by \( \tau_{p,mn}^{(P')} \), which is equivalent to \( \tau_{p,mn}^{(P'')} \). When ignoring the front wall, the delay can simply be calculated as the Euclidean distance between the receiver \( n \) and the virtual target at \( x'_p \) divided by the propagation speed. If the front wall is present, the double refraction at the front wall interfaces has to be considered. In this case, the calculation for \( P'' \) can be performed in the same fashion as for the one-way direct path between the receiver \( n \) and target \( p \) using Snell’s law [30].

B. Received Signal Model

Considering the aforementioned multipath mechanisms, we can find a model that describes the return signal under multipath propagation. Front wall response is not considered and we assume that the measurements contain only the target returns, as mentioned in Section II.

Any round-trip path \( P \) can be described as a combination of two one-way paths, namely, the path \( P'' \) from the transmitter
to the scattering target and the path $\mathcal{P}'$ from the target back to the receiver. Either one-way path can assume different forms, for example, it could be the direct propagation to the target or involve a single reflection at an interior wall. We assume that there exist $R_1$ return paths from a target back to the receiver, which will be denoted as $\mathcal{P}'_{r_1}$, $r_1 = 0, \ldots, R_1 - 1$. The same observation holds for the one-way transmit paths, which are denoted by $\mathcal{P}_{t_2}$, $r_2 = 0, \ldots, R_2 - 1$. Therefore, there exist a total of $R_1 R_2$ round-trip paths. However, as mentioned earlier, we are only considering direct round-trip propagation and first-order and quasi-monostatic second-order multipath returns. Hence, a maximum of $R < R_1 R_2$ combinations are considered for the round-trip path $\mathcal{P}$, i.e., $\mathcal{P}_{r}, r = 0, \ldots, R - 1$.

A function can be established that maps the index $r$ of the round-trip path to a pair of indices of the one-way paths, $r \mapsto (r_1, r_2)$. In the following, we will consider $\mathcal{P}_0$ as the direct round-trip path, i.e., the case without any secondary reflections. This model is illustrated in Figure 2, which depicts three possible return paths, namely, direct propagation and secondary reflections from a side wall and the back wall. Three equivalent paths will also be present for the propagation from the transmitter to the target. Hence, we obtain a total of nine round-trip paths by combining three transmit paths and three return paths, which are depicted in Figure 3. Paths $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3$ and $\mathcal{P}_5$ correspond to first order multipath, $\mathcal{P}_4$ and $\mathcal{P}_8$ are quasi-monostatic second order multipath, while round-trip paths $\mathcal{P}_3$ and $\mathcal{P}_7$ are bistatic second order multipath. The latter are ignored in the model.

As the round-trip path $\mathcal{P}_r$ consists of the one-way paths $\mathcal{P}'_{r_1}$ and $\mathcal{P}'_{r_2}$, the round-trip delay can be expressed as

$$\tau_{pmn}^{(\mathcal{P}_r)} = \tau_{pmn}'^{(\mathcal{P}'_{r_1})} + \tau_{pmn}''^{(\mathcal{P}'_{r_2})}.$$  (15)

The method to obtain the appropriate delays for the indirect one-way paths has been described in Section III-A. For notational simplicity, we denote the round-trip delay between the $m$th transmitter, $r$th receiver, and $p$-th target associated with path $\mathcal{P}_r$ as $\Gamma_{pmn}^{(\mathcal{P}_r)}$.

In a similar manner, the complex amplitudes $\Gamma_{pmn}^{(\mathcal{P}_r)} \in \mathbb{C}$ associated with all the paths, transmitters, receivers, and targets can be addressed. At each reflection and refraction, the traveling wave will undergo some attenuation and possibly a phase shift. For each one-way path, the complex amplitude $\Gamma_{pmn}^{(\mathcal{P}_r)}$ can be derived from the dielectric properties of the front and sidewalls and the corresponding angles of incidence and refraction. A detailed derivation can be found in [16], [18].

In the sequel, two simplifications are made. First, we assume that the transmit and receive arrays are closely spaced and their extents are sufficiently small as compared to the distance to the scene. As such, the incident, reflection, and refraction angles associated with a particular path do not vary much across the arrays, leading to

$$\Gamma_{pmn}^{(\mathcal{P}_r)} \approx \Gamma_{p}^{(\mathcal{P}_r)}, \quad m = 0, \ldots, m - 1, \quad n = 0, \ldots, N - 1, \quad p = 0, \ldots, N_x N_y N_v - 1.$$  (16)

In other words, the complex amplitude for each path depends only on the target position. Second, as the direct round-trip path return from a target is typically the strongest path compared to the associated multipath, the corresponding complex amplitudes are normalized w.r.t. the direct path in order to avoid over-parameterization.

$$g_p^{(r)} = \frac{\Gamma_{p}^{(\mathcal{P}_r)}}{\Gamma_p^{(\mathcal{P}_0)}}, \quad r = 0, \ldots, R - 1, \quad p = 0, \ldots, N_x N_y N_v - 1.$$  (17)

Hence, we assign a single complex path weight $g_p^{(r)}$ to each possible path corresponding to the $p$th target state and a transmitter/receiver combination, with the direct path having the weight $g_p^{(0)} = 1$.

We are now in a position to formulate the target signal model under multipath propagation conditions. The received

Fig. 2. Example for three possible partial return paths.

Fig. 3. Round-trip paths between transceiver and target for the partial paths shown in Fig. 2.
signal is a superposition of delayed and weighted versions of the transmitted signal corresponding to all possible propagation paths \( r = 0, \ldots, R - 1 \). That is,

\[
z = \Psi(0)\sigma(0) + G(1)\Psi(1)\sigma(1) + \cdots + G(R-1)\Psi(R-1)\sigma(R-1),
\]

where \( G(r) = \text{diag}(g_{0}^{(r)}, g_{1}^{(r)}, \ldots, g_{N_{c}N_{r}N_{s} - 1}^{(r)}) \), \( r = 0, \ldots, R - 1 \) are the path weight matrices, and the dictionaries \( \Psi(r) \) are defined according to (5) and (7) with \( \tau_{pmn} \) replaced by \( \tau_{pmn}^{(r)} \). Note that, in (18), we assume an individual target state vector for each path, as the phase and amplitude of the target reflectivity change in general with the bistatic angle and target orientation. Further, we assume the same number of paths for each target state, as a particular path weight can be set to zero if the corresponding path is not available for that target state.

For notational convenience, the path weights can be absorbed into the target state vectors as \( \sigma(r) = G(r)\sigma(r) \), as the weighting imposes only a per target state scaling. The resulting measurement model takes the form

\[
z = \Psi(0)\sigma(0) + \Psi(1)\sigma(1) + \cdots + \Psi(R-1)\sigma(R-1).
\]

Note that (19) is a generalization of the single path propagation model in (6). If the number of propagation paths is set to 1 in (19), then the two models are equivalent.

### C. Apparent Doppler Velocity

In order to study the effect of multipath and motivate its exploitation, we examine the information contained in the multipath returns [31]. A target at position \((x_{p}, y_{p})\) moving with velocity \((v_{xp}, v_{yp})\) has an apparent Doppler velocity. In case of direct propagation, this is approximately the radial velocity component with respect to the midpoint of the baseline of the interrogating transmitter/receiver pair. However, if the wave travels on an indirect path, the apparent Doppler velocity changes. In order to determine the form of the apparent Doppler velocity under multipath propagation, we consider an alternate transmitter/receiver/target geometry for the multipath by reflecting the physical transmitter and/or receiver locations about the secondary reflector (interior wall). For first order multipath, either the transmitter or receiver location is mirrored depending on whether the secondary reflection occurred on transmit or receive. On the other hand, both the transmitter and receiver locations will be mirrored for second order quasi-monostatic multipath propagation. The mirrored locations constitute a virtual transmitter and a virtual receiver. We can now cast the multipath as direct propagation to/from these virtual antenna locations. As such, for apparent Doppler velocity corresponding to multipath propagation, the radial velocity component with respect to the midpoint of the virtual baseline is a relevant measure. For illustration, consider the example in Fig. 4, where multipath occurs on the return path only. The signal travels along \( P'' \) from the physical transmitter (Tx) to the target and along \( P' \) back to the physical receiver (Rx) via reflection at the interior wall. The return path can equivalently be described by direct propagation along \( P' \) to a virtual receiver (vRx) that has been constructed as described above. Now, the midpoint of the baseline formed by the physical transmitter (Tx) and the virtual receiver (vRx) needs to be considered for the apparent Doppler velocity. This velocity may also be approximated using propagation delays. Depending on the transmitter \( m \), the receiver \( n \), the path \( r \), and averaging over the full CPI, the apparent Doppler velocity for the \( p \)th target may be expressed as

\[
v_{D,pmn} = \frac{1}{K - 1} \sum_{k=0}^{K-2} \tau_{pmn}(k+1) - \tau_{pmn}(k) T_{r}.
\]

For illustration, we simulate a target at an arbitrary location within three walls, moving with a velocity \((v_{xp}, v_{yp}) = (1, 0)\text{m/s}\). Hence, the target is solely moving in the cross-range direction. At each assumed target position, the apparent Doppler velocity is color coded in Fig. 5. The surrounding walls are also superimposed on the figure. The velocity pattern is only shown for a single transmitter/receiver pair that represents the centers of the transmit and receive arrays. In the direct propagation case, shown in Fig. 5a, we observe the expected pattern, with zero velocity along broadside and gradually increasing velocity for angles deviating from broadside. However, the pattern is different for an indirect path, reaching the target via reflection at the right side wall, as shown in Figs. 5b,c. Fig. 5b corresponds to a first order multipath that involves direct propagation on transmit and a secondary reflection at the right side wall on receive, whereas Fig. 5c involves a secondary reflection at the right side wall on both transmit and receive. The patterns in Figs. 5b,c are shifted and distorted as compared to Fig. 5a. In particular, the zero velocity line is shifted as compared to that in Fig. 5a. Hence, we obtain additional information on target motion through the first and second order multipath returns. If properly modeled, as described earlier, this property is exploited to improve the velocity estimation.

### IV. Compressive Sensing Based Scene Reconstruction

Benefits of CS are realized when the radar return is undersampled in all four dimensions, i.e., fast time, slow time and transmit/receive elements. For the latter two, most savings are achieved by random omission of some elements, leading to sparse transmit and receive arrays. Random undersampling of slow time does not lead to any benefits in terms of time or cost savings, as long as the first and the last pulses are retained in the CPI. However, reducing the number of pulses within the CPI leads to power savings, which may be desirable in portable applications. Various methods are available to compressively sample in the fast time. Here, we adopt a
random mixing scheme in which each pulse is correlated with a set of random signals and only the corresponding correlation result is sampled. For a detailed discussion of this scheme, the reader is referred to [14], [32].

The compressively sampled version of the radar return in (19) can be expressed as
\[ \tilde{z} = \Phi z = \Phi (\Psi^{(0)} \sigma^{(0)} + \Psi^{(1)} \sigma^{(1)} + \ldots + \Psi^{(R-1)} \sigma^{(R-1)}) , \]
where \( J \) is the number of reduced measurements, and the measurement matrix \( \Phi \in \mathbb{R}^{J \times TMNK} \) represents the undersampling operation. With the aforementioned undersampling considerations, reducing the number of samples along transmit sampling operation. With the aforementioned undersampling
\[ \Phi = (\Phi_1 \otimes I_{N_y K_y T_y}) \cdot (\Phi_2 \otimes I_{M_K K_y T_y}) \cdot (\Phi_3 \otimes I_{M N_T y}) \cdot \text{diag}(\Phi_4^{(0)}, \ldots, \Phi_4^{(M N_K - 1)}) , \]
where \( \otimes \) denotes Kronecker product and \( I_a \) is an identity matrix of dimension \( a \). The total number of reduced measurements is given by \( J = M_d N_d K_d T_d << TMNK \). Each of the matrices \( \Phi_1 \in \mathbb{R}^{M_d \times M_y} \), \( \Phi_2 \in \mathbb{R}^{N_y \times N_y} \) and \( \Phi_3 \in \mathbb{R}^{K_d \times K_d} \) consists of randomly chosen rows from an identity matrix, while random mixing in fast time is achieved by Gaussian random matrices \( \Phi_4^{(i)} \in \mathbb{R}^{T_d \times T} \) with entries drawn form a standard normal distribution. Other random matrices, e.g., drawn from a Bernoul\( \mathcal{I} \)li\( \mathcal{I} \) distribution, can also be considered to achieve a good trade-off between ease of implementation and performance, see [32].

### A. Group Sparse Scene Reconstruction

In order to account for all propagation paths, a high-dimensional model is constructed using (21) as
\[ \tilde{z} = \Phi \tilde{\Psi} \hat{\sigma} \]
where \( \tilde{\Psi} = [\Psi^{(0)} \Psi^{(1)} \ldots \Psi^{(R-1)}] \in \mathbb{R}^{MNKT \times N_x N_y N_r R} \) is the concatenated overcomplete dictionary for all possible paths and the unknown target state vectors are stacked into one tall vector
\[ \hat{\sigma} = \begin{bmatrix} \sigma^{(0)} \ T \\ \sigma^{(1)} \ T \\ \vdots \\ \sigma^{(R-1)} \ T \end{bmatrix} \in \mathbb{C}^{N_x N_y N_r R \times 1} . \]
\[ \]

Given the reduced measurements \( \tilde{z} \) in (23), we aim at recovering the target state information \( \hat{\sigma} \) using sparse reconstruction. If no multipath propagation is present, this can be achieved by standard \( \ell_1 \)-norm minimization as considered in [14]. However, this method is suboptimal in the presence of multipath. Similar to [18], we exploit multipath by utilizing the group sparse structure in the target state information. More specifically, the target state vectors corresponding to each path exhibit a group sparse structure, where the individual groups extend across the paths for each target state. Note that the apparent Doppler velocity for a particular target may differ when observed through different paths. This is captured in the model through different delays \( \tau^{(r)}_p(k) \), which are a function of slow time and are all calculated based on the same coordinate system. In this way, the reconstruction benefits from the additional diversity in the received signal due to different Doppler velocities corresponding to the same target.

The group sparse reconstruction of the unknown vector \( \sigma \) is achieved by solving the mixed \( \ell_2/\ell_1 \)-norm minimization problem
\[ \hat{\sigma} = \arg \min_{\sigma} \| \tilde{z} - \Phi \tilde{\Psi} \hat{\sigma} \|_2 + \lambda \| \hat{\sigma} \|_{1.2} , \]
where
\[ \| \hat{\sigma} \|_{1.2} = \sum_{p=0}^{N_x N_y N_r R - 1} \| \sigma^{(0)}_p , \sigma^{(1)}_p , \ldots , \sigma^{(R-1)}_p \|_2 \]
and \( \lambda \) is a regularization parameter. The convex optimization problem (25) can be solved using SparSA [33] or other available schemes [34]-[36].

Once a solution \( \hat{\sigma} \) is obtained, a composite target state vector corresponding to the scene can be obtained by incoherent combination of the state vectors corresponding to the various paths as
\[ [\hat{\sigma}_{\text{comb}}]_p = \left\| \begin{bmatrix} \sigma^{(0)}_p \\ \sigma^{(1)}_p \\ \vdots \\ \sigma^{(R-1)}_p \end{bmatrix} \right\|_2 , \]
\[ p = 0, \ldots, N_x N_y N_r R - 1 . \]

The final recovery result contains the information about the location and the translatory motion of all targets in the scene. Stationary targets are included in the spatial image corresponding to the zero velocity case. For an in-depth treatment of the group-sparse reconstruction approach, the reader is referred to [18].
B. A Note on the Dictionary Structure

The high-dimensional measurement model of (23) does not make any assumptions about the resolvability of the various multipath arrivals. If the multipath returns are resolvable and successful association can be made between each signal component and its respective dictionary as

\[
\hat{z}^{(r)} = \Phi \tilde{\Psi}^{(r)} \sigma^{(r)}, r = 0, 1, \ldots, R - 1,
\]

then the equivalent high-dimensional model would take the form

\[
\begin{bmatrix}
(\hat{z}(0))^T \\
(\hat{z}(1))^T \\
\vdots \\
(\hat{z}(R-1))^T
\end{bmatrix}^T
= \text{bdiag}\left\{ \Phi \tilde{\Psi}^{(0)}, \Phi \tilde{\Psi}^{(1)}, \ldots, \Phi \tilde{\Psi}^{(R-1)} \right\} \tilde{\sigma}
\]

with \text{bdiag}\{\} denoting the block diagonal matrix operation. Unlike the concatenated dictionary in (23), the combined overcomplete dictionary for the resolvable multipath case has a block diagonal structure. The vector \tilde{\sigma} can again be estimated by exploiting the group sparse property of the target state vectors corresponding to the \( R \) paths.

We note that the block diagonal dictionary form is similar to the multipath-free signal model under widely-separated MIMO operation [37]. The transmitters and receivers, in this case, observe the targets from different aspect angles. For non-isotropic targets, this results in varying radar cross section for different transmitter/receiver pairs. For each pair, a different target state vector \( \sigma^{(r)} \) is assumed. Thus, the overcomplete dictionary exhibits a block-diagonal structure, with the individual dictionaries \( \Phi \tilde{\Psi}^{(r)} \) corresponding to the various transmitter/receiver pairs, and the respective scene. The stacked target state vector exhibits a group sparse structure in this case as well.

In the remainder of this paper, we will use the general model in (23) for the data measurements and subsequent reconstruction of the scene of interest.

C. Reconstruction with Wall Position Uncertainties

A shortcoming of the proposed multipath exploitation approach is the need for prior information on the exact positions of the secondary reflectors. Inaccurate wall locations lead to degradation of the reconstruction result. Note that the returns traveling along a particular path are coherently combined in the measurement model (23). If the actual wall location deviates from the assumed value, the apparent range to the target changes. As such, the assumed propagation delays do not match the true delays, resulting in a perturbed representation by the dictionary \( \hat{\Psi} \). Further, the target may appear to reside at a grid point in the target space different from its true state. This is because the propagation delays could be longer or shorter than anticipated. As a result, the same target may be reconstructed at different grid points in the target state vectors corresponding to the \( R \) paths, thereby violating the group sparse property. Thus, it is imperative to deal with wall location errors and devise a scheme to mitigate their adverse effects.

When considering wall position errors, we express the dictionary matrix \( \hat{\Psi} \) as a function of the assumed wall locations, i.e., \( \hat{\Psi}(\hat{w}) \), where \( \hat{w} = [w_1, w_2, \ldots, w_{R-1}]^T \) is the vector of wall locations. Note that in practice, all of the elements of \( \hat{w} \) will not be distinct, since some of the walls may be common to more than one of the considered \( R \) paths. The true wall locations are denoted as \( w^0 \), with \( \hat{\Psi}(w^0) \) representing the dictionary in the absence of any wall location inaccuracies. We perform the sparse reconstruction using the assumed wall location vector \( \hat{w} \) in the model as

\[
\hat{\sigma}(\hat{w}) = \arg \min_{\sigma} \| \hat{z} - \Phi \hat{\Psi}(\hat{w}) \sigma \|_2^2 + \lambda \| \sigma \|_{1,2}.
\]

We devise a simple approach for restoring the reconstruction quality under wall positioning errors as follows. First, we solve the reconstruction problem in (30) for a number of assumed wall locations \( \hat{w} \in \mathcal{W} \). Thus, in the worst possible case where none of the paths share a wall, the possible wall locations need to be sampled on an \((R - 1)\)-dimensional grid \( \mathcal{W} \) in order to test all possible combinations. Note that this works well for a few walls, but becomes quickly infeasible for a larger number of walls. Second, we apply a suitable quality metric in order to choose the best amongst the different reconstruction results corresponding to the various assumed wall locations.

We consider three different metrics as potential candidates for the proposed reconstruction scheme under wall position errors. As explained above, the sparsity of the reconstructed target states will decrease if there is a model mismatch. Therefore, the first metric is selected to be the sparsity of the recovered solution, i.e.,

\[
\| \hat{\sigma} \|_0 = \#(i | \hat{\sigma}_i \neq 0). \tag{31}
\]

Further, we observe that the signal energy may only be captured adequately in the sparse solution if the model matches well with the ground truth. However, in case of a model mismatch, some of the energy in the received signal is “lost” as it is treated as noise. As such, we opt for the total energy in the solution as the second choice for a quality metric, which is defined as

\[
\| \hat{\sigma} \|_2^2 = \sum_i \hat{\sigma}_i^2. \tag{32}
\]

The third metric is the mean energy (ME) per non-zero element, which is simply a combination of the sparsity and total energy metrics,

\[
\text{ME}(\hat{\sigma}) = \frac{\| \hat{\sigma} \|_2^2}{\| \hat{\sigma} \|_0}. \tag{33}
\]

The ME metric is expected to provide a sharper response, as it combines the first two metrics.

The optimal target state vector is \( \hat{\sigma}(\hat{w}) \), with \( \hat{w} \) being the assumed wall locations that yield the optimum value for a given metric, i.e.

\[
\hat{w} = \arg \min_{\hat{w}} \| \hat{\sigma}(w) \|_0 \tag{34}
\]

\[
\hat{w} = \arg \max_{\hat{w}} \| \hat{\sigma}(w) \|_2^2 \tag{35}
\]

\[
\hat{w} = \arg \max_{\hat{w}} \text{ME}(\hat{\sigma}(w)). \tag{36}
\]

A good metric should reach the highest or the lowest value only for the undistorted reconstruction result corresponding to the correct wall positions.
V. RESULTS

We present simulation and experimental results to show the effectiveness of the proposed multipath exploitation approach both with and without wall position uncertainties. The setups are chosen such that they represent a realistic wideband pulsed TWRI system. The multipath environment is modeled to mimic a typical room behind a concrete exterior wall. In all simulation examples, independent and identically distributed complex circular Gaussian receiver noise with a signal-to-noise ratio of 10 dB is added to the measurements before applying the downsampling operation. All reconstruction results in this section are shown on a 40 dB scale.

A. Simulation Results

Simulations were performed for a wideband pulse-Doppler multistatic radar with a 4-element uniform linear array of length 1 m. Each array element can be used for both transmission and reception, leading to \( M = N = 4 \). A modulated Gaussian pulse, centered around \( f_c = 2 \) GHz, with a relative bandwidth of 50% is transmitted. The PRI is set to 10 ms and \( K = 15 \) pulses are processed coherently. At the receiving side, \( T = 150 \) fast time samples in the relevant interval, covering the target and multipath returns, are taken at a sampling rate of \( f_s = 4 \) GHz. The front wall is modeled with \( d = 20 \) cm thickness and relative permittivity \( \epsilon_r = 7.66 \), and is located parallel to the array at a distance of 3 m. Two side walls are considered at \( \pm 2 \) m in crossrange, each of which causes 3 different multipath returns per target. These are, in total, 4 first order multipath returns and 2 second order quasi-monostatic multipath returns per target, which are all considered to be 6 dB weaker than the direct path. Hence, in total, there are \( R = 7 \) paths per target contributing to the received signal. We assume that the returns from the front wall have been properly suppressed. We neither consider any wall returns nor any multipath from the back wall located at 6 m downrange. The imaged region extends 6 m in crossrange and 4 m in downrange and is centered around a point in the broadside direction of the array at 3 m. The scene of interest is spatially discretized into an \( N_x \times N_y = 32 \times 32 \) pixel grid.

The target velocities are discretized on a \( N_v_x \times N_v_y = 5 \times 7 \) crossrange by downrange grid, spanning target velocity components of \( \pm 0.9 \) m/s.

1) Imaging Results: We consider two stationary targets residing at coordinates \((0.5, 3.7)\) m and \((-1.5, 3.7)\) m and two moving targets at \((0.5, 4.7)\) m and \((-1.5, 4.7)\) m, respectively. The moving targets are assumed to be 8 dB weaker than the stationary targets and possess respective velocities \((-0.45, 0)\) m/s and \((0, 0.3)\) m/s. At first, no returns from the room corners are considered. We assume that all targets are visible via all \( R = 7 \) possible paths. We first show the conventional beamforming results using full measurements in Fig. 6, where the beamformer for each spatial image has been matched to the corresponding velocity pair according to (14). The image appears very cluttered due to the multipath responses and the moving targets cannot be discerned. Next, we show in Fig. 7 the multipath exploitation based group sparse reconstruction using 7% of the full Nyquist sampled measurements, averaged over 20 Monte Carlo runs. The downsampling parameters of (22) are set to \( T_d = 20, M_d N_d = 8 \) and \( K_d = 15 \), performing linear measurements using a Gaussian random mixing matrix in fast time. It is evident that both the locations and the velocities of the four targets have been correctly recovered. The ghost targets have been largely suppressed and only a few weak clutter pixels remain. The clutter can mostly be attributed to the high correlation in the dictionary for neighboring crossrange velocities, leading to a certain “leakage”. Overall, the multipath exploited CS reconstruction features a very clean and accurate high-
resolution image.

We next evaluate the impact of the corner returns on the performance of the proposed scheme. The simulation is repeated with the returns from the two corners in the back of the room included in the received signal. The corner returns are modeled according to (9) with \( N_c = 2 \) and \( L_o = 2 \) m, \( u = 0, 1 \). The CS multipath exploitation result is depicted in Fig. 8. The targets have been clearly detected, but strong clutter is now visible at the two considered corners of the room in the image corresponding to the \((0, 0)\) m/s velocity pair. Since the corner reflectivity varies across the various transmitter-receiver pairs, the imaged corners are smeared. Again, leakage of the corners into the neighboring crossrange velocity images is present in Fig. 8.

In order to assess the wall error estimation procedure, we assume a maximum uncertainty of \( \pm 20 \) cm in the position of the right side wall only. We use assumed positions of the right side wall from 1.8 m to 2.2 m in 2 cm increments. For each assumed wall location, we reconstruct the scene and evaluate the three quality metrics, described in Section IV-C. Fig. 9 plots the values of the three metrics vs. the considered wall positions. We observe that each metric shows a clear maximum or minimum, respectively, at the true wall location of 2 m. In particular, the ME metric yields a very sharp maximum, clearly depicting its higher sensitivity to wall position errors.

2) Velocity estimation: We carry out another simulation which specifically focuses on velocity estimation. First, we show that the additional information on the target velocity contained in the multipath returns can be exploited to improve the velocity resolution. We employ the proposed CS reconstruction scheme to resolve two targets with similar velocities. The targets are fixed at 4 m downrange in the broadside direction of the array, while the room and system parameters are kept the same as in the previous examples. Both targets reside in the same range/crossrange cell, but move in opposing crossrange directions. The velocities differ by only 0.8 m/s. We present the results in Fig. 10a-c for different numbers of multipath returns, namely, the direct propagation path only, direct and first order multipath returns only (5 paths in total per target), and all 7 paths per target as described above. It is evident that the velocity resolution capability improves with the incorporation of an increasing number of multipath returns. If only the direct path is available, the two moving targets cannot be resolved, as seen in Fig. 10a. If all four first order multipath returns are included and exploited, the two targets are resolved, but surrounding clutter pixels may render the velocity estimation difficult (See Fig. 10b). Finally, if all 7 paths are available and exploited, the two moving targets are resolved with accurate velocity estimates, as evident in Fig. 10c.

In order to further quantify the velocity resolution performance as a function of the number of multipath returns, we use the same setup as in the previous example. However, we vary the velocity difference between the two targets from 0.4 m/s to 2 m/s in steps of 0.4 m/s. In addition to \( R = 1, 5, \) and 7, we also consider \( R = 6 \) total number of paths, which corresponds to the case of including only one second order multipath to the \( R = 5 \) case. We repeat the experiment 100 times and use a simplistic detection scheme to get an upper bound on the performance. More specifically, we choose the two strongest pixels and check if they correspond to the true target velocities. While in real life this detection scheme is not feasible, it serves as a suitable metric to provide a fair comparison for the examined cases. The results are depicted in Fig. 10d. Again, we observe that without multipath exploitation, the two velocities cannot be resolved at all, while the resolution capabilities improve with increasing number of exploited multipath returns. For \( R = 7 \), we can resolve even the smallest velocity difference with high probability. The results in Fig. 10 confirm that it is advantageous to exploit the information contained in the multipath returns for improved scene reconstruction.

3) Receiver Operating Characteristic Curves: Finally, we compare different scenarios and algorithms for TWRI by simulating the receiver operating characteristic (ROC) curve. We use the same multistatic radar geometry and scene setup with four targets as described in Section V-A1. The ROC curves for the considered multi-target scene have been calculated in the following manner. Simple amplitude detection is used to first form a binary image. A target pixel is considered to
be correctly detected if the detected pixel coincides with the true target state or lies within a second order neighborhood. This must be fulfilled for both the location and the velocity. Several detected pixels within this neighborhood are treated as one. A false alarm event is defined as a pixel detected outside the second order neighborhood of any target. This corresponds to an unwanted clutter or ghost pixel in the image. 

The simulation results are averaged over 20 Monte Carlo runs and the corresponding ROC curves are averaged on a common false alarm axis. The realizations of the receiver noise as well as the random downsampling matrices change across the various Monte Carlo runs.

In the first simulation, we consider only the target returns and do not include the room corner returns in the received signal. The undersampling parameters are selected as $T_d = 20$, $M_dN_d = 2$, and $K_d = 15$ and the following scenarios are considered:

- **DP only**: Only the direct path is modeled, i.e. $R = 1$ and conventional CS reconstruction is employed.
- **MP, no exploit**: All $R = 7$ paths are modeled. Conventional CS reconstruction is employed, i.e. no multipath exploitation is performed.
- **MP, exploit**: All $R = 7$ paths are modeled. CS based multipath exploitation is employed.

Fig. 11a depicts the corresponding ROC curves. The “DP only” curve can be considered as a benchmark, as no multipath effects are present in this case. As expected, the performance degrades if the multipath returns are present and not exploited in the reconstruction (“MP, no exploit”). Proper exploitation of unresolved returns (“MP, exploit”) results in a performance comparable with the benchmark curve.

In a second simulation, corner returns are added to the received signal. The number of used transceiver-pairs has been increased to $M_dN_d = 8$ to accommodate for the reduced sparsity in the reconstruction. The resulting ROC curves are presented in Fig. 11b. Due to the presence of the corner returns, the performance is severely degraded as compared to Fig. 11a for the “DP only” and “MP, no exploit” cases. However, it becomes clear that exploiting multipath leads to substantial improvements over the “DP only” and “MP, no exploit” cases. In this case, multipath is clearly a benefit rather than a nuisance. By exploiting the power of the target multipath, the reconstructed targets appear stronger as compared to the corner clutter. Hence, the target detection performance improves and the multipath exploitation reconstruction is superior over the “DP only” scenario.

### B. Experimental Results

We show experimental results for a wideband real aperture pulse-Doppler radar with $M = 1$ transmitter and a uniform linear array with $N = 8$ receivers. The data has been recorded at the Radar Imaging Lab, Villanova University, in a semi-controlled lab setup. The transmit waveform is a modulated Gaussian pulse, covering the frequency range of 1.5 to 4.5 GHz. We recorded 768 fast time samples at a sampling rate $f_s = 7.68$ GHz and gated out the early and late returns to clean the data, resulting in $T = 153$ samples.

The transmitter was placed 62 cm away from a side wall and the receive array (element spacing 6 cm) was placed on the other side of the transmitter at a distance (to the first element) of 29.2 cm on the same baseline. No front wall was present in the scene. This is because, if present, the wall EM scatterings...
Fig. 12. Scene setup of a human (ellipse) walking diagonally towards the radar. Smaller stationary object (circle) resides a lower downrange.

Fig. 13. CS reconstruction of the walking human using 20% of the measurements.

The human as a moving target is recovered with approximately correct location and velocities, with the direction of the movement consistent with the ground truth. There is some leakage in the neighboring velocity cell, owing to the high coherence in the measurement matrix and also due to the complex nature of torso and limb movements. Additionally, some residual clutter in the stationary image can be observed. This is attributed to some stationary objects present in the lab.

We also applied the wall error correction method on the real data. The three quality metrics are evaluated while reconstructing the image with varying location of the side wall. The nominal location of the wall is 62 cm. The assumed wall positions are varied from 42 cm to 82 cm using a step-size of 2 cm. The results averaged over 100 Monte Carlo runs for the reconstruction quality metrics are provided in Fig. 14. The peaks/minima are much less pronounced as compared to the simulated data case, refer to Fig. 9. Nonetheless, a minimum in Fig. 14a and maximum in Figs. 14b are visible at 78 cm and 74 cm, respectively. According to the combined ME metric shown in Fig. 14c, the wall is located at 68 cm, which has a percentage error of 9.6% compared to 25.8% and 19.35% for the sparsity and total energy metrics, respectively. The performance drop in the proposed metrics can be attributed to several factors. First, our model ignores higher-order effects of the wave propagation, which causes a model mismatch. Second, we model the antennas as perfect isotropic radiators, while in reality, the employed antennas were directional and their phase center and gain changes considerably with angle and across the broad operational frequency band.

VI. CONCLUSION

In this paper, we presented a forward linear model under multipath propagation for simultaneous localization of stationary and moving targets behind walls. We considered multistatic radar operation, with time-division multiplexing of the transmitters for data acquisition. Based on the presented model, we proposed a group sparse reconstruction approach to solve the inverse problem under reduced data volume. In this way, we are able to recover the locations and velocities of indoor targets from much fewer than Nyquist measurements without multipath ghosting effects. We also proposed a scheme to deal with wall position errors, which enables us to effectively utilize the indirect propagation paths even under imprecise knowledge of the building layout. Supporting simulation and experimental results are provided, which demonstrate that highly-resolved, ghost-suppressed target information is obtained from very few measurements using the proposed scheme.

REFERENCES

Fig. 14. Reconstruction quality metric versus location of right side wall.


