Fault Diagnosis and Accommodation Design for Nonlinear Systems Described by Interpolated LTI Models

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Abstract— The main goal of this paper consists in the development of a new actuator fault-tolerant control dedicated to nonlinear systems. Based on the assumption that the nonlinear system is described by a finite number of interpolated linear time invariant models, the proposed method makes possible the faults compensation for the whole operating range through an extended Interacting Multiple Model. Its principle is based on the simultaneous on-line detection, isolation, estimation of actuator faults and an effective operating regime. Multiple banks of decoupled Kalman filters is used to build an actuator fault-tolerant control in order to compensate the fault effect on the system, which is controlled by a multiple model switching and tuning scheme. The performances of the method are tested in simulation on a nonlinear benchmark.

I. INTRODUCTION

During the last years, fault-tolerant control has received more and more attention [4]. The aim of the fault-tolerant control (FTC) is to adjust or to modify on-line the nominal control laws in order to maintain the safety of the operators and the reliability of the processes. The survey paper of [21] gives the state of the art in the field of the fault accommodation. Moreover, a general approach of fault-tolerant control has been presented by [19].

In the FTC framework, Fault Detection and Isolation (FDI) plays an important role. Usually, faults and failures in systems are detected using analytical redundancy by comparing measured and estimated outputs of the system whatever the design used. The residual relations are elaborated so that it is possible to isolate any kind of faults. A large diversity of advanced methods for automated FDI already exists integrating the robustness against parameters uncertainties and model plant mismatches ([5], [8], and [22]). A short historical view on this development can be seen in [11] and currents developments can be observed in [7]. Very few methods deal with the nonlinear systems. A great number of techniques are established around an operating point or requires an exact knowledge of the nonlinear system ([1], [9]), or as in chapter nine of [22], the nonlinear dynamic systems are described by a finite number of locally linearized models based on the idea of Tagaki-Sugeno fuzzy models. Various studies based on a multiple model method using a single bank of linear Kalman filters, have been developed in order to detect, isolate and estimate an accurate state of a system in the presence of faults/failures around an operating point ([15], [25] and [28]) or the whole operating range of a nonlinear systems ([20]). Among these approaches, some studies on multiple model adaptive estimator and control have been designed for the FTC against actuator or sensor failures but very few methods are dealing with the nonlinear systems, a great number is established around a single operating point.

This paper aims at investigating the design of an actuator fault-tolerant control of nonlinear systems under the assumption that the nonlinear system can be approximated by a finite number of interpolated linear time invariant (L.T.I.) models [12], [16] and controlled by a multiple model switching and tuning scheme [18]. The paper aims and contributes to the problem of multiple actuator faults detection, isolation, estimation and accommodation within the nonlinear systems represented by interpolated LTI models. Based on a multiple bank of decoupled Kalman filters, according to an extension of the interacting multiple model algorithm [25], the mode probabilities isolate the faulty actuator and also determine the effective operating regime. In this paper, the magnitude, the time of occurrence and the localization of the faulty actuator are considered as unknown. Based on the use of failure and identification scheme combined with a control reconfiguration algorithm, a fault-tolerant control for stochastic systems is developed for the whole operating range of the system.

The paper is organized as follows. In section II, a general formulation of the problem is given. Section III is devoted to present a decoupled Kalman filter. According to the FDI algorithm, a switching and tuning control is developed based on the robust scheduled variable generated by an extended interacting multiple model algorithm. A simulation example is given in section IV to illustrate the proposed method. Finally, concluding remarks are given in the last section.

II. BACKGROUND AND PROBLEM FORMULATION

A. Interpolated L.T.I. model approach to nonlinear representation and control

Consider a stochastic discrete nonlinear system described by the following state space representation:

\[
\begin{align*}
X_{k+1} &= F(X_k, U_k) + W_k \\
Y_k &= G(X_k, U_k) + V_k
\end{align*}
\]

where \( X \in \mathbb{R}^n \) is the state space, \( U \in \mathbb{R}^p \) is the input vector, and \( Y \in \mathbb{R}^m \) is the output vector; \( W \in \mathbb{R}^n \) (respectively \( V \in \mathbb{R}^m \)) represents the plant noise vector (respectively the measurement noise vector), \( F(\cdot) \) and \( G(\cdot) \) are smooth nonlinear functions.

Let us assume that the nonlinear system (1) can be modelled as a collection of linear systems with identical structure around \( Z \) operating points [13]. Each operating point is defined by a couple of input-output measurement \((U_{j0}, Y_{j0}) \forall j \in \{1 \cdots Z\}\). The dynamic plant behaviour can be described at the \( j^{th} \) operating point by the following linear stochastic state representation:
\[ X_{k+1} = A_j X_k + B_j U_k + \Delta X_j + w_k^j \]
\[ Y_k = C_j X_k + D_j U_k + \Delta Y_j + v_k^j \]

(2)

where \( \{A_j, B_j, C_j, D_j\} \) are the system matrices around the \( j \)th operating point, \( \Delta X_j \) and \( \Delta Y_j \) are constant matrices depending on the operating point, \( w_j \) and \( v_j \) are independent zero mean white noise sequences with covariance matrices \( Q_j \) and \( R_j \) around the \( j \)th operating point (\( \forall j \in [1 \cdots Z] \)).

As proposed by [2] under the assumption that the dynamic of the system is exactly represented by \( Z \) operating points and span the entire range of operating zone of the nonlinear plant, a model validity function \( \text{mvf}_k^i \) can be used to evaluate the validity of each linear model such that:

- \( \text{mvf}_k^i \rightarrow 1 \): when the \( i \)th model satisfactorily describes the behaviour of the nonlinear system;
- \( \text{mvf}_k^i \rightarrow 0 \): otherwise.

with \( \sum_{k=1}^{Z} \text{mvf}_k^h = 1 \) which implies that as the nonlinear system moves onto an operating point where one of the \( Z \) models becomes more trustworthy than the others, the other models lose their validity.

The probability for each of the assumed models is calculated as a function of the actual and estimated measurements. Then the model considered at sample \( k \) is determined as the conditional mean value among all the set of \( Z \) models. The maximum likelihood estimate is the value which has the highest probability. The implementation of the procedure is realised by means of a set of residual generators. Based on the \( i \)th linear model, each residual generator produces a residual vector, noted \( r_k^i \), which corresponds to the difference between the measured \( Y_k \) and the estimated \( \hat{Y}_k^i \) output. Each residual vector is computed according to the \( i \)th linear model defined as:

\[ \hat{X}_k^i = A_j \hat{X}_k^i + B_j \hat{U}_k + \Delta X_j + K_k^i (Y_k - \hat{Y}_k^i) \]
\[ \hat{Y}_k^i = C_j \hat{X}_k^i + D_j \hat{U}_k + \Delta Y_j \]

(3)

where \( \hat{X}_k^i \) and \( \hat{Y}_k^i \) denote the state and output estimation vectors established with the following linear stochastic state space representation around the \( i \)th operating point (\( \forall i \in [1 \cdots Z] \)). \( K_k^i \) represents the gain of the \( i \)th Kalman filter.

The residuals of the considered filter, around the corresponding operating point, follow a Gaussian distribution. Then assuming stationary property of the residual, the probability distribution function, noted \( \ell_0^i(k) \), is defined as:

\[ \ell_0^i(k) = \frac{\exp \left\{ -0.5 \times r_k^i \times (\Theta_k^i)^{-1} \times r_k^i^T \right\}}{\sqrt{2\pi \times \Theta_k^i}} \]

(4)

where \( \Theta_k^i \) defines the covariance matrix of the residuals \( r_k^i \).

Based on the probability distribution function, a mode probability, noted \( \hat{\mu}_k^i(k) \), can be calculated using Bayesian techniques \( \forall i \in [1 \cdots Z] \):

\[ \hat{\mu}_k^i(k) = \frac{{\ell_0^i(k) \times \tilde{\mu}_k^i(k)}}{\sum_{k=1}^{Z} {\ell_0^i(k) \times \tilde{\mu}_k^i(k)}} \]

(5)

The model validity functions, \( \text{mvf}_k^i \), are computed using (5). Therefore, the probability estimation algorithm can get locked onto one model so that the probability converges to one, while the one associated to the other models converges to zero. The mode probabilities, \( \hat{\mu}_k^i(k) \), are used to isolate the actual operating point and therefore to explain the plant behaviour as the following global model:

\[ \hat{X}_{k+1} = \left( \sum_{k=1}^{Z} \hat{\mu}_k^i(k)\hat{A}_i \right) \hat{X}_k + \left( \sum_{k=1}^{Z} \hat{\mu}_k^i(k)\hat{B}_i \right) \hat{U}_k + \left( \sum_{k=1}^{Z} \hat{\mu}_k^i(k)\Delta \hat{X}_j \right) \]
\[ \hat{Y}_k = \left( \sum_{k=1}^{Z} \hat{\mu}_k^i(k)\hat{C}_i \right) \hat{X}_k + \left( \sum_{k=1}^{Z} \hat{\mu}_k^i(k)\hat{D}_i \right) \hat{U}_k + \left( \sum_{k=1}^{Z} \hat{\mu}_k^i(k)\Delta \hat{Y}_j \right) \]

(6)

where \( \hat{X}_k \) and \( \hat{Y}_k \) represent the state space and the output estimation.

Furthermore, based on the mode probabilities computation and in order to control the considered nonlinear system, a gain scheduler local linear state feedback can be considered as an interpolated controllers’ outputs:

\[ U_k = \sum_{k=1}^{Z} \hat{\mu}_k^i(k) \left( G_k \hat{X}_k^i + U_{i, 0} \right) \]

(7)

where \( G_k \) is the feedback gain matrix of each local controller designed as a LQG controller and synthesised according to the controllability of the pair \( \{A_i, B_i\} \) and under the assumption of the observability condition of the pair \( \{A_i, C_i\} \).

B. The problem in a faulty closed loop system

Various additive and/or multiplicative faults which can affect a system due to abnormal operation or to material aging, are considered. This paper deals with actuator faults described as follows:

\[ U_f^i = \rho_i U_i + U_{i, 0} \]

(8)

where \( U_f \) and \( U_f^i \) represent the normal and fault control actions of the \( i^th \) actuator, respectively. \( U_{i, 0} \) denotes a constant offset and \( \rho_i \leq 1 \) denotes a gain degradation of the \( i^th \) actuator. In this paper, only the reduction of the effectiveness is considered i.e.:

\[ \forall \ell \in [1 \cdots p] \] \( U_f^\ell = \rho_i U_i \) with \( 0 < \rho_i \leq 1 \)

(9)

Various types of failures, defined in the framework of model based fault diagnosis, are included in the actuator fault representation. In the presence of actuator faults, around the \( j^th \) operating point, the equation (2) becomes:

\[ X_{k+1} = A_j X_k + B_j U_k + F_j \hat{f}_k + \Delta \hat{X}_j + w_k^j \]
\[ Y_k = C_j X_k + D_j U_k + \Delta \hat{Y}_j + v_k^j \]

(10)

where \( f \in \mathbb{R}^p \) represents actuator faults vector whose magnitude and the time of occurrence are considered as unknown.
\( F_j \in \mathbb{R}^{m \times p} \) is the actuator faulty distribution matrix around the \( j^{th} \) operating point with \( F_j = B_j \).

The fault effects on the control loop can be dramatic, effectively in the presence of faults (\( f_k \neq 0 \)), under the assumption that the nonlinear system is established around the \( i^{th} \) operating point, according to the \( i^{th} \) linear model, the estimation error
\[
eq (X_k - \hat{X}_k) \in \mathbb{R}^m \quad \text{and the residuals}
\]
\[
= (Y_k - \hat{Y}_k) \in \mathbb{R}^m \quad \text{propagate as:}
\]
\[
eq A_i - K_i C_i \hat{Y}_k + F_k f_k - K_i v_k^{\hat{e}} + w_k^{\hat{e}} + (\Delta Y_{ij} - K_i \Delta Y_{ij}) \xi_{ij}^{\hat{e}}}
\]
\[
= C_i \hat{e}_k + v_k^{\hat{e}} + \Delta Y_{ij} \xi_{ij}^{\hat{e}}}
\]
where \( e_{ij}^{\hat{e}} \in \mathbb{R}^m \) represents the model error vector between the \( i^{th} \) active linear local model, image of the nonlinear system at sample \( k \), and the \( i^{th} \) linear model used in the filter. \( \Delta Y_{ij} \in \mathbb{R}^{m \times m} \), respectively \( \Delta Y_{ij} \in \mathbb{R}^{m \times m} \), are the distribution matrices of structured uncertainty associated to the state equation, respectively the output equation.

The residuals are corrupted by faults and are not zero-mean value although the \( i^{th} \) model exactly matches the nonlinear system behaviour. Then the probability estimation algorithm (4) and (5) can get locked onto one model that is not explaining correctly the dynamic behaviour of the nonlinear system. Effectively, when the nonlinear system operates around the \( j^{th} \) operating point, then the residual, computed with the \( i^{th} \) linear model, has the following properties:
\[
\forall i = j \quad e_{ij}^{\hat{e}} = 0 \quad \Rightarrow \quad r_k^{ij} = 0 \quad \text{if} \quad f_k = 0
\]
\[
\forall i \neq j \quad e_{ij}^{\hat{e}} \neq 0 \quad \Rightarrow \quad r_k^{ij} \neq 0 \quad \text{if} \quad f_k = 0
\]
\[
\forall i, j \quad r_k^{ij} \neq 0 \quad \text{if} \quad f_k \neq 0
\]
\[
(12)
\]
The presence of such faults may lead to performance deterioration, instability of the system or the loss of the process. In an intelligent control scheme, in order to ensure the control objective, a fault detection, isolation, estimation and accommodation module will be developed in order to provide to operators an information about the occurrence and magnitude estimation of a possible fault.

### III. FAULT DETECTION, ISOLATION AND ESTIMATION MODULE: USE OF A DECOUPLED KALMAN FILTER

Isolation filter for enhancing the isolability of faults, the residuals generation having directional properties in response to particular faults is an attractive idea in order to accomplish fault detection and isolation. In this paper, a bank of full-order Kalman filters decoupled from faulty inputs, is proposed. Each decoupled filter generates some residuals sensitive to operating conditions and insensitive to faults. Under the general classical conditions [10], the number of faults is not greater than the number of measurements \( \forall j \quad \text{rank}(C_i F_i) = p \), [13] has proposed a solution to decouple the residuals from faults in parameterizing the Kalman filter gain as follows:

\[
K_i^1 = \omega_i \Xi_i + \overline{K}^1_i \Psi_i
\]
with \( \Xi_i = (C_i F_i)' \), \( \omega_i = A_i F_i \) and \( \Psi_i = \beta_i (I_m - C_i F_i \Xi_i) \).
\[
\beta_i \in \mathbb{R}^{(m \times p) \times m}
\]
is an arbitrary matrix determined so that matrix \( \Psi_i \) is of full row rank.

Then, the filter gain decomposition into two matrices makes possible the generation of a modified residual vector:
\[
\bar{r}_k^i = \begin{bmatrix}
\eta_k^i \\
\gamma_k^i
\end{bmatrix} = \begin{bmatrix}
\Xi_i \\
\Psi_i
\end{bmatrix}_{j} r_k^i = \begin{bmatrix}
\Xi_i r_k^i \\
\Psi_i r_k^i + f_k-1
\end{bmatrix}
\]
(14)
where \( \bar{r}_k^i \) is the residuals output in the fault-free case.

Therefore the component vector \( \gamma_k^i \in \mathbb{R}^{p} \) is only sensitive to the fault magnitude \( f_k \) when the residual is computed with the \( i^{th} \) linear model and the system operates around the \( j^{th} = j^{th} \) operating point. Moreover, the new residuals vector \( \eta_k^i \in \mathbb{R}^{p \times m} \), insensitive to faults has the following properties when the system operates around the \( j^{th} \) operating point:
\[
\forall f_k \quad \eta_k^i \neq 0 \quad \text{if} \quad i = j
\]
\[
\eta_k^i \neq 0 \quad \text{if} \quad i \neq j
\]
(15)
Under stochastic consideration, the residual vector generation is designed by computing the single free parameter \( \overline{K}^1_i \), defined in the algebraic constraint solution. For each \( i^{th} \) linear model, defined in (10), a fault detection filter is computed as:
\[
\hat{X}_{k+1} = A_i \hat{X}_k + B_i U_k + \Delta X_k + (\omega_i \Xi_i + \overline{K}^1_i \Psi_i) \left( v_k - C_i \hat{X}_k \right)
\]
(16)
\[
\hat{Y}_k = C_i \hat{X}_k + \Delta Y_{ij}
\]
(17)
\[
\overline{K}^1_i = A_i P_k \left( C_i P_k C_i' + \overline{V}_i \right)^{-1}
\]
(18)
\[
\bar{r}_{k+1} = \left( A_i - \overline{K}^1_i C_i \right) P_k \left( A_i - \overline{K}^1_i C_i \right)' + \overline{V}_i
\]
with \( \overline{A}_i = (A_i - \omega_i \Xi_i C_i), \overline{C}_i = \Psi_i C_i, \overline{V}_i = \Psi_i R_i \Psi_i', \text{ and} \overline{Q}_i = Q_i + \omega_i \Xi_i R_i \Xi_i' \).

According to their properties, defined in (15), the residuals \( \eta_k^i \in \mathbb{R}^{p \times m} \) could be used to compute the probability distribution function, defined in (4), in order to isolate the actual operating point and therefore to explain the plant behaviour as a global model [27]. However, in order to take into account the reliability of the actuators and to determine the fault probability of each actuator, an extended Interacting Multiple Model algorithm applied to multiple banks of decoupled filters is developed. The extended Interacting Multiple Model generates an accurate state estimation, fault detection and isolation but also indicates which operating regime is active in order to conserve an accurate switching system.

The other components of residual vector \( \bar{r}_k^i \) are exploited for FDI. Effectively, a fault detection and isolation algorithm can be performed by the direct fault magnitude evaluation \( (\gamma_k^i \in \mathbb{R}^{p}) \). Based on the simultaneous robust scheduled variable generation and fault detection, isolation and estimation, a new control law, based on the nominal one, must be computed in order to avoid the fault effect on the system.
IV. SWITCHING AND TUNING CONTROL FOR FAULT ACCOMMODATION

A. Extended IMM approach for Robust Scheduled Variable Generation

In the spirit of fault diagnosis, a basic idea of the approach is to reconstruct the state of the system from the subsets of measurements. The objective is to build, for each bank of decoupled filters, some filters such that each filter is driven by all inputs and all outputs except the \( \ell \)th input variable for each operating regime. \( U^i_\ell \) is not used in the \( i \)th filter due to the fact that \( U^\ell_\ell \) is assumed to be corrupted by the fault [6]. The scheme requires only \((p+1)\times Z\) which is less than the presumed post-fault models as classically considered in the multiple model approach [24]: \( p \) for each actuator fault and one for the fault free case. For each operating regime, each filter is based on the following linear model:

\[
X^i_j(k+1) = A_jX^i_j(k) + B_jU^i_j(k) + B^\ell_jf^\ell_j(k) + \Delta X^i_j + w^i_j(k) \\
Y^i_j(k) = C_jX^i_j(k) + \Delta Y^i_j + v^i_j(k)
\]  

(19)

where \( \forall i \in [1 \ldots Z] \) and \( \forall \ell \in \{0 \ldots p\} \), \( X^i_j(k) \in \mathbb{R}^p \) is the state vector associated to the \( \ell \)th faulty actuator around the \( i \)th operating point. \( B^\ell_j \) is the \( \ell \)th row of the \( B_j \). \( w \) and \( v \) are independent zero mean white noise sequences with covariance matrices \( Q \) and \( R \). For \( \ell = 0 \), the filter is based on system (3) which corresponds to the fault-free case.

In order to produce an accurate estimation of the system state after an actuator fault occurrence, the proposed method consists of applying to the multiple banks of filters an interacting multiple model algorithm. The interacting multiple model algorithm offers a good compromise between the computational, storage requirements and estimation accuracy. Any difference of global or local model has been considered as suggested first by [28] in their multiple model adaptive control developed around a unique operating point. The interacting multiple model algorithm computes a state estimation by a weighted sum of the estimates from a number of filters that are matched to different models of the system established around a single operating point. A complete description of this algorithm is given in [25].

In this paper, an extended interacting multiple model algorithm is developed to make FDI of nonlinear system. In a fault-free case around an operating point, all the residuals of the \( p+1 \) filters are zero mean. Under this consideration, an extension of the classical interacting multiple model algorithm has been considered, in order to provide a reliable fault detection and isolation which depends on the distinguishability between models in the model set. To obtain a successful operation, in terms of probabilities, it is proposed to associate a confidence parameter \( \gamma \) to each model. This concept is driven by the fact that in a fault-free case for a particular operating point, the innovation of each filter is close to zero: this confidence parameter represents a weight in order to distinguish the model in the model set.

The steps of the extended interacting multiple model algorithm are briefly outlined below. At each sampling period, the algorithm has different steps dedicated to each \( i \)th Kalman filter of the \( i \)th bank:

**Interaction:** \( \forall i \in [1 \ldots Z] \) and \( \forall \ell \in \{0 \ldots p\} \), a predicted mode probability is calculated

\[
\mu_i^\ell(k) = \sum_{h=1}^{Z} \sum_{t=0}^{p} \pi_{ih}^\ell \mu_h^\ell(k-1)
\]

where \( \mu_i^\ell(k-1) \) represents the conditional mode probability for each filter established around the \( i \)th operating point according to a fault on the \( \ell \)th actuator and \( \pi_{ih}^\ell \) is an element of the transition probability matrix \( \Pi \) in the proposed fault diagnosis approach, the transition probability matrix \( \Pi \) is considered to be equal to the identity matrix. A mixed state is estimated by

\[
\hat{X}^i_j(k-1) = \sum_{h=1}^{Z} \sum_{t=0}^{p} \hat{X}^i_h(k-1) \mu_i^h
\]

with an associated covariance:

\[
P^i_j(k-1) = \sum_{h=1}^{Z} \sum_{t=0}^{p} P^i_h(k-1) \mu_i^h
\]

\[
+ \sum_{h=1}^{Z} \sum_{t=0}^{p} \left[ \hat{X}^i_h(k-1) - \hat{X}^i_j(k-1) \right] \left[ \hat{X}^i_h(k-1) - \hat{X}^i_j(k-1) \right]^T \mu_i^h
\]

\[
\mu_i^h = \pi_{ih}^\ell \mu_h^\ell(k-1)/\mu_i^\ell\text{ defines the mixing probability.}
\]

**Filtering:** \( \forall i \in [1 \ldots Z] \) and \( \forall \ell \in \{0 \ldots p\} \), a predicted state \( \hat{X}^i_j(k) \) and a covariance matrix \( P^i_j(k) \) are calculated based on the decoupled Kalman filter defined previously in Section III (see eq 16, 17, 18). A residual insensitive to the \( \ell \)th actuator can be computed: \( \eta_i^\ell \in \mathbb{R}^{p-\ell} \) for each operating point.

**Mode probability calculation:** \( \forall i \in [1 \ldots Z] \) and \( \forall \ell \in \{0 \ldots p\} \), based on the likelihood function,

\[
\zeta_i^\ell(k) = \frac{\exp(-0.5 \eta_i^\ell(k)^T P_i^\ell(k)^{-1} \eta_i^\ell(k))}{\sqrt{2\pi P_i^\ell(k)^{-1}}}
\]

a mode probability can be calculated:

\[
\mu_i^\ell(k) = \frac{\nu_i \mu_i^\ell \zeta_i^\ell(k)}{\sum_{h=1}^{Z} \sum_{t=0}^{p} \nu_i \mu_i^h \zeta_h^\ell(k)}
\]

**Combination:** Based on the previous steps, a state estimation around each operating point is computed by a weighted sum of the following form:

\[
\hat{X}^i_j(k) = \sum_{\ell=0}^{p} \mu_i^\ell(k) \hat{X}^i_j(k)
\]

used in the tracking control law, defined in (7): the control law is becoming “robust” against failures and faults.

As proposed in [25], the mode probabilities \( \mu_i^\ell(k) \) provide an indication of the active mode at each sampling period. The mode probabilities is used to isolate the faulty actuator and also the effective operating regime. Moreover, the mode probabilities can be used in a supervision scheme in order to provide information about the occurrence of a possible fault to the operators.
B. Fault accommodation

After the occurrence of an $i^{th}$ actuator fault, the control performance may be rapidly degraded, since the nominal controller is no longer tuned to stabilize the system in the presence of fault. Therefore, the nominal controller has to be reconfigured in order to preserve the basic stability conditions as follows:

$$U_k = U_{k,nom} + U_{k,ad}^{d} = \sum_{h=1}^{n} \mu_k(k) (G_k X_k + U_{k,h}) + \sum_{h=1}^{n} \mu_k(k) U_{k,h}^{ad}$$  \hspace{1cm} (20)$$

where the additional control law $U_{k,ad}^{d}$ must be computed such that the faulty system is as close as possible to the nominal one. The blended representation immediately suggests a natural divide and conquer type of additive control design approach whereby an additive local law is designed for each controller:

$$U^{i} = U^{i,nom} + U^{i,ad}$$  \hspace{1cm} (21)$$

The local additional control law $U^{i,ad}$ must be computed such that the faulty system is as close as possible to the nominal one, therefore:

$$B_i U^{i,ad} + B_i x = 0$$  \hspace{1cm} (22)$$

Using the estimation of the fault magnitude $\hat{f}$ obtained from the fault diagnosis module, the solution of (14) can be obtained by the following relation if matrix $B$ is full row rank:

$$U^{i,ad} = -B_i^+ B_i x$$  \hspace{1cm} (23)$$

where $B_i^+$ is the pseudo-inverse of matrix $B$.

V. APPLICATION

A. Process description

The approach presented in this paper has been applied to the well known three tanks benchmark. As all the three liquid levels are measured by level sensors, the output vector is $Y = [l_1 \ l_2 \ l_3]^T$. The control input vector is $U = [q_1 \ q_2]^T$. The goal is to control the system around three operating points. Thus, 3 linear models have been identified around each of the operating conditions given in Table 1.

<table>
<thead>
<tr>
<th>Operating Point</th>
<th>n\textsuperscript{1}</th>
<th>n\textsuperscript{2}</th>
<th>n\textsuperscript{3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_r^{i}$ (m)</td>
<td>0.20 - 0.15</td>
<td>0.50 - 0.15</td>
<td>0.50 - 0.40</td>
</tr>
<tr>
<td></td>
<td>-0.175</td>
<td>0.325</td>
<td>0.45</td>
</tr>
<tr>
<td>$U_r^{i} \times 10^{-5}$ (m\textsuperscript{3}/s)</td>
<td>1.7509 -</td>
<td>4.6324 -</td>
<td>2.4761 -</td>
</tr>
<tr>
<td></td>
<td>4.0390</td>
<td>1.1574</td>
<td>6.9787</td>
</tr>
</tbody>
</table>

The linearized system is described by a discrete state space representation with a sampling period $T_s = 1s$. Control matrix pair $(A_i, B_i)$ of the augmented plant are controllable. Controllers have been designed in order to levels $l_1$ and $l_2$ can track reference input vector $Y^{r} \in \mathbb{R}^{2}$. Nominal controllers, designed by a LQR+I technique, leading to three feedback gain matrix $K_i$ have been designed in order to achieve satisfying tracking performances [5].

B. Results and comments

The results shown in the following figures are responses with respect to set-point changes. In the simulation, a gaussian noise $(N(0, \sigma^2 = [0.3^2 \ 0.2^2])$ is added to each output signal. The reference inputs correspond to step changes for $l_1$ and $l_2$ which excited the whole behaviour of the nonlinear system.

Firstly, the validation of the tracking control with the multiple models control law is shown in Figure 1 where step responses are considered for a range of 4000s. Reference inputs $Y_r$ are step changes for $l_1$ and $l_2$ of their exactly corresponding operating values. The dynamic responses demonstrate that a tracker is synthesised correctly.

Figure 1. System outputs in fault free case with a classical gain scheduling controller

Figure 2.a. presents the dynamic evolution of the mode probability mechanism, established with three residuals computed by classical three Kalman filters. In this figure, the three mode probabilities evolution $\mu_i^0$ are presented. The selected model is always close to the dynamic behaviour of the nonlinear system according to the considered operating regimes in Table 1.

Figure 2. Three mode probabilities $\mu_i^0$ in fault free case (a)

The consequence of an actuator fault is illustrated in Figure 3. A gain degradation of pump 1 (clogged or rusty pump, ...
equivalent to 80% is considered and appears abruptly at instant 1500s on the system. The dynamic behaviour of the other levels is also affected by this fault.

The control law tries to cancel the static error created by the corrupted input. Consequently, the real output is different from the reference input and the control law is different from its nominal value. Since an actuator fault acts on the system as a perturbation, and in spite of to the presence of the integral error in the controller, the system outputs can not reach again their nominal values: the mode probabilities (in fact the residuals of Kalman filters) are corrupted by the faults as shown in Figure 2.b. Consequently the controller is not correctly selected according to the operating point.

In the same way, the actuator fault-tolerant control method’s ability to compensate faults is illustrated in the presence of the same fault. The figure 4 shows that the probabilities are evolving according to the behaviour of the nonlinear system and robust against actuator faults. Without actuator faults, the probabilities associated to the fault-free case $\mu_i^0$ are equal to one according to the dynamic behaviour of the system (Figure 4.a). However, these probabilities are equal to zero when a fault occurs. Only the mode probabilities associated to the first faulty actuator $\mu_i^1$ is closed to one according to the dynamic behaviour of the system (Figure 4.b.c.d). The fault detection, and isolation module is established directly by the statistical evaluation of the fault-free probabilities (Page-Hinkley test, ...) and schedules a fault compensation loop in order to cancel the fault effect.

It can be noticed that the fault isolation is always available and independently of the operating point. The actuator fault-tolerant control is able to maintain performances as close as possible to nominal ones despite the presence of instruments malfunction.

VI. CONCLUSION

The method developed in this paper emphasizes the importance of the active fault-tolerant control of nonlinear systems based on interpolated LTI representation where an extended Interacting Multiple Model algorithm applied to multiple banks of decoupled filters has been developed in order to generate an accurate state estimation, fault detection and isolation. This method is suitable for actuator faults on the whole operating range of the system with the opportunity to take into account the reliability of the components. The proposed method requires an exact knowledge of the number of linear models describing the complete dynamic behaviour of the nonlinear system in the fault-free case. The performances and the effectiveness of the fault-tolerant control based on multiple model approach have been illustrated in simulation on a three-tank system.

**REFERENCES**


