Digraphs of degree 3 and order close to the Moore bound

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ABSTRACT

It is known that Moore digraphs of degree $d > 1$ and diameter $k > 1$ do not exist (see [20] or [5]). Furthermore, for degree 2, it is shown that for $k \geq 3$ there are no digraphs of order ‘close’ to, i.e., one less than, Moore bound [18]. In this paper, we shall consider digraphs of diameter $k$, degree 3 and number of vertices one less than Moore bound. We give a necessary condition for the existence of such digraphs and, using this condition, we deduce that such digraphs do not exist for infinitely many values of the diameter.

Keywords — digraphs, Moore bound, diameter, degree.

1. Introduction

By a digraph we mean a structure $G = (V, A)$ where $V(G)$ is a nonempty set of distinct elements called vertices; and $A(G)$ is a set of ordered pairs $(u, v)$ of distinct vertices $u, v \in V$ called arcs.

The order of a digraph $G$ is the number of vertices in $G$, i.e., $|V(G)|$. An in-neighbor of a vertex $v$ in a digraph $G$ is a vertex $u$ such that $(u, v) \in G$. Similarly, an out-neighbor of a vertex $v$ is a vertex $w$ such that $(v, w) \in G$. For $S \subset V(G)$
denote by $N^-(S)$ (respectively $N^+(S)$) the set of all in-neighbours (respectively out-neighbours) of elements of $S$. The in-degree (respectively out-degree) of a vertex $v \in G$ is the number of its in-neighbours (respectively out-neighbours) in $G$. If in a digraph $G$, the in-degree equals the out-degree ($= d$) for every vertex, then $G$ is called a \textit{dirregular} digraph of degree $d$.

A \textit{walk} $W$ of length $k$ in $G$ is an alternating sequence $(v_0a_1v_1a_2...a_kv_k)$ of vertices and arcs in $G$ such that $a_i = (v_{i-1}, v_i)$ for each $i$. A \textit{closed} walk has $v_0 = v_k$. If the arcs $a_1, a_2, ..., a_k$ of a walk $W$ are distinct, $W$ is called a \textit{trail}. If, in addition, the vertices $v_0, v_1, ..., v_k$ are also distinct, $W$ is called a \textit{path}. A \textit{cycle} $C_k$ of length $k$ is a closed trail of length $k > 0$ with all vertices distinct (except the first and the last).

The \textit{distance} from vertex $u$ to vertex $v$ in $G$, denoted by $\delta(u, v)$, is defined as the length of the shortest path from vertex $u$ to vertex $v$. Note that in general $\delta(u, v)$ is not necessarily equal to $\delta(v, u)$. The \textit{diameter} $k$ of a digraph $G$ is the maximum distance between any two vertices in $G$.

Let one vertex be distinguished in a dirregular digraph of degree $d$ and diameter $k$, having $n$ vertices. Let $n_i, i = 0, 1, ..., k$ be the number of vertices at distance $i$ from the distinguished vertex. Then,

$$n_i \leq d^i \quad \text{for } i = 0, 1, ..., k$$  \hspace{1cm} (1)

Hence,

$$n = \sum_{i=0}^{k} n_i \leq 1 + d + d^2 + ... + d^k$$  \hspace{1cm} (2)

If the equality sign holds in (2) then such a digraph is called a \textit{Moore} digraph. The right-hand side of (2) is called the \textit{Moore bound}.

It is well known that except for trivial cases (for $d = 1$ or $k = 1$) Moore digraphs do not exist (see [20] or [5]). The trivial cases are the cycles $C_{k+1}$ of length $k + 1$ and the complete digraphs $K_{d+1}$ on $d + 1$ vertices. The problem of how ‘close’ to the Moore bound the order of a dirregular digraph of diameter $k \geq 2$ and degree $d \geq 2$ can be is an interesting problem. Several results have been obtained. For instance, in [18] it is proved that for degree 2 there is no dirregular digraph of diameter $k \geq 3$ whose order is one less than the Moore bound (i.e., the ‘defect’ is 1). Furthermore, for degree 2 and diameter $k > 2$, it has been shown that dirregular digraphs with defect 2 do not exist for most values of $k$ [19]. For other related results see also [2], [9], [10], [11], [13], [14], [17] and [21].

The corresponding problem in undirected graphs has been studied extensively by several authors and many results have been obtained (see [1], [3], [6], [8] and [12]).

Throughout this paper, we shall consider only dirregular digraphs $G$ of degree $d \geq 2$, diameter $k \geq 3$ and number of vertices $n$ one less than the Moore bound, i.e., $n = d + d^2 + ... + d^k$. Such digraphs have the characteristic property that for every
vertex $v \in G$ there is a unique vertex $\text{rep } v \in G$ such that there are two walks of lengths $\leq k$ from $v$ to $\text{rep } v$ in $G$. An interesting property that we find in such digraphs is that the set of all out(in)-neighbours of $\text{rep } v$ is the same as $\text{rep}$ of the set of all out(in)-neighbours of $v$ for each $v$ in $G$ (see Theorem 1). The main results of this paper are Theorems 2 and 4. In Theorem 2 we prove that a diregular digraph of degree $d \geq 2$, diameter $k \geq 3$ and order $n = d + d^2 + \ldots + d^k$ cannot satisfy the condition that for every vertex $v$ in $G$, $\text{rep } v$ is $v$ itself. In other words, for $k \geq 3$ and $d \geq 2$ there is no digraph $G$ of order one less than the Moore bound with every vertex in a cycle of length $k$. In Theorem 4 we prove that for degree 3 and diameter $k \geq 3$ there is no digraph $G$ with order $n = 3 + 3^2 + \ldots + 3^k = \frac{3}{2}(3^k - 1)$ if $k + 1$ does not divide the number of arcs in $G$, i.e., $3n = \frac{3}{2}(3^k - 1)$.

2. Results

We consider a diregular digraph $G$ of diameter $k$, degree $d$ with number of vertices

$$n = d + d^2 + \ldots + d^k.$$  \hspace{1cm} (3)

We shall call such a digraph a $(k, d)$-digraph.

From the definition of a $(k, d)$-digraph it can be easily seen that the following propositions hold.

**Proposition 1** For any two vertices $x, y$ of a $(k,d)$-digraph $G$ there is at most one walk of length $l < k$ from $x$ to $y$. In particular, $G$ contains no cycle of length $l$ ($C_l$).

**Proposition 2** For every vertex $x$ of a $(k,d)$-digraph there exists exactly one vertex $y$ so that there are two walks of lengths $\leq k$ from $x$ to $y$.

The vertex $y$ is called the repeat of $x$ (rep $x$). Note that if $x \in C_k$, then rep $x = x$, the two walks of Proposition 2 have lengths 0 and $k$, and $x$ is called a self-repeat. Furthermore,

**Proposition 3** No vertex of a $(k,d)$-digraph is contained in two $C_k$'s.

2.1. General properties

Let rep $S$ denote the set of repeats of all elements of a set $S$.

**Theorem 1** For every vertex $v$ of a $(k,d)$-digraph we have: (a) $N^+(\text{rep } v) = \text{rep } N^+(v)$ and (b) $N^-(\text{rep } v) = \text{rep } N^-(v)$. 

3
Proof. Let \( v \) be a fixed vertex in such a digraph \( G \). We will count in two ways the number of all walks of length \( \leq k + 1 \) from \( v \) to any \( y \) in \( G \).

1) Since \( G \) is a regular digraph with outdegree \( d \), there are exactly \( d^i \) walks of length \( i \) beginning at \( v \) for every \( i = 0, 1, 2, \ldots, k + 1 \). Thus there exist exactly

\[
1 + d + d^2 + \ldots + d^{k+1}
\]
walks of lengths \( \leq k + 1 \) beginning at \( v \).

2) The second way of counting is a little bit more complicated. There is one (trivial) walk of length zero from \( v \) to \( v \). For each vertex \( y \) there are at least \( d \) non-trivial walks of lengths \( \leq k + 1 \) from \( v \) to \( y \). Usually, for each \( x \in N^{-}(y) \) there is one walk of length \( \leq k \) from \( v \) to \( x \) (\( G \) has diameter \( k \)), and \( d \) walks of lengths \( \leq k + 1 \) are obtained. However, if \( y \in N^{+}(\text{rep } v) \), then there is one further walk of length \( \leq k \) from \( v \) to \( \text{rep } v \) (by the definition of \( \text{rep } v \)) and hence altogether \( d + 1 \) non-trivial walks of lengths \( \leq k + 1 \) from \( v \) to \( y \).

Now let \( y = \text{rep } v \); \( v_i \in N^{+}(v) \). Then we have one walk of length \( \leq k \) from \( v_j \) to \( y \) if \( j \neq i \) and two such walks if \( j = i \). Hence also in this case there are \( d + 1 \) non-trivial walks of lengths \( \leq k + 1 \) from \( v \) to \( y \).

Summarizing, we see that there are \( d + 1 \) non-trivial walks of lengths \( \leq k + 1 \) from \( v \) to \( y \) if \( y \in N^{+}(\text{rep } v) \) or \( y = \text{rep } N^{+}(v) \), and \( d \) otherwise. Thus,

\[
1 + d + d^2 + \ldots + d^{k+1} \geq 1 + nd + |N^{+}(\text{rep } v) \cup \text{rep } N^{+}(v)|
\]

Hence,

\[
d \geq |N^{+}(\text{rep } v) \cup \text{rep } N^{+}(v)|
\]

As \( |N^{+}(\text{rep } v)| = |\text{rep } N^{+}(v)| = d \), we see that \( N^{+}(\text{rep } v) = \text{rep } N^{+}(v) \), as desired. The proof of \((b)\) is similar. \( \square \)

Theorem 2 For \( k \geq 3 \) and \( d \geq 2 \) there is no \((k, d)\)-digraph with every vertex in a \( C_k \).

Proof. Assume such a digraph \( G \) exists. Then \( \text{rep } x = x \) for all \( x \) and therefore the adjacency matrix of \( G \) fulfills the matrix equation

\[
A^k + A^{k-1} + \ldots + A + I = J + I,
\]

\[i.e.,\]

\[
A^k + A^{k-1} + \ldots + A = J.
\]

where \( A \) is the adjacency matrix of \( G \), \( I \) is the \( n \times n \) identity matrix and \( J \) is the \( n \times n \) matrix with all its entries equal to 1.
From now we shall consider only the case \( d > 1 \), for \( d = 1 \), such digraphs exist only if either \( b = a \) (De Bruijn digraphs [7]) or \( b = a + 1 \) (Kautz digraphs [15, 16]). As we have \( a = 1 \) and \( b = k \geq 3 \), no such digraph can exist. Nevertheless, Bosák’s proof is rather long and thus, for the reader’s convenience, we give another simpler proof which is based on a proof technique similar to that from [5]. The eigenvalues of \( J \) are \( n \) (with multiplicity 1) and 0 (with multiplicity \( n - 1 \)). The eigenvalues of \( A \) are \( d \) (this corresponds to \( n \)) and some of the roots of

\[
x^k + x^{k-1} + \ldots + x = 0
\]

(6)

The roots of Eq. (6) are 0 and \( e^{i2\pi q/k} \) for \( q = 1, 2, \ldots, k - 1 \) (the roots of \( x^k = 1, x \neq 1 \)).

Let us denote the eigenvalues of \( A \) other than \( d \) by \( x_1, x_2, \ldots, x_{n-1} \). Since \( G \) has no cycle of length less than \( k \),

\[
\text{trace}(A^p) = 0, \quad \text{for } p = 1, 2, \ldots, k - 1,
\]

Hence,

\[
d^p + \sum_{j=1}^{n-1} x_j^p = 0 \quad (1 \leq p \leq k - 1)
\]

(7)

Using conjugate Eq. (7) for \( p = 1 \) and then Eq. (7) for \( p = k - 1 \) we get

\[
-d = \sum_{j=1}^{n-1} \overline{x}_j = \sum_{j=1}^{n-1} x_j^{k-1} = -d^{k-1}
\]

(since \( d \) is real and if \( x = e^{i2\pi q/k} \) then \( \overline{x} = e^{-i2\pi q/k} = e^{i2\pi((k-1))} = x^{k-1} \); also \( 0 = \overline{0} = 0^{k-1} \)).

Thus \( d = d^{k-1} \) which is fulfilled only if \( d = 1 \) or \( k = 2 \).  

2.2. The case of degree 3

From now on we shall consider only the case \( d = 3 \) and \( k \geq 2 \).

In the following three lemmas, we will assume that the \((k, 3)\)-digraph \( G \) contains a cycle of length \( k \), denoted by \( C^0_k \).

We shall label the vertices of \( G \) by 0, 1, 2, ..., \( n - 1 \). Obviously we can suppose that \( N^+(0) = \{1, 2, 3\} \) and that \( (0, 3) \in C^0_k \).

For all \( i = 1, 2, 3 \), and \( j = 1, \ldots, k \), define \( L^i_j = \{x \in V(G) : \delta(i, x) = j - 1\} - \{0\} \). Let \( L_j = L^1_j \cup L^2_j \cup L^3_j \) and \( \Delta_i = \bigcup_{j=1}^{k} L^i_j \). Then \( G \) contains a subdigraph drawn as in Figure 1, which contains an arc \((x, y)\) joining two vertices of \( \Delta_i \) iff \( x \in L^i_{j-1} \) and \( y \in L^i_j \) for some \( j \).

Repeated applications of Theorem 1 yield the following lemma.
Lemma 1 If $a \in L_j$ then $\text{rep } a \in L_j$, for $j = 1, 2, \ldots, k$.

Our aim is to show that each vertex is a selfrepeat. Therefore the following assumption will be examined:

$$\text{rep } 1 = 2$$ \hspace{1cm} (8)

Then owing to Theorem 1 and the fact that 3 is a selfrepeat we have

**Proposition 4** $\text{rep } 2 = 1$.

**Proposition 5** $a \in \Delta_1$ implies $\text{rep } a \in \Delta_2$.

**Proposition 6** $b \in \Delta_2$ implies $\text{rep } b \in \Delta_1$.

**Proposition 7** $\Delta_1 \cup \Delta_2$ contains no selfrepeat.

**Proposition 8** For all $i = 1, 2, 3$, and $j = 1, 2, \ldots, k$, $(u, v) \not\in G$ whenever $u \in L_i^k$ and $v \in L_j^i$.

Since we have to reach 1 from 2, there exists a vertex $x_0 \in L_2^3$ such that $(x_0, 1) \in G$.

Lemma 2 If $y \in C_3^0$ then $(x_0, y) \not\in G$.

**Proof.** If for a vertex $z \in C_3^0$, $z \neq 3$ we have $(x_0, z) \in G$ then $\text{rep } x_0 = 1$, a contradiction with Proposition 4. Now suppose $(x_0, 3) \in G$ and denote by $v$ the third out-neighbour of $x_0$. Due to Prop. 8, $v \not\in \Delta_2$, thus $v = \text{rep } x_0$ and so due to Prop. 6, $v \in \Delta_1$. Let the out-neighbours of $v$ be $v_1, v_2, v_3$. According to Prop. 8, one of $v_i$'s is equal to 2 and the remaining two are repeats of $v$ which is a contradiction with Proposition 2. \hfill $\Box$
Lemma 3 \( \text{rep} \ 1 = 1 \).

Proof. For a contradiction suppose (8) holds. We know that \((x_0, 1) \in G\); let the remaining out-neighbours of \(x_0\) be \(x_1\) and \(x_2\). Not both \(x_1\) and \(x_2\) can be from \(\Delta_1\) (see Proposition 2). Hence at least one, say \(x_1\), belongs to \(\Delta_3\) (see Prop. 7). Due to Lemma 2, \(x_1 \notin C^0_k\); thus \(x_1 \in \Delta_3 - C^0_k\). Denote by \(z\) this vertex of \(L^3_{j-1}\) for which \((z, x_1) \in G\) and by \(u\) and \(v\) denote the remaining two vertices such that \((z, u), (z, v) \in G\) (see Fig. 2).

![Figure 2:](image)

Now, we have to reach \(z\) from \(x_0\). We shall distinguish two cases:

a) we reach \(z\) from \(x_0\) via \(x_1\),

b) we reach \(z\) from \(x_0\) via \(x_2\).

In case (a) we have a path \(P\) of length \(k - 1\) from \(x_1\) to \(z\) (see Proposition 1) which together with arc \((z, x_1)\) form a \(C^1_k\). \(C^1_k\) is different from \(C^0_k\), because \(x_1 \notin C^0_k\). However, due to Proposition 3 we have \(3 \notin C^1_k\). But in \(\Delta_3\) each \(C_k\) contains 3 (see Prop. 8). Thus \(P\) (and also \(C^1_k\)) must contain a vertex of \(\Delta_1 \cup \Delta_2\), a contradiction with Prop. 7.

In case (b) we have a path \(Q\) from \(x_2\) to \(z\) of length \(k - 1\) (due to Prop. 6: \(x_1 \neq \text{rep} x_0\)). Now, we have to reach \(u\) and \(v\) from \(x_0\). It cannot be done via \(x_1\) because then \(u\) (or \(v\)) would be a repeat of \(z\), a contradiction with Lemma 1. So we have to reach both \(u\) and \(v\) from \(x_0\) via \(x_2\). However, the paths do not go via \(z\), because \(Q\) is of length \(k - 1\) (see Proposition 1). Thus we have a path \(P_1\) (respectively \(P_2\)) of length at most \(k - 1\) from \(x_2\) to \(u\) (respectively \(v\)) not containing \(z\). Hence there are two paths of lengths \(\leq k\) from \(x_2\) to \(u\) (respectively \(v\)), namely \(Q \cup (z, u)\) and \(P_1\) (respectively \(Q \cup (z, v)\) and \(P_2\)). Thus \(u = \text{rep} x_2\) and \(v = \text{rep} x_2\), a contradiction with Proposition 2. \(\square\)

Lemma 4 Let \(k \geq 2\). If a \((k, 3)\)-digraph \(G\) contains a \(C_k\), then each vertex of \(G\) is contained in a \(C_k\).

Proof. Suppose \(G\) contains a vertex \(x\) such that \(x\) lies in a \(C_k\) but at least one of
its out-neighbours is contained in no $C_k$. Putting $x = 0$ we get a contradiction with Lemma 3.

2.2.1. Degree 3 and $k \geq 3$

**Theorem 3** Let $k \geq 3$. In a $(k, 3)$-digraph there is no cycle of length $k$.

**Proof.** Apply Lemma 4 and Theorem 2. \qed

In this section we shall focus on a $(k, 3)$-digraph $G$ without $C_k$ and prove that no arc can be contained in two $C_{k+1}$’s. For a contradiction suppose

$$(0, 3) \text{ is contained in two } C_{k+1} \text{’s.}$$

Then we have two arcs from $L^3_k$ to 0 and
two of $\Delta_i$’s have a common vertex, say, $a$.

Obviously without loss of generality we can suppose $a \in \Delta_2$. Then we have two essentially different cases as shown in Figure 3.

![Figure 3](image)

**Proposition 9** $rep \ 3 = 0$, $rep \ 0 = a$ and $\{1, 2, 3\} = N^+(a)$.

**Proof.** See Theorem 1. \qed

**Corollary 1** $rep (L^3_j) = L_{j-1}$, for $j = 2, ..., k$.

**Proof.** Apply Theorem 1 $(k - 1)$ times. \qed

**Lemma 5** Let $x \in L^2_k - \{a\}$. If $(x, y) \in G$ then $y \notin \Delta_2$.  

8
**Theorem 4** Suppose arc \((x, y) \in G\) where \(y \in \Delta_2\). Then \(y \neq 2\) because \(C_l \notin G\) for \(l \leq k\). Moreover, \(y\) is the repeat of 2, which together with Prop. 9 implies the existence of an arc \((a, y)\). Thus we have three arcs from \(\Delta_2\) to \(y\). But then we can not reach \(y\) from 1 or 3. \(\Box\)

As we have to reach 1 from 2, there exists a vertex \(x_0 \in I^2_k\) such that \((x_0, 1) \in G\). Since \(k > 2\), we have

\[x_0 \neq a.\]  

**Lemma 6** \((x_0, 3) \notin G\).

**Proof.** Suppose \((x_0, 3) \in G\). Denote by \(v\) the third out-neighbour of \(x_0\). Obviously \(v \neq 0\) and due to Lemma 5, \(v \notin \Delta_2\), thus \(v = \text{rep} \ x_0\). Denote by \(v_1, v_2, v_3\) the out-neighbours of \(v\). All vertices of \(\Delta_1\) and \(\Delta_3\) are reached from \(x_0\) via 1 and 3. We have \(v_1 \in \Delta_2, v_1 \neq a\) (because \(a \neq \text{rep} \ x_0\)), and one of the out-neighbours of \(v\) must be 2 (otherwise we can not reach 2 from \(x_0\)). Then the remaining two \(v_i\)'s are repeats of \(v\), which is a contradiction. \(\Box\)

**Lemma 7** No arc of \(G\) is contained in two \(C_{k+1}\).

**Proof.** Denote by \(x_1\) and \(x_2\) the remaining two out-neighbours of \(x_0\). Due to Lemma 5, \(x_1, x_2 \notin \Delta_2\) because \(x_0 \neq a\). At most one of them can belong to \(\Delta_1\), namely the repeat of \(x_0\). Thus at least one, say \(x_1\), belongs to \(\Delta_3\). If both \(x_1, x_2\) belong to \(\Delta_3\), then \(x_1\) is chosen so that \(x_1 \neq \text{rep}\ x_0\). Due to Lemma 6, \(x_1 \neq 3\). If \(x_1 \in I^3_j\), denote by \(z\) that vertex of \(I^3_{j-1}\) for which \((z, x_1) \in G\) and by \(u, v\) denote the remaining two vertices adjacent from \(z\) (see Fig. 4).

Now, \(z\) cannot be reached from \(x_0\) via \(x_1\), because \(C_l \notin G, l \leq k\). Thus we have a path \(Q\) of length \(k - 1\) from \(x_2\) to \(z\) (not shorter, because we have \(x_1 \neq \text{rep} \ x_0\)). If \(u\) or \(v\) are reached from \(x_0\) via \(x_1\), then \(\text{rep} \ z = u\) or \(v\), a contradiction with Corollary 1. Thus we have to reach both \(u\) and \(v\) from \(x_0\) via \(x_2\). Indeed, we can not reach them via \(z\), because \(Q\) is of length \(k - 1\). Hence there exists a path \(P_1\) (respectively \(P_2\)) of length \(\leq k - 1\) from \(x_2\) to \(u\) (respectively \(v\)). Hence we have two paths of lengths \(\leq k\) from \(x_2\) to \(u\) (respectively \(v\)), namely \(P_1\) and \(Q \cup (z, u)\) (respectively \(P_2\) and \(Q \cup (z, v)\)), i.e., \(u = \text{rep} \ x_2, v = \text{rep} \ x_2\), a contradiction. \(\Box\)

**Theorem 4** If \(k \geq 3\) then the arc-set of any \((k, 3)\)-digraph can be decomposed into cycles of length \(k + 1\).

**Proof.** Obviously if there is no \(C_k\) in such a digraph then every arc is contained in some \(C_{k+1}\). Now Lemma 7 applies. \(\Box\)

The following theorem is an immediate consequence of Theorem 4.

**Theorem 5** If a \((k, 3)\)-digraph exists for \(k \geq 3\) then \(k + 1\) divides \(\frac{3^k}{2} - 1\).
Corollary 2 If \( k \) is odd then no \((k, 3)\)-digraph exists.

**Proof.** By induction on \( k \) we see that \( 3^k - 1 = 4t + 2 \) for an integer \( t \) (because \( 3^{k+2} - 1 = 9(3^k - 1) + 8 = 9(4t + 2) + 8 = 4(9t + 6) + 2 \)). Thus \( \frac{9}{2}(3^k - 1) \) is odd but \( k + 1 \) is even. \( \square \)

Unfortunately, for \( k \) even we have only the following assertion.

Corollary 3 If \( 27 \) divides \((k + 1)\) then no \((k, 3)\)-digraph exists.

**Remark.** We have checked the divisibility condition in Theorem 5. Using a computer we found 1256 feasible \( k \)’s in the interval from 3 to 10,000.

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**References**


