Posted price selling and online auctions

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A R T I C L E   I N F O

Article history:
Received 31 August 2012
Available online 13 January 2015

JEL classification:
C70
D44
D61

Keywords:
Online auctions
Posted price selling
Buy-it-now
Allocative inefficiency
Random matching
eBay auctions

A B S T R A C T

In an auction-style listing at eBay, sellers have the option to set a posted price (also known as buy-it-now price), which allows buyers to instantly purchase an item before the start of the auction. This paper provides a rationale for such a selling mechanism. When many identical items are offered for sale and there are many buyers, random matching between auctions and the bidders can cause allocative inefficiency. We show that, with the buy-it-now option, some high valuation buyers buy the item before the start of the auction. In the case of a single seller with many items for sale, this not only reduces the allocative inefficiency, but also increases the seller’s expected revenue. In the case of many competing sellers, if sellers choose between the strategies of (i) auction only or (ii) auction with buy-it-now option, the option of buy-it-now will be used with positive probability in any equilibrium.

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1. Introduction

Online auctions allow vendors to utilize a combination of posted price selling and auction.1 Pinker et al. (2003) believe that most business transactions take place through one of three mechanisms: (i) a posted price sale, (ii) negotiations or (iii) an auction.2 They argue that the choice of mechanism depends mainly on the size of the associated transaction cost and the level of uncertainty about the correct price.3 At eBay, sellers can use a posted price listing or an auction-style listing.4 In an auction-style listing, a seller has the option to include a “buy-it-now” price – i.e., a posted price, at which an

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1 See Ockenfels et al. (2006) and Krishna (2010) for an excellent literature review. Shunda (2009) presents a model where sellers who augment online auctions with a buy price earn higher revenue.

2 In a very interesting empirical study, Joo et al. (2012) have discussed bidding strategies associated with the selling mechanism of Name Your Own Price (NYOP). They suggest that in pursuit of high quality products, hagglers tend to place many bids. NYOP is mostly used within the context of perishable products. The empirical work of Wood et al. (2005) suggests that online retailers can also increase their revenue by starting with a lower price and running shorter auctions.

3 Popkowski Leszczyc et al. (2009) argue that posted prices (i.e., Buy-Now prices) serve as an external reference point to the bidders. The posted prices can influence the bidder valuation and hence the bidding behavior.

4 At present, five selling mechanisms are available at eBay: Auction-style listings, Fixed price listings, Classified Ads, Motors National listings, Motors Local Market (see http://pages.ebay.com/help/sell/formats.html).

http://dx.doi.org/10.1016/j.geb.2014.11.005
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item can be sold immediately. In the case of “buy-it-now” auction-style listings, the posted price selling ends immediately if anyone places a bid on the item, or before the bids reach an unspecified reserve price. This is equivalent to the option of posted price selling before the start of the auction. In the following, we refer to an auction without “Buy It Now” option as auction only, and an auction with “Buy It Now” option as a buy-it-now auction.

Since sellers can extract more rent from buyers by using auctions, why do sellers at eBay use the option of posted price selling before the auctions start? In this paper, we provide another rationale for the popularity of posted price selling before the start of auctions, as in eBay buy-it-now auctions. We take into account a unique feature of online auction markets: at any given point of time, a number of similar (or identical) items are available for sale at eBay. This creates competition not only among buyers but also among different auctions, which contributes to allocative inefficiency.

Allocative inefficiency has been highlighted by McAfee (1993) and Peters and Severinov (1997) who consider non-online competing auctions that involve many sellers, each with a single unit for sale, and many bidders, each with unit demand. They have shown that the equilibrium involves bidders randomizing over all available sellers. It is therefore possible that some auctions may have many bidders, while other auctions may have only a few or no bidders and hence some profitable trades may not be realized.

Owing to the availability of online search engines, one might think that the problem of random matching at competing auctions will not exist because buyers have the ability to quickly locate and bid at the auctions having few or no bidders. However, in reality, random matching remains a serious problem in online auctions because online auctions run over several days. Continuous monitoring several auctions can be very costly, at least in terms of time. At the time of placing a bid, a prospective buyer cannot predict how many others will subsequently bid against him/her. Bidding on a unit with fewer bids, or bidding multiple times in small increments, as described by Anwar et al. (2006), will not be very effective unless one is bidding at the last minute of the auction. However, if many bidders bid at the last minute of the auction, matching between bidders and units becomes random. Consequently, random matching between buyers and sellers and allocative inefficiency continues to exist in online auctions. In this paper, we show that the option of posted price selling before the start of auctions can (a) reduce the allocative inefficiency and (b) also raise the expected revenue of the sellers.

The argument is that in the case of a buy-it-now auction, a properly set buy-it-now price will attract some high valuation buyers and therefore buy-it-now sale may screen out some high valuation buyers before the start of auctions. Consequently, fewer buyers will participate in auctions, which will reduce the problem of random matching and therefore the extent of allocative inefficiency. We use a very simple model where a number of identical items are offered for sale. There are many buyers but they can be divided into two categories – high and low valuation buyers. In the case of a single seller with many items for sale, we show that, compared to selling that relies only on auctions, posted price selling before the start of auctions (i.e., buy-it-now auctions) not only raises the seller’s revenue but also reduces allocative inefficiency. When identical items are listed for sale by a number of sellers, each of whom has a single unit for sale, if the sellers can implicitly or explicitly collude, the result is the same as for a single seller who offers many units. In a non-cooperative environment involving competing sellers, we show that if sellers can choose between the selling strategies of (i) auction only or (ii) buy-it-now auction, the strategy where all sellers use auction only cannot be an equilibrium strategy. This implies that, in the case of competing sellers, buy-it-now option will always be used with a positive probability in any equilibrium (provided the equilibrium exists).

Thus, we provide a rationale for posted price selling prior to auctions, as in buy-it-now auctions, without explicitly considering a number of other factors such as the effect of risk aversion and buyer impatience. To the best of our knowledge, only a few studies have considered the issue of random matching in online auctions with the option of posted price selling.

The rest of this paper is organized as follows. Section 2 contains a brief review of the related literature. Section 3 contains a simple theoretical model involving a single seller with many units for sale. Section 4 contains a discussion of the case for competing sellers, each with one homogeneous unit for sale. The issue of allocative efficiency associated with the selling mechanism of buy-it-now auctions is discussed in Section 5. The last section contains some concluding remarks.

2. Review of related literature

A number of studies have attempted to provide explanations for the simultaneous use of posted price selling and online auctions. As far as studies that consider the simultaneous use of posted price selling and online auctions are concerned, Budish and Takeyama (2001) consider a single seller and two types of risk-averse buyers and show that the English auction with a buy price can raise the revenue of the seller. By making use of a model in which buyers make their auction-participation decisions using an estimate of their expected discount, Etzion et al. (2006) show that the simultaneous use of

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5 In a very interesting study, based on 700 auctions involving almost identical products, Anderson et al. (2008) found that some sellers converted their auctions into posted price selling by setting the initial bid to equal the “buy-it-now” price. Furthermore, “buy-it-now” option was used frequently by sellers with higher ratings, whereas posted prices selling mostly involved used items.

6 We use “buyer” and “bidder” interchangeably throughout this paper.

7 Roth and Ockenfels (2002) and Ockenfels and Roth (2006), among others, have considered various aspects of the last minute bidding at eBay.

8 The recent work of Haruvy and Popkowski Leszczyc (2010) can also be used to argue that significant random matching occurs during online auctions. Their empirical work suggests that, despite relatively little cost associated with searching, bidders rarely switched between auctions.

9 For a theory of random matching, please refer to Aliprantis et al. (2007).
posted price selling and online auctions leads to a significant increase in sellers' revenue. The buyers in this study take into account the fact that the level of competition is higher in the online auction market. The model has a number of unique features, including the arrival rate of the buyers to the website. This study focuses on buyers' bidding strategies. By taking a buyer's discounting of the expected utility of auctions into account, Sun (2008) shows that the achievement of market segmentation is a rationale for the simultaneous use of posted price selling and online auctions. Sun argues that, in the case of a posted price sale, there is little or no uncertainty regarding the price of the product but the number of units sold is subject to uncertainty. The reverse is true in the case of auctions and hence there is no dominant selling mechanism. Sun further argues that the choice of selling mechanism depends on factors such as the seller's inventory cost and the buyer's discount factor. Sun's analysis is based on near-optimal approximation of the seller's profits.10 Based on data collected on compact disk sales, Hammond (2013) argues that differences across buyers do not explain the simultaneous use of auctions and posted price selling. Hammond argues that the simultaneous use of auction and posted price sale decreases the level of competition among the sellers. Sellers with high value items prefer posted price sale, even though it leads to fewer sales, because the items can be sold at a higher price.

As far as studies that focus on eBay's use of posted price selling before auctions are concerned, Mathews (2004) considers the case of one seller with a single unit and several bidders, focusing on the role of impatience of both bidders and seller. Mathews argues that impatient bidders are willing to pay a premium. As a result, the auction ends earlier and sellers receive higher revenue. Reynolds and Wooders (2009) focus on the consequences of bidder risk aversion. They consider the case of one seller with a single unit and several bidders. In order to avoid the risk associated with auctions, buyers are prepared to pay a premium. As a result, the seller's revenue increases. In other words, the main determinant of the simultaneous use of posted price selling and auctions is the degree of buyer risk aversion. Bauner (2015) uses a structural model to examine the behavior of competing buyers and sellers at eBay auctions. Bauner's work suggests that buyers benefit from eBay's Buy It Now option but sellers are not very attracted to the idea of a hybrid mechanism as it leads to a significant increase in competition between sellers.

Without appealing to risk aversion and impatience, Kirkegaard and Overgaard (2008) justify the option of buy-it-now auctions by considering two sequential sellers, each with a single unit for sale. Realizing that future auctions can depress the revenue generated by today's auction, the early seller introduces posted price selling. This strategy increases the revenue of the first seller but its effect on the revenue of the second seller is negative. Onur and Tomak (2009) used a game theoretic approach to examine the interplay between online bidder behavior pertaining to the strategy of last minute bidding and a posted price sale. They consider a single seller with a single unit for sale and two types of buyers. Onur and Tomak argue that the Buy It Now option, at the beginning of the auction, is a dominant strategy for the seller.

Our paper differs from previous studies in that we take some specific features of eBay into account. For example, in our model, a number of auctions involving similar or identical units take place simultaneously. We consider a single seller with many identical units for sale and there are many buyers with two types of valuations. This allows us to derive some interesting analytical results. In addition, we discuss the case of many competing sellers, each with a single unit for sale.

3. A simple model involving a single seller

Consider an auction market with one seller who offers $N$ identical items for sale to $\theta N$ buyers. The number of buyers can be greater or less than the number of units – i.e., $\theta$ can be greater than or less than 1. Each buyer demands only one unit and $N$ is large.11 The buyers can be divided into two groups: low valuation and high valuation buyers. The seller's valuation of each unit is $m$; whereas $L$ and $H$, respectively, are the valuations of low and high valuation buyers. We assume that $m < L < H$. Furthermore, $\alpha$ and $(1 - \alpha)$, respectively, are the proportions of high and low valuation buyers, where $0 < \alpha < 1$.

The auction site eBay allows sellers to utilize both posted price listings and auction-style listings. In the case of auction-style listings, a seller can also set a reserve price for the auction. The buyers cannot observe the reserve price, but they can see if a listing has a reserve price. If bidding fails to reach the reserve price, the seller is not obligated to sell the listed item.

We assume that the seller can choose (i) posted price listing at a fixed price or (ii) buy-it-now auction-style listing with a buy-it-now price $P$ and a reserve price $r$ for auction. When $P > r$, the selling mechanism of auction only can be viewed as a special case of buy-it-now auctions. We assume that buyers cannot observe the reserve price. As the items that are offered for sale are identical, we assume that the seller selects an identical $(P, r)$ for each unit at buy-it-now auctions.

In the case of buy-it-now auctions, the buyers decide whether or not to buy at price $P$ in stage 1. Some units may be sold in stage 1. In stage 2, each of the unsold items is simultaneously offered for sale at a separate auction. All auctions take the form of a second price auction and the buyers participating in stage 2 bid their true valuations.

Using the Internet search engines, buyers can locate all units that are offered for sale in stage 1. If more than one buyer attempts to buy the same unit, then the unsuccessful buyer can easily switch to another unsold unit. If the number of buyers in stage 1 is more than the number of units, all units will be sold but the units are randomly assigned to the buyers. In

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10 Wang et al. (2008), among others, have argued that posted price selling increases seller's profit. However, none of the existing studies focused on the issue of allocative inefficiency arising at online auctions.

11 We assume a large $N$ to simplify our calculations in the following. The results presented in this paper also hold without this assumption.
stage 2, due to the search and monitoring costs associated with auctions, the unsuccessful buyer cannot immediately switch to another auction. The widely used strategy of bidding at the last minute (Bajari and Hortacsu, 2004 provide evidence of such behavior) also results in random matching between the buyers and the units. Therefore, it is reasonable to assume that, in stage 1, buyers can switch without any cost until they get a unit or until no unit is available. However, in stage 2, buyers are randomly matched to auctions.

A buy-it-now auction can be viewed as a strategic game where, in order to maximize his/her expected revenue, the seller chooses a buy-it-now price $P$ for stage 1 and a reserve price $r$ for stage 2. We define a seller’s revenue from a unit as the payment he/she receives minus his/her valuation $m$. Buyers cannot observe the auction reserve prices. After observing the buy-it-now price, buyers decide whether to bid only in stage 2 or also try to buy in stage 1. Buyers who fail to buy in stage 1 can bid in stage 2. In the following, we refer to these choices as (i) buying through buy-it-now sale or (ii) buying only through auction in stage 2. The payoff of a buyer from bidding in stage 2 depends on the number of buyers who participate in auctions and the number of units that are available for auction.

While the reserve price is not known to the buyers, the choice of the reserve price can have several effects. A unit remains unsold if bidding on it fails to reach the reserve price. The choice of the reserve price also determines the selling price if there is only one bid on the unit. As we consider the case of second price auctions, a unit is sold at the reserve price if it receives only one bid that is higher than the reserve price.

As the buyers cannot observe the reserve price, their choices cannot depend explicitly on the reserve price. However, the reserve price affects the seller’s expected revenue. We now consider the best choice of the reserve price $r$ for the seller. As all auctions in the stage 2 run independently, for any given numbers of high and low valuation bidders, we first consider the optimal choice of the reserve price for a seller with a single unit for sale. The optimal choice can be stated by means of a lemma as follows:

**Lemma 1.** Irrespective of the number of high or low valuation bidders, the optimal reserve price for a seller, with a single unit for sale through auction only, is either $L$ or $H$.

In order to prove Lemma 1, we only need to consider how an item will be sold at a given reserve price, with all possible combinations of high and low valuation buyers bidding on it. It is clear that a unit with a reserve price $r > H$ cannot be sold. A unit with a reserve price $L < r \leq H$ will be (i) sold at price $H$, if at least two high valuation bidders bid on it; (ii) sold at price $r$ if there is only one high valuation bidder; (iii) not sold if it has no high valuation bidder. A unit with reserve price $r \leq L$ will be (i) sold at price $H$ if it has at least two high valuation bidders; (ii) sold at price $L$ if it has only one high valuation and at least one low valuation bidder, or it has at least two low valuation bidders; (iii) sold at price $r$, if it has only one high valuation bidder, or it has only one low valuation bidder; (iv) unsold if it has no bidders.

Therefore, if $r < L$, irrespective of the choices of the buyers, the seller’s revenue will be higher if he/she raises the reserve price to $L$. If $r > L$, the seller’s revenue will be higher if he/she raises the reserve price to $H$. Accordingly, the seller’s optimal choice of the reserve price is either $r = L$ or $r = H$.

In the case of a seller with multiple units for sale, more than one unit may be available for auction in stage 2 as independent auctions. As the units are identical, we assume that the seller will use a symmetric selling strategy for all units and hence the optimal reserve price would also be either $L$ or $H$.

Realizing the seller’s possible choices for the reserve price and after observing the buy-it-now price $P$ set by the seller, the buyers select their strategies, concerning whether to buy in stage 1. Let’s consider the choice of the buyers if $P \leq L$. If $P \leq L$, all low valuation buyers would try to buy through buy-it-now in stage 1. This follows from the fact that their payoff from bidding in stage 2 will be zero but, when $P < L$, buying through buy-it-now in stage 1 will result in a strictly positive payoff. In the case of $P = L$, the payoff of low valuation buyers will be zero irrespective of whether or not they buy in stage 1. We assume an indiff erence rule in that if buyers are indifferent between buy-it-now and bidding at stage 2 then they choose buy-it-now. Therefore, if $P \leq L$, all low valuation buyers will attempt to buy in stage 1. As the reserve price can only be either $L$ or $H$, the payoff of a high valuation buyer from bidding in stage 2 will be either $0$ or $H - L$. The expected payoff of a high valuation buyer from buying through buy-it-now in stage 1, when $P \leq L$, is strictly higher than buying in stage 2. This implies that, when $P \leq L$, all buyers will attempt to buy using buy-it-now in stage 1. Thus, in stage 1, either all units will be sold (if $\theta \geq 1$) or each buyer will be able to buy one unit (if $\theta < 1$). Accordingly, a buy-it-now auction with $P \leq L$ is equivalent to the strategy of posted price sale (without auction) at price $P$. It is clear that out of all posted price sales with $P \leq L$, posted price sale at $P = L$ is the optimal as it maximizes the seller’s revenue. Therefore, irrespective of the reserve price (i.e., $L$ or $H$), the strategy of buy-it-now auction with $P \leq L$ is dominated by the strategy of posted price sale only at $P = L$.

In the case of a buy-it-now auction with $P > L$, low valuation buyers cannot buy in stage 1. We have already established that the auction reserve price can only be either $L$ or $H$. In the case of $P > L$ and $r = H$, low valuation buyers would not be able to buy any unit and hence all units will be sold to high valuation buyers. Compared to the strategy of buy-it-now auction with $P > L$ and $r = H$, if the seller uses the strategy of posted price sale only at price $H$, the seller’s revenue will be higher.

In the case of $P > L$ and $r = L$, all low valuation buyers will participate only in bidding in stage 2. Some high valuation buyers may choose to buy through buy-it-now in stage 1, whereas others will bid in stage 2. These cases are examined in detail in Sections 3.1 and 3.2.
For all strategies that involve posted price sale only, the optimal posted price for a seller is either \( P = L \) or \( P = H \). The seller’s revenue when \( P < L \) is less than the revenue for \( P = L \), because all units sold at \( P < L \) can also be sold at \( P = L \). The seller’s revenue for any price such that \( P > L \) is no more than the revenue when \( P = H \). This follows from the fact that, in the case of \( P > L \), the units are sold only to high valuation buyers and the seller can maximize his/her revenue by setting \( P = H \). Accordingly, when the seller can choose between a posted price listing and an auction-style listing, the seller’s optimal pure strategy involves either (i) posted price sale at price \( L \) without auction, or (ii) posted price sale only at price \( H \) without auction, or (iii) buy-it-now auction with posted price \( P = P^* > L \) and reserve price \( r = L \), where \( P^* \) is the optimal buy-it-now price when \( r = L \) and \( P > L \).

In the following, we first consider the choices of the buyers and the seller when the selling mechanism of buy-it-now auction with \( r = L \) and \( P > L \) is used. We then compare this strategy with strategies (i) and (ii) as specified in the above.

3.1. Buyers’ choice when the seller uses buy-it-now auctions with \( r = L \) and \( P > L \)

As indicated earlier, buyers cannot observe the reserve price, but rational buyers know that the reserve price selected by the seller can only be \( L \) or \( H \), and the seller’s optimal choice can only be (i), or (ii), or (iii) as described in the previous section. Therefore, in any equilibrium, buyers can assume that the reserve price at a buy-it-now auction is \( L \). When \( P > L \) and \( r = L \), all low valuation buyers have no choice but to bid in stage 2. On the other hand, high valuation buyers have two choices: try to buy instantly at buy-it-now price in stage 1 or participate only in bidding in stage 2. Suppose that \( s \) is the proportion of high valuation buyers who participate only in bidding, whereas \( 1 - s \) is the proportion of high valuation buyers who participate in buy-it-now sale in stage 1. Another way of describing this situation is to state that high valuation buyers use a mixed strategy: with probability \( 1 - s \), they choose to buy in stage 1 and with probability \( s \), they choose to only participate in bidding in stage 2.

It is clear that when \( \alpha \theta \geq 1 \) (i.e., the number of high valuation buyers is more than the number of units), the best strategy for the seller is to use posted price sale at price \( H \) without auctions. Accordingly, when examining the strategy of buy-it-now auction with \( P > L \) and \( r = L \), we assume that \( \alpha \theta < 1 \). In such a case, any high valuation buyer who wants to buy in stage 1 can buy a unit with probability one. Which of the available units will be sold in stage 1 is randomly determined and the probability that a unit will be sold in stage 1 is \( \alpha \theta (1 - s) \).

The proportion \( s \) of high valuation buyers who choose to bid only in stage 2 is determined by the buy-it-now price \( P \). Given the high valuation buyers’ choices, the seller chooses \( P \) to maximize his/her expected revenue.

The choice of a high valuation buyer is determined by comparing the payoff from buying through buy-it-now sale in stage 1 with the payoff from buying only through bidding in stage 2. For a given buy-it-now price, a high valuation buyer’s payoff from buying in stage 1 is \( H - P \). If a high valuation buyer chooses to buy only through bidding, he/she will be randomly matched to one of the remaining units in stage 2. The expected payoff of a high valuation buyer depends on the strategy used by all other high valuation buyers.

At the end of stage 1, \( [1 - \theta \alpha (1 - s)]N \) units will be available for bidding at auctions in stage 2. The \( \theta \alpha s N \) high valuation buyers and all low valuation buyers will participate in bidding. Each bidder can only be matched to one unit and can only bid his/her true valuation once. With random matching, the probability that a high valuation bidder will place a bid on a unit is

\[
\frac{1}{[1 - \alpha \theta (1 - s)]N}
\]

Since there are \( \theta \alpha s N \) high valuation buyers, for each unit, the number of high valuation bidders who place a bid on it can be described by the binomial distribution as follows:

\[
B\left( \theta \alpha s N, \frac{1}{[1 - \theta \alpha (1 - s)]N} \right)
\]

Similarly, for each unit, the number of low valuation bidders who place a bid on it can be described by the binomial distribution as follows:

\[
B\left( \theta (1 - \alpha) N, \frac{1}{[1 - \theta \alpha (1 - s)]N} \right)
\]

When \( N \) is large, the above binomial distributions can be approximated by Poisson distributions \( Q(\lambda_h) \) and \( Q(\lambda_l) \) where:\[12\]

\[
\lambda_h = \frac{\theta \alpha s}{1 - \theta \alpha (1 - s)}, \quad \lambda_l = \frac{\theta (1 - \alpha)}{1 - \theta \alpha (1 - s)}
\]  

Each high valuation buyer who chooses to buy only in stage 2 faces two possibilities:

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12 Kuliti (1999), among others, uses a similar approximation.
(i) With probability \(e^{-\lambda h}\), there are no other high valuation bidders competing against him/her for the unit on which he/she bids. With a reserve price \(L\) for the auction, whether or not there are low valuation bidders competing against him/her, the payoff will be \(H - L\); and

(ii) With probability \(1 - e^{-\lambda h}\), there are other high valuation bidders competing against him/her and hence the payoff will be 0.

The expected payoff of a high valuation buyer who chooses to buy only through bidding is \((H - L)e^{-\lambda h}\), which is a decreasing function of \(s\). The lowest possible expected payoff is achieved when all other high valuation buyers also buy through auctions (i.e., when \(s = 1\) and the expected payoff is then \((H - L)e^{-\lambda h(1)} = (H - L)e^{-\theta a}\).

Similarly, the highest possible expected payoff is achieved when \(s = 0\) (i.e., when no other high valuation buyers buy only through auction). In such a case, the expected payoff is \((H - L)e^{-\lambda h(0)} = H - L\).

If \(P > H - (H - L)e^{-\theta a}\), buying only through bidding in stage 2 always leads to a higher payoff for a high valuation buyer compared to buying in stage 1. In such a case, no high valuation buyer will buy in stage 1. The selling mechanism becomes auction-only.

If high valuation buyers use a mixed strategy, their expected payoff from the two alternatives must be equal, which leads to Eq. (2) as follows:

\[
(H - L)e^{-\lambda h} = H - P
\]

Since \((H - L)e^{-\lambda h}\) is a strictly decreasing function of \(s\), Eq. (2) determines a one-to-one correspondence between \(P\) and \(s\), where \(P \in (L, H - (H - L)e^{-\theta a})\) and \(s \in (0, 1)\). As \(P\) decreases, more high valuation buyers will choose to buy through buy-it-now in stage 1. The seller’s optimal choice of \(P\) is discussed in the next subsection.

### 3.2. Seller’s optimal choice of \(P\) for buy-it-now auction with \(P > L\) and \(r = L\)

Anticipating the buyers’ response, the seller chooses \(P\) such that his/her expected revenue is maximized. If \(P > H - (H - L)e^{-\theta a}\), all buyers will buy in stage 2 (i.e., \(s = 1\)). For a buy-it-now price \(P \in (L, H - (H - L)e^{-\theta a})\), a proportion \((1 - s)\) of high valuation buyers will purchase in stage 1, where \(s\) is determined by Eq. (2). Since \(N\) units are available and only \(\theta a(1 - s)N\) high valuation buyers participate in stage 1, the probability that a unit will be sold in stage 1 at price \(P\) is \(\theta a(1 - s)\). Accordingly, the probability that a unit will be offered for bidding at auctions in stage 2 is \(1 - \theta a(1 - s)\).

The revenue of the seller when a unit is sold in stage 1 is \(P - m\). Taking into account Eq. (2), the revenue can be written as

\[
R_1 = P - m = H - (H - L)e^{-\lambda h} - m
\]

However, when a unit is offered for auction in stage 2, there are three possibilities:

(i) With probability \(1 - e^{-\lambda h} - e^{-\lambda h}\lambda h\), there is more than one high valuation bidder. The unit is sold at price \(H\) and hence the seller’s revenue from the sale of the unit is \(H - m\).

(ii) With probability \(e^{-\lambda h}e^{-\lambda h}\) there are no bidders. The unit is not sold and hence the revenue is zero.

(iii) For all other possibilities, with probability \(e^{-\lambda h} + e^{-\lambda h}\lambda h - e^{-\lambda h}e^{-\lambda h}\), the unit is sold at price \(L\) and hence the revenue of the seller is \(L - m\).

Based on these three possibilities, the expected revenue of the seller from a unit offered for auction in stage 2 is as follows:

\[
R_2 = (1 - e^{-\lambda h} - e^{-\lambda h}\lambda h)(H - m) + (e^{-\lambda h} + e^{-\lambda h}\lambda h - e^{-\lambda h}e^{-\lambda h})(L - m)
= (H - m) - (H - L)e^{-\lambda h} - (H - L)e^{-\lambda h}\lambda h - (L - m)e^{-\lambda h-\lambda t}
\]

Using the payoff from a unit that is sold in stage 1 or offered for sale in stage 2 and the associated probabilities, the expected revenue \(R\) of the seller from a unit is \(R = \theta a(1 - s)R_1 + [1 - \theta a(1 - s)]R_2\), which can be written as follows:

\[
R = \theta a(1 - s)[H - m - e^{-\lambda h}(H - L)] + [1 - \theta a(1 - s)][(H - m) - e^{-\lambda h}(H - L) - e^{-\lambda h}\lambda h(H - L) - e^{-\lambda h-\lambda t}(L - m)]
= (H - m) - (1 + \theta a)e^{-\lambda h}(H - L) - (1 - \theta a + \theta a s)e^{-\lambda h-\lambda t}(L - m)
\]

The presence of a one-to-one correspondence between \(P\) and \(s\) allows us to express the seller’s expected revenue in terms of \(s\). The one-to-one correspondence between \(s\) and \(P\) implies that the seller’s choice of the buy-it-now price determines the value of \(s\) (i.e., the proportion of high valuation buyers who choose to buy only in stage 2). When \(s = 1\), no unit is sold in stage 1. This is equivalent to a selling mechanism that relies solely on auction. By making use of this information and Eq. (5), the following observation can be made. At \(s = 1\), the seller’s expected revenue increases as \(s\) decreases. This can be seen by differentiating Eq. (5) with respect to \(s\) and evaluating the derivative at \(s = 1\) as follows:
\[
\frac{\partial R}{\partial s}\bigg|_{s=1} = -\theta^3 \alpha^3 e^{-\lambda_h} (H - L) - \theta^2 \alpha e^{-\lambda_h} - \lambda_t (L - m) < 0
\] 

Eq. (6) shows that starting from a situation where the buy-it-now price is set too high such that no one is able to buy in stage 1 and hence the sale relies only on auction, the seller’s expected revenue increases as the buy-it-now price is lowered. Lowering the buy-it-now price induces some high valuation buyers to buy in stage 1. Based on Eq. (6), we have the following proposition.

**Proposition 1.** Compared to the selling mechanism of auction only (with reserve price \( r = L \)), the seller’s expected revenue will be higher from buy-it-now auctions (with buy-it-now price \( P > L \) and reserve price \( r = L \)).

It should be noted that Proposition 1 holds only for reserve price \( r = L \). However, from Lemma 1, if the seller relies only on auctions (without the option of buy-it-now), the optimal reserve price is either \( L \) or \( H \). If the seller selects reserve price \( H \) for all units (we assumed earlier that the seller uses the same selling mechanism for all units), the seller can be better off by using the strategy of posted price sale at price \( H \) without auctions.

Using Eq. (5), the optimal value of \( s \) can be determined, which due to one-to-one correspondence between \( s \) and \( P \) determines the optimal (i.e., expected revenue maximizing) value of \( P \). It has to be kept in mind that it is possible to present the expression for expected revenue (i.e., Eq. (5)) as a function of \( P \) and determine the profit maximizing value of \( P \) and then work out the optimal value of \( s \). The first order condition that maximizes the expected revenue is as follows, where \( \lambda'_h \) and \( \lambda'_t \) are the first order derivatives of the relevant variables with respect to \( s \).

\[
\frac{\partial R}{\partial s} = \left[ -\theta \alpha + (1 + \theta \alpha s) \lambda'_h \right] e^{-\lambda_h} (H - L) + \left[ -\theta \alpha + (1 - \theta \alpha + \theta \alpha s) (\lambda'_h + \lambda'_t) \right] e^{-\lambda_h} - \lambda_t (L - m) = 0
\]

Eq. (7) can be used to determine the expected revenue maximizing value of \( s \). However, an important question to consider is whether or not the optimum exists. Since the revenue function \( R \) is a smooth function of \( s \) over interval (0, 1), the existence of equilibrium can be established by evaluating the value of Eq. (7) at \( s = 0 \) and \( s = 1 \).

\[
\frac{\partial R}{\partial s}\bigg|_{s=0} = \left[ -\theta \alpha + \lambda'_h(0) \right] e^{-\lambda_h} (H - L) + \left[ -\theta \alpha + (1 - \theta \alpha) (\lambda'_h(0) + \lambda'_t(0)) \right] e^{-\lambda_h} - \lambda_t (L - m)
\]

As \( 0 < \theta \alpha < 1 \), we have

\[-\theta \alpha + \lambda'_h(0) = \frac{\theta^2 \alpha^2}{1 - \theta \alpha} > 0 \quad \text{and} \quad -\theta \alpha + (1 - \theta \alpha) (\lambda'_h(0) + \lambda'_t(0)) = \frac{\theta \alpha}{1 - \theta \alpha} > 0\]

Therefore, the first order derivative of the expected revenue with respect to \( s \) is positive at \( s = 0 \). We already showed in Eq. (6) above that this derivative is negative at \( s = 1 \). Therefore, there exists at least one value \( s^* \) in the interval (0, 1) such that the expected revenue for the seller (i.e., \( R \)) reaches its maximum. We denote such revenue maximizing value(s) of \( P \) by \( P^* \). However, the revenue maximizing \( P^* \) is not necessarily unique. In the following, \( P^* \) can be any such revenue maximizing buy-it-now price. This leads to Proposition 2 as follows.

**Proposition 2.** Among all possible choices of buy-it-now auctions such that \( P > L \) and \( r = L \), there exists an optimal buy-it-now price \( P = P^* \) which maximizes the seller’s revenue. At the optimum, \( L < P^* < H \) and at least some high valuation buyers buy through buy-it-now at price \( P^* \) in stage 1.

It is worth noting that, irrespective of whether \( \theta < 1 \) or \( \theta > 1 \), Propositions 1 and 2 hold as long as \( \alpha \theta < 1 \). Some numerical examples can be used to illustrate the above propositions. Suppose that \( H = 10, L = 6, m = 0, \theta = 2 \). When \( \alpha = 0.3 \), among all possible buy-it-now auctions with \( P > L \) and \( r = L \), the optimal buy-it-now price \( P^* = 7.088 \) and 69% of the high valuation buyers choose to buy through buy-it-now. The seller’s expected revenue from each unit sold is 6.311. When the seller relies only on auctions without buy-it-now option, his/her revenue from each unit is 5.676, which is 11.2% lower compared to the revenue from the selling mechanism of buy-it-now auctions.

As the proportion of high valuation buyers decreases to \( \alpha = 0.2 \), the optimal buy-it-now price \( P^* \) decreases to 6.498 and 77% of the high valuation buyers buy through buy-it-now. The seller’s expected revenue from each unit is 5.816. When the seller relies only on auctions without buy-it-now option, the expected revenue from each unit is 5.434. As the proportion of high valuation buyers (i.e. \( \alpha \)) increases to 0.4, the optimal buy-it-now price increases to 7.682, and 71% of the high valuation buyers choose to buy through buy-it-now. The seller’s expected revenue from each unit is 7.026. However, when the seller relies exclusively on auctions, the expected revenue from each unit decreases to 5.953.

### 3.3. The optimal strategy for a single seller

From the discussion at the beginning of Section 3, the seller’s optimal choice can be one of the following three strategies: posted price sale at price \( L \) without auction, posted price sale at price \( H \) without auction, and buy-it-now auction with \( P = P^* > L \) and \( r = L \).
Which of the above three strategies is optimal depends on the parameters $H$, $L$, $m$, $\theta$ and $\alpha$. As indicated earlier, if $\theta \alpha \geq 1$ such that the number of high valuation buyers is more than the number of units, posted price sale at price $H$ is optimal.

In the case of $\theta \alpha < 1$ so that the number of high valuation buyers is less than the number of units, we consider the possibilities of $\theta < 1$ (i.e., the number of units is more than the number of buyers) and $\theta > 1$ (i.e., the number of units is less than the number of buyers). If $\theta < 1$ such that the number of units available for sale is more than the number of buyers, for posted price sale at price $L$, only $\theta N$ units will be sold at price $L$ and the expected revenue per unit is $\theta(L - m)$. If the seller uses the strategy of posted price sale at price $H$, only $\alpha \theta N$ units will be sold and the expected revenue per unit is $\alpha \theta (H - m)$. On the other hand, if the seller uses the strategy of buy-it-now auction at price $P^*$ and $r = L$, the expected revenue per unit sold as shown in Section 3.2 is $R(P^*)$. When the number of buyers is more than the number of units (i.e., $\theta > 1$) and the seller uses the strategy of posted price sale at price $L$, all units will be sold at posted price $L$. In this situation, the seller's revenue per unit is $L - m$. If the seller uses the strategy of posted price sale at price $H$, only $\alpha \theta N$ units will be sold and the seller's expected revenue per unit is $\theta \alpha (H - m)$. Finally, if the seller uses the strategy of buy-it-now auction, the expected revenue per unit remains $R(P^*)$, where $P^*$ is the optimal buy-it-now price.

The seller's expected revenue for the three strategies when $\theta \alpha < 1$ and $\theta \geq 1$ can be summarized in Table 1.

<table>
<thead>
<tr>
<th>Seller's strategy</th>
<th>Seller's expected revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posted price sale at price $L$ without auction</td>
<td>$L - m$</td>
</tr>
<tr>
<td>Posted price sale at price $H$ without auction</td>
<td>$\theta \alpha (H - m)$</td>
</tr>
<tr>
<td>Buy-it-now auction at price $P = P^* &gt; L$ and $r = L$</td>
<td>$R(P^*)$</td>
</tr>
</tbody>
</table>

While it is easy to compare $L - m$ and $\theta \alpha (H - m)$, there is no explicit formula that allows one to compare the seller's revenue in all three cases. We can provide an intuitive comparison. If $H$ and $L$ are very close to each other (and both are much higher than $m$) and the proportion of high valuation buyers is high, then it is possible that the strategy of posted price sale at price $L$ is the seller's best strategy. If $L$ and $m$ are close to each other but are much smaller than $H$ and the proportion of high valuation buyers is higher, it is possible that the strategy of posted price sale at price $H$ is the seller's best strategy. In most of the cases in-between, a buy-it-now auction with $P^* > L$ and reserve price $r = L$ is the seller's best strategy.

Consider the numerical examples in Section 3.2 with parameters $H = 10$, $L = 6$, $m = 0$, $\theta = 2$. The expected revenue from the strategy of posted price sale at price $L$ is always $6$. However, the expected revenue from the strategy of posted price sale at $H$ is $\theta \alpha (H - m) = 20 \alpha$. When $\alpha = 0.2$, 0.3 and 0.4, respectively, the seller's expected revenue per unit is 4, 6 and 8. In the numerical example in Section 3.2, we showed that when $\alpha = 0.2$, 0.3 and 0.4, respectively, the expected revenue of seller from the optimal strategy of buy-it-now auction is 5.816, 6.311 and 7.026. Therefore, when $\alpha = 0.2$ (i.e., the proportion of low valuation buyers is relatively high), the optimal strategy is to use posted price sale at price $L = 6$. When $\alpha = 0.4$ (i.e., the proportion of high valuation buyers is relatively high), the optimal strategy is to use posted price sale at price $H = 10$. When $\alpha = 0.3$, the optimal strategy is to use the buy-it-now auctions with $P^* = 7.088$ and $r = 6$.

4. The optimal strategy of $N$ competing sellers each with one identical unit for sale

The analysis presented above is based on the assumption that there is a single seller with $N$ identical units for sale. In this section, we consider the case of $N$ sellers, each with a single homogeneous unit for sale. As the units are identical, we assume that $m$ is the valuation of each seller. We also maintain our assumption that $m < L < H$ i.e., the valuation of the sellers is less than the valuation of the buyers.

If the sellers can collude (implicitly or explicitly), all of the above analysis will still be applicable. However, in a non-cooperative environment, the sellers are likely to compete against each other. As the competing sellers cannot observe each other’s auction reserve prices, the sellers cannot compete by setting different reserve prices. However, in the case of posted price selling, sellers can compete against each other by setting different posted prices. Furthermore, in the case of buy-it-now auctions, sellers can compete by setting different buy-it-now prices. The equilibrium involving $N$ competing sellers depends on the value of parameters $H$, $L$, $m$, $\theta$ and $\alpha$.

For $N$ competing sellers, we can identify some strategies that are Nash equilibria in some special cases. For example, when the number of high valuation buyers is more than the number of sellers (i.e., $\theta \alpha \geq 1$), posted price sale at price $H$ is a pure strategy Nash equilibrium for $N$ competing sellers. It is interesting to note that posted price sale at price $H$ is also the best strategy for a single seller with $N$ units for sale. If all competing sellers use such a strategy, all units will be sold to high valuation buyers at price $H$ and there is no profitable deviation for any seller. However, when $\theta \alpha < 1$ and $0 < \alpha < 1$, the strategy of using posted price sale at $P = H$ is no longer a Nash equilibrium. If all sellers use this strategy,

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13 In a related study, Peters and Severinov (2006) examine the properties of a perfect Bayesian equilibrium involving a large number of sellers each with a single homogenous unit for sale. They show that, depending of the reserve price, the equilibrium can involve a uniform price. However, they do not consider posted price selling.
the expected revenue of each seller is $\theta \alpha (H - m)$, where $\theta \alpha$ is the probability that a unit can be sold. However, if a seller deviates by using the strategy of buy-it-now auction with a buy-it-now price $H - \varepsilon$ (or the seller simply lowers the price to $H - \varepsilon$ without the option of auction, where $\varepsilon$ is very small), his/her unit will be sold for sure. Because $\theta \alpha < 1$, the expected revenue of the deviating seller will increase from $\theta \alpha (H - m)$ to $H - m - \varepsilon$. Hence, the seller will be strictly better off by deviating from posted price sale at $P = H$.

Similarly, by considering possible profitable deviation, we can prove that the optimal strategy of buy-it-now auction with $L < P^* < H$ and $r = L$ for a single seller in Proposition 2 is not a symmetric pure strategy Nash equilibrium for competing sellers.

It is difficult to identify the equilibrium for all possible parameter values. Even if we restrict the sellers’ strategies to posted price sale only, the Betrand-style price competition can be complicated for different parameters of $\theta$ and $\alpha$. A seller may use complicated mixed strategies that involve posted price selling, auction only, or buy-it-now auction with an appropriate buy-it-now price. As this paper focuses on the effect of the option of buy-it-now auction, we now assume that a seller can only choose between the strategy of auction only or auction with buy-it-now option. In the following, we prove that for competing sellers, the selling mechanism where all sellers use auction only cannot be an equilibrium strategy. Therefore, in any competitive equilibrium (provided the equilibrium exists), the option of buy-it-now with $P < H$ will be used with positive probability and some high valuation buyers will choose to buy through buy-it-now.

**Proposition 3.** For competing sellers, if sellers could choose between the strategies of (i) auction only or (ii) buy-it-now auction, the situation where all sellers use the selling mechanism of auction only cannot be the equilibrium strategy.

**Proof.** We prove Proposition 3 by contradiction. Specifically, we show that, if all sellers use the strategy of auction only, a seller can be better off by switching to the strategy of buy-it-now auction.

We first explore the sellers’ choice if they choose auction-only. If a seller chooses auction-only, he/she only needs to set a reserve price. As the buyers and competing sellers cannot observe the reserve price selected by a seller, the reserve price set by one seller cannot influence the behavior of the buyers and other sellers. Although the reserve price cannot be observed by the buyers and other sellers, a seller’s revenue depends on his/her choice of the reserve price. Consequently, each seller must choose his/her optimal reserve price.

From Lemma 1, we know that each seller will either set $r = L$ or $r = H$ for his/her auction. In the following, we attempt to compare the sellers’ revenue for these two choices of the reserve price.

When the selling strategy of auction only is used, buyers are randomly matched to units, as in the single seller case in Section 3 when $s = 1$. In this case, the number of high valuation and low valuation buyers, respectively, bidding on a unit follows a Poisson distribution with mean $\lambda_h = \theta \alpha$ and $\lambda_l = \theta (1 - \alpha)$.

When a seller chooses the reserve price $r = H$ and if at least one high valuation buyer bids on the item, the seller’s revenue $R_H$ is $H - m$ and zero otherwise. Accordingly, the expected revenue of the seller is

$$R_H = (1 - e^{-\lambda_h})(H - m)$$

If a seller sets his/her reserve price $r = L$, the expected revenue of the seller $R_L$ is the same as revenue $R_2$ in Eq. (4) with $r = L$ and $s = 1$ as follows:

$$R_L = R_2 = (H - m) - (H - L)e^{-\lambda_h} - (H - L)e^{-\lambda_h}\lambda_h - (L - m)e^{-\lambda_h} - \lambda_l$$  \hspace{1cm} (9)

We can see that it is possible that $R_H \geq R_L$ for some parameter values but $R_L \geq R_H$ for other parameter values; $H, L, m, \theta$ and $\alpha$ are the relevant parameters. We also notice that if the reserve price $r = L$ (or $r = H$) is the best choice for a seller, it is also the best choice of all the other sellers. Thus, if all sellers rely on auction only, the equilibrium can only be either: (i) all sellers choose the reserve price $H$ or (ii) all sellers choose the reserve price $L$.

When all sellers set reserve price $r = H$ at auctions, low valuation buyers would not be able to buy any unit but high valuation buyers would be able to buy a unit at price $H$. Accordingly, the expected payoff of all buyers is zero. However, if a seller switches to the strategy of buy-it-now auction with a buy-it-now price $P = H - \varepsilon$ and reserve price $H$, where $\varepsilon$ is small such that $H - \varepsilon - m > R_H = (1 - e^{-\lambda_h})(H - m)$, the unit offered for sale by this seller will be sold to a high valuation buyer for sure. This follows from the fact that at this slightly lower buy-it-now price, the payoff of a high valuation buyer is strictly positive. Furthermore, the deviating seller can earn higher revenue by switching to the above buy-it-now auction. Therefore, the selling strategy of auctions only with reserve price $r = H$ cannot be an equilibrium for competing sellers.

When all sellers set reserve price $r = L$ at auctions, a seller’s expected revenue is $R_L = R_2$, as in Eq. (9). In this case, the payoff of a low valuation buyer is zero. However, the payoff of a high valuation buyer is either zero, or $H - L$ (when no other high valuation bidders compete against him/her). The probability that no other high valuation bidders competing

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14 In fact, the strategy of auction only can be considered as a special case of buy-it-now auction with $P > H$.

15 We are grateful to a reviewer for his/her suggestions regarding this proposition and its proof.

16 If the sellers are indifferent between $r = H$ and $r = L$ (and so might use a mixed strategy), we assume that the sellers will choose only one of the two strategies.
against him/her is $e^{-\lambda s}$. A high valuation buyer’s expected payoff is $\theta > 1$, $\alpha < 1$ and $0 < \alpha < 1$.

For strategies that could be the seller’s optimal choice for different parameter values, the strategy of posted price selling at $P = H$ guarantees that only high valuation buyers are able to buy the available units. However, the drawback of this strategy is that some units cannot be sold. The strategy of posted price sale at $P = L$ guarantees that the maximum number of units will be sold (i.e., all units will be sold if $\theta > 1$). These two selling mechanisms eliminate the random matching arising from competing auctions. The strategy of buy-it-now auctions with an appropriate $P^* > L$ and $r = L$ allows some high valuation buyers to buy through buy-it-now before the remaining buyers bid at auctions.

To study the allocative efficiency with or without the option of buy-it-now, we consider the likelihood of a unit being allocated to a high valuation buyer and the likelihood of a unit being unsold.

Without the option of buy-it-now, a unit is allocated to a high valuation buyer only if at least one high valuation buyer bids on it. Given that there are $\alpha N$ high valuation buyers and $N$ units, the number of high valuation buyers matched to a unit can be approximated by a Poisson distribution. The probability that a unit has no high valuation bidder is $e^{-\alpha}$. Accordingly, the probability that a unit is allocated to a high valuation buyer is $Q(s) = 1 - e^{-\alpha s}$.

In the case of buy-it-now auctions with $P > L$ and $r = L$, either all units are sold in stage 1, or some units are available for sale in stage 2. In the first case, all units are allocated to high valuation buyers and hence we only need to consider the second case where $1 - \alpha \theta (1 - s) > 0$. Since a proportion $s$ of the high valuation buyers choose to buy only through auction in stage 2, a unit is allocated to a high valuation buyer if:

(i) It is sold through buy-it-now with probability $\alpha \theta (1 - s)$ or
(ii) It is sold through auction and at least one high valuation buyer bids on it [the probability of this event is $(1 - \alpha \theta (1 - s))(1 - e^{-\lambda s})]$.

Therefore, the probability that a unit is allocated to a high valuation buyer can be written as follows:

$$Q(s) = \theta \alpha (1 - s) + (1 - \theta \alpha (1 - s))(1 - e^{-\lambda s})$$

(11)

The first order derivative of $Q$ with respect to $s$ is as follows:

$$\frac{\partial Q}{\partial s} = -\theta \alpha + \theta \alpha (1 - e^{-\lambda s}) + (1 - \theta \alpha (1 - s))e^{-\lambda s} \lambda_h$$

$$= -\theta \alpha e^{-\lambda s} + (1 - \theta \alpha (1 - s))e^{-\lambda s} \left[ \frac{\theta \alpha (1 - \theta \alpha)}{(1 - \theta \alpha (1 - s))^2} \right]$$

$$= \theta \alpha e^{-\lambda s} \left[ -1 + \frac{1 - \theta \alpha}{1 - \theta \alpha (1 - s)} \right] = -\theta \alpha \lambda_h e^{-\lambda s} < 0$$

(12)
Eq. (12) shows that there is a negative relationship between the proportion of high valuation buyers who choose to buy only through bidding in stage 2 and the probability that a unit will be sold to a high valuation buyer. This implies that an increase in the proportion of high valuation buyers who buy through buy-it-now in stage 1 leads to an increase in the probability that a unit is allocated to a high valuation buyer.

Similarly, without the option of buy-it-now, a unit is unsold only if no buyer bids on it. The probability that a unit is unsold is \( e^{-\lambda t} \). With the option of buy-it-now with \( P > L \) and \( r = L \), either all units are sold in stage 1, or some units are available for auction in stage 2 and \( 1 - \theta \alpha (1 - s) > 0 \). In the first case, all units are sold and hence we only need to consider the second case. In the second case, the probability that a unit is unsold in stage 2 is \( e^{-\lambda_{h} t} e^{-\lambda_{l} t} \). Thus, the probability that a unit is not sold is

\[
Q_{u}(s) = (1 - \theta \alpha (1 - s)) e^{-\lambda_{h} t} e^{-\lambda_{l} t}
\]

The first order derivative of \( Q_{u}(s) \) with respect to \( s \) is as follows:

\[
Q_{u}'(s) = \theta \alpha e^{-\lambda_{h} - \lambda_{l}} - (1 - \theta \alpha (1 - s)) e^{-\lambda_{h} - \lambda_{l}} \frac{\partial (\lambda_{h} + \lambda_{l})}{\partial s}
\]

\[
= \theta \alpha e^{-\lambda_{h} - \lambda_{l}} \left[ \frac{\theta (1 - \alpha (1 - s))}{1 - \theta \alpha (1 - s)} \right]
\]

where \( \frac{\partial (\lambda_{h} + \lambda_{l})}{\partial s} = \frac{\theta (1 - \alpha (1 - s))}{(1 - \theta \alpha (1 - s))} \).

For \( 0 < \alpha < 1 \), we get \( 1 - \theta \alpha (1 - s) > 0 \). Eq. (14) shows that \( Q_{u}'(s) \) is positive for \( 1 - \theta \alpha (1 - s) > 0 \). In other words, an increase in the proportion of high valuation bidders who only participate in bidding in stage 2 increases the probability that a unit will be unsold. This implies that as more high valuation bidders choose buy-it-now in stage 1, the probability that a unit will be unsold decreases. By making use of Proposition 2, and Eqs. (12) and (14), we get the following proposition.

**Proposition 4.** With the option of buy-it-now (with \( L < P < H \) and \( r = L \)), more units will be allocated to high valuation buyers and fewer units will be unsold compared to auctions without the option of buy-it-now.

When the seller uses the option of buy-it-now, some high valuation buyers buy in stage 1 and hence fewer buyers participate in bidding at auctions in stage 2, which results in relatively less random matching. As a result, more units are sold to high valuation buyers and fewer units will be unsold.

For \( H = 10, L = 6, m = 0, \theta = 2 \), we know that if \( \alpha = 0.3 \), the optimal strategy for the seller is to use buy-it-now auction with \( P^* = 7.088 \) and \( r = 6 \). If there is no buy-it-now option and the seller relies only on auctions, the probability that a unit is allocated to a high valuation buyer is 0.451, and the probability that a unit is unsold is 0.135. With the option of buy-it-now at \( P^* = 7.088 \), the probability that a unit is allocated to a high valuation buyer increases to 0.573. At the same time, the probability that a unit is unsold decreases to 0.039.

6. Concluding remarks

Online sellers use a number of selling strategies. The existing empirical and theoretical studies have examined various aspects of posted price selling at auctions. Online auctions involve a situation where many similar or identical units are simultaneously offered for sale. This creates allocative inefficiency because the matching between the buyers and the units available for auction can be random. By focusing on the problem of random matching, this paper attempts to provide a new rationale for the option of buy-it-now at eBay auctions.

By making use of a simple theoretical model, where a single seller offers to sell a number of identical units to many buyers with either high or low valuations, this paper shows that the option of buy-it-now helps to single out some high valuation buyers before bidding at auctions. This strategy reduces the problem of random matching and improves allocative efficiency at online auctions. In addition, compared to the selling mechanism that involves auctions only, the seller’s expected revenue increases. We also show that for competing sellers, each with a single unit for sale, if the sellers choose between the strategies of (i) auction only or (ii) auction with buy-it-now option, the option of buy-it-now will be used with positive probability in any equilibrium (provided the equilibrium exists) and some buyers will buy through buy-it-now.

Acknowledgments

This paper has greatly benefited from extremely useful comments and suggestions received from two anonymous reviewers. The authors are also grateful to an advisory editor and the editor (Professor David Parkes) of Games and Economic Behavior and Robert Alexander for helpful suggestions and encouragement. However, the authors are solely responsible for all remaining errors and imperfections.
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