Insufficient Hartree–Fock Exchange in Hybrid DFT Functionals Produces Bent Alkynyl Radical Structures

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Supporting Information

ABSTRACT: Density functional theory (DFT) is often used to determine the electronic and geometric structures of molecules. While studying alkynyl radicals, we discovered that DFT exchange-correlation (XC) functionals containing less than ~22% Hartree–Fock (HF) exchange led to qualitatively different structures than those predicted from ab initio HF and post-HF calculations or DFT XCs containing 25% or more HF exchange. We attribute this discrepancy to rehybridization at the radical center due to electron delocalization across the triple bonds of the alkynyl groups, which itself is an artifact of self-interaction and delocalization errors. Inclusion of sufficient exact exchange reduces these errors and suppresses this erroneous delocalization; we find that a threshold amount is needed for accurate structure determinations. Below this threshold, significant errors in predicted alkyne thermochemistry emerge as a consequence.

SECTION: Molecular Structure, Quantum Chemistry, General Theory

Computational modeling plays an important role in combustion studies and predicting properties of fuels. This modeling requires accurate thermochemical properties of species such as bond dissociation energies (BDEs), which are often obtained from ab initio quantum chemistry methods. However, before reliable BDEs can be calculated, molecular structures of the relevant species must be obtained with reasonable accuracy.

Alkynyl radicals are participants in soot formation and interstellar chemistry, but details of their properties and reactivity have been challenging to elucidate. For example, accurate electronic structures for these radicals are often difficult to characterize, and the associated alkyne photo-dissociation branching ratios have been the subject of ongoing controversy. Ethynyl (2Σ+) and 1-propynyl (2Σg+) ground-state geometries are established from experiment, and their symmetries signify that the R−C−C angles in these two radicals are linear. We are not aware of experimental studies characterizing geometries of larger alkynyl radicals. However, one might expect that, like ethynyl and 1-propynyl, larger radicals also have locally linear R−C≡C• structures.

While investigating BDEs of fuel molecules, we found that some commonly used density functional theory (DFT) exchange-correlation (XC) functionals predict qualitatively different structures for these radicals. Ab initio Hartree–Fock (HF) exchange reduces these errors and suppresses this erroneous delocalization; we find that a threshold amount is needed for accurate structure determinations. Below this threshold, significant errors in predicted alkyne thermochemistry emerge as a consequence.

DFT XCs (including PBE, X3LYP, and B3LYP) produce structures with noticeably nonlinear bond angles of ~160°.

This spurious bending was observed previously in calculations using the B3LYP XC functional. Theoretical studies reporting bent 1-propynyl structures note that single-reference methods converge to a low-lying 2E state, which is then subject to symmetry breaking and Jahn–Teller distortion. Eisfeld demonstrated the need for detailed multireference configuration interaction calculations to obtain the correct ground state electronic structure of 1-propynyl. However, multireference effects cannot be fully responsible for bent geometries, as some single reference methods also predict linear R−C−C bond angles. We investigated this subject further to alert the chemical community that alkynyl geometries from some commonly used XC functionals are qualitatively inaccurate due to erroneous electron delocalization, and we show that this artifact is a consequence of insufficient inclusion of exact exchange. One serious outcome of predicting bent instead of linear geometries for these radicals is that significant errors arise in predictions of alkyne BDEs, as we demonstrate here.

Geometry optimizations and vibrational frequency calculations on the four alkynyl radicals shown in Figure 1 were run using the GAMESS-US computational chemistry program. To allow full geometric relaxation, symmetry was not imposed.
in calculations. We optimized geometries using restricted open-shell HF (ROHF),\textsuperscript{13,14} restricted open-shell MP2 (ROMP2),\textsuperscript{15,16} and unrestricted DFT (U-DFT) with a selection of nine different XC functionals, each consisting of different mathematical constructions and amounts of exact exchange. Restricted open-shell DFT and unrestricted HF were not used because the former introduces formally unjustified constraints into the Kohn–Sham determinant,\textsuperscript{24} and the latter suffers from much larger spin contamination errors than U-DFT.\textsuperscript{25} In all calculations, U-DFT converged to geometries with minimal spin contamination ($S^2 = 0.75–0.79$). Table 1 shows the XC functionals used in this study along with the percentage of HF exchange ($X$) they contain. The 6-311G(2d,p)\textsuperscript{26} Pople basis set and the Dunning basis sets\textsuperscript{27–29} cc-pVDZ, cc-pVTZ, cc-pVQZ, aug-cc-pVDZ, and aug-cc-pVTZ were used for geometry optimizations and frequency calculations. See Supporting Information for details of algorithms and optimization parameters used.

After determining the molecular structures, BDEs were calculated using our complete basis set extrapolated multi-reference singles and doubles configuration interaction scheme (MRSDCI/cc-pVQZ) previously described.\textsuperscript{8} This scheme uses complete active space self-consistent field (CASSCF)\textsuperscript{30} wave function references for the MRSDCI calculations. The minimal version of this method was used for the BDEs ($D_x$ and $D_{298}$) reported below. Here, the CASSCF active space at the equilibrium geometry is composed of the two electrons involved in the breaking bond and their corresponding bonding and antibonding orbitals. The corresponding active space at the dissociation limit (taken here to be 10 Å separation) is composed of two singly occupied orbitals, one on each radical fragment (here, the alkynyl radical and an H atom). Only valence electrons are correlated in the MRSDCI/cc-pVQZ calculations. The computed frequencies are used to calculate zero point energies (ZPEs) and thermal corrections (TCs) to the BDEs using the ideal gas, rigid rotor approximation.\textsuperscript{31}

Single reference \textit{ab initio} ROHF and ROMP2 calculations on alkynyl radicals ($R$–C≡C•; $R$ = H, CH$_3$, C$_2$H$_5$, C$_3$H$_7$) predict ethynyl and 1-propynyl to be linear, while bond angles for 1-butynyl (ROHF = 179.2°; ROMP2 = 176.4°) and 1-pentynyl (ROHF = 179.1°; ROMP2 = 175.1°) were linear to within 1–5°. This is consistent with the expectation that the triply bonded carbon atoms should be sp-hybridized. The differences in ROMP2 energy between fully linear and the optimized nearly linear geometry were very small: 0.03 kcal/mol for 1-butynyl and 1-pentynyl, confirming the linear/quasilinear geometry predicted with ROHF theory (and the flat nature of the potential energy surface near 180°). By contrast, Figure 1 shows the four alkynyl radicals optimized at the U-DFT-B3LYP/6-311G(2d,p) level. Note that these would be the same geometries obtained from the popular CBS-QB3 method.\textsuperscript{32} All structures have $R$–C=C bond angles of ~163° except the ethynyl radical, which is predicted to be linear. Results were robust with respect to using bent or linear initial guesses for all four radicals, and tests on the representative case of 1-propynyl verified that tighter energy and geometry convergence criteria and denser DFT grids did not change the predicted structures (see Supporting Information). Geometry optimizations using Dunning-type basis sets larger than cc-pVDZ and up to cc-pVQZ and aug-cc-pVTZ all converged to the same geometries for the representative cases of ethynyl and 1-propynyl. Thus, these structures are not likely to be artifacts of basis set choice. The differences in B3LYP/6-311G(2d,p) energy between fully linear and the optimized bent geometries were significantly larger than in the ROMP2 cases: 2.03 kcal/mol for 1-propynyl and 1.93 kcal/mol for 1-butynyl and 1-pentynyl, suggesting that the bent structure in each case is a robust minimum.

We then optimized the structures of the four alkynyl radicals with the U-DFT XC functionals listed in Table 1 using the 6-311G(2d,p) basis set. Several of the selected DFT XC functionals share common features. The hybrid-GGA PBE0 functional is the same as the GGA PBE functional except the former has 25% HF exchange ($X = 25\%$) while the latter has none. Hybrid-GGA BH&HLYP ($X = 50\%$) and hybrid-GGA B3LYP ($X = 20\%$) are likewise very similar to each other except for their value of $X$, as are hybrid-meta-GGA M06 ($X = 27\%$) and M06-2X ($X = 54\%$). However, hybrid-GGAs X3LYP ($X = 21.8\%$), B97-1 ($X = 21\%$), and hybrid-meta GGA

![Figure 1. U-DFT-B3LYP/6-311G(2d,p) structures for alkynyl radicals. This model chemistry predicts alkynyl structures with bond angles of 180° for ethynyl and ~163° for the other radicals.](image)

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<th>Table 1. DFT XC Functionals Used In This Study</th>
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TPSSH (X = 10%) have different formulations more distinctive from other functionals. Despite the differences in functionals, calculations on alkynyl radicals yielded two categories of minimum energy structures: those that are linear or near linear (with an R–C−C bond angle of 176.2−180°), and those that are noticeably bent (with an R–C−C bond angle of 160.4°−165.4°).

An explanation emerges upon plotting the R–C≡C• bond angle against X, the percentage of HF exchange (Figure 2).

Here, one clearly sees that all methods that utilize ≥25% HF exchange (DFT with PBE0, M06, BH&HLYP, or M06-2X XC functionals; ROHF; and ROMP2) predict essentially linear alkynyl radical structures regardless of whether the XC functional is a hybrid-GGA or a hybrid-meta-GGA. However, the DFT XCs with less than this amount of HF exchange (PBE, TPSSH, B3LYP, B97-1, and X3LYP) predict bent 1-propynyl, 1-butynyl, and 1-pentynyl radical structures with an R–C−C bond angle of ~163° regardless of whether the XC functional is a pure GGA, hybrid-GGA, or a hybrid-meta-GGA. For ethynyl, all XC functionals found linear structures except for PBE, for which the structure was bent (165.4°) due again to a very flat potential energy surface: the energy difference between linear and bent structures is only 0.08 kcal/mol (see Supporting Information). The very flat potential energy surface arising from PBE indicates it is not suitable for structural determinations of these radicals.

Insight into the XC trend above may be gleaned by examining the electronic spin density differences (α−β) from Löwdin populations at the terminal carbon, C1, and at the other triple bonded carbon, C2. For a radical localized at C1, α−β will be 1.0 since this site has an excess of 1 electron. For a case involving radical delocalization across the triple bond, α−β on C1 will be less than 1.0, and a significant build-up of spin should be found on C2. Our results for spin density differences are shown in Figure 3. The observed trend here matches that seen in Figure 2. All methods that predict linear structures have spin density differences of ~1.0 at C1 and ~0.0 at C2, indicating a localized radical on C1. For the bent cases, around 30% percent of the spin density migrates from the terminal C1 to the adjacent C2 (see Supporting Information for a full table of spin density difference values). Thus, we attribute the bending to delocalization of the lone electron, which causes rehybridization of both C atoms to sp3, leading to bending at C2. The delocalization is caused by the well-known problem with pure DFT XC functionals of self-interaction and delocalization errors.43 These errors have been implicated in several unphysical DFT predictions (see refs 44, 45, and the references therein), but the full extent of their manifestation in common applications is not established. The self-interaction and delocalization errors produce extraneous electron repulsion (the radical electron interacts with itself) not canceled by exact exchange interactions. To minimize this spurious extra electron repulsion, the radical electron spreads out.

Based on results shown in Figure 3, it is clear that the values of X, the spin orbital populations, and the structures of 1-propynyl, 1-butynyl, and 1-pentynyl are interrelated as explained above. Ethynyl is an exception, however. Figure 3 shows little to no electron delocalization in ethynyl, and all DFT XC functionals (except for PBE) predict linear ethynyl structures. Since we see little evidence of electron delocalization to the alkyl group (see Supporting Information), another factor seems to keep ethynyl linear.

Our best explanation for ethynyl’s tendency to stay linear while the others bend for a certain class of XC functionals invokes chemical bonding principles. In these alkynyl radicals, an sp-hybridized C2 is either bound to an H atom (in ethynyl) or to an sp3 carbon (in 1-propynyl, 1-butynyl, or 1-pentynyl). In the latter cases, the sp3 carbon would prefer to bond to C2 when C2 is hybridized with substantial p character to maximize its overlap. The spurious radical electron delocalization induces rehybridization (i.e., symmetry-breaking) to incorporate more p overlap to C2 (sp → sp3), thereby producing bent structures. On the other hand, the H atom, which forms bonds with only an s orbital, would prefer to bond to C2 when C2 is hybridized with substantial s character. In this case, the character of the normally sp-hybridized C2 remains intact and that bonding interaction appears to be enough to counteract the self-interaction/delocalization error driving force in most DFT XC functionals and maintain a linear ethynyl structure.

Ultimately, our goal was to ascertain the significance of these spuriously bent structures on predictions of their thermochemistry. Table 2 shows the energy components needed to calculate (MRSDCI/cc-pVQZ) alkene RCC-H BDEs, using geometries from calculations incorporating different quantities of exact exchange (B3LYP, M06-2X, and HF as representatives) for ethyne, propyne, butyne, and their corresponding radicals. Table 2 includes energy contributions due to differences in TCs and ZPEs (reported as the sum of the TCs/ZPEs of the separated radical fragments minus the TC/ZPE of the molecule at equilibrium) as well as their Dv values, the electronic energy
In conclusion, we have identified a class of molecules where change upon bond dissociation. As mentioned earlier, B3LYP predicts a linear structure for ethynyl and bent structures for 1-propynyl and 1-butyln, while ROHF and M06-2X calculations predict linear structures for all three alkynyl radicals. Bent and propynyl and 1-butynyl, while ROHF and M06-2X calculations predicts a linear structure for ethynyl and bent structures for 1-butynyl. As mentioned earlier, B3LYP leads not only to incorrect structures but also to ~3 kcal/mol errors in thermochemistry, as demonstrated here for alkynyl C–H bond energies. We would not be surprised to find similar inaccuracies for other classes of molecules susceptible to spurious electron delocalization.

**REFERENCES**
