Objective

Solve the localization task despite clutter in a non-polygonal dynamic environment using a range finder.

Localization involves:
Solving a correspondence problem in which measurements in the robot’s local coordinate system must be matched to map elements in global coordinates.
Motivation

• Self-localization is an essential task for autonomous navigation.

• Limitations of common localization approaches:
  
  ▪ Feature-based matching approaches assume structured environments and polygonal models, e.g. line-to-line via Split-and-Merge.
  
  ▪ Point-to-point matching approaches are more susceptible to misalignments due to occlusions. These methods often seek the alignment of consecutive scans to solve a robot tracking problem and thus are less suitable for global localization.

Geo-referenced Satellite Image and Radar Scan

Concepción Bay, 36° 42’ S - 73° 05’ W

Satellite Image (reference)
Landsat, resolution 15 m
UTM Zone 18H

Radar Scan (measurement)
Range resolution 7.5 m
Bearing resolution 0.5°
Matching the Measurement to the Reference

Before Matching

After Matching

General Localization Scheme

**Odometry**
- IMU/Compass
- Internal States

**Position Prediction**
- (Motion Model)

**Position Update**
- (eg. EKF Estimation)
- Pose Estimate
- Position/Heading Observation (inferred from matching)

**Range Finder**
- Raw or Interpreted Sensor Data
- (Perception)
- Environment Description

**Scan Matching**
- Predicted Position

**Maps DB**
- External States
Shape Recognition Using the Hausdorff Distance

• Minimize the **dissimilarity** (largest deviations) between the reference set \( A \) (model) and the measurement set \( B \) (scan).

  \[
  \rightarrow \text{Translate, rotate and scale } B \text{ until the } \textit{best match} \text{ with respect to } A \text{ is found.}
  \]

Hausdorff Distance (HD)

The **Hausdorff distance** between to sets of points:

\[
A \overset{def}{=} \{a_1, a_2, \ldots, a_p\} \\
B \overset{def}{=} \{b_1, b_2, \ldots, b_q\}
\]

is defined as

\[
H(A, B) \overset{def}{=} \max(h(A, B), h(B, A))
\]

where

\[
h(A, B) \overset{def}{=} \max_{a \in A} \min_{b \in B} ||a - b||
\]

is the **directed Hausdorff distance**.

Thus the **Hausdorff distance** is the “greatest distance between closest” points from \( A \) to \( B \) and viceversa.
Hausdorff Distance (HD)

Step 1: \( d_B(a) \stackrel{def}{=} \min_{b \in B} \|a - b\| \quad \forall a \in A \)

The Hausdorff distance ensures that every point in set \( \psi \) will be at most at a distance \( \psi(\psi,\psi) \) from set \( \psi \).

In order words, \( \psi(\psi,\psi) \) yields a measure of the largest "deviation" of set \( \psi \) from \( \psi \).

All point in \( \psi \) are at most a distance \( \psi(\psi,\psi) \) from \( \psi \).

\[ \textbf{Step 2:} \quad h(A, B) \stackrel{def}{=} \max_{a \in A} d_B(a) \]

Averaged Partial Hausdorff Distances

Define the mapping that returns the distance from a point \( x \) to the closest point in some set \( \Omega \) as:

\[ d_\Omega : x \rightarrow d_\Omega(x) = \min_{\omega \in \Omega} \|x - \omega\| \]

The partial HD of the \( K \) best matching points in the measurements set \( B \) to the model set \( A \) can then be defined recursively for \( K = q, q - 1, q - 2, \ldots, 2, 1 \) as:

\[ h_K(B, A) = \max_{b \in B^K} d_A(b) \]

where

\[ B^K = B^{K+1} - \{b^*_K+1\} \]

\[ b^*_K = \arg \max_{b \in B^K} d_A(b) \]

and with initial values:

\[ B^{q+1} = B, \quad b^{q+1}_* = \{\emptyset\} \]
Averaged Partial Hausdorff Distances

Note that:

\[ h_q(B, A) = h(B, A) \]

Hence, \( h_K(B, A) \Rightarrow \exists K \) measurements in \( B \) within a distance \( h_K(B, A) \) from \( A \).

The average of partial Hausdorff distances, also called modified Hausdorff distance (despite not being formally a distance), is simply defined as:

\[
\bar{h}_K(B, A) \overset{\text{def}}{=} \frac{1}{K} \sum_{i=1}^{K} h_i(B, A)
\]
Final Match

Distance Transform Map (L1-norm)
Distance Transform Map (L1-norm)

Robustness of the Approach

Ladar distance measurements
Reference map
## Simulation Results: Matching Accuracy

<table>
<thead>
<tr>
<th>Noise Percentage [%]</th>
<th>Noise Level $\sigma$ [pixels]</th>
<th>Final Match Position Error [pixels]</th>
<th>Final Match Heading Error [°]</th>
<th>Initial $h_K(A, B)$ [pixels]</th>
<th>Final $h_K(A, B)$ [pixels]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2.67</td>
<td>88.99</td>
<td>0.65</td>
</tr>
<tr>
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<td>0</td>
<td>0.33</td>
<td>88.66</td>
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<td>0</td>
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<tr>
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<td>4.65</td>
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<tr>
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<td>20</td>
<td>1.41</td>
<td>1.33</td>
<td>87.65</td>
<td>8.36</td>
</tr>
</tbody>
</table>

1. **Low final matching error** when percentage of spurious/noisy measurements is below the specified threshold (30%).
2. Averaged partial HD increases proportionally to the number of noisy samples and the magnitude of the noise.

**The matching approach is robust to spurious measurements!**
Averaged Partial Hausdorff Distances vs. Iteration

Partial Hausdorff Distances
Motion Model

The state space model can be stated as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta} \\
\dot{v}_R \\
\dot{v}_L \\
\dot{\beta}_R
\end{bmatrix}
= \begin{bmatrix}
\frac{v_R + v_L}{2} \sin(\theta) + \xi_x \\
\frac{v_R + v_L}{2} \cos(\theta) + \xi_y \\
v_R - v_L \xi_\theta \\
u_1 + \xi_{u_1} \\
u_2 + \xi_{u_2} \\
u_3 + \xi_{u_3}
\end{bmatrix}
\overset{\text{def}}{=} f(x, u)
\]

where \( x, y \) are the global position coordinate,
\( v_R, v_L \) are the right/left wheel velocities,
\( \theta \) is the heading angle wrt the \( x \)-axis,
\( \beta_R \) is the range bias,
\( \xi \)'s are zero-mean, i.i.d., Gaussian disturbances.

Observation Model

The observation model (output of the matching process) is given by:

\[
\begin{bmatrix}
z_1 \\
z_2 \\
z_3 \\
z_4
\end{bmatrix}
= \begin{bmatrix}
x + \zeta_x \\
y + \zeta_y \\
\theta + \zeta_\theta \\
\beta_R + \zeta_{\beta_R}
\end{bmatrix}
\overset{\text{def}}{=} h(x)
\]

where \( \zeta \)'s are assumed to be zero-mean, i.i.d., Gaussian noises.
ActivMedia® Pioneer 3-AT

Seminar on Visual Servoing – UFSC – 2011
M. Torres-Torriti
Initial Measurement

Final Matching
Estimated Trajectory

Estimated Position Error
Performance Results (using Sick PLS-101)

- Position Accuracy < Sensor Resolution (7 cm)
- Precision < Map Resolution (15 cm)
- Computation Time:
  - In Matlab: 30 s on first iteration, .1 s on following iterations.
  - Computational complexity for $N_s$ samples:
    
    $W \cdot N_s \log(N_s)$
    
    where $W$ is the largest axis of the reference map.
Conclusions

- The proposed approach for scan-to-map matching is very accurate and precise thanks to its robustness to occlusions or unspecified environment elements.

- Accuracy is mostly limited by the resolution of the rasterized maps.

- Precision is affected by the amount of occlusions and objects that do not appear in the reference map, as well as the number $K$ of samples used in the modified HD.

- Accurate estimates of the robot’s position, heading and velocity can be obtained in real-time.

Ongoing Research

- Improving the scan-matching technique to reduce the computation time by introducing multi-scale techniques.

- Developing methods to adjust the MHD threshold dynamically.

- Extending the approach to MCL in order to improve the robustness under multiple matching solutions.

- Extending the technique to solve the SLAM problem.
Range Finder Measurements Model

The range finder measurement model is given by:

\[
\begin{bmatrix}
    z_{iw}^x \\
    z_{iw}^y
\end{bmatrix} =
\begin{bmatrix}
    (r_i^s + \beta_r) \cos(\theta_i^s + \theta + \beta_\theta) + x + \eta_x \\
    (r_i^s + \beta_r) \sin(\theta_i^s + \theta + \beta_\theta) + y + \eta_y
\end{bmatrix}
\]

where:
- \( r_i^s, \theta_i^s \) are the \( i \)-th range and bearing measurements in (local) sensor coordinates
- \( x, y \) are the ship’s position coordinates in the global frame
- \( \theta \) is the ship’s heading in the global coordinates
- \( \beta_r, \beta_\theta \) are range and bearing biases
- \( \eta_x, \eta_y \) are zero-mean Gaussian noises
HD-computed and Filtered Trajectory

The data corresponds to a sequence of 54 radar scans taken over a time interval of 135 seconds from a patrol boat in the Concepcion Bay, Chile (36° 42’ S - 73° 05’ W) sailing East (90° heading) with diminishing speed from 12 to 2 knots.

Estimated Heading
Estimated Velocity

![Graph showing estimated speed and log datum over scans]

Estimated Range Bias

![Graph showing filtered range bias and computed range bias over scans]
Estimated Bearing Bias

-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0 1 0 2 0 3 0 4 0 5 0 6

degrees

scan #

filtered angular bias
computed angular bias