by an iterative feasibility test over a grid in $\alpha$ and the domain of attraction is potentially larger than $\mathcal{O}_\infty$.

It would be interesting to get some selection rule in order to link the choice of $\alpha$ to the size of $\mathcal{C}_1$. We believe that its choice must be a consequence of a trade-off between the size of the polyhedral region $\mathcal{R}_k(\alpha) = \{x \in \mathbb{R}^n : |Kx| \leq \rho(\alpha)\}$ that gives the admissible region and the conservatism of the maximal invariant set $\mathcal{C}_1$ for the related polytopic model. Moreover, it would also be interesting to study if it is possible to avoid the usage of LDI exploiting a saturated feedback obtained with different design techniques as, for instance, the ones reported in Refs [4,5]. Obviously the optimality should be revisited and the inclusion of state constraints might not be straightforward.

References


Discussion on: “Improved MPC Design based on Saturating Control Laws”

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In the paper by Limon et al., the authors proposed a design method of stabilizing MPC design for constrained linear systems. Using a polytopic differential inclusion of the saturated closed-loop system, the authors have computed a quadratic terminal cost and a polytopic invariant. As a result, a stabilizing MPC controller can be designed. In MPC, the proposed method is an interesting approach. Although the paper is well organized and clearly written, one problem of computing the convex invariant set seems to be probably considered as an open approach. For this reason and for the further discussion, the problem statement and further work are stated in this discussion.

1. An Outline of the Problem Statement

Consider a discrete-time linear system described by the following state space equation:

$$x^+ = Ax + Bu$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $x^+$ are the current system state, the current control input and the successor state, respectively [1]. We assume that the pair $(A, B)$ is stabilizable and the full state vector is measurable. The system is subject to hard constraints on the state and control defined respectively by the sets $X$ and $U$. That is,

$$x_k \in X, \quad u_k \in U$$

for all $k \geq 0$, where $x_k$, $u_k$ denote, respectively, the system state and the control input applied at sampling time $k$. Set $X$ is a polyhedron containing the origin in its interior, and set $U$ is given by

$$U = \{u \in \mathbb{R}^m : |u| \leq \rho\}$$

where vector $\rho = \{\rho(1), \ldots, \rho(m)\} \in \mathbb{R}^m$ is such that $\rho > 0$ [1].

The authors studied model predictive control schemes that stabilize system (1) in the presence of the above state and control constraints, where each component of the control law can be presented as

$$-\rho(i) \leq u(i) \leq \rho(i), \quad i = 1, \ldots, m.$$

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This constraint is equivalently expressed as
\[
u(\alpha) = \text{sat}(K(\alpha)x) = \alpha(x)(\alpha)K(\alpha)x, 0 < \alpha(x) \leq 1
\]
(4)
\[
\alpha(x) = \begin{cases}
\frac{\rho(x)}{K(\alpha)x} & \text{if } K(\alpha)x > \rho(x) \\
1 & \text{if } -\rho(x) \leq K(\alpha)x \leq \rho(x) \\
-\frac{\rho(x)}{K(\alpha)x} & \text{if } K(\alpha)x < -\rho(x)
\end{cases}
\]
(5)
where \(K(\alpha)\) is a component of the state feedback gain matrix defined by (5). Defining both \(\alpha \in R^n\) as a vector for which the \(i\)th entry is \(\alpha(i)(i = 1, \ldots, m)\) and a diagonal matrix \(D(\alpha) = \text{diag}((\alpha))\), the closed-loop system can be computed by the following polytopic model
\[
x^+ = \sum_{j=1}^{2^m} \lambda_j A_j(\alpha)x
\]
(6)
\[
A_j(\alpha) = A + BD_j(\alpha)K
\]
(7)
\[
\sum_{j=1}^{2^m} \lambda_j = 1
\]
where \(\lambda_j \geq 0\) and \(\alpha(j) \leq \alpha(i) \leq 1\). \(D_j(\alpha)\) are diagonal matrices whose diagonal elements can assume the value 1 or \(\alpha(i)\). Then under Theorem 2 in Ref. [1] system (6) is asymptotically stable and satisfies the constraints if there exists at least one vertex matrix \(A_j(\alpha)\) such that \((K, A_j(\alpha))\) is observable, where the following LMI is satisfied.
\[
A_j(\alpha)^TPA_j(\alpha) - P + K^TD_j(\alpha)^T RD_j(\alpha)K + Q < 0
\]
(8)
\(R, Q\) and \(P\) are positive definite matrices with appreciating orders.

However, in the paper of Ref. [1], \(\alpha\) is supposed to be given. The problem of choosing \(\alpha\) seems worth to have a discussion. The objective in this discussion is to discuss how to select \(\alpha\) for the above plant and also the plant with structured uncertainty.

2. Some Approaches to Overcome this Problem

In general, considering a fixed vector \(\alpha\) and the given data, but it may be impossible to find a feasible solution for (8). The authors [1] suggested that the suitable \(\alpha\) can be obtained by an iterative LMI feasibility test of (8) on a grid in \(\alpha\). In order to limit the grid, a lower bound for \(\alpha\) is determined, by considering the maximum value of \(\alpha\), denoted \(\alpha^*\). For 1-input or 2-input cases, the proposed method can determine \(\alpha\) as smaller as possible. Considering the general multi-input system, iterative schemes in Ref. [2] are suggested. The above methods are obviously better than the simple way being to apply trial and error procedure to select \(\alpha\). In this case, two issues have to be considered, namely, (1) determining the initial vector \(\alpha\); (2) choosing the grid. The two issues can be considered as open problems. However, in this discussion, we attempt to provide one new option for future research in this approach of concerning with plants with structured uncertainty. We summarized our proposal as follows.

In practice, a structured uncertainty problem arises frequently when the plant model receives the effect of uncertainty caused by the variation of parameter values. Then, by considering robust stability problem for a family of polynomials, the upper bound of \(\alpha(i)\) can be obtained if equation related with \(\alpha(i)\) has a solution for ensuring robust stability for the family of polynomials. In discrete time approach, robust Schur stability conditions for a polytope of polynomials and for interval polynomials can been found in Refs [3,4]. For guaranteeing robust Schur stability, we can obtain \(\alpha(i)\) related equation. That is, for ensuring the robust stability of the following polytopic model
\[
x^+ = \sum_{j=1}^{2^m} \lambda_j A_j(\alpha)x
\]
we can obtain \(\alpha(i)\) related equations. The new issue can be considered as: how to solve the above-mentioned equations for obtaining \(\alpha(i)\). In this case, if the robust stability problem can be reduced to an eigenvalue problem [3], there exists the possibility to obtain the bounds of \(\alpha(i)\). Concerning the conditions of the existence of the bounds, the further work will be on this approach.

As final remark, in continuous time approach, the design parameter can be obtained for plants with structured uncertainty [5] by using Kharitonov Theorem and plants with Affine linear structured uncertainty [6] using Edge Theorem. In practice, these results may be useful even if the approach is different to discrete time approach. However, there are some remarks stated as follows. Kharitonov’s Theorem for robust Hurwitz stability of a family of interval polynomials is not a corresponding result for Schur stability. In this case, all the exposed edges of the polytope must be stable. Since the polytope framework provides a more general setting than one considered in Kharitonov’s Theorem, for this case discrete time result can be obtained from extending the result of continuous time. In Kharitonov’s Theorem, it is assumed that the coefficient variations
are independent, and the polytope formulation allows for linear dependencies. When the plant is known, the suggestion in this discussion is still work.

References

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