Passive Forces in Fixturing and Grasping

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Abstract

Analysis and characterization of contact forces are important in fixture design and robotic grasping since they define the object stability during fixturing or grasping. This paper presents a description of passive forces arising at the normal and frictional contacts by passive physical means. The passivity nature requires application of the minimum norm principle to solve a constrained quadratic optimization in order to determine the passive contact force. The model reveals some intricate properties of the passive contact forces, including internal passive forces. Further, a notion of hybrid force closure is considered to characterize the passive nature of the hybrid forces. The hybrid force closure conditions and their implications in practice are illustrated with an example.

1 Introduction

In robotic grasping of a multi-fingered hand, the usual notion is that the fingers are able to actively produce contact forces to balance any disturbing wrench acting on the grasped object. The active contact forces require sophisticated sensing and force control of each robot finger. However, an entirely different situation may arise when the grasping elements are simple devices such as locator buttons of a fixture. In such an application, physical processes at the contact such as rigid body kinematics and friction provide passive forces for the balance. These contact forces are said to be passive forces, since they have no active control or coordination. Another important application concerns a robotic manipulator that relies on its environment for manipulating an object. The manipulator may push the object against a wall for stabilizing the object. Here the wall plays a role of grasping but the contact forces between the object and the wall are passive as reaction forces to the manipulator’s action. In all of these examples, object stabilization and/or manipulation is achieved by passive means, in part or in whole, and the notion of force passivity is important.

Force passivity brings a higher level of complexity for analysis and planning of fixturing and grasping. A general problem is that the passive forces are not pre-determined, especially in the presence of friction. They must be solved as the solution to a statically indeterminate system. It is even more difficult to predict the manipulation effects that the frictional forces can make. Another implication concerns with force closure. Passive forces would render the notion of passive force closure or hybrid passive and active force closure (or hybrid closure).

In this paper we first discuss the notions of force passivity and passive (and hybrid) force closure in fixturing and grasping. We then present a solution method for the determination of passive contact forces within the rigid body model and Coulomb friction model. The method is based on the application of the minimum norm principle with frictional forces as constraints. The paper focuses on two areas of discussions contributing to the general understanding of force passivity, (1) the concept of passive frictional contact forces and (2) conditions for passive and hybrid force closure and their strong implications in fixture design and hybrid manipulation.
2 Background

The essential requirement of fixturing concerns with the kinematic concepts of localization and force closure, which have been extensively studied in recent years. There are several formal methods for fixture kinematic analysis based on the assumptions of rigid workpiece and fixture and frictionless workpiece-fixture contacts [1, 2]. For the analysis of workpiece-fixture contact forces a comprehensive approach is to consider the workpiece-fixture system as an elastic system. This system can be analyzed with a finite element model [3, 4, 5, 6].

A conceptually simpler approach is the rigid body model approach where the workpiece and the fixture are assumed to be completely rigid [7, 1, 2]. A general problem of the approach is that the workpiece-fixture system usually is statically indeterminate, especially in the presence of friction [8, 4]. It is not unusual in the literature that the frictional forces are ignored so that the issue of static indeterminacy is avoided, in spite of the significant impact that the frictional forces can make.

Another important issue related to frictional contacts of a fixture is passive force closure. Since fixture locator elements are simple physical contacts that are preloaded against the workpiece with initial clamping forces, it is the physical processes of mechanics at the frictional contacts to determine passive balance of the workpiece against any external force. Unlike in a multi-fingered robot hand, force closure in a fixture is achieved by passive means, without active control or coordination of the contact forces. This results in a more restrictive condition than that for active force closure often discussed in the literature of robotic grasping [9, 10, 11]. The passivity of the locator forces may severely limit the possibility for force closure [12]. Again, the usual approach to frictionless cases of passive force closure ignores the significance of friction and often yields impractical solutions [13, 14, 15].

3 Active and Passive Contact Forces

3.1 Contact Force Terminology

For the purpose of analysis of contact forces in this paper, the basic elements of a fixture or a grasp are represented by passive and active forces. An active force element means that the force can be actively and precisely controlled in time. It requires sophisticated contact-force sensors and force controllers to coordinate the actions of a set of active forces. For a multi-fingered robot hand, the finger contact forces can be modelled to be active. A clamp in a fixture can is said to be active, since it is typically engaged manually or pneumatically. However, in fixtures other elements such as locators are said to be passive, since they are simple devices that are preloaded against an object with initial applied forces. Passive force elements include the conventional locator pins or buttons that are used essentially for a unique localization of the workpiece with respect to a fixture reference frame.

Another important application concerns passive objects that help establish a robotic grasp of an object to be manipulated. The passive contact between the object and its environment together with the active frictional forces from the robot fingers provide stabilization of the grasped object. In other words, the robot may push the grasped object against the environment walls to utilize the passive contact forces from the walls as a strategy for manipulating the object. In this case, robotic manipulation is achieved by a combination of active and passive means.

3.2 Frictional Contact

Within the framework of rigid body model, we describe each contact (passive or active) with a point contact model with Coulomb friction for clarity [1, 2]. As shown in Fig. 1, the frictional contact produces three force components on the workpiece,

\[ f = [f_t, f_b, f_n]^T = [\beta_1 t, \beta_2 b, \beta_3 n]^T \]  

(1)

with force intensities \((\beta_1, \beta_2, \beta_3)\) for the tangent and normal directions respectively. Here, the inward surface unit normal of the workpiece is represented by \(n\), while \(t\) and \(b\) represent two orthog-
3.3 Passive Forces

For a passive contact $i$ on the object at position $\mathbf{r}_i$ the contact wrench $\mathbf{P}_i$ (force and moment) exerted on the object is described as

$$\mathbf{P}_i = \sum_{k=1}^{N_i} \mathbf{w}_{ik} \beta_{ik} \quad (2)$$

where $N_i$ is the number of force components of the contact $i$ and usually $N_i = 3$ for frictional contact, and $\mathbf{w}_{ik}$ is given as

$$\mathbf{w}_{i1} = \begin{bmatrix} t \\ \mathbf{r} \times t \end{bmatrix} , \quad \mathbf{w}_{i2} = \begin{bmatrix} b \\ \mathbf{r} \times b \end{bmatrix} , \quad \mathbf{w}_{i3} = \begin{bmatrix} \mathbf{n} \\ \mathbf{r} \times \mathbf{n} \end{bmatrix} \quad (3)$$

for the three forces respectively.

Suppose that there are $n$ passive contacts applied on the grasped object. Then, the total passive force is given by

$$\mathbf{P} = \sum_{i=1}^{n} \mathbf{P}_i = \sum_{i=1}^{n} \sum_{k=1}^{N_i} \mathbf{w}_{ik} \beta_{ik} = \sum_{r=1}^{N} \mathbf{w}_r \beta_r = \mathbf{Gp} \quad (5)$$

where

$$\mathbf{G} = \begin{bmatrix} \mathbf{w}_1 , \cdots , \mathbf{w}_N \end{bmatrix} \quad (6)$$

$$\mathbf{p}^T = [\beta_1 , \beta_2 , \cdots , \beta_N] \quad (7)$$

$$N = N_1 + N_2 + \cdots + N_n \quad (8)$$

with $N$ being the total number of independent passive force components and $\mathbf{p}$ representing the force intensities of all the passive force components rearranged into a single vector.

3.4 Active Forces

Similarly, for an active contact $j$ on the object at position $\mathbf{r}_j$ the contact wrench $\mathbf{F}_j$ (force and moment) exerted on the object is described as

$$\mathbf{F}_j = \sum_{k=1}^{M_j} \mathbf{w}_{jk} \alpha_{jk} \quad (9)$$

where $M_j$ is the number of force components of the contact $j$ and usually $M_j = 3$ for frictional contact, and $\mathbf{w}_{jk}$ is given similarly for the three forces respectively.

Suppose that there are $m$ active contacts applied on the grasped object. Then, the total active force is given by

$$\mathbf{F} = \sum_{j=1}^{m} \mathbf{F}_j = \sum_{j=1}^{m} \sum_{k=1}^{M_j} \mathbf{w}_{jk} \alpha_{jk} = \sum_{r=1}^{M} \mathbf{w}_r \alpha_r = \mathbf{Ha} \quad (10)$$

where

$$\mathbf{H} = [\mathbf{w}_1 , \cdots , \mathbf{w}_M] \quad (11)$$

$$\alpha^T = [\alpha_1 , \alpha_2 , \cdots , \alpha_M] \quad (12)$$

$$M = M_1 + M_2 + \cdots + M_m \quad (13)$$

with $M$ being the total number of independent active force components and $\alpha$ representing the force intensities of all the active force components rearranged into a single vector.

3.5 Coulomb’s Friction Model

We assume the standard Coulomb’s friction model for each active and passive contact. Each contact is considered to be unilateral, and the contact force $\beta_{i3}$ or $\alpha_{j3}$ can only push on the object. Thus, each contact must satisfy the inequality

$$\beta_{i3} \geq 0 \quad \text{and} \quad \alpha_{j3} \geq 0 \quad (14)$$

Further, the friction law requires that

$$\beta_{i1}^2 + \beta_{i2}^2 \leq (\mu_i \beta_{i3})^2 \quad (i = 1 , 2 , \cdots , n) \quad (15)$$

$$\alpha_{j1}^2 + \alpha_{j2}^2 \leq (\mu_j \alpha_{j3})^2 \quad (j = 1 , 2 , \cdots , m) \quad (16)$$

for every passive contact and active contact with corresponding friction coefficient $\mu_i$ and $\mu_j$ respectively. For clarity we shall denote the friction constraints by $PC_i$ and $FC_j$ for a passive and an active contact respectively, and by $PC = PC_1 \cap PC_2 \cap \cdots \cap PC_n$ and $FC = FC_1 \cap FC_2 \cap \cdots \cap FC_m$ for all contacts respectively.
4 The Method of Minimum Norm Solution

4.1 The Force Equations

Let \( Q \) represent all external wrench (force and moment) vectors applied on the grasped object. Then, the static equilibrium equation of the object is given as

\[
Gp + Ha = Q
\]  

(17)

For grasping or fixturing analysis we must determine the active forces represented by \( a \) to be applied for a static equilibrium and perhaps a force closure, together with the passive forces \( p \) as a solution to the force equation. For a general three-dimensional object we must have sufficient number of force components (active and passive), i.e., \( N + M \geq 6 \). In the presence of friction, the grasping system represented by Eq. 17 is usually statically undeterminate.

4.2 The Minimum Norm Principle

In a practical application of grasping or fixturing the active forces and the passive forces play different roles. Active forces can be regarded as independent forces whose actions can be precisely coordinated. Passive forces are reactive forces whose actions depend on the active forces in addition to the friction cone constraints. In the problem of force analysis this dependency must be specified precisely and meaningfully based on a sound physical principle. The dependency in effect becomes an additional constraint on the passive forces that satisfy the static equilibrium.

In this paper, it is proposed that the passivity of all passive contact forces is governed by the principle of minimum norm \([16]\). Within the framework of the rigid body model this principle essentially states that of all possible passive forces for a rigid body at equilibrium subjected to active forces and prescribed loading the unique force solution compatible to the equilibrium renders a minimum norm on the intensities of all the passive forces. In other words, the passivity of the contact system is completely described as an optimality condition given as

\[
\text{Minimize } \|p\| = (p^T p)^{1/2}
\]  

(18)

It should be pointed out that the minimum norm principle is equivalent to the principle of minimum complementary energy for an elastic contact system \([17, 18]\), if we consider it to be linear and with contact elasticity defined by a compliance matrix \( C \). In that case, the complementary energy is defined by \( U = \frac{1}{2}p^T Cp \). Thus, the minimum norm principle characterizes the passive contact in a similar sense but under the simpler provision of rigid body contact.

Thus, the contact force solution is represented by a quadratic minimization with equality and inequality constraints:

\[
\text{Minimize } p = (p^T p)^{1/2}
\]  

subject to

\[
Gp + Ha = Q
\]  

(20)

\[
p \in PC
\]  

(21)

\[
a \in FC
\]  

(22)

The linear equality constraints of Eq. 20 describe the equilibrium state. The inequality constraints of Eqs. 21 and 22 maintain that all of the contacts are unilateral and obey Coulomb’s friction law.

However, the objective of quadratic optimization is a function of partial variables only, i.e., of passive forces \( p \) only. Thus, for the solution we must distinguish if the passive forces along are sufficient to satisfy the equilibrium equation. This leads to the following discussions of two situations of different solution procedures.

5 Passive Contact Forces

First, we consider the situation that the passive contact matrix \( G \) is of full rank of 6. In this case, if \( N = 6 \) the contact system is said to be passively determinate, and if \( N > 6 \) the system is said to be passively over-determinate. In either case, there exists sufficient number of independent passive forces \( p \) to satisfy the static equilibrium, and the active forces \( a \) can be regarded completely arbitrary.

A passively over-determinate contact system is typical in workpiece fixturing. For a general three dimensional workpiece, its fixture must have at least six locators and one clamp for the localization and force stabilization requirements, i.e., \( n \geq 6 \) and \( m \geq 1 \). In the general frictional case, there exist at least 18 passive contact force components at
the locators. The clamping forces can be considered to be independent active forces applied on the workpiece. Under the unilateral and frictional inequality constraints (Eqs. 21 - 22), the minimum norm principle (Eq. 19) would reveal a number of intricate properties of the solution as follows.

5.1 The Specific Solution

When the contact system is passively over-determinate or passively determinate, the dependent variables in the equilibrium equation are the passive forces $p$ only. Thus, we have

$$Gp = Q - Ha$$

(23)

If all of the passive forces and the active forces are within their respective friction cones, i.e., $p \in \text{int}(PC)$ and $a \in \text{int}(FC)$, then it is said that all of the inequality constraints are inactive. In this case, the minimum norm solution for Eq. 23 is easily obtained as

$$p_s = G^+(Q - Ha)$$

(24)

directly in terms of the minimum-norm generalized inverse of matrix $G$: $G^+ = G^T(GG^T)^{-1}$, which is also known as the right pseudo-inverse such that $GG^+ = I$ [16, 9]. This is the specific solution to the passively (over) determinate linear system (Eq. 23), which is effectively unconstrained. Here, it is given that $(GG^T)$ is not singular.

It is well known that the unconstrained linear system attains its minimum norm with the specific solution and its homogeneous solution vanishes [16]. The system of passive contact forces is essentially linear in this case where at each passive contact its normal contact force exists and its friction forces lie strictly inside the friction cone. From an optimization point of view, it can be said that the solution satisfies the Kuhn-Tucker (K-T) conditions as a minimum point.

5.2 Passive Internal Forces

However, when any of the passive contacts becomes non-reactive and/or when its limit friction is reached, one or more inequality constraints become active. Then, the solution to Eq. 23 with all relevant constraints has to be solved as a minimum norm solution [16], i.e., $\min \|p\|$, with a numerical procedure. Accordingly, the minimum-norm solution is in the form of

$$p = p_s + p_h$$

(25)

The first term is the specific solution of Eq. 24, and the second term is said to be the homogeneous solution.

According to the linear algebra, the specific solution is a projection of the minimum-norm solution $p$ defined as

$$p_s = Wp$$

(26)

by the projection matrix $W = G^+G = G^T(GG^T)^{-1}G$. The homogenous solution $p_h$ is the other orthogonal projection given as

$$p_h = (I - W)p = \alpha - G^-(Q - Ha)$$

(27)

Thus, in using the common terminology of robotics, the homogenous component shall be referred to as the passive internal forces among the passive contacts.

Therefore, in reaction to the external force $Q$ and the active grasping forces $a$, the specific passive force $p_s$ is generated at the passive contacts to balance the combined applied forces only, while the homogenous passive force component $p_h$ is to solely maintain their unilateral and frictional constraints. The constraint satisfaction is achieved at the cost of increasing the contact force intensities, since the passive internal forces generate no net force, i.e., $Gp_h = 0$.

The complete set of relationships between the external force, the active forces and the passive forces are summarized in Fig. 2. Let denote the inequality constraint (Eq. 21) by $\mathcal{C}(G)$. These constraints define a region in the solution space $\mathbb{R}^N$ of $p$ ($N \geq 6$). If the passive constraint region does not intersect with the row space of $G$ as illustrated in Fig. 2, then at least one of the constraints is active. The solution $p$ is split into two components $p_s$ and $p_h$ as projections of $p$ into the row space and the null space of $G$ as described by Eqs. 26 and 27, respectively. Further, $G$ transforms the row space component $p_s$ into the combined force vector ($R = Q - Ha$) in the column space $\mathbb{R}^C$ of $G$, while it transforms the null space component $p_h$ into zero. If the constraint region $\mathcal{C}(G)$ contains
$p_s$ in the row space, then the specific passive solution $p_s$ is sufficient as a minimum norm solution $p = p_s$ and $p_h = 0$. No passive internal forces are generated and all of the inequality constraints are inactive as discussed in Section 5.1.

![Figure 2: The mapping and projection of force vectors.](image)

It should be further pointed out that passive internal forces described above are passive forces as a result of their reaction to the applied load including the active forces $a$. The force intensity vector $p_h$ of the passive internal forces lies in the null space of $G$ as a fixed vector specified by Eq. 27 (Fig. 2). The internal passive force may not span the entire null space. This passive nature of the contact forces has a strong implication for achieving the requirement of force closure in an application such as fixture design as to be discussed in next section.

### 5.3 Active Internal Forces

In the case of a passively over-determinate or determinate system, the active force $a$ could be actively controlled and arbitrarily specified like those of a multi-fingered hand [19, 7]. Thus, if $M > 6$ for the active contact $6 \times M$ matrix $H$, then there also exist a set of statically indeterminate components of the active force $a$ called internal active force. If a set of orthonormal basis vectors which span the null space of $H$ are assembled into the column of the matrix $M$, then the active internal forces can be described in terms of the $(M - 6) \times 1$ vector $\lambda$ as

$$a_h = M \lambda$$  \hspace{1cm} (28)

where the elements of $\lambda$ represents the magnitudes of the internal forces among the active contacts.

### 6 Hybrid Force Closures

An important functional requirement of fixturing and grasping is force-closure, where the passive and active contacts are able to balance any and all external forces. This means that the object is completely immobile, and will not move in response to any applied force.

The force closure requirement is a century-old concept [20]. While it has been extensively studied in recent years, the conventional notion of force closure was originally formulated for multi-fingered robot hands [13, 9]. It is an association of active force closure, since it requires the contact forces of the fingers be actively changed and coordinated. However, when passive contacts are involved, force closure is partly achieved by passive means, without active control or coordination of the passive contact forces. Here, this is referred to as hybrid force closure.

The literature on active force closure is only partially relevant for studying passive force closure of fixtures. The analysis results and computation algorithms on friction-based active force closure (e.g., [9, 10, 11, 21]) do not generally apply to the hybrid passive/active system described by Eqs. 19-22. Particularly, the geometric conditions for active force closure are necessary but not sufficient for hybrid force closure [12, 22]. The passivity of the internal forces as governed by Eq. 27 may severely limit the possibility for force closure.

Passive force closure of fixtured rigid bodies has been studied mainly for frictionless cases corresponding to $\mu_i = 0$ ($i = 1, \ldots, n$) and $\mu_j = 0$ ($j = 1, \ldots, m$) [13, 14, 15]. This sort of passive force closure does not rely on friction and is easy to compute (by solving a linear program). However, frictionless force closure is only sufficient for the existence of frictional force closure. It can be practically impossible to use it to design force closure fixtures with a reasonable number of contacts such as six locators and one clamp.

In this paper, we classify force closure into three main classes, active force closure ($AC(\mu)$), hybrid force closure ($HC(\mu)$), and completely passive closure (or form closure). A completely passive contact system is equivalent to a form closure system [20, 12]. In the kinematics of closure analysis, a form closure system is equivalent to a com-
pletely active force closure without any friction, i.e., AC(0).

Form closure or frictionless active closure AC(0) characterizations are the most conservative conditions, so their use in grasping and fixturing is overly limited. On the other hand, frictional active force closure AC(μ) conditions are overly general – they are necessary but not sufficient for the existence of hybrid force closure HC(μ) [12, 22]. In other words, active force closure AC(μ) does not automatically imply hybrid force closure HC(μ). These important force closure subsets satisfy

\[ \text{AC}(0) \subseteq \text{HC}(\mu) \subseteq \text{AC}(\mu) \] (29)

and they are illustrated in Fig. 3.

![Figure 3: The important subsets of force closure.](image)

These relationships have strong implications in practice, especially in fixture design. Fig. 4 shows an example 3D workpiece fixtured by six locators and a clamp, where the arrows of a solid end circle indicate the locators and an arrow of an open end circle indicates the clamp. When consider frictionless case (\(\mu = 0\)), the fixture is not a force closure (or form closure) AC(0). When the friction coefficient is assumed to be \(\mu = 0.2\) for all the contacts, the fixture is found to be a hybrid force closure HC(\(\mu\)). However, the contact force at one locator (\(L_1\)) is found to be vanished, creating a situation of locator release under clamping. Clearly, this situation cannot be predicted based on the analysis for frictional active force closure AC(\(\mu\)).

![Figure 4: A fixture of a locator release and hybrid force closure.](image)

### 7 Conclusion

An important characteristic of a fixture or a robotic grasp is the notion of passive forces and hybrid force closure. In this paper, a rigid body model with Coulomb friction for force analysis of a grasp is presented. Passive contact forces are defined in addition to actively controlled grasping force, along with the unilateral and frictional constraint. Passivity of the passive forces requires that the minimum norm principle is invoked to solve the overall model as a constrained quadratic optimization problem. It yields a solution of all the contact forces, including internal forces in the passive and the active contacts.

It is also shown that nature of force closure of a hybrid (passive and active) contact system is quite different from that of the active force closure of a multi-fingered robot hand. Geometric conditions for hybrid force closure are more restrictive. However, hybrid force closure conditions are more general than frictionless closure conditions. Its analysis and characterization provide a broader set of solutions and have a strong implication in design and planning of fixtures and robotic grasps.

### Acknowledgement

This research work is supported in part by the Hong Kong Research Grants Council (Earmarked Grant CUHK4217/01E), the Chinese University of Hong Kong (Direct Research Grant 2050254), the Min-
istry of Education of China (a Visiting Scholar Grant at the State Key Laboratory of Manufacturing Systems in Xi’an Jiaotong University), and the Natural Science Foundation of China (NSFC) (Young Overseas Investigator Collaboration Award 50128503).

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