Linear Risk Sharing in Intergenerational Pension

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Abstract

We introduce and analyze a novel collective defined contribution plan (CDC) which guarantees upon retirement at least a target benefit as a lump sum. The guarantee is provided by the remaining working generations under a pre-determined linear intergenerational risk sharing (IRS) rule. Through a simulation-based study, we show that the CDC scheme consistently outperforms the comparable individual DC scheme in terms of risk-adjusted performance. An extensive sensitivity analysis indicates that this outperformance is robust, especially in a 'double-hit' scenario where the underlying market dynamics and the demographics are severely worse than expected. Our work indicates that guaranteed retirement benefits can be organized via a CDC scheme with IRS in a way that is both beneficial for all generations and resilient.

Keywords: Collective pension scheme, sharing rules, defined contribution, defined benefit

1 Introduction

Nowadays, the two most prevalent types of occupational pension plans are defined benefit (DB) and defined contribution plans (DC). Under a typical DB plan, the employer promises the employee a retirement benefit which hinges on the employee's years of service and salary. In the past, DB plans were popular (see e.g. Gustman et al. (1994)). However, as market and demographic conditions worsened and accounting regulations tightened, employers increasingly struggled to make good on their costly promises (see e.g. Baily and Kirkegaard (2008) and Chen et al. (2022)). As a result, their support of DB pension plans waned and DC plans became widely available instead (see e.g. Gale et al. (2006)). This plan design transfers the pension's risks, mostly investment and longevity risk, from the employer to the employees. Thereby, the employee's retirement savings from the DC plan are, especially close to retirement, extremely vulnerable with respect to potential drawdowns of the financial market as the retirement benefit depends on the realized investment returns.

CDC schemes. As a third option in occupational pensions, some legislators allowed so-called *collective defined contribution plans* (CDC), for example in the Netherlands (Westerhout et al. (2021)) or in the UK (Mirza-Davies (2022)). Under a stylized CDC plan, retirement savings of multiple generations are pooled in one pension fund and invested collectively in the financial market. Contingent on the pension plan's financial market performance, the employees' contributions and retirees' benefits may be adjusted.

By saving collectively for retirement, CDC schemes provide the opportunity of intergenerational risk sharing (IRS) of, for instance, investment risk. Thereby members may undertake collectively a riskier investment strategy which will potentially yield higher payoffs (e. g. Gollier (2008), Cui et al. (2011) among others). Additionally, they benefit from economies of scale (Broeders et al. (2016)).

A lot of research on utility-maximizing CDC designs has been conducted in the past (see e.g. Gollier (2008) and Cui et al. (2011)). However, Donnelly (2017) points out that the eventual objective of pension plans is rather the actual retirement benefit than, for example, the expected utility associated thereto. Therefore, this manuscript focuses on designing a CDC scheme which decreases the incertitude about the retirement benefit for the participating generations. The CDC scheme aims to be, first of all, beneficial for all generations. That is, we take – similar to Donnelly (2017) – the perspective of the individual generations as opposed to the viewpoint of a social planner.[1] Second,

[1] In fact, CDC schemes, developed from a social-planning viewpoint, do not necessarily entail a

the plan aims to be resilient under adverse market and demographic conditions.

Our CDC scheme. To decrease the aforementioned incertitude, our CDC design includes two features: First, the CDC scheme guarantees at least a target benefit as a lump sum upon retirement. This *guarantee* is provided by the remaining working generations. However, as we impose a relatively strict reserve requirement, the pension fund's asset suffice in the vast majority of scenarios to pay at least the target benefit without any intervention of the remaining generations. To ensure that the remaining generations obtain the same protection from future generations at their retirement, we assume compulsory enrollment. Second, we suggest implementing a CDC scheme which is easy to communicate with the plan's beneficiaries. Therefore, we adopt a linear sharing rule where the working generations share the investment risk pro rata to an individual reserve requirement.

More precisely, our fund works as follows. At fund set-up, the pension fund manager calculates the fund's required reserve as the α-quantile of the discounted sum of all target benefits. Subsequently, all working generations need to provide collectively the fund's required reserve. The pension fund manager fixes a collective investment strategy, taking the generations' risk aversions into account. At the beginning of each subsequent year, the pension fund manager verifies if the accumulated fund's assets suffice to cover the required reserve. If the fund's assets undercut the required reserve due to bad portfolio performance, the pension fund manager raises the working generations' contributions to the fund. Otherwise, the surplus is paid out to the generations which can be considered as a reduction of contributions. Upon reaching its retirement age, each generation leaves the pension fund with at least the target benefit. In case that the accumulated fund's assets do not suffice to deliver the target benefit, the remaining generations pay the lacking amount. To determine each generation's share in a total required contribution, the pension fund manager resorts to the aforementioned pre-determined linear sharing rule.

This fund design – to our knowledge – has not been studied in the literature so far. In fact, the literature CDC schemes with guarantees is relatively scarce. Gollier (2008) proves that a CDC scheme with a minimal return of 0% on contributions is the 'second-best-solution' (compared to a CDC scheme without guarantee) but still welfare-improving. Van Binsbergen et al. (2014) advocate a CDC scheme with intergenerational guarantees which are traded on so-called *Pension Guarantee Exchanges*.

benefit for all generations although they are overall utility-maximizing. For example, under the CDC scheme of Gollier (2008) all generations are only better off joining the scheme as long as the fund's assets exceed a so-called minimal sustainable reserve.

Two characteristics of the scheme are inspired by the literature. First, Cui et al. (2011) and Gollier (2008) show that CDC schemes require mandatory enrollment in order to function for all generations. Moreover, in some countries such as the Netherlands enrollment in Pillar II pensions is compulsory by law.[2] Second, Cui et al. (2011) show that CDC plans lose attractiveness for future generations if they pile up huge deficits over time. Clearly, our scheme with its annual rebalancing of assets and reserves precludes such a deficit accumulation.

Performance Assessment. In order to evaluate the performance of the proposed scheme, we run a number of simulations against two benchmarks. Our first benchmark is an individual DC scheme (IDC) where each generation saves on its own in a life-cycle manner and constructs a put option to guarantee itself at least the target benefit upon retirement. Our second benchmark is the same CDC scheme but with a non-linear risk sharing rule. It is based on the famous Shapley value (Shapley (1953)) and therefore dubbed *Shapley risk sharing*. This risk sharing rule attributes the highest share in contributions to younger working generations and the smallest share to older working generations. Therefore, cashflows associated to the Shapley risk sharing exhibit a life-cycle pattern with higher volatility during younger age and lower volatility as retirement approaches.

To compare the different schemes (CDC Linear, IDC, CDC Shapley) we then consider the distribution of the net present value (NPV) of all benefits and contributions for each generation. Three key performance indicators are used: the expected net present value as a measure of the average benefit, the 95% Value at Risk as a tail risk measure, and the Sharpe ratio of the NPV distribution as a risk-adjusted performance measure.

Main Results. Our main results are that our proposed CDC scheme with linear risk sharing is, first, beneficial for all generations and, second, resilient. First, the proposed CDC scheme is beneficial in the sense that it exhibits for all generations the best risk-adjusted performance in terms of Sharpe ratio compared to the IDC scheme and the CDC scheme with Shapley risk sharing. In addition, it delivers the lowest tail risk for the younger generations. An extensive sensitivity analysis indicates that these observations, especially the superior risk-adjusted performance, are robust with regard to all main model parameters.[3] Second, the proposed CDC scheme's risk-adjusted performance remains superior under an adverse 'double-hit' scenario where we stressed the

[2]Note that although mandatory, Dutch occupational pension schemes cover only between 85% and 90% of the workforce. This is mainly due to a minimal required wage (*Franchise*) and the fact that self-employed workers do not have to enroll (Frericks (2013)).

[3]More precisely, we verify the performance of the CDC scheme with respect to demographic change, investment strategies, the quantile level α and the market Sharpe ratio.

assumptions regarding the underlying market dynamics and the demographic change jointly. More precisely, even if the decrease in the size of the future workforce is much more severe than the European Union predicts for its member states, the main results about the superiority of our CDC scheme still hold. In this sense, we deem our CDC scheme as resilient. In a nutshell, our work indicates that guaranteed retirement benefits, even under adverse demographic conditions, may be organized via a CDC scheme with intergenerational risk sharing in a mutually beneficial and resilient fashion (from an *ex ante* perspective).

These results are also meaningful from a practical point of view due to two reasons. First, providing some kind of guarantee in a CDC scheme may help to win support for necessary pension reform. In fact, new pension plans which obtained approval from all relevant stakeholders in Europe recently, include some guarantee component. For example, the Royal Mail Group and unions agreed that the UK's very first CDC scheme contains a guaranteed lump sum benefit upon retirement, besides a variable income during retirement.[4] A particularly interesting example is the public occupational DC pension fund which was created in Greece in 2022, where inflation-adapted returns are guaranteed for all generations (Altiparmakov (2022)). Second, our results indicate that the conversion of existing IDC schemes to CDC schemes by implementing relatively simple risk sharing rules, such as our linear risk sharing rule, can be expected to be welfare improving in that all generations strike a better balance between risk and reward.

Structure. The remainder of this paper is structured as follows. Section 2 describes the fund set-up and our financial market assumptions. Section 3 follows with a detailed description of the risk sharing rules and the two alternatives. Section 4 presents and discusses our main results. Finally, Section 5 concludes and gives an outlook.

2 The Fund Set-Up

In this section, we describe the pension fund's set-up. If not mentioned otherwise, we work on a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\mathcal{F} = (\mathcal{F}_t)_{t\geq 0}$. We first address the generations in the fund.

Fund Generations. This part follows closely Chen et al. (2021). We set up the fund at $t = 0$, where t denotes from now on the current *calendar year*. Let $\tau_w \in \mathbb{N}$ be the

[4]The *Royal Mail Collective Plan* is expected to launch in 2023.

age of work begin and $\tau_R \in \mathbb{N}$ be the age of retirement. Consequently, we consider a generation with age τ in the calendar year t to be a *working generation* if and only if

$$\tau_w \leq \tau < \tau_R. \tag{2.1}$$

We identify each generation uniquely via the *calendar year of retirement* $i \in \mathbb{N}$:

$$i := i(\tau, t) := \tau_R - \tau + t, \quad \tau \in \mathbb{N}, \quad t \in \mathbb{N}. \tag{2.2}$$

Given a retirement age of $\tau_R = 65$, let us consider the example of a generation of age $\tau = 35$ in the calendar year $t = 0$. According to (2.2), this generation will retire in calendar year $i = 65 - 35 + 0 = 30$. Ten years later, at $t = 10$, the generation is of age $\tau = 45$ but still retires in calendar year $i = 65 - 45 + 10 = 30$. Hence, the calendar year of retirement i is for every generation fixed. Therefore, we use it to identify any given generation from now on, where a generation with a smaller i represents an older one. We denote with I the *set of all working generations.*

Moreover, the *calendar year of work begin* i_w in which the generation starts to work is given as

$$i_w = \; i - (\tau_R - \tau_w).$$

Note that $i_w < 0$ means merely that the generation has already started to work before the fund is set up at $t = 0$. With the calendar year of retirement as the identifier at hand, we pin down the *set of all working generations at time* t as

$$I_t^w := \{i \in I | i_w \leq t < i\}. \tag{2.3}$$

Note that later on a risk sharing rule will rely on all possible collections $J_t \subset I_t^w$ of working generations that work at time t. We refer to J_t as a *subfund.*

Fund Investment. Every generation i joins the fund to receive at least a lump sum benefit of $L_i > 0$ upon retirement. Younger generations take the time value of money into account, that is, they demand a higher lump sum benefit than older generations. Therefore, in order to make the lump sums comparable, we define the present value of the lump sum benefit upon entering the fund as $\bar{L}_i = e^{-r \cdot i} L_i$ where $r > 0$ denotes the deterministic risk-free interest rate. We assume throughout this paper that only two assets are available for investment to deliver the lump sum benefit upon retirement.

First, the fund may invest in a risk-free asset B which satisfies

$$\mathrm{d}B_t = rB_t\mathrm{d}t, \quad B_0 \text{ given.}$$

Any remaing part of the fund's reserve is then invested in a risky asset S with instantaneous rate of return $\mu > 0$ and volatility $\sigma > 0$ satisfying

$$\mathrm{d}S_t = \mu S_t\mathrm{d}t \ + \ \sigma S_t\mathrm{d}W_t, \quad S_0 < \infty \text{ given,}$$

where $(W_t)_{t\in[0,T]}$ denotes a standard Brownian motion under the real-world probability measure P. This financial market model has become one of the standard models since the pioneering work of Samuelson (1969).

The pension fund manager stipulates the fund's investment strategy within the described financial market. The fraction invested in the risky asset hinges on the working generation's relative risk aversions where we denote with $\gamma_t^i, i \in I_t^w$ the relative risk aversion of the working generation i at time t. We assume that older generations have a higher relative risk aversion level compared to younger ones, i. e.,

$$\gamma_t^j < \gamma_t^i, \quad i, j \in I_t^w, \quad j > i. \tag{2.4}$$

This choice reflects that workers usually invest a smaller fraction of their wealth in risky assets as they grow older. Numerous studies support this assumption. For instance, Jianakoplos and Bernasek (2006) detect a negative effect of age on the willingness to take risk based on data from multiple Surveys of Consumer Finances provided by the U.S. Federal Reserve.[5]

Depending on the working generations' risk aversions, the pension fund manager fixes for the fund $J_t \subset I_t^w$ the investment in the risky asset to be

$$\pi_t^{J_t} = \frac{\mu - r}{\sigma^2 \gamma^{J_t}} \tag{2.5}$$

with a fund risk aversion γ^{J_t} given as the weighted geometric mean $\gamma^{J_t} = \prod_{i\in J_t}(\gamma_t^i)^{w_i}$ where $w_i = \frac{\bar{L}_i}{\sum_{j\in J_t}\bar{L}_j}$. The weighting factor w_i takes the size of the respective working generation into account. Consider for example a big working generation that consequently demands a higher discounted lump sum benefit $\bar{L}_i$ for the entire generation compared to the other generations. Hence, its associated weighting factor w_i is the biggest. As a result, the generation's risk aversion influences the strongest the common

[5]Other pertinent studies are for example Bucciol and Miniaci (2011) or McInish (1982).

risk aversion γ^{J_t}.[6]

Fund Reserve & Assets. The investment strategy determines how much capital needs to be provided to set up the fund. For each generation $i \in J_t \subset I_t^w$, we discount its target lump sum benefit L_i under the future financial market development, taking the fund's investment strategy into account. We then define the *required reserve* R_t^J for the fund J_t as the quantile P_α of the sum of discounted lump sum benefits, that is,

$$R_t^J = P_\alpha \left(\sum_{i \in J_t} \prod_{s=t}^{i-1} e^{-(\pi_s^J \mu + (1-\pi_s^J) r - \frac{1}{2}(\pi_s^J)^2 \sigma^2) - \pi_s^J \sigma (W_{s+1} - W_s)} \cdot L_i \right), \quad t = 0, 1, 2 \ldots \tag{2.6}$$

where we call $\alpha \in (0,1)$ the *quantile level*. We write $R_t^J \equiv R_t^{J_t}$ and $\pi_t^J \equiv \pi_t^{J_t}$ for the sake of brevity of notation to which we shall adhere to in the following. Note that due to definition (2.3), it follows from $i \in J_t$ that $t \leq i - 1$.

Similar reserving principles have been considered by Vanduffel et al. (2003) and Dhaene et al. (2005). The interpretation is as follows. Under the assumption of no future capital injections and no risk sharing, investing the required reserve R_t^J ensures with probability α that the fund can meet its liabilities. In other terms, the target lump sums may be paid out to the generations upon retirement with probability α.

The generations in any fund $J_t \subset I_t^w$ provide collectively at the beginning of every calendar year t the required reserve R_t^J. The reserve is invested in the financial market according to the fund's investment strategy π_t^J. Therefore, the dynamics of the *fund's assets* can be described as

$$\mathrm{d}A_t^J = \underbrace{(1 - \pi_t^J) A_t^J r \, \mathrm{d}t}_{\text{risk-free investment}} + \underbrace{\pi_t^J A_t^J \, (\mu \, \mathrm{d}t + \sigma \, \mathrm{d}W_t)}_{\text{risky investment}}, \quad t \in [s, s+1], \quad A_0^J = R_0^J, \tag{2.7}$$

$$A_{t+}^J = R_t^J, \quad t \geq 0 \tag{2.8}$$

where $s = 0, 1, 2 \ldots$, and $A_{t+}^J := \lim_{\epsilon \to 0} A_{t+\epsilon}^J$ denotes the right continuous limit. Note that (2.8) stipulates a *rebalancing* of the fund's asset according to the required reserve at the beginning of each calendar year. For instance, if the fund's assets undercut the required reserve at the beginning of the calendar year, the working generations need to put additional money into the fund. We shall refer to this as *deficit* in the fund in the following. It is also possible that the assets exceed the required reserve. In such a case, we shall speak of a *surplus* in the fund. Any surplus is paid out to the

[6] We use this investment strategy because it is simple to implement and flexible enough to include different risk aversions and different sizes of generations. Note that the risk-aversion level is chosen similar to the one in Jensen and Nielsen (2016).

generations.[7] Note that upon retirement of one generation, the generation's share in the fund assets constitutes its lump sum benefit as explained in the following paragraph.

Fund Cashflows. To determine how to share the fund reserve R_t, any fund deficit (or surplus) $A_t - R_t$ or the assets A_t at retirement among the working generations $J_t \subset I_t^w$, we stipulate a *risk sharing rule*:

Definition 2.1. *A risk sharing rule is a collection of functions* $(\Phi^{i,J_t})_{i\in J_t}$ *with* $\Phi^{i,J_{(\cdot)}}(\cdot) : \{0,1,\ldots,T\} \times \mathbb{R} \to \mathbb{R}, (t,x) \mapsto \Phi^{i,J_t}(x)$ *which distributes a money amount* x *among the working generations, i. e.,*

$$\sum_{i\in J_t} \Phi^{i,J_t}(x) = x.$$

In particular, the risk sharing rule determines how the remaining generations provide the guarantee of the target benefit L_i to the retiring generation i. This guarantee at time $t = i$ is given by the remaining generations $j \in J_{i-1}\backslash\{i\}$ via a *protection payment* $\chi_i^{i,J_{i-1}\backslash\{i\}} = (L_i - \Phi^{i,J_{i-1}}(A_i))^+$ where $(X)^+ := \max\{X, 0\}$. In other terms, if the share in the fund $\Phi^{i,J_{i-1}}(A_i)$ of the retiring generation i is below the target benefit L_i, the remaining generations pay the difference. At the same time, generation i contributes to the protection payment for any retiring generation prior to its own retirement. Let us assume, for instance, that the share of generation $i = 20$ in the fund $\{20, 30, 40\}$ amounts to $\Phi^{20,\{20,30,40\}} = 9500$ euros which is below the target benefit $L_{20} = 10000$ euros. Therefore, generation $i = 20$ receives a protection payment of $\chi^{20,\{30,40\}} = 500$ euros from the remaining generations 30 and 40.

Given a risk sharing rule, we define the cashflows C_t^i at time $t \in \{i_w, \ldots, i\}$ from the point of view of the working generation i under collective investment:

$$C_t^i := \begin{cases} -\Phi^{i,J_t}(R_t), & \text{if } t = i_w, \\ \Phi^{i,J_t}(A_t - R_t), & \text{if } i_w < t < i \text{ and } t \notin J_{t-1}, \\ \Phi^{i,J_{t-1}}(A_t) - \Phi^{i,J_{t-1}\backslash\{t\}}(\chi_t) - \Phi^{i,J_t}(R_t), & \text{if } i_w < t < i \text{ and } t \in J_{t-1}, \\ \Phi^{i,J_{t-1}}(A_t) + \chi^{i,J_{t-1}\backslash\{i\}}, & \text{if } t = i. \end{cases}$$

The interpretation is as follows. When starting to work ($t = i_w$), the generation needs to pay its share in the required reserve. At the beginning of each following calendar

[7] Alternatively, surpluses may fund a *solidarity reserve* which is used to smooth subsequent slumps of the financial market. Such solidarity reserves are common practice in the Netherlands (Westerhout et al. (2021)).

year, the generation has to pay (or receives) its part in the difference between the assets and the required reserve given no other generation retires ($t \notin J_{t-1}$). If another generation retires ($t \in J_{t-1}$), the assets of the fund are distributed among the participating generations and adjusted – if necessary – for the share in the protection payment to the retiring generation. Consequently, the retiring generation leaves the fund and a new generation starts to work (i. e., $J_{t-1} \neq J_t$). Therefore, generation i has to pay for its share in the required reserve of the new fund J_t. Upon retirement ($t = i$), generation i leaves the fund with its share in the fund's assets and – if necessary – with the protection payment from the remaining generations so that its benefit upon retirement is at least the target benefit L_i. Note that $\Phi^{i,J_{t-1}}(A_t) + \chi^{i,J_{t-1}\setminus\{i\}} \geq L_i$ indeed.

Paying their entire share in the required reserve at once at the beginning of their working career might not be possible for all participants. Therefore, we annuitize in the following the initial contribution $\Phi^{i,J_{i_w}}(R_{i_w})$ to smooth the generations' cashflows over time which brings our setting closer to the usual practice. That is, we find an annual contribution, denoted as $\tilde{\Phi}^{i,J_{i_w}}(R_{i_w})$, whose present value equals the initial contribution:

$$\Phi^{i,J_{i_w}}(R_{i_w}) = \sum_{k=0}^{i-1} e^{-rk} \cdot \tilde{\Phi}^{i,J_{i_w}}(R_{i_w}).$$

Subsequently, we define the *annual cashflows* $\tilde{C}_t^i$ as

$$\tilde{C}_t^i = \begin{cases} -\tilde{\Phi}^{i,J_{i_w}}(R_{i_w}), & \text{if } t = i_w, \\ C_t^i - \tilde{\Phi}^{i,J_{i_w}}(R_{i_w}), & \text{if } i_w < t < i, \\ C_t^i, & \text{if } t = i. \end{cases}$$

Fund Example. In the following simulation study, we consider a fund with a minimum required age of $\tau_w = 35$ and where working generations retire at the age of $\tau_R = 65$. At $t = 0$, the fund starts with three working generations which retire in 10, 20 and 30 years, respectively. When one generation retires, a new working generation of age $\tau_w = 35$, which retires in 30 years, joins the fund. Consequently, we consider a *three-generation fund* which comprises for any $t \geq 0$

- an *old* generation O which retires in 10 years,
- a *middle* generation M which retires in 20 years, and
- a *young* generation Y which retires in 30 years.

The generations' relative risk aversion develops according to

$$\gamma_t^i = \begin{cases} \gamma_O = 10, & \text{if } i - t \leq 10, \\ \gamma_M = 5, & \text{if } 10 < i - t \leq 20, \\ \gamma_Y = 2, & \text{if } 20 < i - t \leq 30, \end{cases}$$

where $i \in I_t^w, t \geq 0$. Thereby, each generation's risk aversion increases as time goes by and specifically takes three values in the entire working period. For instance, during the last ten years before retirement each generation has a relative risk aversion of an old generation which is assumed to be $\gamma_O = 10$. Clearly, this definition respects the requirement laid out in (2.4).

Besides these choices, we assume the following parameters in our simulation study: The market return amounts to $\mu = 6.5\%$, the risk-free interest rate is $r = 3\%$, and the volatility $\sigma = 17.5\%$ which results in a market Sharpe ratio of 20%. All working generations desire the same discounted target benefit $\bar{L}_i = 10\,000$ euros. The required fund reserve is calculated at a quantile level of $\alpha = 90\%$ at any time $t \in \{0, 1, 2, \ldots\}$. Our simulation comprises 10 000 scenarios. We refer to this parameter setting as the *baseline case* throughout this text.[8]

3 Linear Risk Sharing & Alternatives

To share the investment risk, we promote the use of the following risk sharing rule:

Linear Risk Sharing. This rule shares the investment risk between the generations by splitting any money amount x proportionally with respect to the generations' required reserve if they were to set up the fund on their own. This is, we compute generation's i share – denoted by $a_{i,t}$ – as

$$a_{i,t} = \frac{R_t^i}{\sum_{j \in I_t^w} R_t^j} \tag{3.1}$$

where $R_t^i \equiv R_t^{\{i\}}$ denotes the required reserve of a fund that only contains generation i. We then set $\Phi^{i,I_t^w}(x) = a_{i,t} \cdot x$ to which we refer as *linear risk sharing* throughout the

[8]We note that the fraction invested in the risky asset of $\pi^{\{10,20,30\}} \approx 25\%$ is lower than the average of the fraction if the generations invested on their own (which is approximately 30%). Therefore, our results hinge less on good stock market returns. In other words, we take a relatively conservative approach.

remainder of the paper.[9] Note that the proportion $a_{i,t}$ changes over time as the required reserve evolves. The evident advantage of this rule is that it is easy to communicate and to implement. Moreover, the rule can be deemed as reasonable in that benefits and reserves are shared among the generations based on the risk, quantified by the reserve requirement, they bear if they were to save for their retirement on their own.

To gain intuition for this rule, Table 1 displays how the difference between fund assets $A_t \equiv A_t^{I_t^w}$ and required reserve $R_t \equiv R_t^{I_t^w}$ is shared under linear risk sharing among the generations for two scenarios ω_A and ω_B at time $t = 5$. Note that we refer in the following to scenarios with a surplus, that is $A_t - R_t > 0$, in the fund as *favorable scenarios*. On the other hand, we deem scenarios with a deficit, that is $A_t - R_t \leq 0$, as *unfavorable*.

	Scenario ω_A	**Scenario ω_B**
$A_t - R_t$	-499.68	499.03
$\Phi^{10,I_t^w}(A_t - R_t)$	$-158.57(\approx 32\%)$	$158.36(\approx 32\%)$
$\Phi^{20,I_t^w}(A_t - R_t)$	$-163.15(\approx 33\%)$	$162.94(\approx 33\%)$
$\Phi^{30,I_t^w}(A_t - R_t)$	$-177.96(\approx 36\%)$	$177.73(\approx 36\%)$

Table 1: Sharing of the difference between fund assets A_t and required reserve R_t among the three working generations at time $t = 5$ according to the linear risk sharing rule with the generations' respective share $a_{i,t}$ given in brackets. Parameter choice as in the baseline case. All numbers are given in euros.

In both scenarios, favorable (scenario ω_A) and unfavorable (scenario ω_B), the generations share the difference between assets and required reserve almost equally among each other. This observation is backed by Figure 1 which gives for every simulated outcome of $A_t - R_t$ the share of the respective generations $\Phi^{i,I_t^w}(A_t - R_t)$ at $t = 5$. A kernel density estimator indicates that the chosen scenarios lie within a likely range.

Considering the whole time period in the fund of the working generation $i = 30$, Figure 2 visualizes the distribution of the resulting annual cashflows $\tilde{C}_t^{30}$ under linear risk sharing before (Figure 2a) and at retirement (Figure 2b).

[9] This risk sharing rule resembles a capital allocation principle known as the *haircut allocation principle* (see e.g. Dhaene et al. (2012)).

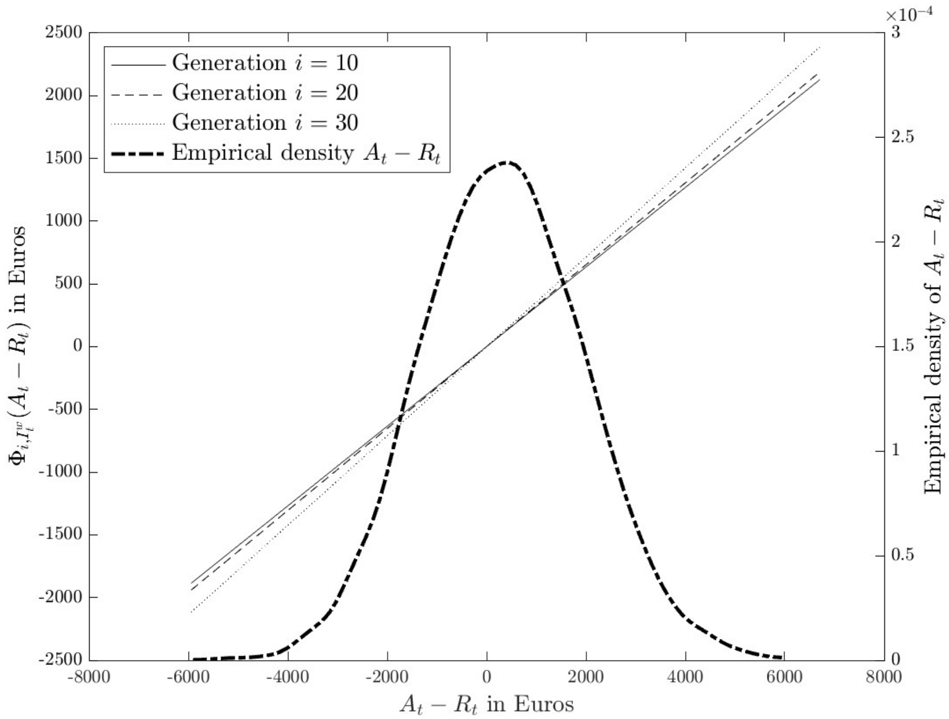
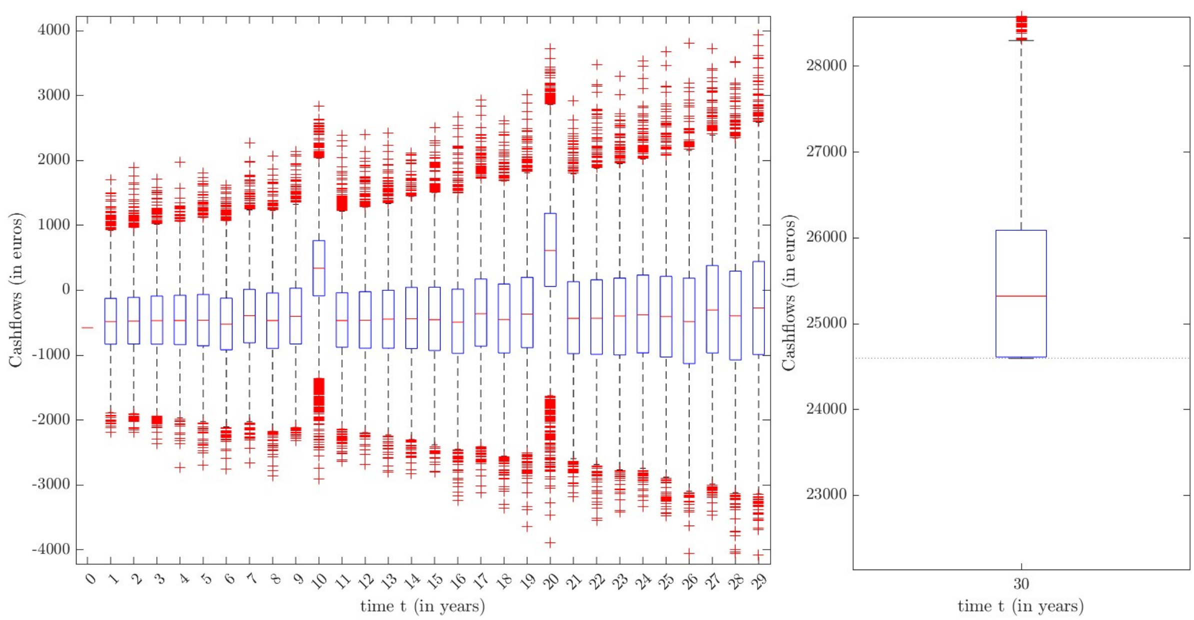


Figure 1: The shares $\Phi^{i,I_t^w}(A_t - R_t)$ of all generations at time $t = 5$ under linear risk sharing for 10,000 simulated scenarios in the difference between fund assets A_t and required reserve R_t. Parameter choice as in the baseline case.



(a) cashflow pattern before retirement

(b) cashflow at retirement

Figure 2: Annual cashflows $\tilde{C}_t^{30}$ of generation $i = 30$ (a) before and (b) at retirement under linear risk sharing. Parameter choice as in the baseline case.

The generation has to pay the annual contribution to the required reserve $\tilde{\Phi}^{30,I_0^w}(R_0)$ of approximately 577 euros each year. Starting from $t = 1$, the contribution is either reduced or increased depending on whether the fund has accumulated a surplus or deficit. Note that for all points in time, the generation receives a discount on its contribution in the median scenario. The discounts may be substantial in the sense that the generation even receives money from the fund in some years before retirement. For example, in the best-case scenario at time $t = 29$ the fund pays out 4,000 euros to the generation. On the other hand, if the financial market plummets, the contribution to the fund in the worst-case scenario amounts to 4,000 euros. We note that adjustments to contributions in CDC schemes have been observed in the past. For example, contributions in Dutch CDC schemes increased by 40% during the period from 2007 to 2020 (Westerhout et al. (2021)). In practice, large adjustments are frequently spread over several years, for instance in the UK or in the Netherlands.[10]

At points in time where other generations retire, we remark two peculiarities. First, the distribution is shifted in positive y-direction such that in the median scenario the generation receives money from the fund. This is due to the fact that the share of generation $i = 30$ in the required fund reserve shrinks, when the new generation with a bigger target benefit joins. For example, at $t = 10$, the share of generation $i = 30$ decreases from 14,778 euros to 14,040 euros. The second peculiarity is that the distribution of cashflows exhibits a fatter lower tail. This is due to the protection payments which are payable by generation $i = 30$ to the retiring generation in case of a drawdown of the financial market. Lastly, we remark that the volatility of the cashflows increases over time. Younger generations demand for a nominally higher target benefit than older generations as they anticipate future inflation. As a result, the required fund reserve increases. Hence, the fund has more money for investment at its disposal.

Upon retirement at $t = 30$, the generation is guaranteed to receive at least the target benefit of $L_{30} = 24{,}596$ euros as Figure 2b depicts. If the financial market's performance is not sufficient to fund this lump sum, the remaining generations $i = 40$ and $i = 50$ pay the difference so that the generation $i = 30$ receives exactly L_{30}. If the performance is sufficient, generation $i = 30$ leaves the fund with its share determined by the linear risk sharing rule. In total, the mean net present value of the protection payments that generation $i = 30$ pays (at $t = 10$ and $t = 20$) and receives (at $t = 30$) is close to zero (95% confidence interval: [−3.76 euros, 3.44 euros]). That is, protection payments from and to generation $i = 30$ level each other out.

[10] In the UK's first CDC scheme, the Royal Mail Collective Plan, decreases in income of more than 5% may be spread over two or three years.

In the following, we benchmark the proposed fund design under linear risk sharing with two alternatives that the pension fund manager could offer the generations. First, she may provide the alternative to each generation to invest on its own, i. e. save individually for its retirement. We refer to this option as *individual defined contribution* (IDC) scheme. This alternative allows us to determine whether our proposed fund design sets an incentive for all generations to join the fund. Second, the pension fund manager could propose to use a non-linear risk sharing rule. For this, we opt for a non-linear risk sharing rule based on the Shapley value from Shapley (1953) (hereafter called *Shapley risk sharing rule*). Shapley sharing is widely used in several cases (even beyond economics), since it can be seen as a natural way to share contributions to a collective action.[11] Although, the practical implementation of this rule may be complicated, they are designed to fairly divide the gains and losses among different groups of people that act collectively. We use this second option, to assess the performance of the linear risk sharing rule as the centerpiece of our fund design.

IDC scheme. To make the cashflows comparable to the CDC scheme with linear risk sharing, we implement also a guarantee under the IDC scheme. Naturally, the generation needs to finance this guarantee by itself.[12] In our approach within a complete market, we assume that each generation constructs a put option with strike price L_i and maturity $T = i$ when starting to work at $t = i_w$. The cost, payable at $t = 0$ by the generation, amounts to the put option's risk-neutral price $p_{i_w} = \mathbb{E}_Q[e^{-r(i-i_w)}(L_i - A_i^i)^+]$ where Q denotes the risk-neutral measure. We stipulate that the working generation invests

$$\pi_t^i = \frac{\mu - r}{\sigma^2 \gamma_t^i} \tag{3.2}$$

in the risky asset depending merely on its own relative risk aversion γ_t^i. This portfolio is consistent with the one found by Merton (1969) for an investor who maximizes her expected utility with constant relative risk aversion in our financial market. Note that the fraction in (3.2) invested in the risky asset decreases as the relative risk aversion increases with age. In other words, the IDC scheme invests in a life-cycle manner.

[11] Considering the Shapley value for allocation problems has a long tradition and dates back to at least Littlechild and Owen (1973) who apply it in the context of allocating an airport's maintenance costs to different plane types.

[12] We note that as the guaranteed payment is fully covered by each generation on its own, the IDC scheme is somewhat handicapped in comparison with the CDC scheme. Yet, we shall see in Section 4.1 that this handicap is not decisive in determining which scheme is superior.

[13] For a further discussion of life-cycle investments and their optimality we refer the interested reader to Graf (2017) and Bosserhoff et al. (2022).

Each generation sets up a fund on its own by providing the required reserve. Consequently, any subsequent deficit or surplus of the fund is borne entirely by (or paid out to) the working generation. Hence, generation i faces the following cashflows under the IDC scheme:

$$C_t^i := \begin{cases} -R_t^i - p_{i_w}, & \text{if } t = i_w, \\ A_t^i - R_t^i, & \text{if } i_w < t < i, \\ A_t^i + (L_i - A_t^i)^+, & \text{if } t = i, \end{cases}$$

where $R_{i_w}^i$ stands for the initial required reserve of a fund that only contains generation i. Again, we smooth the generations' cashflows over time by annuitizing the initial contribution $C_{i_w}^i = -R_{i_w}^i - p_{i_w}$ to get the annual cashflows $\tilde{C}_t^i$.

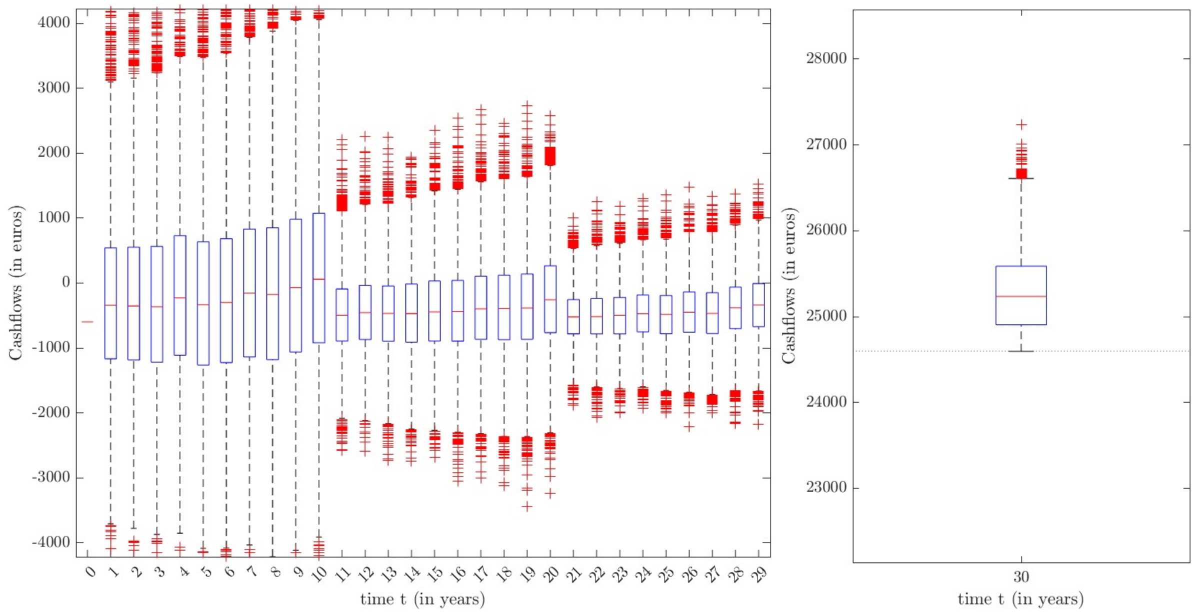
Table 2 shows the performance under the IDC scheme for the same two scenarios ω_A and ω_B at $t = 5$ as in Table 1. The youngest generation, which is the least risk averse, invests now a higher fraction in the risky asset than the CDC scheme does. Consequently, its losses and gains in the respective scenario outstrip the ones under linear risk sharing. On the other hand, the oldest generation $i = 10$, which is the most prudent generation, invests a smaller fraction in the risky asset than the CDC scheme does. As a result, its gains and losses under the IDC scheme are smaller than under linear risk sharing.

	Scenario ω_A	**Scenario ω_B**
$\sum_{i \in I_t^w} A_t^i - R_t^i$	-617.71	685.56
$A_t^{10} - R_t^{10}$	-57.89	92.66
$A_t^{20} - R_t^{20}$	-128.29	181.73
$A_t^{30} - R_t^{30}$	-431.54	411.18

Table 2: Difference between assets A_t^i and required reserve R_t^i for each working generation at time $t = 5$ under IDC scheme. Parameter choice as in the baseline case. All numbers are given in euros.

[13] In fact, our collective investment strategy could be seen as a blend of different life-cycle investment strategies.

Figure 3a depicts the resulting annual cashflows of working generation $i = 30$ under the IDC scheme before retirement.



(a) cashflow pattern before retirement

(b) cashflow at retirement

Figure 3: Annual cashflows $\tilde{C}_t^{30}$ of generation $i = 30$ (a) before and (b) at retirement under the IDC scheme. Parameter choice as in the baseline case.

We assume that the generation demands the target benefit L_{30} also under the IDC scheme. This results in an annual contribution of approximately 596 euros which is slightly bigger than under linear risk sharing, mostly due to the put option's cost of 15 euros in total. Depending on the financial market's performance, the contribution is again reduced or increased. In the median scenario, the generation receives a discount. We remark that, contrary to the fund with linear risk sharing, the volatility of the annual contribution declines over time. This is due to the fact that the generation is increasingly risk averse which triggers a reduced investment in the risky asset according to (3.2).

At retirement, the generation receives at least the target benefit of $L_{30} = 24{,}596$ euros as Figure 3b shows. This time, in case of insufficient financial market performance, the generation exercises the put option purchased at $t = 0$ to receive the target benefit.

This closes our description of the IDC scheme as the first alternative. We move on to the second alternative that the pension fund manager might offer: A fund with a non-linear risk sharing rule.

Shapley Risk Sharing. A non-linear risk sharing rule may be defined based on a concept which attracted a lot of attention in the field of game theory: The *Shapley value* presented first by Shapley (1953). Formulated as a risk sharing rule in our context, it stipulates the individual share of generation i in the fund's surplus (or deficit) x at time t depending on its marginal contributions $\Delta^i_{J_t}$ thereto:

$$\Phi^{i,I_t^w}(x) = \sum_{J_t \subset I_t^w,\, i \in J_t} \frac{(|J_t|-1)!(N-|J_t|)!}{N!} \underbrace{(x^{J_t} - x^{J_t/\{i\}})}_{=:\Delta^i_{J_t}},$$

where N is the number of working generations and x^{J_t} denotes the fund's surplus (or deficit) which is due if we only consider the fund $J_t \subset I_t^w$. In other terms, each generation needs to contribute its average marginal contribution to the money amount where the average is taken over all possible subfunds $J_t \subset I_t^w$. Note that we define $x^{\{\}} \equiv 0$.

In Table 3, we apply the Shapley risk sharing rule to the same scenarios ω_A and ω_B as before. This time, in scenario ω_A, the biggest share (approximately 59%) is borne by the young generation $i = 30$ while the old generation $i = 10$ needs to provide the smallest part. In return, the young generation also receives the lion's share (approximately 62%) in scenario ω_B when there is a surplus to distribute among the generations. In this case, the old generation gets the smallest part of the surplus.

	Scenario ω_A	**Scenario ω_B**
$A_t - R_t$	-499.68	499.03
$\Phi^{10,I_t^w}(A_t - R_t)$	$-28.32(\approx 6\%)$	$17.48(\approx 4\%)$
$\Phi^{20,I_t^w}(A_t - R_t)$	$-133.44(\approx 27\%)$	$144.10(\approx 29\%)$
$\Phi^{30,I_t^w}(A_t - R_t)$	$-337.92(\approx 68\%)$	$337.45(\approx 68\%)$

Table 3: Sharing of the difference between assets A_t and required reserve R_t among the three working generations at time $t = 5$ according to the Shapley risk sharing rule with the generations' respective share given in brackets. Parameter choice as in the baseline case. All numbers are given in euros.

This is due to the fact that the proportion invested in the risky asset increases when the young generation $i = 30$ joins the fund (e.g. from $\pi_t^{\{10,20\}} = 16.16\%$ to $\pi_t^{I^w} = 24.62\%$). Thereby, the fund's exposure to investment risk grows. In favorable scenarios as scenario ω_B, where the underlying financial market performs well, a bigger investment in the risky asset proves beneficial, yielding a higher surplus and thus a positive marginal contribution. On the other hand, in an unfavorable scenario, the deficit increases due to the stronger position in the risky asset. The marginal contribution is then negative. Therefore, the young generation takes always the biggest share compared to the other generations – in surpluses *and* deficits. For further details, we refer the interested reader to Table 7 in the appendix. It provides an example calculation of the Shapley value for generation $i = 10$ and $i = 30$ in scenario ω_B. Figure 4 depicts the described pattern for all simulated scenarios at $t = 5$. In a nutshell, under Shapley risk sharing, the older a generation is at a given time t, the less it pays to and the less it receives from the fund. In other words, older generations experience smaller adjustments of their annual contributions than younger generations if the financial market over- or underperforms.

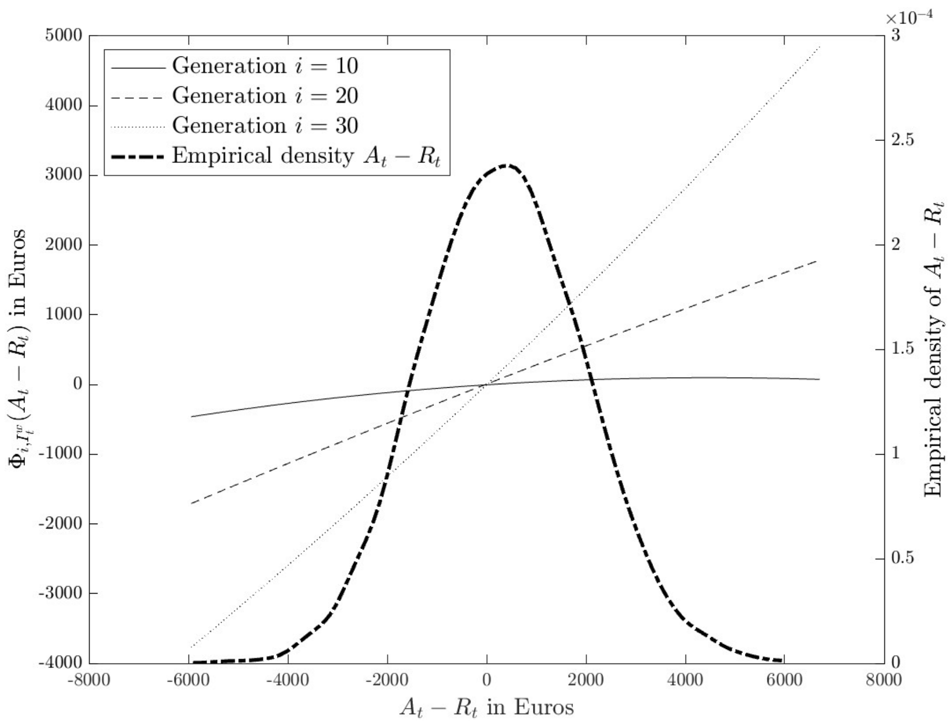
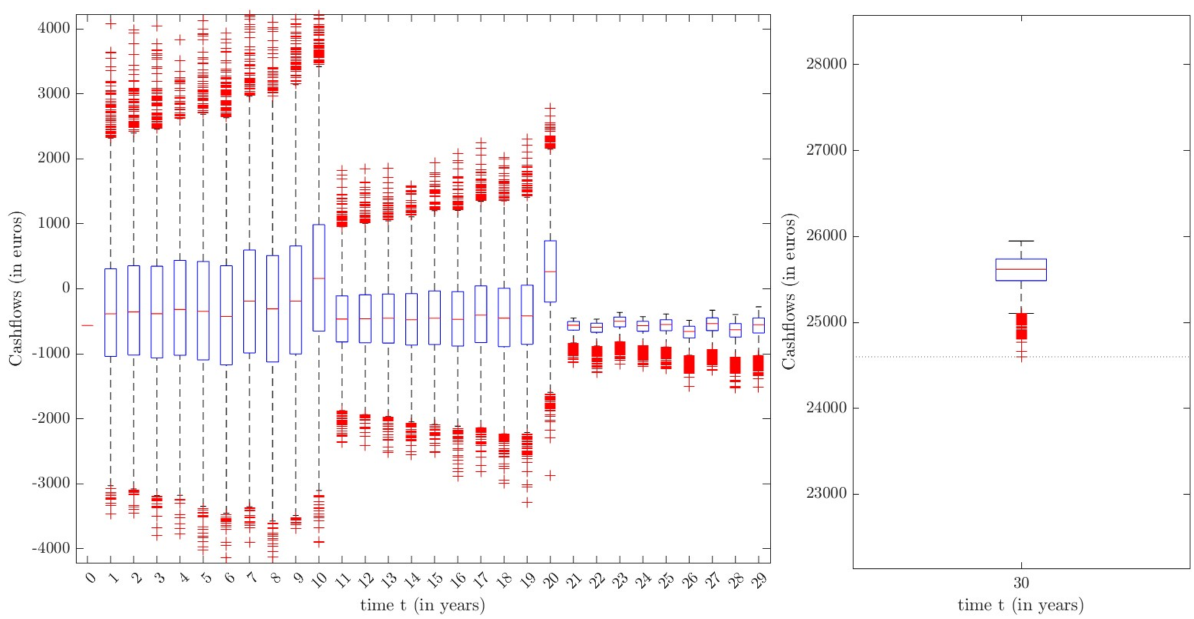


Figure 4: The shares $\Phi^{i,I_t^w}(A_t - R_t)$ of all generations at time $t = 5$ under Shapley risk sharing for 10,000 simulated scenarios of the difference between fund assets A_t and required reserve R_t. Parameter choice as in the baseline case.

Considering the entire time period until retirement, the aforementioned observation entails a decreasing volatility of the generation's annual cashflows over time as Figure 5 displays for generation $i = 30$. During the first ten years (when the generation is young) the generation experiences high volatility in the cashflows whereas in the last ten years (when the generation is old) before retirement the volatility is comparably low (Figure 5a). In fact, the volatility for the old generation is that low so that, upon retirement, only in outlier scenarios the guarantee must be triggered to balance the bad financial market performance out (Figure 5b).



(a) cashflow pattern before retirement (b) cashflow at retirement

Figure 5: Annual cashflows $\tilde{C}_t^{i30}$ of generation $i = 30$ (a) before and (b) at retirement under Shapley risk sharing. Parameter choice as in the baseline case.

4 Results

This section compares the CDC scheme with linear risk sharing to the two introduced alternatives, the IDC scheme and the CDC scheme with Shapley risk sharing. Our approach is twofold. First, we analyze the schemes' performances in the baseline case parameter setting. Second, we perform a sensitivity analysis with respect to the underlying market dynamics, risk aversion, quantile level and demographics. In particular, we elaborate on a 'double-hit' scenario where underlying market dynamics and demographics are unfavorable.

To take contributions and benefits into account, we compute the net present value

(NPV) of each cashflow scenario of generation i:

$$\text{NPV}_\text{i} = \underbrace{C^i_{i_w}}_{\substack{\text{share in the fund's}\\\text{initial reserve}}} + \underbrace{\sum_{t=i_w+1}^{i-1} \frac{C^i_t}{e^{r(t-i_w)}}}_{\substack{\text{share in the fund's}\\\text{deficits and surpluses}}} + \underbrace{\frac{C^i_i}{e^{ri}}}_{\substack{\text{benefit upon}\\\text{retirement}}}.$$

To evaluate the schemes across all scenarios for every generation i, we choose three key performance indicators (KIPs) based on the net present value. First, to measure the average benefit we consider the *expected net present value* ($\mathbb{E}[\text{NPV}_i]$). Second, we inspect the *95% Value at Risk* (V@R_i) to measure risk. Yet, investors try to balance average benefit and undertaken risk when they make investment decisions and never focus solely on one of these measures. Hence, to take the average benefit *and* risk into account, we compute the *Sharpe ratio* (SR_i) of the net present value defined as

$$\text{SR}_i = \frac{\mathbb{E}[\text{NPV}_i]}{\sqrt{\text{Var}(\text{NPV}_i)}}.$$

where $\text{Var}(\text{NPV}_i)$ denotes the variance of the NPV. We perform our analysis under the real-world measure P unless otherwise mentioned.

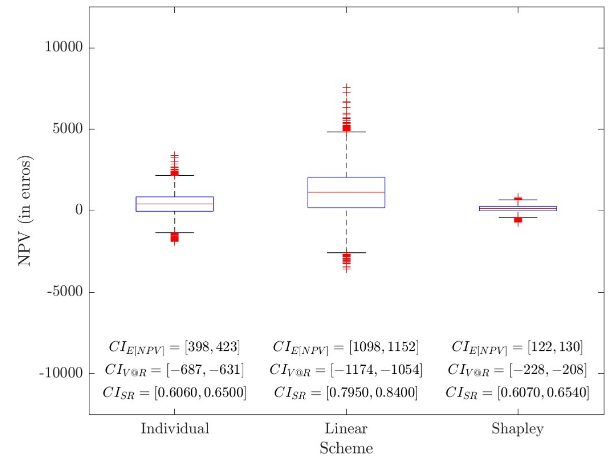
4.1 Baseline case

Figure 6 compares the empirical distributions of the net present values for the first four generations $i \in \{10, 20, 30, 40\}$. Note that generation $i = 40$ joins the fund only upon retirement of generation $i = 10$ in calendar year $t = 10$. We include the first four generations to explore stability over time in our analysis. For each generation, a box plot depicts the distribution of the net present values under the IDC scheme and the CDC scheme with either linear or Shapley risk sharing. Beneath each box plot, the 95% Monte Carlo confidence intervals for the expected net present value, the 95% Value at Risk and the Sharpe ratio are stated.

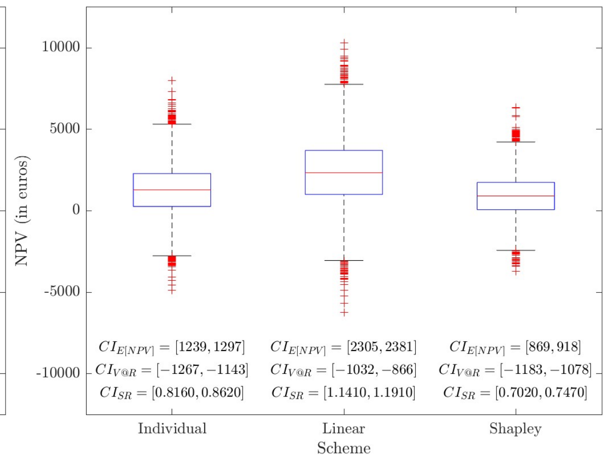
As for the expected NPV, the CDC scheme with linear risk sharing performs better than the IDC scheme for the oldest generations $i = 10$ and $i = 20$. This comes unsurprisingly out of two reasons: First, and most important, the CDC scheme invests a higher fraction in the risky asset than the two IDC schemes do. On average, this pays off due to the positive equity premium. Second, these generations have to support either no ($i = 10$) or only one ($i = 20$) generation while being protected by two generations upon retirement. In other terms, they get the guarantee of their target lump sum without providing a respective support. Yet this is not decisive as it only increases the expected NPV in comparison with the IDC scheme by approximately 75 euros ($i = 10$) and 45 euros ($i = 20$) respectively. Note, that this effect is imperceptible under Shapley risk sharing since the guarantee under that scheme is hardly triggered as discussed above with the help of Figure 5b. On the other hand, the younger generations $i = 30$ and $i = 40$ achieve the highest expected NPV under the IDC scheme, which is also expected. Indeed, IDC induces higher investment positions in the risky asset, which yields a higher return on average. However, the risk-adjusted criterion below indicates that this higher NPV comes with much higher risk when a generation invests individually. Besides the endogenous guarantee, risk-sharing decreases the volatility of all young generations' cashflows.

In particular, the higher expected NPV under the IDC scheme for generations $i = 30$ and $i = 40$ comes at the cost of an increased tail risk, measured by the 95% Value at Risk. In fact, the generations $i = 20, i = 30$ and $i = 40$ obtain the lowest Value at Risk under linear risk sharing whilst the oldest generation $i = 10$ is exposed to the lowest Value at Risk under Shapley risk sharing. The latter observation fits to Figure 5a which displays the lowest volatility in cashflows for generation $i = 30$ when it is the oldest generation in the fund for $t \in [20, 30]$. The first observation hints towards a favorable trade-off between risk and return.

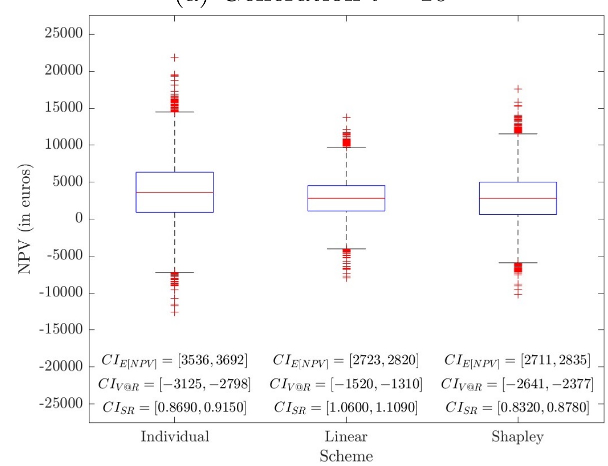
In the baseline case, such a favorable trade-off indeed exists as Figure 6 depicts. Adjusting the expected NPV for the undertaken risk by considering the Sharpe ratio, *all* generations are better off under linear risk sharing. That is, our fund design under linear risk sharing incentivizes in terms of risk-adjusted performance all generations to join. In this sense, we deem our proposed CDC scheme as beneficial for all generations. Next, we analyze via a sensitivity analysis whether the presented argumentation holds for a variety of parameter choices.



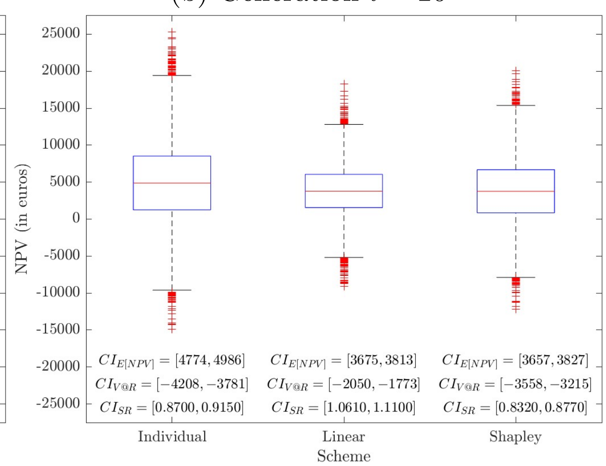
(a) Generation $i = 10$



(b) Generation $i = 20$



(c) Generation $i = 30$



(d) Generation $i = 40$

Figure 6: The distribution of net present values (NPVs) for generation $i = 10$, generation $i = 20$, generation $i = 30$ and generation $i = 40$ compared across schemes. Below each box plot, we provide 95% confidence intervals for $\mathbb{E}$[NPV], Sharpe ratio (SR), and V@R. Parameter choice as in the baseline case. Confidence intervals are obtained via Monte Carlo simulation.

4.2 Sensitivity Analysis

In the following, we explore how sensitive our results in the previous subsection are with respect to our four most important parameter choices.

First, we vary the market Sharpe ratio $(\mu - r)/\sigma$ to evaluate the fund's performance under more adverse or more favorable market conditions. To do so, we keep the risk premium stable at $\mu - r = 3.5\%$ and vary the market's volatility σ. Note that an altered market Sharpe ratio also triggers a change in the fraction invested in the risky asset according to the definition in (2.5).

Second, we scrutinize the effect of the life-cycle risk aversion $(\gamma_O, \gamma_M, \gamma_Y)$. In one scenario, we consider a parameter choice of constant risk aversion. In that case, we choose the risk aversion parameters such that the fraction invested in the risky asset equals $\pi_O = \pi_M = \pi_Y = 25\%$ which corresponds to the average equity investment of DB plans in Europe (see Ganatra et al. (2019)).

Third, we assess the impact of the quantile level α. Fourth, we analyze the demographic change's repercussions on our fund system as it is a well-known fact that the working age population in most Western countries shrinks. For instance, the European Commission (2020) predicts that by 2070 the size of the working age population in its member states decreases by 18% compared to 2020 which corresponds to an annual decrease of 0.4%. To incorporate this finding in our analysis, we adopt a simple model. Under the assumption that smaller working generations demand a smaller lump sum, we compute

$$L_i = \bar{L} \cdot (1 + d)^i, \tag{4.1}$$

where we set $\bar{L} = 10,000$ euros as in the baseline case and $d = -0.4\%$. For completeness, we also consider a scenario where the working generations increase in size over time with $d = 0.4\%$.

The resulting Monte Carlo confidence intervals for all three schemes are shown in Table 4 (expected net present value $\mathbb{E}[\text{NPV}_i]$), Table 5 (95% Value at Risk V@R_i) and Table 6 (Sharpe ratio SR_i) for the first four generations. The scheme with dominating confidence intervals is marked in bold unless the confidence intervals overlap. If the confidence intervals overlap, the one with highest midpoint is printed in italics.

Expected NPV. First, Table 4 confirms for a wide range of parameter choices that the older generations $i = 10$ and $i = 20$ prefer linear risk sharing over the IDC scheme with respect to the expected NPV. Merely, the case of constant relative risk aversion presents an exception. Here, the older generations prefer Shapley risk sharing over the other two schemes. Second, Table 4 also backs the previous subsection's result that the younger generations $i = 30$ and $i = 40$ prefer the IDC scheme over linear risk sharing in terms of expected NPV. Yet, below it turns out again that these higher expected values come with much higher risk.

In addition, Table 4 presents meaningful links between the parameters and the expected NPV. For instance, a higher market Sharpe ratio increases the expected NPV. This is due to the fact that under this circumstance the fund invests a bigger fraction of its money in the risky asset according to (2.5). As the risk premium is positive, this yields a higher NPV on average.

Moreover, if a higher quantile level α is imposed, the expected NPV rises in almost all cases. The reason for this is simple: A higher quantile level α implies a stricter reserve requirement, i. e., a higher reserve and therefore more money available for investment. This means that the nominal amount of money invested in the risky asset increases. Consequently, the NPV increases on average. Only for the older generations $i = 10$ and $i = 20$ under Shapley risk sharing this effect is outweighed by the following. The more capital is available for investment, the higher is the lost return in the average scenario for the youngest generation because the fund invests in general less in the risky asset than the youngest generation would do on its own. This prudent investment strategy is due to the older generations' higher risk aversion. Therefore, the marginal contributions of the older generations to subfunds $J_t \subset I_t^w$ which contain the youngest generation are negative which translates into a negative Shapley value in total. For more details on this, we refer the interested reader to Table 8 in the appendix where the Shapley value of generation $i = 10$ is calculated under a quantile level of $\alpha = 99\%$ in the scenario ω_B.

Lastly, we stress that the demographics, modeled via the target benefit, do not change which generation prefers which scheme. However, if future working generations are bigger than today's, the expected NPV increases for all generations under all schemes – except for generation $i = 10$ under Shapley risk sharing due to the very same reason as mentioned for the quantile level α.

Value at Risk. Table 5 shows that also the results regarding the Value at Risk are remarkably stable regarding our parameter choice. In general, the oldest generation $i = 10$ prefers Shapley risk sharing whereas the other generations prefer linear risk sharing. In other words, all generations under almost all parameter choices prefer the fund with some risk sharing rule over the IDC scheme in terms of tail risk. Only if risk attitudes get more disparate ($\gamma_O = 11.43, \gamma_M = 2.29, \gamma_Y = 1.43$), generation

Expected NPV $\mathbb{E}[\mathrm{NPV_i}]$

Parameter	$i = 10$			$i = 20$			$i = 30$			$i = 40$		
baseline case: $r = 0.03, \mu = 0.065, \sigma = 0.175$ s.t. $(\mu - r)/\sigma = 0.2, \alpha = 0.9, \bar{L}_i = 10000\ \forall i, [\gamma_O, \gamma_M, \gamma_Y] = [10, 5, 2]$												
Market Sharpe ratio $(\mu - r)/\sigma$ with $\mu - r = 0.035$	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
0.15	[220, 239]	**[746, 787]**	[69, 75]	[689, 732]	**[1523, 1580]**	[488, 525]	**[1965, 2080]**	[1529, 1601]	[1519, 1612]	**[2652, 2809]**	[2062, 2165]	[2049, 2176]
0.175	[303, 325]	**[917, 964]**	[93, 100]	[944, 995]	**[1899, 1966]**	[665, 708]	**[2698, 2834]**	[2087, 2172]	[2077, 2185]	**[3642, 3827]**	[2816, 2937]	[2800, 2950]
0.2	[398, 423]	**[1098, 1152]**	[122, 130]	[1239, 1297]	**[2305, 2381]**	[869, 918]	**[3536, 3692]**	[2723, 2820]	[2711, 2835]	**[4774, 4986]**	[3675, 3813]	[3657, 3827]
0.225	[506, 535]	**[1292, 1352]**	[156, 166]	[1571, 1637]	**[2737, 2823]**	[1099, 1154]	**[4469, 4644]**	[3429, 3538]	[3415, 3554]	**[6033, 6272]**	[4628, 4782]	[4608, 4799]
0.25	[628, 660]	**[1497, 1564]**	[198, 209]	[1939, 2012]	**[3194, 3289]**	[1356, 1415]	**[5484, 5675]**	[4196, 4316]	[4180, 4333]	**[7401, 7665]**	[5662, 5832]	[5641, 5850]
Risk aversion $[\gamma_O, \gamma_M, \gamma_Y]$	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
$[11.43, 2.29, 1.43]$ s.t. $\pi_O = 10\%, \pi_M = 50\%, \pi_Y = 80\%$	[346, 369]	**[1978, 2048]**	[−628, −587]	[2344, 2473]	**[2749, 2856]**	[1099, 1217]	**[6027, 6289]**	[3919, 4059]	[3888, 4097]	**[8131, 8495]**	[5287, 5487]	[5242, 5524]
$[10, 5, 2]$ s.t. $\pi_O = 11\%, \pi_M = 23\%, \pi_Y = 57\%$	[397, 423]	**[1098, 1152]**	[122, 130]	[1239, 1297]	**[2305, 2381]**	[870, 918]	**[3536, 3692]**	[2723, 2820]	[2711, 2835]	**[4774, 4986]**	[3675, 3813]	[3657, 3827]
$[4.57, 4.57, 4.57]$ s.t. $\pi_O = 25\%, \pi_M = 25\%, \pi_Y = 25\%$	[914, 973]	[288, 343]	*[959, 1015]*	[1876, 1956]	[1439, 1520]	*[1877, 1957]*	*[2821, 2922]*	[2769, 2869]	[2770, 2869]	*[3807, 3948]*	[3738, 3877]	[3738, 3876]
Quantile α	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
0.8	[386, 411]	**[888, 939]**	[157, 167]	[1186, 1241]	**[1845, 1919]**	[875, 922]	**[3226, 3365]**	[2556, 2647]	[2546, 2660]	**[4353, 4545]**	[3450, 3579]	[3436, 3591]
0.85	[391, 417]	**[973, 1025]**	[141, 150]	[1209, 1266]	**[2059, 2134]**	[874, 922]	**[3359, 3505]**	[2629, 2723]	[2617, 2736]	**[4534, 4735]**	[3547, 3679]	[3532, 3693]
0.9	[398, 423]	**[1098, 1152]**	[122, 130]	[1239, 1297]	**[2305, 2381]**	[869, 918]	**[3536, 3692]**	[2723, 2820]	[2711, 2835]	**[4774, 4986]**	[3675, 3813]	[3657, 3827]
0.95	[407, 433]	**[1311, 1366]**	[88, 94]	[1282, 1342]	**[2783, 2860]**	[857, 907]	**[3819, 3989]**	[2872, 2973]	[2857, 2991]	**[5156, 5388]**	[3874, 4020]	[3854, 4037]
0.99	[422, 449]	**[1821, 1879]**	[15, 19]	[1367, 1432]	**[3811, 3893]**	[824, 875]	**[4434, 4639]**	[3190, 3302]	[3170, 3324]	**[5988, 6264]**	[4302, 4465]	[4275, 4486]
Demographic change	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
Decrease by 0.4% p.a.	[382, 407]	**[1029, 1080]**	[129, 137]	[1143, 1197]	**[2073, 2142]**	[816, 861]	**[3136, 3274]**	[2334, 2418]	[2408, 2518]	**[4067, 4248]**	[3025, 3140]	[3121, 3265]
Stable	[397, 423]	**[1098, 1152]**	[121, 129]	[1239, 1297]	**[2305, 2381]**	[869, 918]	**[3536, 3692]**	[2723, 2820]	[2711, 2835]	**[4774, 4986]**	[3675, 3813]	[3657, 3827]
Increase by 0.4% p.a.	[414, 441]	**[1171, 1228]**	[113, 120]	[1342, 1405]	**[2560, 2644]**	[924, 977]	**[3986, 4161]**	[3174, 3285]	[3050, 3190]	**[5600, 5849]**	[4457, 4622]	[4281, 4482]
All parameter changes ceteris paribus												

Table 4: Sensitivity analysis of expected net present value $\mathbb{E}[\mathrm{NPV}_i]$ of the first four generations regarding assumptions with respect to the market (Sharpe ratio $(\mu - r)/\sigma$), the life-cycle risk aversion $(\gamma_O, \gamma_M, \gamma_Y)$, the quantile level α and demographic change (Lump sums L_i). Estimates given as 95% Monte Carlo confidence intervals based on 2000 runs. Scheme with highest mean net present value printed in bold unless the confidence intervals overlap. If the confidence intervals overlap, the one with highest midpoint is printed in italics. All changes ceteris paribus with respect to baseline case.

$i = 10$ prefers the IDC scheme instead of Shapley risk sharing. Furthermore, the case of constant relative risk aversion stands out again where all generations favor Shapley risk sharing. Most importantly, for all but one parameter choices the tail risk for the generations $i = 30$ and $i = 40$ is reduced under linear risk sharing in comparison with the IDC scheme. Hence, a favorable risk-return trade-off is once again possible.

Sharpe ratio. Table 6 shows that linear risk sharing exhibits a favorable risk-return trade-off in general. All generations, generations $i = 30$ and $i = 40$ in particular, obtain the highest Sharpe ratio under linear risk sharing for all parameter choices except for constant relative risk aversion. Hence, our argument that our fund design incentivizes all generations to join in terms of risk-adjusted performance is rather robust.

In addition, Table 6 identifies the market Sharpe ratio and the demographic change as the determinants of the Sharpe ratio under linear risk sharing. First, all generations enjoy a higher Sharpe ratio as the market Sharpe ratio increases. Second, a shrinking working age population decreases the Sharpe ratio in comparison with a stable or even growing workforce.

Next, we shall stress our assumptions regarding these parameters simultaneously in a double-hit scenario regarding the crucial generations $i = 30$ and $i = 40$.

Double-hit scenario. In practice, common stress tests, for instance under Solvency II, include 'double-hit' scenarios. Under these scenarios, the respective model is stressed regarding two parameter choices simultaneously. In this paper, we consider a scenario where the underlying market dynamics and the demographics are severely worse than expected in the baseline case. We calculate for a given market Sharpe ratio the *maximal allowed annual decrease* d^* in the working age population such that the third generation still prefers linear risk sharing over the IDC scheme in terms of risk-adjusted performance measured by the Sharpe ratio. A similar analysis can be conducted for generation $i = 40$ which yields similar results.

Figure 7 depicts for a given market Sharpe ratio the resulting maximal allowed annual decrease d^*. Unsurprisingly, a better market performance (higher market Sharpe ratio) allows a higher annual decrease in the working age populations as compared with worse market performance. In our baseline case with a market Sharpe ratio of 20%, the maximal allowed annual decrease d^* amounts to $d^* \approx -4.80\%$. This is thus more than ten times higher than the European Commission (2020) predicts (see page 24). Even in the case of worse market performance with a market Sharpe ratio of only 15%, we can allow for a decrease of up to $d^* \approx -2.90\%$ per year which is still remarkably

Value at Risk $V@R_i$

Parameter	$i=10$			$i=20$			$i=30$			$i=40$		
baseline case: $r=0.03, \mu=0.065, \sigma=0.175$ s.t. $(\mu-r)/\sigma=0.2, \alpha=0.9, \bar{L}_i=10000\ \forall i, [\gamma_O,\gamma_M,\gamma_Y]=[10,5,2]$												
Market Sharpe ratio $(\mu-r)/\sigma$ with $\mu-r=0.035$	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
0.15	[−589, −547]	[−967, −879]	**[−195, −181]**	[−1183, −1090]	*[−984, −860]*	[−1059, −981]	[−2997, −2750]	**[−1662, −1504]**	[−2503, −2304]	[−4042, −3721]	**[−2241, −2029]**	[−3374, −3115]
0.175	[−644, −595]	[−1079, −976]	**[−212, −195]**	[−1246, −1138]	**[−1024, −882]**	[−1137, −1045]	[−3126, −2837]	**[−1638, −1452]**	[−2624, −2390]	[−4213, −3837]	**[−2206, −1961]**	[−3535, −3234]
0.2	[−687, −631]	[−1174, −1058]	**[−228, −208]**	[−1267, −1143]	**[−1032, −866]**	[−1183, −1078]	[−3125, −2798]	**[−1520, −1310]**	[−2641, −2377]	[−4208, −3781]	**[−2050, −1773]**	[−3558, −3215]
0.225	[−717, −654]	[−1253, −1126]	**[−243, −220]**	[−1246, −1107]	**[−1001, −817]**	[−1196, −1080]	[−2996, −2630]	**[−1317, −1084]**	[−2558, −2270]	[−4029, −3550]	**[−1775, −1464]**	[−3451, −3065]
0.25	[−734, −664]	[−1318, −1178]	**[−255, −230]**	[−1182, −1029]	**[−932, −730]**	[−1176, −1048]	[−2737, −2331]	**[−1026, −771]**	[−2378, −2055]	[−3679, −3160]	**[−1388, −1046]**	[−3207, −2784]
Risk aversion $[\gamma_O,\gamma_M,\gamma_Y]$	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
$[11.43, 2.29, 1.43]$ s.t. $\pi_O=10\%, \pi_M=50\%, \pi_Y=80\%$	**[−598, −550]**	[−1030, −884]	[−2464, −2365]	[−3129, −2861]	**[−1897, −1667]**	[−3835, −3587]	[−5259, −4735]	**[−2232, −1928]**	[−5118, −4681]	[−7126, −6381]	**[−3017, −2603]**	[−6911, −6333]
$[10, 5, 2]$ s.t. $\pi_O=11\%, \pi_M=23\%, \pi_Y=57\%$	[−687, −631]	[−1174, −1058]	**[−228, −208]**	[−1267, −1143]	**[−1032, −866]**	[−1183, −1078]	[−3125, −2798]	**[−1520, −1310]**	[−2641, −2377]	[−4208, −3781]	**[−2050, −1773]**	[−3558, −3215]
$[4.57, 4.57, 4.57]$ s.t. $\pi_O=25\%, \pi_M=25\%, \pi_Y=25\%$	[−1577, −1449]	[−2080, −1961]	**[−1447, −1322]**	[−1702, −1524]	[−2066, −1894]	*[−1635, −1462]*	[−1558, −1344]	[−1566, −1352]	*[−1543, −1330]*	[−2098, −1813]	[−2117, −1824]	*[−2074, −1793]*
Quantile α	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
0.8	[−670, −615]	[−1315, −1206]	**[−275, −252]**	[−1207, −1092]	[−1372, −1211]	*[−1128, −1024]*	[−2750, −2454]	**[−1438, −1237]**	[−2359, −2116]	[−3705, −3318]	**[−1938, −1675]**	[−3173, −2864]
0.85	[−678, −622]	[−1261, −1150]	**[−251, −235]**	[−1233, −1113]	[−1211, −1046]	*[−1148, −1046]*	[−2910, −2599]	**[−1473, −1269]**	[−2476, −2224]	[−3917, −3516]	**[−1987, −1716]**	[−3341, −3014]
0.9	[−687, −631]	[−1174, −1058]	**[−228, −208]**	[−1267, −1143]	**[−1032, −866]**	[−1183, −1078]	[−3125, −2798]	**[−1520, −1310]**	[−2641, −2377]	[−4208, −3781]	**[−2050, −1773]**	[−3558, −3215]
0.95	[−701, −645]	[−1022, −901]	**[−185, −169]**	[−1315, −1189]	**[−654, −486]**	[−1244, −1136]	[−3482, −3123]	**[−1604, −1384]**	[−2911, −2629]	[−4691, −4223]	**[−2171, −1864]**	[−3925, −3555]
0.99	[−726, −667]	[−610, −486]	**[−193, −180]**	[−1422, −1284]	**[170, 347]**	[−1376, −1265]	[−4291, −3866]	**[−1814, −1564]**	[−3516, −3187]	[−5786, −5225]	**[−2449, −2111]**	[−4728, −4302]
Demographic change	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
Decrease by 0.4%	[−660, −607]	[−1105, −996]	**[−237, −217]**	[−1170, −1055]	**[−939, −791]**	[−1088, −992]	[−2771, −2481]	**[−1350, −1169]**	[−2318, −2086]	[−3585, −3221]	**[−1751, −1516]**	[−3003, −2710]
Stable	[−687, −631]	[−1174, −1058]	**[−228, −208]**	[−1267, −1143]	**[−1032, −866]**	[−1183, −1078]	[−3125, −2798]	**[−1520, −1310]**	[−2641, −2377]	[−4208, −3781]	**[−2050, −1773]**	[−3558, −3215]
Increase by 0.4%	[−715, −657]	[−1249, −1125]	**[−217, −198]**	[−1373, −1238]	**[−1133, −949]**	[−1286, −1173]	[−3523, −3154]	**[−1712, −1472]**	[−3009, −2710]	[−4936, −4435]	**[−2403, −2067]**	[−4220, −3814]
All parameter changes ceteris paribus												

Table 5: Sensitivity analysis of the 95% Value at Risk $V@R_i$ of the first four generations regarding assumptions with respect to the market (Sharpe ratio $(\mu - r)/\sigma$), the life-cycle risk aversion $(\gamma_O, \gamma_M, \gamma_Y)$, the quantile level α and demographic change (Lump sums L_i). Estimates given as 95% Monte Carlo confidence intervals based on 2000 runs. Scheme with highest Value at Risk printed in bold unless the confidence intervals overlap. If the confidence intervals overlap, the one with highest midpoint is printed in italics. All changes ceteris paribus with respect to the baseline case.

Sharpe-Ratio $\mathrm{SR_i}$

Parameter	$i=10$			$i=20$			$i=30$			$i=40$		
baseline case: $r=0.03, \mu=0.065, \sigma=0.175$ s.t. $(\mu-r)/\sigma=0.2, \alpha=0.9, \bar{L}_i=10000\ \forall i, [\gamma_O,\gamma_M,\gamma_Y]=[10,5,2]$												
Market Sharpe ratio $(\mu-r)/\sigma$ with $\mu-r=0.035$	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
0.15	[0.451, 0.492]	**[0.718, 0.763]**	[0.451, 0.495]	[0.609, 0.652]	**[1.004, 1.053]**	[0.523, 0.566]	[0.651, 0.694]	**[0.794, 0.838]**	[0.623, 0.665]	[0.651, 0.694]	**[0.794, 0.839]**	[0.623, 0.665]
0.175	[0.529, 0.571]	**[0.756, 0.801]**	[0.529, 0.574]	[0.713, 0.757]	**[1.073, 1.123]**	[0.612, 0.656]	[0.76, 0.804]	**[0.927, 0.974]**	[0.727, 0.771]	[0.76, 0.804]	**[0.927, 0.974]**	[0.727, 0.771]
0.2	[0.606, 0.65]	**[0.795, 0.84]**	[0.607, 0.654]	[0.816, 0.862]	**[1.141, 1.191]**	[0.702, 0.747]	[0.869, 0.915]	**[1.06, 1.109]**	[0.832, 0.878]	[0.87, 0.915]	**[1.061, 1.11]**	[0.832, 0.877]
0.225	[0.684, 0.728]	**[0.833, 0.879]**	[0.686, 0.736]	[0.92, 0.968]	**[1.208, 1.26]**	[0.793, 0.839]	[0.978, 1.026]	**[1.193, 1.245]**	[0.937, 0.985]	[0.979, 1.025]	**[1.193, 1.245]**	[0.937, 0.984]
0.25	[0.762, 0.807]	**[0.873, 0.918]**	[0.767, 0.818]	[1.023, 1.074]	**[1.275, 1.329]**	[0.886, 0.934]	[1.087, 1.137]	**[1.325, 1.379]**	[1.043, 1.093]	[1.088, 1.136]	**[1.325, 1.379]**	[1.044, 1.091]
Risk aversion $[\gamma_O,\gamma_M,\gamma_Y]$	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
$[11.43, 2.29, 1.43]$ s.t. $\pi_O=10\%, \pi_M=50\%, \pi_Y=80\%$	[0.607, 0.65]	**[1.076, 1.126]**	[−0.599, −0.56]	[0.702, 0.745]	**[0.975, 1.023]**	[0.367, 0.409]	[0.869, 0.915]	**[1.052, 1.101]**	[0.707, 0.751]	[0.869, 0.914]	**[1.052, 1.101]**	[0.707, 0.75]
$[10, 5, 2]$ s.t. $\pi_O=11\%, \pi_M=23\%, \pi_Y=57\%$	[0.606, 0.65]	**[0.795, 0.84]**	[0.607, 0.654]	[0.816, 0.862]	**[1.141, 1.191]**	[0.702, 0.747]	[0.869, 0.915]	**[1.06, 1.109]**	[0.832, 0.878]	[0.87, 0.915]	**[1.061, 1.11]**	[0.832, 0.877]
$[4.57, 4.57, 4.57]$ s.t. $\pi_O=25\%, \pi_M=25\%, \pi_Y=25\%$	[0.605, 0.648]	[0.2, 0.239]	**[0.657, 0.7]**	[0.865, 0.911]	[0.68, 0.722]	*[0.883, 0.929]*	*[1.064, 1.112]*	[1.057, 1.106]	[1.062, 1.111]	*[1.064, 1.113]*	[1.056, 1.106]	[1.062, 1.111]
Quantile α	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
0.8	[0.605, 0.647]	*[0.662, 0.706]*	[0.623, 0.67]	[0.818, 0.864]	**[0.948, 0.995]**	[0.725, 0.77]	[0.884, 0.93]	**[1.059, 1.108]**	[0.853, 0.899]	[0.884, 0.93]	**[1.058, 1.108]**	[0.853, 0.899]
0.85	[0.605, 0.649]	**[0.716, 0.76]**	[0.614, 0.661]	[0.817, 0.864]	**[1.041, 1.089]**	[0.717, 0.761]	[0.877, 0.924]	**[1.06, 1.109]**	[0.844, 0.89]	[0.878, 0.923]	**[1.06, 1.11]**	[0.844, 0.89]
0.9	[0.606, 0.65]	**[0.795, 0.84]**	[0.607, 0.654]	[0.816, 0.862]	**[1.141, 1.191]**	[0.702, 0.747]	[0.869, 0.915]	**[1.06, 1.109]**	[0.832, 0.878]	[0.87, 0.915]	**[1.061, 1.11]**	[0.832, 0.877]
0.95	[0.608, 0.651]	**[0.925, 0.972]**	[0.572, 0.62]	[0.815, 0.861]	**[1.335, 1.391]**	[0.676, 0.721]	[0.856, 0.902]	**[1.06, 1.109]**	[0.813, 0.859]	[0.857, 0.902]	**[1.06, 1.109]**	[0.814, 0.859]
0.99	[0.608, 0.652]	**[1.233, 1.286]**	[0.136, 0.178]	[0.81, 0.856]	**[1.722, 1.788]**	[0.62, 0.664]	[0.831, 0.877]	**[1.053, 1.102]**	[0.779, 0.823]	[0.832, 0.876]	**[1.053, 1.102]**	[0.779, 0.823]
Demographic change	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley	IDC	Linear	Shapley
Decrease by 0.4% p.a.	[0.606, 0.65]	**[0.793, 0.838]**	[0.61, 0.658]	[0.816, 0.862]	**[1.136, 1.187]**	[0.71, 0.755]	[0.869, 0.915]	**[1.047, 1.096]**	[0.837, 0.883]	[0.87, 0.915]	**[1.048, 1.096]**	[0.837, 0.882]
Stable	[0.606, 0.65]	**[0.795, 0.84]**	[0.607, 0.654]	[0.816, 0.862]	**[1.141, 1.191]**	[0.702, 0.747]	[0.869, 0.915]	**[1.06, 1.109]**	[0.832, 0.878]	[0.87, 0.915]	**[1.061, 1.11]**	[0.832, 0.877]
Increase by 0.4% p.a.	[0.606, 0.65]	**[0.796, 0.841]**	[0.601, 0.648]	[0.816, 0.862]	**[1.144, 1.195]**	[0.693, 0.738]	[0.869, 0.915]	**[1.073, 1.122]**	[0.827, 0.873]	[0.87, 0.915]	**[1.073, 1.123]**	[0.827, 0.872]
All parameter changes ceteris paribus												

Table 6: Sensitivity analysis of Sharpe ratio $\mathrm{SR_i}$ of the first four generations regarding assumptions with respect to the market (Sharpe ratio $(\mu-r)/\sigma$), the life-cycle risk aversion $(\gamma_O,\gamma_M,\gamma_Y)$, the quantile level α and demographic change (Lump sums L_i). Estimates given as 95% Monte Carlo confidence intervals based on 2000 runs. Scheme with highest Sharpe ratio printed in bold unless the confidence intervals overlap. If the confidence intervals overlap, the one with highest midpoint is printed in italics. All changes ceteris paribus with respect to baseline case.

higher than the EU's prediction. Therefore, we conclude that our scheme is resilient, in the sense that the incentive to join the CDC scheme with linear risk sharing due to a better risk-adjusted performance is stable for all generations.

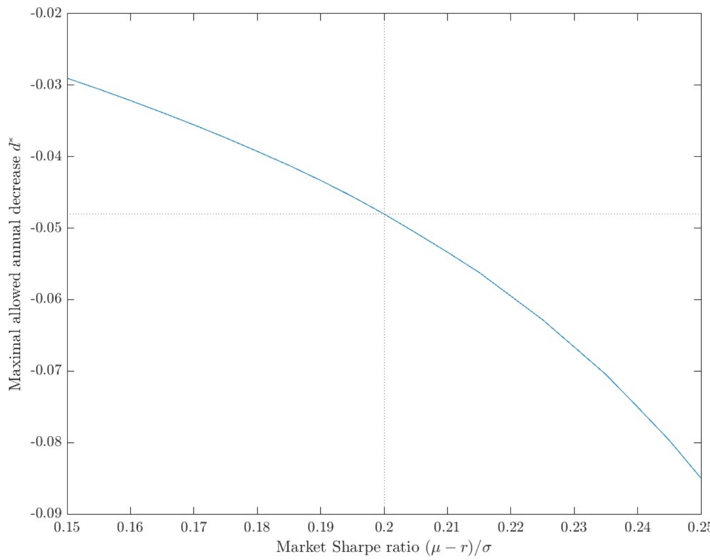


Figure 7: The maximal allowed annual decrease d^* in the working age population such that the third generation $i = 30$ still prefers linear risk sharing over the IDC scheme in terms of Sharpe ratio SR_{30}.

5 Conclusion & Outlook

In this paper, we propose a novel collective defined contribution (CDC) plan which guarantees upon retirement at least a target benefit as a lump sum. The guarantee is provided by the remaining working generations. We impose a relatively strict reserve requirement so that in the vast majority of scenarios the remaining generations do not need to intervene. To determine the generation's share in the required reserves, we propose to use a pre-determined linear sharing rule in which the share that each generation pays is pro rata to its required reserve. We benchmark this CDC scheme with a comparable individual defined contribution (IDC) scheme and a CDC scheme with the non-linear Shapley risk sharing rule.

A simulation-based study indicates that the proposed CDC scheme leads to a somewhat lower expected net present value for younger generations in comparison with the IDC scheme. However, at the same time, it reduces the tail risk for these generations. The combination of both features culminates in the first main result that in terms of risk-adjusted performance the CDC scheme with linear risk sharing outperforms the IDC scheme and the CDC scheme with Shapley risk sharing for all generations. Hence our CDC scheme with linear risk sharing is beneficial for all generations. An extensive sensitivity analysis shows that this outperformance is robust with respect to the main model parameters such as quantile level, market Sharpe ratio or the underlying demographics. In particular, this outperformance proves stable in a 'double-hit' scenario where we stressed assumptions regarding the underlying market dynamics and demographics simultaneously. More precisely, even if the decrease in the size of the future workforce is much more severe than the European Union predicts for its member states, the main result about the superiority of our CDC scheme still hold. In this sense, we deem our CDC scheme as resilient which is our second main result.

We recognize that our CDC scheme is fairly stylized so that there are extensions in at least three dimensions possible. First, one extension of our study is to ameliorate the intergenerational fairness of our fund design. Although our results indicate that the proposed fund design is beneficial for all generations in terms of risk-adjusted performance, the older generations profit especially as they get the guarantee although they do not provide respective support to other generations. Therefore, future research may explore how to distribute benefits more evenly among the generations to enhance intergenerational fairness. For example, one could introduce a maximum target benefit upon retirement. If the retiring generation's share is bigger than this threshold, the remaining generations receive the exceeding amount. Second, we did not discuss discontinuity risk in our study. For instance, under UK law, CDC schemes need to prove their viability in cases where the regulator withdraws their authorization or where the pension fund's sponsor decides to discontinue the CDC scheme (Swift et al. (2021)). In our case, such trigger events would leave all working generations without the guarantee that they provided to other generations before. Third, in our fund design, mostly working generations share the investment risk as the retired generations are protected by the minimum target benefit. Cui et al. (2011) show for their CDC scheme that including both retirees and working generations in intergenerational risk sharing increases welfare gains. One possible way to do so, is to provide the retired generations besides the lump sum benefit with a variable income. This variable income may then be adjusted according to the fund's financial performance. A similar mechanism is intended for the *Royal Mail Collective Plan* in the UK.

A Shapley Value Computation

Generation $i = 10$				Generation $i = 30$			
$J_t \subset I_t^w$	π_t^J	$A_t^J - R_t^J$	$\Delta_{J_t}^{10}$	$J_t \subset I_t^w$	π_t^J	$A_t^J - R_t^J$	$\Delta_{J_t}^{30}$
$\{10, 20, 30\}$	24.62%	499.03	12.48	$\{10, 20, 30\}$	24.62%	499.03	333.89
$\{20, 30\}$	36.14%	486.55		$\{10, 20\}$	16.16%	165.14	
$\{10, 30\}$	25.56%	322.39	-88.78	$\{10, 30\}$	25.56%	322.39	229.73
$\{30\}$	57.14%	411.18		$\{10\}$	11.43%	92.66	
$\{10, 20\}$	16.16%	165.14	-16.59	$\{20, 30\}$	36.14%	486.55	304.83
$\{20\}$	22.86%	181.73		$\{20\}$	22.86%	181.73	
$\{10\}$	11.43%	92.66	92.66	$\{30\}$	57.14%	411.18	411.18
$\{\}$	–	0		$\{\}$	–	0	
$\Phi^{10,I_t^w}(A_t - R_t) = 17.48$				$\Phi^{30,I_t^w}(A_t - R_t) = 337.45$			

Table 7: Exemplary computation of the share $\Phi^{10,I_t^w}(A_t - R_t) = 17.48$ euros of generation $i = 10$ and $\Phi^{30,I_t^w}(A_t - R_t) = 337.45$ euros in the difference between fund assets and required reserve $A_t - R_t = 449.03$ euros at time $t = 5$ in scenario ω_B. Parameter choice as in the baseline case, $\alpha = 90\%$ in particular. All numbers are given in euros.

Table 7 gives an exemplary computation of the shares of generation $i = 10$ and $i = 30$ in the fund's surplus of $A_t - R_t = 499.03$ euros in scenario ω_B. We start by examining the working generation $i = 10$. For every subfund $J \subset I_t^w$ which contains working generation $i = 10$, we calculate the generation's marginal contribution $\Delta_{J_t}^{10}$ to the respective surplus by comparing $A_t^J - R_t^J$ with $A_t^{J/\{10\}} - R_t^{J/\{10\}}$. For example, the surplus at $t = 5$ increases from 486.55 euros to 499.03 euros when considering $J_t = \{10, 20, 30\}$ instead of $J_t/\{10\}$=$\{20,30\}$. Hence, we say that the working generation $i = 10$ contributes $\Delta^{10}_{J_t} = 12.48$ euros to the surplus of the subfund J_t. Averaging these marginal contributions gives then the Shapley value as the share $\Phi^{10,I^w_t}(A_t - R_t) = 17.48$ euros for the working generation $i = 10$. Compared to this, the young generation $i = 30$ exhibits a much higher share in the surplus of $\Phi^{30,I^w_t}(A_t - R_t) = 337.45$ euros.

Table 8 depicts the same computation for generation $i = 10$ for a quantile level of $\alpha = 99\%$. Compared to Table 7, its share $\Phi^{10,I^w_t}(A_t - R_t)$ turns negative, owing to the generation's negative marginal contributions to subfunds J_t which contain the youngest generation $i = 30$.

Generation $i = 10$			
$J_t \subset I^w_t$	π^J_t	$A^J_t - R^J_t$	$\Delta^{10}_{J_t}$
$\{10, 20, 30\}$	24.62%	516.23	-81.04
$\{20, 30\}$	36.14%	597.27	
$\{10, 30\}$	25.56%	314.59	-360.09
$\{30\}$	57.14%	674.68	
$\{10, 20\}$	16.16%	468.60	184.15
$\{20\}$	22.86%	284.44	
$\{10\}$	11.43%	105.61	105.61
$\{\}$	–	0	
$\Phi^{10,I^w_t}(A_t - R_t) = -21.13$			

Table 8: Exemplary computation of the share $\Phi^{10,I^w_t}(A_t - R_t) = -21.13$ euros of generation $i = 10$ in the difference between fund assets and required reserve $A_t - R_t =$ 516.23 euros at time $t = 5$ in scenario ω_B. Parameter choice as in the baseline case except for a quantile level of $\alpha = 99\%$. All numbers are given in euros.

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