Implementation of FPGA based Fast DOA Estimator using Unitary MUSIC Algorithm

Minseok Kim, Koichi Ichige and Hiroyuki Arai
Graduate School of Engineering, Yokohama National University
Tokiwadai 79–5, Hodogaya-ku, Yokohama, 240–8501, Japan
Email: mskim@arailab.dnj.ynu.ac.jp

Abstract—This paper proposes the practical implementation of DOA estimation system using FPGA (Field Programmable Gate Array) that is a key technique in the realization of the DOA-based adaptive array antenna for cellular wireless basestation. It incorporates spectral unitary MUSIC (Multiple Signal Classification) algorithm, which is one of the representative super resolution DOA estimation techniques [1]. This paper describes the way of DSP design and real hardware implementation of the unitary MUSIC algorithm. This system achieves the high performance in the eigenvalue decomposition (EVD) and MUSIC angular spectra computation with Cyclic Jacobi processor based on CORDIC (Coordinate Rotation Digital Computer) [2] and spatial DFT (Discrete Fourier Transform), respectively. All DSP functions are computed by only fixed-point operation with finite bit-length in order to meet the requirements of fast processing and low power consumption due to the simplified and optimized architecture.

I. INTRODUCTION

Exploiting adaptive array antenna technologies, the wireless system capacity will be dramatically increased and the harmful effects by multipath fading can be combated as well. From the theoretical point of view, many useful algorithms of adaptive array antenna techniques need DOAs (Directions Of Arrival) of desired and interferer signals in advance. Of course, the practical researches often have used Wiener-solution based algorithms like LMS and RLS employing a temporal reference signal instead of DOA informations, while the DOA-based systems exploit the exact DOAs in a beamformer to separate the desired signal from interferers spatially. However the DOA-based systems have many advantages over the conventional temporal reference based solutions. For example, they are more applicable to the downlink solution thanks to the exact directional information. And the performance of DOA-based beamforming is superior to that of other types of algorithms for small angular spread, while it has time-consuming task of DOA estimation [3]. In order to implement such a DOA-based system, the most time-consuming DOA estimation step should be processed as fast as possible. But such processing has been very difficult to realize in the practical systems from the lack of cost effective digital processing devices to solve the hard computational burden. We believe that general Von Neumann architecture processors can never usually meet the requirements of the fast and compact architecture and low power consumption at the same time.

Thus, in this paper, the FPGA based DSP (Digital Signal Processing) design and hardware implementation of the fast DOA estimator will be presented. It can be applied to cellular wireless basestation for DOA-based beamforming and a realtime DOA monitoring system usefully, if it is tuned up appropriately corresponding to the environment. It incorporates unitary MUSIC (Multiple Signal Classification) algorithm, which is one of the representative super resolution DOA estimation techniques. MUSIC based algorithm has many advantages in the real hardware implementation due to its simplicity compared with other well-known subspace based techniques like ESPRIT. However, there still remains the computational complication of the complex number arithmetic, which is a great distress to the fast and compact computation. With a unitary transform, the eigendecomposition of the correlation (or covariance) matrix in the MUSIC algorithm can be achieved with real number only [1] [6]. The unitary MUSIC processor (UMP) performs all DSP functions with only fixed-point operation with finite bit-length in order to meet the requirement of fast processing and low power consumption by simplified and optimized architecture. This system performs the fast computation of EVD and MUSIC angular spectra with Cyclic Jacobi processor based on CORDIC (Coordinate Rotation Digital Computer) [2] and spatial DFT (Discrete Fourier Transform), respectively.

II. UNITARY MUSIC DOA ESTIMATOR

MUSIC algorithm is a kind of DOA (Direction Of Arrival) estimation technique based on eigenvalue decomposition, which is also called subspace-based method [5]. It is well known for the implementation simplicity as well as the capability of estimating DOA in much higher resolution than any other conventional methods.

We assume the basic model of a narrowband signal $s(n)$. The signals received at $K$ antenna array spaced by half wavelength can be written by linear combination of $L$ incident signals from far-field and white Gaussian noise as

$$X(n) = A \cdot S(n) + N(n), \quad (1)$$

where $S(n)$ is a signal matrix and $X(n)$ is a $K \times 1$ vector of array output at any sampling time $n$, which is called a snapshot. The columns of $A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_J)]$ are the steering vectors. The correlation matrix of $X(n)$ is given by

$$R_{xx} = E[X(n)X^H(n)] = AR_{ss}A^H + \sigma^2 I, \quad (2)$$

where $R_{ss}$ is a signal matrix and $X(n)$ is a $K \times 1$ vector of array output at any sampling time $n$, which is called a snapshot. The columns of $A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_J)]$ are the steering vectors. The correlation matrix of $X(n)$ is given by

$$R_{xx} = E[X(n)X^H(n)] = AR_{ss}A^H + \sigma^2 I, \quad (2)$$
but, in reality, the correlation matrix is approximated by uniform averaging by some number of snapshots as

$$R_{xx}(n) \approx \frac{1}{\text{snapshots}} \sum_{n=1}^{\text{snapshots}} X(n)X^H(n),$$  \hspace{1cm} (3)

where $E[\cdot]$ and superscript $H$ denote expectation and hermitian operator, respectively. And $R_{ss} = E[S(k)S^H(k)]$ is signal covariance matrix and $\sigma^2$ is noise variance.

Since $R_{xx}$ is a positive definite hermitian matrix, the correlation matrix of $X(n)$ can be decomposed to signal and noise subspaces. The noise subspace eigenvectors of corresponding eigenvalue of $\sigma^2$ lies in the nullspace of $A^H$, that is to say, they are orthogonal to the signal subspace, and eventually orthogonal to the signal steering vectors. Using this principle, the MUSIC spectrum is computed using noise subspace eigenvectors as

$$P_{MU} = \frac{a^H(\theta) a(\theta)}{a^H(\theta) E_N E_N^H a(\theta)},$$  \hspace{1cm} (4)

where $E_N$ is the matrix whose column vectors are noise subspace eigenvectors. In the spatial MUSIC algorithm, the peaks appear at the corresponding angles to DOAs of incident signals, since they are reciprocal of the nulls. Exploiting null peaks appear at the corresponding angles to DOAs of incident signals, since they are reciprocal of the nulls. Exploiting null steering toward DOAs makes the super-resolution estimation available.

Generally the correlation matrix in Eqs.(2)-(3) is complex-valued. It is certain that the EVD with complex-valued correlation matrix should be high computational burden. Reducing the computational complexity via unitary transform allows real-valued eigenvalue decomposition of the transformed real number correlation matrix [1]. Since the EVD process has a large portion of whole computational load of MUSIC based algorithms, real-valued eigenvalue decomposition can provide the fast and compact computation. If the steering vectors are arranged conjugate centro-symmetric as

$$a(\theta_l) = \left[ e^{j\pi(2l-1)\sin \theta_1}, \cdots, e^{-j\pi(K-1)\sin \theta_1} \right]^T,$$  \hspace{1cm} (5)

the correlation matrix $R_{xx}$ becomes centro-Hermitian. The real-valued correlation matrix $\hat{R}_{xx}$ can be obtained via any unitary transform $Q$ as

$$\hat{R}_{xx} = R_{xx}(Q^H R_{xx} Q),$$  \hspace{1cm} (6)

The unitary transform $Q$ can be chosen as

$$Q = \frac{1}{\sqrt{2}} \begin{pmatrix} I & jI \\ II & -jII \end{pmatrix},$$

$$Q = \frac{1}{\sqrt{2}} \begin{pmatrix} I & 0 \\ 0 & \sqrt{2}jI \end{pmatrix},$$  \hspace{1cm} (7)

according to the even and odd number of arrays respectively, where the vector $0 = [0, 0, \cdots, 0]^T$, and $I$ and $II$ are the identity matrix and column flipped identity matrix in the left-right direction, respectively. In Eq.(6), the selection of real part only provides FB (Forward-Backward) averaging [1].

### III. DSP Design Concepts for Dominant Procedures

The unitary MUSIC computational flow is involved in 4 steps largely; Estimation of correlation matrix including unitary transform and spatial smoothing if needed, EVD (Eigen Value Decomposition) of the correlation matrix, Computation of MUSIC spectrum and 1-dimensional local maximum detection. In this section, the DSP concepts for the dominant procedures in the unitary MUSIC algorithm will be described.

#### A. Eigenvalue Decomposition via CORDIC based Jacobi Processor

In our former work, the circuit design of EVD computation processor for MUSIC DOA estimator was studied [2]. It used CORDIC based Jacobi method, and it was suitable for hardware implementation for fast parallel processing. Cyclic Jacobi processor computes real symmetric eigenvalue problems by applying a sequence of orthonormal rotations to the left and right sides of the target $K \times K$ real symmetric correlation matrix $R_{yy}$ as

$$E^T \cdot R_{yy} \cdot E = D,$$  \hspace{1cm} (8)

where $W_{pq}$ is an orthonormal plane rotation over an angle $\theta$ in the $(p,q)$ plane whose elements are $w_{pp} = \cos \theta$, $w_{qq} = \sin \theta$, $w_{qp} = -\sin \theta$, $w_{qp} = \cos \theta (p > q)$. $J$ is the multiple rotation of $W_{pq}$’s in the cyclic-by-row manner of $(p,q)$ which is called a Jacobi sweep, and the superscript $T$ and subscript $K$ denote transposition and array length, respectively. This system employed the hardware friendly CORDIC (COordinate Rotation Digital Computer) algorithm for vector rotators and arctangent computers to solve Eq.(8), which were the basic processing unit of this design. As far as the fixed-point operation is applied, of course there exist the approximation errors. But when it was implemented with above 16-bit precision, we could get the reasonable performance. In this EVD processor, the number of iterations and the computation bit-length are 4 Jacobi sweeps and 16-bit long, respectively.

#### B. MUSIC Spectrum Computation via Spatial DFT

In spectral MUSIC algorithm, in order to find out DOA angles of the incident signals, the angular spectrum should be computed after the EVD step. Of course, there exist another alternative solutions for direction finding problem in MUSIC based algorithms. It is the technique based on root finding of the MUSIC polynomial, which called root-MUSIC [6]. However complex number coefficient polynomial is very complicated and not suitable to solve with the dedicated circuit computer with the fixed-point operation like FPGAs.

For fast DSP implementation on FPGAs, a simple iterative algorithm should be the best solution. To compute the angular MUSIC spectrum, spatial DFT (Discrete Fourier Transform) technique is very attractive due to the well-known performance guarantee as well as the simplicity. This section will describe...
how to apply DFT to the computation of the MUSIC angular spectrum.

The simple continuous spatial signal model of ULA (Uniform Linear Array) is given typically by

\[ x_d = U(t) \cdot \exp(-j2\pi \cdot f_{spa} \cdot d), \]  

(9)

where \( U(t) \) includes all time varying components and complex amplitude, and \( d \) and \( f_{spa} \) are the distance from the first reference antenna element and spatial frequency of \( \sin \theta / \lambda \), respectively. By applying spatial \( P \)-point DFT, the discrete spatial frequency distribution function can be obtained by

\[ X_d[k] = \frac{1}{P} \sum_{m=0}^{P-1} x_d[m] \cdot e^{-j\frac{2\pi}{P} m \cdot k}, \]  

(10)

where \( k \) and \( m \) is the indices of the discrete spatial frequency and discrete distance, respectively. When the antenna spacing \( D_{spacing} = \lambda / 2 \), eventually the discrete wavefront \( \theta \) can be computed from the relation of Eqs.(11)–(12).

\[ f_{spa, discrete} = \frac{\sin \theta}{\lambda} = \frac{k}{P \cdot D_{spacing}} \]  

(11)

\[ \theta = \sin^{-1} \left( \frac{k}{P/2} \right) \]  

(12)

From above, in the MUSIC algorithm, it is certain that the spatial DFT of the noise subspace eigenvectors as Eq. (10) provide the distribution of the spatial frequency. But if DFTed with only a few spatial samples at antenna array, the resolution of the spatial spectrum becomes very coarse. Thus any estimation will not be available from the coarse spectrum. In that regards, the interpolation of the spectrum should be taken into consideration. According to digital signal processing theory, the DFT spectrum can be generated fine and smoothly by adding a few number of zeroes to the spatial data of the noise eigenvector elements. The spectrum generated by the spatial DFT is completely equivalent to that by steering mainbeam toward whole directions as Eq. (4) mentioned in Sect.II.

Instead of finding peaks in the MUSIC spectrum written in Eq. (4), local minima (LM) detection of the DFT spectrum as Eq. (10), which is equivalent to the denominator of Eq. (4), can be applied for implementation simplicity. Rearranging the spectrum of Eq. (10) the concrete discrete wavefronts (DOAs) are obtained from Eq. (12) by

\[ \theta = \sin^{-1} \left( \frac{l - P/2}{P/2} \right), \]  

(13)

As shown in Eq. (13), the discrete wavefronts from spatial DFT spectrum are not uniformly spaced. That, however, may be of no concern in the practical sectorized basestation configuration. From Eq. (13), \( \theta \) is an inverse sinusoidal function of \( l \). In the region between \(-30\) to \(30\) degrees, the function of Eq. (13) can be approximated by linear function whose gradient is given by derivative of Eq. (14) at \( l = P/2 \) as

\[ \frac{d\theta}{dl} \bigg|_{l=P/2} = \frac{1}{P/2} \cdot \frac{\pi}{180} = 0.4476 \text{ deg}. \]  

(14)

In this linear region, the angular spacing can be regarded as almost uniform and the estimation resolution is inversely proportional to DFT length \( P \). When \( P \) is 256, the estimation resolution is about 0.4476 degree.

IV. PRACTICAL DSP DESIGN AND HARDWARE LEVEL SIMULATION

In this time, DOA estimation under stationary case without any fading was assumed for the simple implementation, and the system was designed to classify highly correlated (coherent) signals via spatial smoothing technique for easy experiment with single wave source only. In unitary MUSIC, with the forward only spatial smoothing of the correlation matrix, backward spatial smoothing can be achieved simultaneously as mentioned in Sect.II. The first step of the unitary MUSIC procedures is to transform the input data vector \( X \) to \( Y \) with a unitary transform \( Q \) as written by

\[ Y_i = Q^H X_i, \]  

(15)

where \( X_i \) and \( Y_i \) are the divided \( M \) sub-vectors and the corresponding transformed sub-vectors for spatial smoothing, respectively. From above unitary transform, the correlation matrix \( R_{yy} \) is given by

\[ R_{yy}(n) = \beta R_{yy}(n-1) \]  

\[ + (1 - \beta) \sum_{i=1}^{M} \text{Re}\{Y_i(n)Y_i^H(n)\}, \]  

(16)

where \( \beta \) is an appropriate real smoothing factor and \( M \) is the number of sub-matrices. It was implemented by first-order IIR (Infinite Impulse Response) exponential averaging filter.

Next step, the correlation matrix is eigen-decomposed by the EVD processor. As described in Sect.III-A, CORDIC based Jacobi EVD processor was incorporated in this system. It had only simple iterative process of vector rotation after obtaining optimal rotation angle. After the EVD step, the reciprocal MUSIC spectrum written in Eq.(4) of Sect.II is computed via
spatial DFT of the noise eigenvectors returned to complex values by the unitary inverse transform as

\[ P_{MU,\text{reciprocal}} = \sum_{i=L+1}^{K} |DFT\{\mathbf{Q} \cdot \mathbf{E}_i\}|^2, \tag{17} \]

where \( \mathbf{E}_i \) is the i-th eigenvector belonging to the noise subspace, \( K \) and \( L \) are the number of antenna elements and the number of waves, respectively.

Figure 1 shows a hardware level simulation result. Hardware level simulations performed the direct measurements with only DSP part of a real hardware to evaluate validity efficiently of the system avoiding working the whole system components. We used the input data computed by an offline PC in advance and obtained the results with real hardware operation. With these hardware level simulations, we could verify the function of the digital signal processor. In this simulation, it was assumed that 2 coherent (or fully correlated) waves were arriving at 4 ULA antennas from the DOAs of -15 and 20 degrees, respectively. And two waves were same powers and the input SNR was 10 dB. For the spectrum computation, the FFT (Fast Fourier Transform) of 256 points including 3-dimensional data of the noise eigenvector’s elements (1 dimension was used for spatial smoothing) and 253 zeroes, was applied.

In final step, we found out the DOAs by the detection of the LM (Local Minimum) points in the reciprocal MUSIC spectrum as shown in Fig.1. It could be easily implemented with whole memory scanning circuit. In this case, as shown in Fig.1, the index numbers of the DFT spectrum corresponding to the LM points below any appropriate threshold level are 95 and 170. With these index values, the concrete discrete wavefronts could be obtained as -14.94 and 19.16 degrees respectively from the relation of the spatial DFT index (or discrete frequency number) and discrete wavefront (DOA angle) as Eq. (13)

V. FPGA IMPLEMENTATION AND PERFORMANCE

Not only the theoretical design, we also tried to implement it on 2 FPGAs (EP20K600, Altera) which had about 1.2 million equivalent gates and 80 Kbytes internal memory block totally. The whole block diagram of the DSP procedures described in previous sections is shown in Fig.2. It is involved in 4 major procedure sections including Correlation Matrix Section, EVD Section, FFT Section and LM Detection Section. The bit precision of every section is also shown in this figure. For the present, it was assumed that the exact number of waves were predetermined and known already from any other process.

In our evaluation testbed as shown in Figs.3 and 5, the RF (Radio Frequency) signals are down-converted to IF (Intermediate Frequency) signals centered at 10 MHz in analog DC (Downconversion) receiver, and then digitized by ADCs (Analog to Digital Converters) at the rates of 40 MSPS. The 4 times oversampled IF signals are digitally down-converted once again to complex baseband and then downsampled by \( L \)-times, where \( L \) is an appropriate integer number. The FPGAs perform the digital signal processing of the unitary MUSIC algorithm.

Table I illustrates the roughly estimated performance of the dominant core functions, where LEs (Logic Elements) means the number of occupied logic blocks in FPGAs and \( f_{\text{max}} \) is the maximum clock frequency at which normal operation can be guaranteed. And the minimum computation time \( t_{\text{min}} \) is calculated by required clks \( \times f_{\text{max}} \). In this time, we assumed that less than 2 coherent/incoherent waves arrive at only 4-element uniform linear array antenna. For spectrum generation, 256-point radix-4 complex FFT was employed [7],
and the FFT with 256 spatial data composed of \( N \) elements of the noise eigenvector and \( (256 - N) \) zeroes interpolates the spectrum fine and smoothly. All computations were performed by fixed-point arithmetic with 12-bit input data from ADCs.

On the other hand, the estimation accuracy of this system depends on so many factors that the proper assessment has some difficulties in detail analysis. For example, the effect of finite bit-length and bit-truncation by scaling in the fixed-point operation, the estimation errors caused by non-uniform discrete wavefront, and so forth. Thus, in order to assess them totally, we would better evaluate the overall accuracy. As [4], the standard deviation of the estimated DOA when the single wave impinging at 0 degree from broadside was one of good overall performance assessment methods of the estimation accuracy. Fig.6 shows the hardware level simulation results, where the squared, diamonded and triangled line at 0, 30 and 60 degrees respectively were processed by UMP, and the circled line was obtained by an offline PC with 64-bit floating-point operation. This measurement included 1000 trials(bursts) data of 32 snapshots. The source wave was a CW signal. In this result, it is clear that the estimation accuracy is below 2 degree if the input SNR is greater than 5 dB in the linear region between -30 and +30 degrees. And it can be also seen that the UMP has a good performance for the offline PC in spite of the compact fixed-point operation.

VI. CONCLUSION

In this paper, the FPGA design of the fast DOA estimator using the unitary MUSIC algorithm was proposed and its real hardware implementation was also introduced. The unique features of this system are the fast and compact computation of the EVD and MUSIC angular spectrum generation with Cyclic Jacobi processor based on CORDIC and spatial DFT, respectively. All DSP functions are computed by only fixed-point operation with finite bit-length in order to meet the requirement of fast processing and low power consumption due to the simplified and optimized architecture.

REFERENCES


