Guided modes in a two-dimensional metallic photonic crystal waveguide

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Abstract

Guided modes in a two-dimensional metallic photonic crystal waveguide are studied. The guided modes in the photonic crystal waveguide are related to those in a conventional metallic waveguide. There exists a cutoff frequency and consequently a mode gap at low frequencies (starting from zero frequency) in the photonic crystal metallic waveguide.

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Photonic band gap (PBG) crystals, which are periodic arrangements of dielectric or metallic materials, can suppress electromagnetic wave propagation within a frequency band (i.e., a stop band) [1–4]. If a line defect is introduced in a photonic crystal, the electromagnetic wave whose frequency is in the gap can be guided along the line defect without appreciable losses. Thus, such a line defect is called a photonic crystal waveguide, or a PBG waveguide. Recently, both theoretical simulations [5–7] and experimental studies [8] have shown that PBG waveguides can efficiently transmit electromagnetic waves, even for 90° bends with zero radius of curvature. Therefore, PBG waveguides are of interest as an alternative to conventional waveguides.

Many studies have been carried out concerning guided modes in two-dimensional (2D) dielectric PBG crystals (see e.g. [9–11]). However, very few studies can be found in the literature concerning guided modes in metallic photonic band gap (MPBG) waveguides. Danglot et al. [12] have presented a modal analysis of a T-stub guiding structure patterned on a two-dimensional metallic photonic crystal. Nevertheless, it has been suggested that periodic metallic structures have important applications [13,14]. It is thus of scientific and technical interest to study the properties of MPBG waveguides.

In the present Letter, we study guided modes in a two-dimensional metallic photonic band gap crystal. The relationship between a MPBG waveguide and a conventional metallic waveguide is also studied. It is shown that the guided modes in a MPBG waveguide correspond to those in a conventional metallic waveguide, and the difference between them is mainly...
due to the loss of the translational symmetry in a MPBG waveguide. We also find that there exists a cutoff frequency in a MPBG waveguide, which causes a guided mode gap. Furthermore, the cutoff frequency corresponds to the cutoff frequency in a conventional metallic waveguide.

For simplicity, in the present Letter we only consider a square array of metallic rods in air. The radius of the metallic rods is $R = 0.2a$, where $a$ is the lattice constant of each unit cell. We use copper as the inclusion material and $\sigma = 5.80 \times 10^5$ S/m is used for the copper conductivity [17]. Only the E-polarization is considered here since metallic rods are almost transparent for the H-polarization. It has been shown that there are two band gaps for such a metallic photonic crystal [15]. The low-frequency band gap starts from zero frequency to $0.529 \times (2\pi c/a)$, and the high-frequency band gap is between the frequencies $0.733 \times (2\pi c/a)$ and $0.862 \times (2\pi c/a)$. A waveguide is introduced here by removing one or several rows of metallic rods in the photonic crystal.

The eigenfrequencies and patterns of the guided modes are calculated with a finite-difference time-domain method (details can be found in Ref. [15]), which is applicable for both dielectric and metallic inclusions. We choose a rectangular supercell, which is much larger than a lattice unit cell, as the computational domain. The waveguide is located at the center of the computational domain. The length of the supercell corresponds to the periodicity of the metallic materials in the direction of the wave guide ($x$ direction), while the width (in $y$ direction) of the supercell is usually chosen to be more than 10 lattice constant. Due to the periodicity in the $x$ direction of the waveguide, we use the periodic condition for the numerical boundary treatment in this direction. If one uses the periodic condition also in the $y$ direction, pseudo-guided modes, which are eigenmodes for the supercell but not localized modes inside the waveguide, may be introduced into the supercell. To avoid this, we surround the computation domain with perfectly matched layers (PML) [16] in the $y$ direction. The FDTD algorithm starts with an initial field distribution. As the FDTD time evolution proceeds, only the true guided modes remain in the computational domain, and the pseudo guided modes will eventually vanish. We refer to Ref. [15] for details.

Fig. 1 shows the eigenfrequencies of the guided modes in the MPBG waveguide obtained by removing one row of metallic rods in the $(10)$ direction of the photonic crystal, as shown by the inset at the right-bottom side of the figure. The filled circles are for the even modes and the open circles are for the odd modes. For comparison, the band structure for the perfect photonic crystal (without the waveguide) is also shown in the same figure by the gray areas and the thick lines. One can see from this figure that there are two even-guided modes and one odd-guided mode in the high-frequency band gap. (Here the parity of the guided modes is distinguished by the symmetry of the electric field with respect to the central plane of the waveguide, i.e., the $x$ axis; see e.g. Ref. [9].) In the low-frequency band gap, there is only one even guided mode. One can notice that there exists a cutoff frequency in the MPBG waveguide, i.e., there exists a mode gap between zero and...
the cutoff frequency. For E-polarization modes, the electric field $E_z$ of the guided modes are almost zero near the boundaries of the MPBG waveguide since the guided modes can not propagate in the otherwise perfect photonic crystal. These properties including the cutoff frequency and zero electric field at the boundaries, etc., are similar to those for a conventional metallic waveguide.

Now we consider a 2D metallic waveguide with width $b$, and assume that the fields have harmonic time dependence $e^{i\omega t}$. For the E-polarization case, it is easy to obtain the following solution of Maxwell’s equations for the spatial part of the electric field, $E_z(x,y)$, with the boundary conditions $E_z(x,y) = 0$ at $y = \pm b/2$,

$$E_z(x,y) = \sin[m\pi(y/b + 1/2)]e^{jk_x x},$$

where $k_x$ is the wave vector in the guiding direction. In order to relate the metallic waveguide to a periodic structure and then compare it with a MPBG waveguide, we impose an artificial periodicity (with period $d$) in the $x$ direction. Then the solution becomes

$$E_z(x,y) = \sin[m\pi(y/b + 1/2)]e^{i(2\pi n/d + k_x x)},$$

where $k$ is the irreducible wave vector in the Brillouin zone. Thus, one obtains the following eigenfrequencies for the guided modes in such a 2D metallic waveguide,

$$\omega = (2\pi c/d)\sqrt{(md/2b)^2 + (n + kd/2\pi)^2}. \quad (3)$$

The cutoff frequency is given by $\omega_c = \pi c/b$ corresponds to the case when $m = 1$, $n = 0$, and $k = 0$. From Eq. (2) one sees that the even modes correspond to $m = \pm 1, \pm 2, \pm 3, \ldots$ and the odd modes correspond to $m = \pm 2, \pm 4, \pm 6, \ldots$. The eigenfrequencies of the guided modes in the metallic waveguide with a width $b = 1.8a$ and an artificial period $d = a$ are shown in Fig. 1. The solid lines are for the even modes and the dot lines are for the odd modes. These lines are labeled by $[m,n]$ according to Eq. (3). From Fig. 1 one can notice that the guided modes in a MPBG waveguide can be related (roughly) to those in a metallic waveguide (with an effective width). For the example of Fig. 1, the effective width is $b = 1.8a$. The cutoff frequency is 0.278$(2\pi c/a)$ for the MPBG waveguide, while it is 0.273$(2\pi c/a)$ for the metallic waveguide. They are also in a good agreement. However, some odd modes, e.g., the $[2,0]$ modes, which exist in the corresponding metallic waveguide, can not be found in the MPBG waveguide. We think the reason is that these modes are too close to the band structure (propagating modes) of the perfect photonic crystal (without the waveguide).

Eigen frequencies of the guided modes in a MPBG waveguide in the (11) direction of the crystal are shown in Fig. 2. The waveguide is obtained by removing one row of metallic rods. All these modes are similar to those in a metallic waveguide with a waveguide width $b = \sqrt{2}a$ and an artificial period $d = \sqrt{2}a$. However, no odd mode can exist in this MPBG waveguide. From the inset of Fig. 2, one sees that the boundary of the MPBG waveguide has a zig-zag shape, while the effective width $b = \sqrt{2}a$ is very small. Therefore, there are few guided modes with one or more nodes (i.e., $m > 1$) inside the waveguide (note that $m = 2$ for the two odd modes in the metallic waveguide).

Fig. 3(a) and (b) show the eigenfrequencies of guided modes for a wider MPBG waveguide in the
(10) and (11) directions of the photonic crystal, respectively. The waveguide is obtained by removing two rows of metallic rods. From this figure one sees that the modes in the (10) direction are similar to those in a metallic waveguide with a width $b = 3\sqrt{2}a/2$ and an artificial period $d = \sqrt{2}a$. From Fig. 3 one sees that almost all the even and odd modes, which exist in the corresponding metallic waveguides, can be found in the MPBG waveguides. Since the waveguide is wider, the cutoff frequency is smaller, i.e. the mode gap is smaller.

One thing we should point out is that the bands at a crossing point (band crossing occurs in metallic waveguides) may repel each other when continuous translational symmetry is lost as the metallic boundaries are replaced by the MPBG materials. In particular, from Figs. 1, 2, and 3 one can see that the repulsion only occurs at a crossing point with the same symmetry. This is consistent with the qualitative group-theoretical analysis in Ref. [9] for dielectric PBG waveguides.

In summary, we have studied guided modes in a typical two-dimensional metallic photonic band gap crystal. The guided modes in a MPBG waveguide have been related to the guided modes in a conventional metallic waveguide. There also exists a cutoff frequency in a MPBG waveguide. Due to the loss of the translational symmetry in a MPBG waveguide, the band curves with the same symmetry may repel each other at a crossing point.

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References