The Bank as Grim Reaper: Debt Composition and Bankruptcy Thresholds

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Abstract

We offer a model and evidence that private debtholders play a key role in setting the endogenous asset value threshold below which corporations declare bankruptcy. The model, in the spirit of Black and Cox (1976), implies that the recovery rate at emergence from bankruptcy on all of the firm's debt taken together is related to the pre-bankruptcy share of private debt in all debt. Empirical evidence supports this and other implications of the model. Indeed, debt composition has a more economically material empirical influence on recovery than all other variables we try taken together. This special role of private debt in the capital structure has important implications for debt pricing models, capital structure, and risk management.

Keywords: credit risk, recovery rates, bankruptcy, debt default
JEL Codes: G12, G33, G32

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We offer a model and evidence showing that the composition of corporate debt strongly influences corporate bankruptcy decisions and recovery rates on debt of bankrupt corporations. Our work is in the spirit of structural and strategic models of default, but the locus of strategic behavior is different. In such models, the firm defaults when the value of its assets falls below a threshold. Implicitly or explicitly, debtholders recover the threshold value of assets, perhaps less a haircut for deadweight costs of bankruptcy. In early structural models of default, such as Merton (1974) or Longstaff and Schwartz (1995), the threshold is exogenous. In models of strategic default, such as Leland (1994) or Fan and Sundaresan (2000), equity holders choose the threshold endogenously to maximize the value of their claims.

Our model is a generalization of the first-passage model of Black and Cox (1976). In our model, a firm's private debtholders ("banks" for simplicity) endogenously choose the bankruptcy threshold value of assets. Private debt has covenants that give banks the right to force a distressed firm into bankruptcy, even if the firm has made all debt payments. The firm's public debt is junior and has no material covenants. Because private debt is also senior, the bank has an incentive to foreclose only when the borrower's asset value drops to the neighborhood of the loan's face value, which can be well below the insolvency value of assets. Therefore, the lower the bank loan share in total debt, the lower the asset value of the borrower at bankruptcy and the lower the recovery to debt as a whole.[1]

The locus of strategic behavior in structural models depends on the interpretation of the asset-sale restrictions that are invariably attached to debt contracts.[2] The branch of the literature in the spirit of Leland (1994) takes a strict view of these restrictions: Coupon payments must be financed out-of-pocket by equity holders (or via new equity issues), so equityholders default when the value of continuation of their call option on assets is below the required "new money" payment. A looser interpretation of asset-sale restrictions would constrain only attempts to divert assets. Even when (net) asset returns are negative, firms typically generate substantial (gross) cashflows. So long as enough cashflows can be used to make required debt payments, equityholders may be able to retain control well past the point of insolvency without having to make payments out-of-pocket. If asset-sale restrictions *never* bind on coupon payments, equityholders will never voluntarily default—all bankruptcies will be forced by banks. We follow the latter, extreme interpretation mainly for simplicity and to complement the well-developed literature that flows from the opposite view.

Debt composition has only recently been considered in the literature on structural models of default, as earlier models have assumed a single class of debt. Hackbarth et al. (2007) examine optimal capital structure in a model in which firms can issue bank debt, public bonds, and equity. The special quality of bank debt in their model is the ability to renegotiate outside formal bankruptcy. Bank debt offers a better tradeoff between tax shields and bankruptcy costs, whereas non-renegotiable public debt offers higher debt capacity.[3] In our model, the special role of bank debt derives instead from the strong covenants that typically are attached to loans but not to public

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[1] Non-bankruptcy defaults and renegotiations of debt contract terms are a material source of credit losses. Indeed, a number of models of strategic default, such as Mella-Barral and Perraudin (1997) and Hackbarth et al. (2007), focus on such events. We do not consider them because we believe understanding of bankruptcy payoffs is an important step in understanding non-bankruptcy defaults. Bargaining out of bankruptcy is likely to be influenced by expectations about bankruptcy timing and outcomes.

[2] Lando (2004, §2.13.2) discusses the fundamental role of assumptions on asset-sale restrictions in structural models of credit risk.

[3] Bourgeon and Dionne (2007) extend the Hackbarth et al. (2007) model to allow banks to adopt a mixed strategy in which renegotiation is sometimes refused ex-post in order to raise debt capacity ex-ante.

bonds. So far as we are aware, ours is the first model to explore the implications for bankruptcy thresholds and recovery rates of this ubiquitous feature of private debt.[4]

Our focus throughout this paper is on the endgame phase of the firm's life (bankruptcy and recovery). Upon the onset of severe financial distress, the costs of altering debt composition or raising new equity are likely to be high, and so it is reasonable to take the firm's capital structure as fixed. For firms with assets well above the insolvency value, however, debt composition should be a material endogenous decision for the firm's owners. For example, by choosing bank debt share, the firm can influence the states of the world in which deadweight costs of bankruptcy are incurred, just as the choice of leverage influences the incidence of such costs in the existing capital structure literature. As our model takes equityholders as passive, it is not well-suited to analysis of the firm's ex-ante choice of capital structure. An extension of the model to allow for active equityholders and explicit transfer of control rights, along the lines of Broadie et al. (2007), is left for future work.

Empirically, we find a robust, economically and statistically significant relationship between recovery and bank debt share of total debt at default. A marginal one percentage point increase in bank debt share improves recovery at emergence from bankruptcy ("ultimate recovery") on all the firm's debt taken together ("total" or "firm-level" recovery) by about one-quarter percentage point or more. That is, an increase from a small amount of bank debt to all bank debt would be associated with an increase in recovery rate of about 25 percentage points, other things equal, which is large relative to the sample mean recovery of about 50 percent. Moreover, we find evidence of the loan coupon interest rate effects implied by our model, of some non-linearities it predicts, and of the high recovery rates on loans it predicts. Mean and median recoveries on loans are 85 and 99.5 percent, respectively, and at least 70 percent of bank debt receives approximately a full recovery. Our empirical findings cannot easily be explained by maturity effects or variations in deadweight costs of bankruptcy.

To address concerns that empirical findings might be due to mechanisms other than our model, we compare predictions of three stylized alternatives with the data and with our model's predictions. The data do not support the alternatives as well as our model, though the alternatives are not necessarily rejects. Different models may be complementary in that each is able to explain some (but not all) bankruptcies.

Our model offers some insight into why average bond recoveries are relatively low, a fact that has been difficult to comfortably reconcile with structural models of default. In models with an exogenous threshold, a common assumption for the threshold has been the asset value boundary between solvency and insolvency, in which case firm-level recovery should be not far from 100 percent.[5] Jumps in asset value (Zhou, 2001), accounting uncertainty (Duffie and Lando, 2001), liquidation costs (Fan and Sundaresan, 2000; Mella-Barral, 1999) or (closely-related) asset specificities that imply a large reduction in value when assets are transferred to new owners (Baird and Jackson, 1988) no doubt play a role, but the magnitudes required to produce an average recovery rate near 50 percent seem implausibly large. In contrast, for reasonable parameter values, such a recovery rate is broadly consistent with our model, given that the empirical mean bank debt share

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[4] As noted previously, we use the terms "banks," "bank debt" and "bank loans" as a convenient shorthand for senior debt with strong covenants. Such terminology does not perfectly represent historical patterns of debt structure in the U.S. Corporate bank loans frequently were most senior in firms' debt structures and had the strongest covenants. Privately placed bonds sometimes had such features, and publicly issued bonds rarely did. In 2006–07, a notable share of risky loans was issued without strong covenants.

[5] Practitioner models of default often assume an exogeneous boundary of this sort, e.g. Moody's KMV Credit-Monitor (Crosbie and Bohn, 2003).

is near one-third.

Our findings imply that debt composition matters for debt pricing and credit risk management. Credit spreads and economic capital charges are roughly linear in expected loss-given-default (one minus the recovery rate), so errors in the specification of recovery are potentially costly.[6] And yet, in models and empirical studies of debt pricing, recovery is almost invariably treated as an afterthought. Expected recovery rates are typically assumed to be homogeneous within very broad debt classes (e.g., senior unsecured bonds). Our results indicate that expected recovery rates on individual debt instruments ought to be conditioned on debt composition and on more sophisticated treatments of seniority than the traditional debt classes. In applied settings, a simple rule of thumb could be based on the finding in this paper of a one-quarter-percentage-point improvement in recovery per additional percentage point of bank debt share, combined with adjustments for relative seniority of individual debt instruments.

Our assumptions that covenants give creditors rights to call loans to distressed borrowers and that such rights are attached to loans, not bonds, are realistic. Nash et al. (2003) characterize *bond* covenants as restricting financing, investing and restructuring activities. A common feature of such covenants is that they are violated only when the borrower takes a forbidden action, such as selling a large share of its assets.[7] An increase in the borrower's probability of bankruptcy does not by itself trigger a violation. Chava et al. (2004) find that only 4 percent of nonfinancial corporate bonds have a leverage or net worth covenant. In contrast, Carey (1996) and Sufi (2006) find that around 70 percent of bank loan agreements contain financial ratio covenants, such as interest-coverage, debt-to-cash-flow, and leverage ratios, and Carey (1996) offers evidence that such covenants are more likely to appear in loans to riskier borrowers. Dichev and Skinner (2002) and Chava and Roberts (2006) offer evidence that such covenants are customized to be relatively tight, that is, trigger values of ratios are close to those reported by the borrower at the time the loan is made. The very existence of customized covenants in loan contracts is evidence of a role for banks in setting the default boundary–if such covenants did not influence the states of the world in which bankruptcy occurs, why would such effort be expended on crafting them?

There is evidence as well that loan covenants do provide banks with significant control rights over weak borrowers. Loan covenant violations are likely to accompany an increase in the probability of borrower default. Dichev and Skinner report that borrower financial performance is much worse than average in quarters when a net worth covenant is violated. Beneish and Press (1993) and Chava and Roberts (2006) report that resolution of covenant violations commonly includes fees paid to the lenders, increases in interest rates, and incorporation of additional covenants into the credit agreement. Chen and Wei (1993) report that measures correlated with distance-to-default predict whether a covenant violation is resolved by a waiver or by the lender calling the loan.

The existing recovery literature is largely empirical and has related debt characteristics to recoveries (for example, Altman and Kishore, 1996) or has examined sources of systematic variation in recoveries (e.g., Frye, 2000a,b; Altman et al., 2005; Acharya et al., 2007). For a survey of this literature, see Schuermann (2005). Nearly all of this literature studies recovery at the level of the individual debt instrument. From the perspective of our paper, an individual defaulted

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[6] For example, in Basel II, capital charges under the Internal Ratings-Based approach are proportional to expected loss-given-default (Basel Committee on Bank Supervision, 2005, ¶272).

[7] Some papers that use "covenants" to motivate model assumptions, such as Fan and Sundaresan (2000), focus on the borrower's promise to pay interest and principal on schedule. Legally this is a covenant, but it appears in all U.S. corporate debt contracts and is not viewed as a customizable contract-design feature by practitioners, as are other covenants.

instrument is a collar option on the underlying firm-level recovery with strike prices that depend on the instrument's position in the firm's capital structure. Linear regression models of instrument recovery do not account for the nonlinearity of option returns. More importantly, seniority and collateral status are only rough proxies for the strike prices. By focusing mainly on firm-level recovery, we avoid these specification issues.[8]

Our model and some comparative statics are presented in Section 1. Section 2 summarizes testable implications and stylized alternative models, and describes the data and measures we use in empirical analysis. Results are presented in Section 3. Concluding remarks follow.

1 Model

We model loan contracts in which covenants permit the bank to foreclose on the borrower and force repayment through the bankruptcy process. In the simplest version of the model, we assume that the bank is effectively able to foreclose at will. We derive the bank's optimal choice of "foreclosure threshold." So long as the borrower's asset value remains above this threshold, the borrower is permitted to continue. Upon first-passage across this threshold, the bank forecloses. In the full version of the model, we recognize that covenant violation is needed for foreclosure. We introduce a contractually-specified "covenant threshold" that serves as an upper bound on the foreclosure threshold and also triggers payment of penalty fees by the borrower to the bank in exchange for forbearance.

Our model is an extension of a model in Black and Cox (1976) for perpetual corporate debt with continuous coupons. These assumptions remove time-dependence in the value of debt, which simplifies both the solution of the model and analysis of comparative statics. We also assume there is no restriction on asset sales. When asset sales are restricted, we are led to strategic bankruptcy by equityholders as in Leland (1994) and Leland and Toft (1996). The focus of our model is on the bank's role in initiating bankruptcy, so we therefore assume that assets may be sold freely for the purpose of paying debt coupons. To avoid diversion of assets to equityholders, we assume that debt contracts specify a fixed dividend rule. The borrower's capital structure is assumed fixed with no possibility of raising new equity or debt.

The baseline model is presented in Section 1.1. This model is identical to the model of Black and Cox (1976) except that the foreclosure boundary is chosen endogeneously by the bank. Comparative statics for the baseline model are explored in Section 1.2. The primary interest here is how the share of bank debt in total firm debt influences the distribution of recoveries at the estate level. In Section 1.3, we extend the model to allow for a stochastic shock to firm value upon bankruptcy. The full model is developed in Section 1.4. This allows for a firm-value boundary above which the bank cannot foreclose and below which the bank receives a waiver fee so long as the bank forbears.

1.1 Baseline model

The firm is financed by debt and equity. Without loss of generality, we assume that the total face value of debt is 1. This unit of debt is divided into a single loan with face value λ and a single class

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[8] Among extant empirical studies, only Hamilton and Carty (1999) examine firm-level recovery as we do. They split their sample into firms with and without publicly issued debt and find that the latter have smaller firm-level recoveries on average, which is broadly consistent with our findings. They attribute the difference to larger deadweight costs of bankruptcy due to bargaining frictions associated with more complex capital structures, which is quite different from our explanation.

of bonds with face value $1-\lambda$. The bond is junior to the loan, and (for simplicity) only the loan has covenants that permit foreclosure. The loan receives continuous coupon c and the bond receives continuous coupon γ. Equity receives a continuous dividend of $\delta+\rho V_t$, where V_t is the firm's asset value at time t. We take these parameters as nonnegative constants, and assume $0 \leq \rho < r \leq c$. For notation convenience, let $\mathcal{C}$ be the rate of fixed cash outflows per unit time, i.e.,

$$\mathcal{C} = c\lambda + \gamma(1-\lambda) + \delta.$$

To keep the focus on credit risk, we assume riskfree interest rates are fixed at r. The asset value (*cum* coupons and dividends) follows a geometric Brownian motion with fixed variance σ^2. Under the risk-neutral measure, we have

$$dV_t = V_t((r-\rho)dt + \sigma\, dZ_t) - \mathcal{C}dt \tag{1}$$

In the event of bankruptcy at time t, coupon and dividend payments are frozen. We assume that the legal claims of debtholders accrue at the riskfree rate during court proceedings.[9] Settlement occurs after a fixed length of time τ, and the bank receives $\min\{\exp(r\tau)\lambda, V_{t+\tau}\}$. As Chapter 11 implies a change in control over the firm's assets, we allow for a change in the level of asset volatility (to $\tilde{\sigma}$) at bankruptcy. The standard fixed-maturity, zero coupon Merton (1974) formula can be used to price the recovery value at bankruptcy, which is given by

$$B(V) = M(V, \lambda, \sqrt{\tau\tilde{\sigma}^2}) \tag{2}$$

where

$$M(V, D, s) \equiv V\Phi\left(-\frac{1}{s}\log(V/D) - \frac{s}{2}\right) + D\Phi\left(\frac{1}{s}\log(V/D) - \frac{s}{2}\right). \tag{3}$$

Our depiction of the bankruptcy process generalizes the treatment in the existing literature (e.g., in Leland, 1994), where it is assumed that settlement is immediate (i.e., that $\tau = 0$).

Applying Black and Cox (1976, eq. 18), the valuation equation for the loan satisfies the second-order ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 F'' + ((r-\rho)V - \mathcal{C})F' - rF + c\lambda = 0, \tag{4}$$

for which the general solution is

$$F(V) = \frac{c\lambda}{r} - A_1 \cdot \psi(V; \alpha, \beta, \zeta) - A_2 \cdot \psi(V; 1-\beta, 1-\alpha, \zeta) \tag{5}$$

where A_1, A_2 are constants that are determined by boundary value conditions. The function ψ is given by

$$\psi(V; a, b, \zeta) = (\zeta V)^{-a} \cdot {}_1F_1\left(a, a+b, -1/(\zeta V)\right) \tag{6}$$

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[9] It is more typically (but not universally) observed in practice that the claim on a defaulted loan accrues interest at its *contractual* rate while in bankruptcy, whereas the claim on a defaulted bond need not accrue any interest. We impose accrual at r as it simplifies the analysis somewhat. Results are unaffected over the empirically plausible range of parameter values.

where ${}_1F_1$ is the confluent hypergeometric function. The constants α, β and ζ are given by

$$\begin{aligned}\alpha &= \sqrt{\left(\frac{1}{2}-\frac{r-\rho}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}-\left(\frac{1}{2}-\frac{r-\rho}{\sigma^2}\right)\\ \beta &= \alpha+2-\frac{2(r-\rho)}{\sigma^2}\\ \zeta &= \frac{\sigma^2}{2}\frac{1}{\mathcal{C}}\end{aligned}$$

By re-writing the equation for α as

$$\alpha-\frac{2(r-\rho)}{\sigma^2}=\sqrt{\left(\frac{1}{2}+\frac{r-\rho}{\sigma^2}\right)^2+\frac{2\rho}{\sigma^2}}-\left(\frac{1}{2}+\frac{r-\rho}{\sigma^2}\right)\geq 0,$$

we can bound parameters $\alpha \geq 2(r-\rho)/\sigma^2 > 0$ and $\beta \geq 2$.

Let κ denote the foreclosure threshold. Given a choice of κ, the boundary conditions to equation (4) are $F(\kappa)=B(\kappa)$ and $F(\infty)=\lambda c/r$. Given bounds on α and β, it is straightforward to show that $\psi(V;1-\beta,1-\alpha,\zeta)$ increases without bound as $V\to\infty$. Therefore, to satisfy the boundary conditions on $F(V)$, we must have $A_2=0$. For $\kappa>0$, the solution to A_1 is

$$A_1=\left(\lambda\frac{c}{r}-B(\kappa)\right)\frac{1}{\psi(\kappa;\alpha,\beta,\zeta)}$$

which implies

$$F(V;\kappa)=\lambda\frac{c}{r}-\left(\lambda\frac{c}{r}-B(\kappa)\right)\cdot\frac{\psi(V;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)} \tag{7}$$

where we have written $F(V;\kappa)$ to emphasize the dependence on κ. For the special case of $\kappa=0$, see Black and Cox (1976, eq. 19).

We now allow the bank to choose κ. For simplicity, we assume in the baseline model that the bank can foreclose at will. As the bank's right to foreclose is a perpetual American option, the optimal foreclosure threshold sets the marginal exercise value equal to the marginal continuation value, i.e.,[10]

$$\frac{\partial B(\kappa)}{\partial\kappa}=\left.\frac{\partial F(V;\kappa)}{\partial V}\right|_{V=\kappa} \tag{8}$$

An equivalent approach with perhaps more intuitive appeal is to find the threshold at which the bank is indifferent between foreclosure and forbearance. This indifference point solves the first order condition

$$\mathcal{F}(\kappa)\equiv\left.\frac{\partial F(V;\kappa)}{\partial\kappa}\right|_{V=\kappa}=0. \tag{9}$$

For this model, we have

$$\mathcal{F}(\kappa)=B'(\kappa)-\left(\lambda\frac{c}{r}-B(\kappa)\right)\Xi(\kappa;\alpha,\beta,\zeta) \tag{10}$$

where

$$\Xi(\kappa;\alpha,\beta,\zeta)\equiv\frac{-\psi'(\kappa;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)} \tag{11}$$

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[10] We are grateful to Pascal François for bringing clarity on this point.

and will henceforth usually be abbreviated as $\Xi(\kappa)$. It is easily verified that conditions (8) and (9) are equivalent.

The first order condition is easily evaluated. The derivative of B is $M_1(y, \lambda, \sqrt{\tau\tilde{\sigma}^2})$, where M_i denotes the partial derivative of M with respect to its i^{th} parameter, which simplifies to

$$M_1(V, D, s) = \Phi\left(-\frac{1}{s}\log(V/D) - \frac{s}{2}\right).$$

The derivative of ψ also simplifies:

$$\psi'(y; a, b, \zeta) = -a\zeta(\zeta y)^{-(a+1)} \cdot {}_1F_1\left(a+1, a+b, -1/(\zeta y)\right) = -a\zeta\psi(y; a+1, b-1, \zeta)$$

where the last equality follows from FWC 07.20.20.0024.01.[11] The optimal κ^* does not have closed-form solution in general, but numerical solution using standard routines for one-dimensional non-linear roots is straightforward. For the limiting case of deterministic recovery, we have

Proposition 1 *When $\tau = 0$ or $\tilde{\sigma} = 0$, the optimal foreclosure threshold is $\kappa^* = \lambda$.*

Proof is given in Appendix A. A A finite positive solution to the first order condition always exists. So long as there are positive fixed cashflows to investors other than the bank, there cannot be a corner solution at $\kappa = 0$, because

Proposition 2 $\mathcal{F}(0) = 1 - \frac{c\lambda}{\mathcal{C}}$.

Observe that this expression is strictly positive if $\gamma(1-\lambda) > 0$ or $\delta > 0$. As κ increases towards infinity, $\mathcal{F}(\kappa)$ converges to zero from below, i.e.,

Proposition 3 $\lim_{\kappa\to\infty} \mathcal{F}(\kappa) \nearrow 0$

Proof of these two propositions is outlined in Appendix B. The Intermediate Value Theorem implies that there must be a finite positive κ such that $\mathcal{F}(\kappa) = 0$ and $\mathcal{F}'(\kappa) < 0$.

Our main variable of interest in this paper is total recovery as a share of total debt claims. Measured by post-default market price, recovery rates for all debtholders, the bank and the bondholders are given by

$$\begin{aligned} R &= M(\kappa^*, 1, \sqrt{\tau\tilde{\sigma}^2}) && \text{(12a)} \\ R_\ell &= \frac{1}{\lambda}M(\kappa^*, \lambda, \sqrt{\tau\tilde{\sigma}^2}) && \text{(12b)} \\ R_b &= (R - \lambda R_\ell)/(1-\lambda) && \text{(12c)} \end{aligned}$$

respectively. Note that we express recovery rates as a share of the present discounted value of the legal claim. This definition of recovery is known as the Recovery of Face Value (RFV) convention (see Schönbucher, 2003, §6), and cleaves most closely to practice in bankruptcy court and accounting treatment (Guha, 2003).

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[11] FWC refers to the website `functions.wolfram.com`.

In our data, recovery is measured at emergence. As this recovery is obtained under the physical measure, we must replace r in equation (2) with the drift μ under the physical measure and accrue to the date of emergence. Thus, we have

$$\begin{aligned} R^e &= M(\exp(\tau(\mu - r))\kappa^*, 1, \sqrt{\tau\tilde{\sigma}^2}) && \text{(13a)} \\ R^e_\ell &= \frac{1}{\lambda} M(\exp(\tau(\mu - r))\kappa^*, \lambda, \sqrt{\tau\tilde{\sigma}^2}) && \text{(13b)} \\ R^e_b &= (R^e - \lambda R^e_\ell)/(1-\lambda) && \text{(13c)} \end{aligned}$$

for all debtholders, the bank and the bondholders, respectively.

1.2 Comparative statics for the baseline model

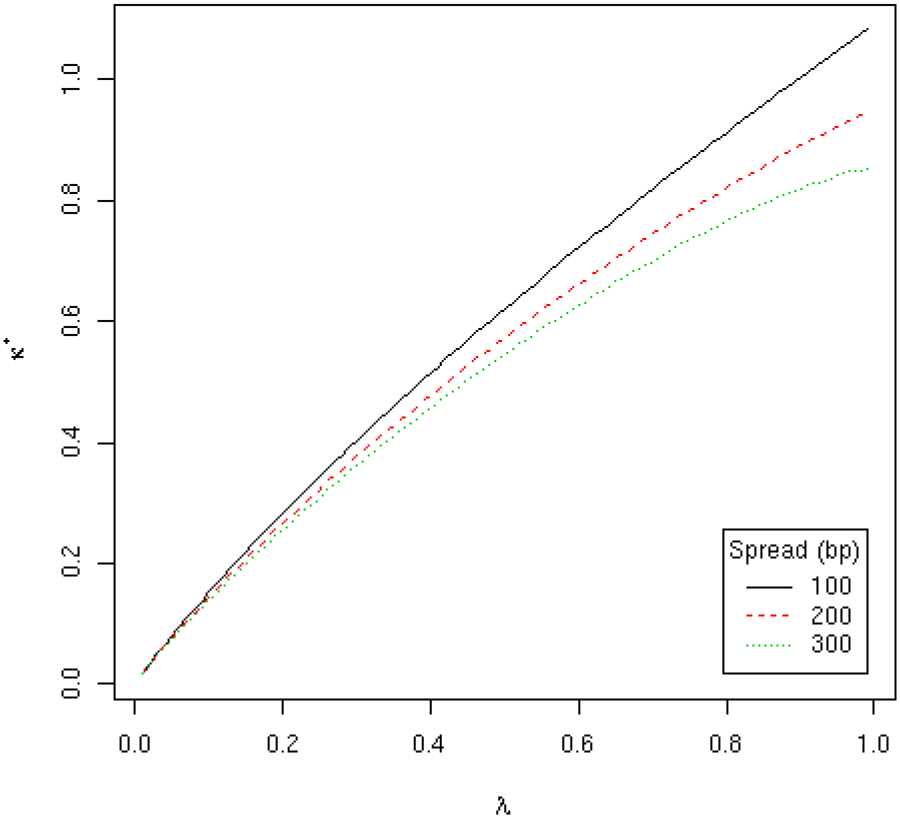
In this section, we examine how the optimal foreclosure threshold and recovery rates vary with changes in model parameters. We recognize that some of these parameters were determined endogenously at the time of contracting as functions of other parameters. The loan coupon rate, for example, would surely have depended on the loan share λ and the asset volatility σ. At the time of contracting, therefore, the total derivative of κ^* with respect to, say, σ would need to account both for the direct effect of σ on bank incentives and indirect effects through the effect of σ on coupon c and other endogeneous variables. By contrast, this paper takes the perspective of a firm in severe distress. The firm's asset value has changed since the time of contracting–fallen substantially, most likely–and therefore the endogeneous relationships that at one time bound c to λ and σ no longer pertain. Empirically, one can observe in a data set of defaulted firms a wide range of combinations of these variables. It is for this reason that we examine each of our comparative statics as partial derivatives, i.e., with all other parameters held fixed.

Comparative statics for the parameters of main interest are by no means straightforward even in parsimonious versions of the model, and so we resort to numerical exercises. We begin with the influence of bank share λ and loan coupon c on the optimal foreclosure threshold. We expect the bank's choice of κ^* to increase with its share of total debt, as the bank forecloses to protect its own stake. When there is no uncertainty on recovery in bankruptcy, as in Proposition 1, $\kappa^* = \lambda$. For modest degrees of recovery risk, we might expect a roughly linear relationship. The influence of loan coupon works through two channels. A higher loan coupon increases the cashflow of the loan, but also drains the firm's assets at a higher rate. Intuition suggests that the first channel should have a positive first-order effect on the marginal continuation value of the loan (which pushes κ^* down), whereas the negative effect of the second channel should have a second-order impact.

Both of these predicted relationships are supported by our numerical results. Figure 1 shows a roughly linear (slightly concave) relationship between λ and κ^*, and that κ^* decreases with c. This pattern is robust over a wide range of parameter values. The negative relationship between bankruptcy threshold and coupon rate stands in contrast to the positive relationship predicted by Leland (1994) and other strategic default models.

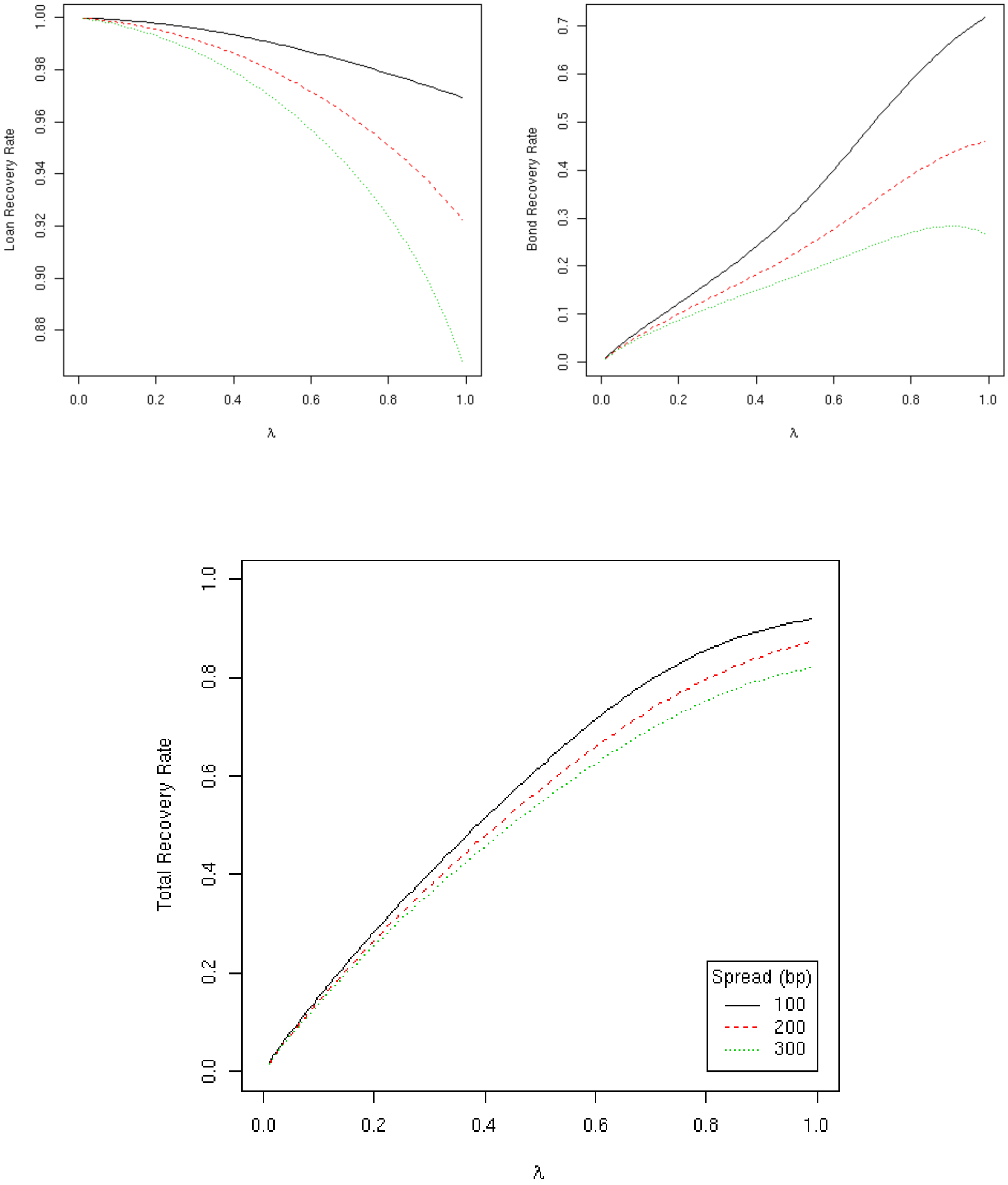
As shown in the bottom panel of Figure 2, total recovery displays the same comparative statics. Somewhat more complicated is the influence of λ on recoveries at the instrument level. In absolute dollar terms, loan recovery increases with λ. However, as depicted in the upper left panel, the loan recovery rate (i.e., as a share of λ) in general is decreasing with bank share. The bond recovery rate is generally increasing with λ, as seen in the upper right panel, but in absolute terms may be humped-shaped in λ.

Figure 1: Effect of debt composition on foreclosure threshold



Spread is $c - r$, measured in basis points. Parameters: $r = 0.05$, $\gamma = 0.08$, $\delta = \rho = 0$, $\sigma = \tilde{\sigma} = 0.2$, $\tau = 1$.

Figure 2: Effect of debt composition on recovery



Recovery rates at emergence. Parameters: $r = 0.05$, $\mu = 0.1$, $c = 0.07$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

Intuition suggests that the bank's optimal κ^* should increase with pre-bankruptcy volatility σ. For any fixed foreclosure threshold, higher σ reduces the expected first-passage time to the threshold, and so reduces the present discounted value of future cashflows received by the bank before bankruptcy. This lowers the marginal continuation value of the foreclosure option and so the optimal κ^* increases. The effect of post-bankruptcy volatility $\tilde{\sigma}$ is ambiguous. For any fixed κ, the loan's recovery value $B(\kappa)$ is decreasing in $\tilde{\sigma}$. At low levels of volatility, an increase in $\tilde{\sigma}$ should cause the bank to raise the foreclosure threshold in order to protect its recovery. At very high levels of volatility, however, protection of recovery becomes too expensive in terms of foregone loan coupons.

These relationships are displayed in the top panel of Figure 3. The foreclosure threshold is everywhere non-decreasing in σ, but the effect is small over the empirically plausible range of $\sigma \in [0.1, 0.3]$ and for $\tilde{\sigma} < 0.3$. As was observed for the loan coupon, the "survival time" channel appears to have only a second-order effect on the optimal foreclosure rule. The effect of $\tilde{\sigma}$ on κ^* is much larger in magnitude, but ambiguous in sign. As shown in the bottom panel, this non-monotonicity is even more apparent at higher values of λ. Similar patterns are observed in the comparative statics for total recovery with respect to the volatility parameters.

Comparative statics for bond coupon (γ) and the fixed rate of equity dividends (δ) can be signed analytically. These two parameters enter the first order condition for the foreclosure threshold through the ζ parameter of the ψ function, so we begin with the lemma:

Lemma 1

$$\zeta \cdot \frac{\partial}{\partial \zeta} \Xi(V; \alpha, \beta\,\zeta) = \Xi(V; \alpha, \beta\,\zeta) + V \cdot \Xi'(V; \alpha, \beta\,\zeta) > 0.$$

This is proved in Appendix C. It follows that

$$\frac{\partial}{\partial \gamma}\Xi(V) = \frac{\partial \zeta}{\partial \gamma} \cdot \frac{\partial}{\partial \zeta}\Xi(V) = -\frac{\sigma^2}{2}\frac{(1-\lambda)}{(c\lambda + \gamma(1-\lambda) + \delta)^2} \cdot \frac{\partial}{\partial \zeta}\Xi(V) = -\frac{(1-\lambda)}{\mathcal{C}}\zeta\frac{\partial}{\partial \zeta}\Xi(V) < 0.$$

Writing the first order condition for κ^* as $\mathcal{F}(\kappa; \gamma)$ to emphasize its dependence on γ, we have

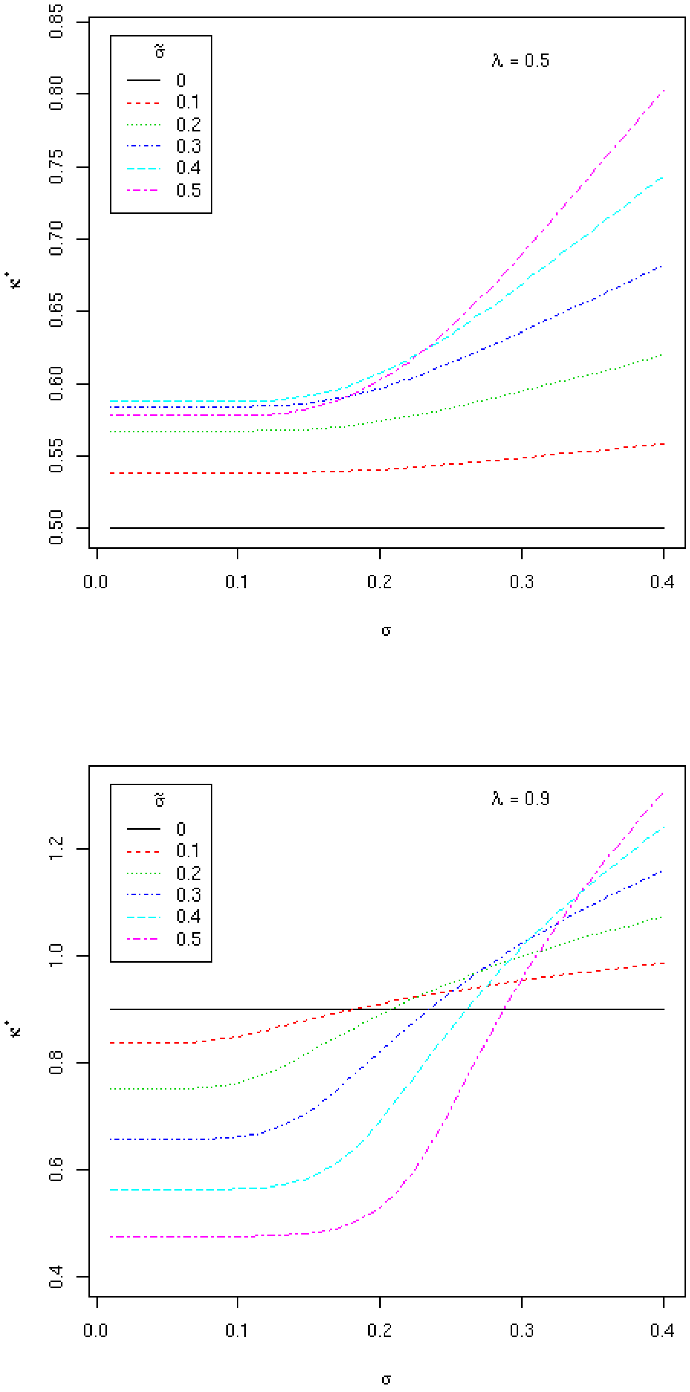
$$\frac{\partial \kappa^*}{\partial \gamma} = -\left.\frac{\partial \mathcal{F}(\kappa;\gamma)/\partial \gamma}{\partial \mathcal{F}(\kappa;\gamma)/\partial \kappa}\right|_{\kappa=\kappa^*}$$

The numerator is

$$\frac{\partial \mathcal{F}(\kappa;\gamma)}{\partial \gamma} = -\left(\lambda\frac{c}{r} - B(\kappa)\right)\frac{\partial}{\partial \gamma}\Xi(\kappa) > 0,$$

as $B(\kappa) \leq \lambda$ for all κ. In order for κ^* to maximize loan value, the second order condition requires $\partial\mathcal{F}/\partial\kappa < 0$ at $\kappa = \kappa^*$. This implies that κ^* is increasing in γ. Parallel arguments show that κ^* is increasing in δ. The intuition for these results is straightforward and based on the "survival time" channel discussed above. An increase in γ or δ increases the rate at which firm assets are drained by subordinated claimants, and so reduces the present discounted value of future cashflows to the bank.

Figure 3: Effect of volatility on foreclosure threshold



Parameters: $r = 0.05$, $c = 0.07$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

1.3 Extension: Stochastic bankruptcy cost

The event of foreclosure can often impart a shock to asset value. Besides the legal costs associated with bankruptcy, franchise value might be sacrificed and certain contracts might be invalidated at foreclosure. In some cases, bankruptcy can help the firm escape a crippling labor contract or pension liability, so the shock need not be negative. We extend the model of the previous section to allow for a foreclosure shock.

We model bankruptcy costs as a multiplicative shock to asset value that is realized immediately following foreclosure by the bank. We assume that the shock is distributed logNormal(χ, η^2). The recovery value $B(V)$ is now

$$B(V) = M(\exp(\chi + \eta^2/2)V, \lambda, \sqrt{\tau\tilde{\sigma}^2 + \eta^2}) \tag{14}$$

It is only through altering the recovery value that χ and η affect the optimal choice of κ.

For this extended model, Proposition 1 generalizes to:

Proposition 1' *There exists* $\underline{\chi} < 0$ *such that the optimal foreclosure threshold is* $\kappa^* = e^{-\chi}\lambda$ *when* $\eta^2 = \tau\tilde{\sigma}^2 = 0$ *and* $\chi \geq \underline{\chi}$.

The proof is a straightforward but tedious extension of the proof in Appendix A. The intuition remains the same as for the baseline case: for bankruptcy shocks that are not too negative, the bank forecloses when the borrower's asset value is just sufficient for the bank to recover fully its own principal.

Proposition 2 also generalizes:

Proposition 2' $\mathcal{F}(0) = \exp(\chi + \eta^2/2) - \frac{c\lambda}{\mathcal{C}}$.

Proposition 3 holds without change. Therefore, the optimal κ^* is always finite but the corner solution $\kappa^* = 0$ may arise when $\chi < -\eta^2/2$. When χ is negative and large in magnitude, it may become too costly (in terms of foregone interest revenue) to protect recoveries, and so κ^* goes to zero. When χ is positive and large, the bank is able to obtain full recovery at a low foreclosure threshold, and so the foreclosure threshold tends to zero in this case too. More formally, in Appendix D we show

Proposition 4

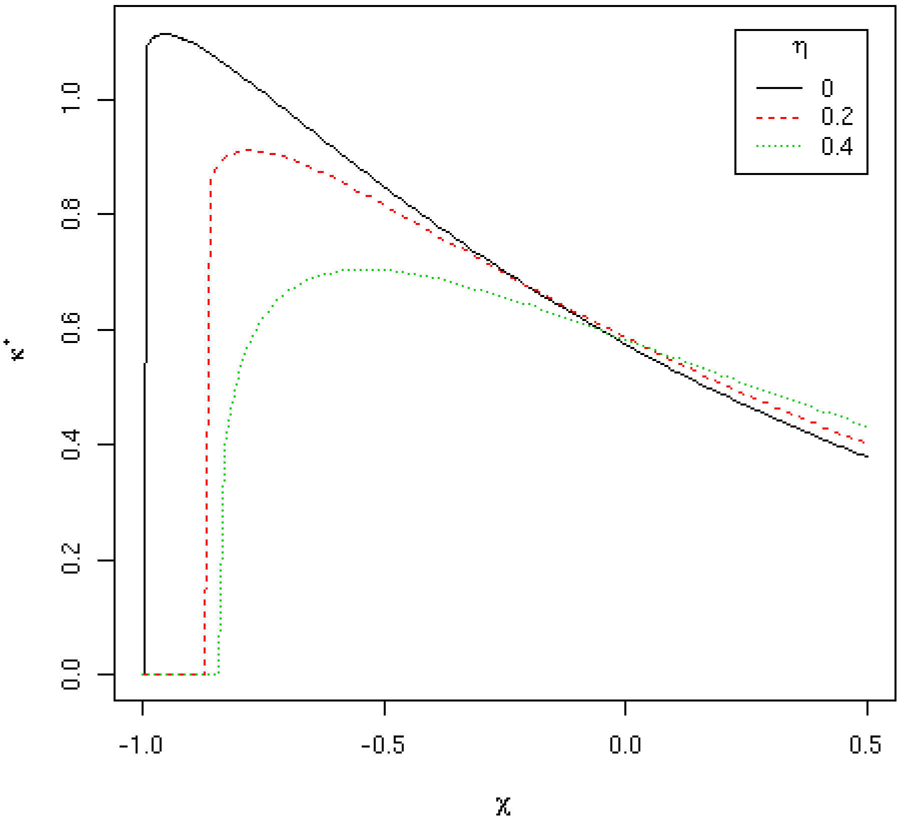
$$\lim_{\chi \to -\infty} \kappa^* = \lim_{\chi \to \infty} \kappa^* = 0.$$

Thus, κ^* is non-monotonic in χ.

The effects of both shock parameters on κ^* are explored numerically in Figure 4. We see that κ^* increases smoothly as χ decreases, and then drops rapidly to zero at extreme values of χ. Similar to $\tilde{\sigma}$, the effect of shock volatility η is non-monotonic. All else equal, higher η reduces the bank's recovery and so compounds the effect of a large negative χ. Therefore, larger η increases the "turning point" in the relationship between χ and κ^*.

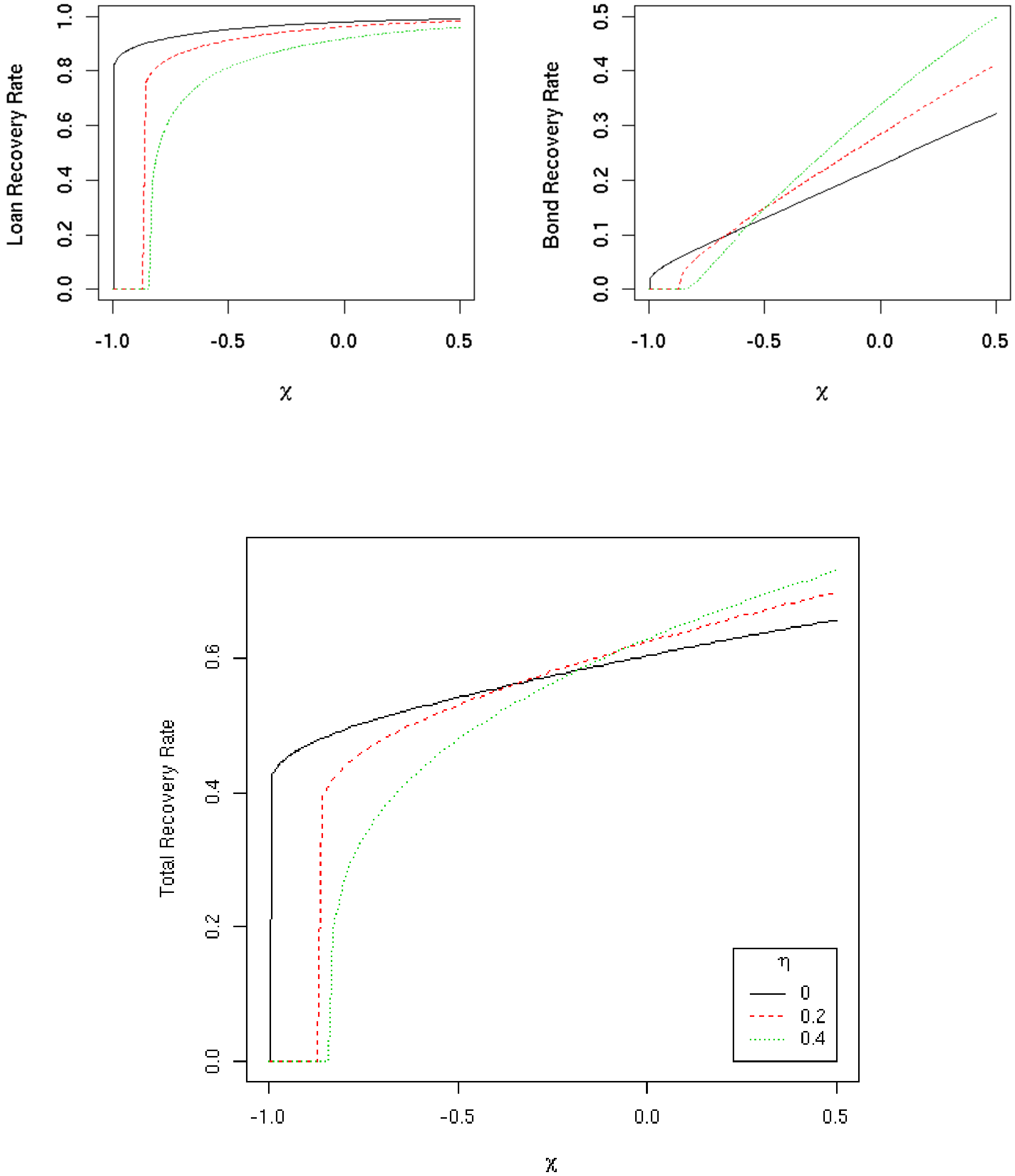
The effect on recoveries at emergence is seen in Figure 5. The loan recovery rate (upper left panel) is strictly increasing in χ and decreasing in η. Comparing the loan recovery panel of Figure 5 with that of Figure 2, we see that this extension to the baseline model allows for materially lower loan recovery rates. The total recovery rate (bottom panel) is also increasing in χ, but the effect of η is non-monotonic. The bond recovery rate (upper panel) is qualitatively similar to the total recovery rate.

Figure 4: Effect of bankruptcy shock on foreclosure threshold



Parameters: $\lambda = 0.5$, $r = 0.05$, $c = 0.07$, $\gamma = 0.08$, $\sigma = \tilde{\sigma} = 0.2$, $\delta = \rho = 0$, $\tau = 1$.

Figure 5: Effect of bankruptcy shock on recovery



Recovery rates at emergence. Parameters: $\lambda = 0.5$, $r = 0.05$, $\mu = 0.1$, $c = 0.07$, $\gamma = 0.08$, $\sigma = \tilde{\sigma} = 0.2$, $\delta = \rho = 0$, $\tau = 1$.

1.4 Covenant boundary and waiver fees

In Appendix E, we introduce a finite covenant boundary ν. Whenever $V_t \leq \nu$, the borrower is considered to be in violation of covenants and the bank has an option to foreclose at will. Whenever $V > \nu$, covenants are satisfied and the bank cannot foreclose. Loan contracts may specify a fee to be paid to the bank when a covenant violation is waived, and in other cases something similar might be achieved by renegotiation at the time of covenant violation. For simplicity, we assume that a waiver penalty of w is added to the coupon rate c whenever $\kappa < V \leq \nu$.

These changes to the setup add realism and richness at the cost of increased complexity. The model's broad implications are not altered. The higher is w, the lower is the recovery rate for both debt classes. The effect on the loan's loss given default of varying w can be quite large on a relative basis, even if not terribly large on an absolute basis.

2 Empirical Strategy, Data and Measures

We offer evidence that bankruptcy outcomes are consistent with our model. Prominent alternatives cannot easily explain all of the evidence.

2.1 Testable implications

Our model has six testable implications. Total (firm-level) recovery rates are: 1) increasing in the share of bank debt in total debt (λ) and 2) decreasing in the coupon interest rate on bank debt (c)s. Seniority and the presence of financial covenants determine whether an instrument is bank debt, not whether it is a loan or the investor is a bank. Thus, 3) only the ratio of loans-with-covenants to total debt should be positively related to total recovery rates, not the ratio of loans-without-covenants to total debt. Our model also has implications for recovery rates on debt instruments of each type: 4) Bank debt recovery rates are slowly decreasing in bank debt share, and 5) bond recovery rates are increasing. Both relationships are conditional on bank debt's coupon interest spread over the risk-free rate. Finally, 6) a large fraction of firms should display high recovery rates on bank debt because banks act to protect themselves, whereas recovery rates on bonds should be lower because bondholders are passive and junior.

Our model's implications for the relationship between recovery and the firm's asset value volatility are untestable. Over the realistic range of values of pre-bankruptcy volatility of the firm's assets, our model implies a weakly positive non-linear relationship that is conditional on post-bankruptcy volatility. Recovery rates are also related to post-bankruptcy volatility, but the sign is unstable (see Figure 3). We cannot observe post-bankruptcy volatility (we would need time series of market prices of the firm's equity and all of its debt, which are rarely observable after bankruptcy is declared). Moreover, in our model, commonly used measures of pre-bankruptcy volatility are measured with an error that is correlated with bank debt share.[12] Overall, asset volatility is not a useful variable for testing the realism of our model or for distinguishing it from other models.[13]

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[12] Common measures implicitly assume a fixed bankruptcy threshold value of assets, such as the insolvency threshold. If the threshold varies across firms, volatility will be measured with error, and in our model, the error will be correlated with bank debt share and thus with recovery rates. If we regress recovery rates on measured volatility, any empirical relationship may simply reflect the endogenous error.

[13] To satisfy curiosity, we added Moody's KMV's measure of asset volatility as an explanatory variable to our primary specification (for a reduced sample that was matchable to Moody's database). For several variants of the

Our model's testable implications hold when other parameters of the model are held fixed. The impact of sample variation in such parameters on point estimates and confidence intervals is particularly likely to be material in regressions on loan and bond recovery rates (less so for total recovery rates; see Figures 2 and 5). Moreover, our model cannot explain all bankruptcies because some involve firms with no bank debt. As noted previously, different models may describe different bankruptcies. The impact on estimates if data are from a mixture of different data generating mechanisms varies by testable hypothesis, as described below.

As noted previously, we assume that bank debt's share of total debt is exogenous when firm value is close to the bankruptcy threshold. In earlier periods, bank debt share is an important choice variable (for example, it influences the states of the world in which bankruptcy occurs). It might be viewed as not important in its own right, but as a summary representation of the characteristics of the firm at the time it made debt structure decisions. Such an interpretation is consistent with our model and with a view that future research on the determinants of debt structure is likely to be fruitful. Our evidence can inform such research.

2.2 Testing strategy

To test the first three implications of our model, we examine parameter estimates from variants of a regression of the form

$$R = a_0 + a_1\lambda + a_2 c + a_n \text{Controls} + \epsilon$$

where R is the ultimate recovery rate on all debt of the firm taken together ("firm-level" recovery), λ is the share of bank debt in total debt, c is the spread over the risk-free rate that the firm pays on bank debt, and Controls is a vector of control variables and other variables of interest taken from the empirical literature on recoveries. If our model explains a substantial share of bankruptcies, we expect $a_1 > 0$ and $a_2 < 0$. If our model explains all bankruptcy decisions and recoveries, we would expect $a_1 \approx 1$.

To test implications 4 through 6, we provide summary statistics about loan and bond recovery rates and we examine the relationship of such recovery rates to bank debt share using separate regressions for loans and bonds that are otherwise similar to the one above.

2.3 Alternatives

Some ideas in the literatures on default, bankruptcy, recovery rates, and debt pricing potentially have empirical implications that match some of our model's implications, but not all. The number of extant models is too large for each to be considered in detail here. We consider two stylized alternatives as well as variants of Leland (1994).

Random threshold: A stylized alternative in the spirit of Merton (1974) or Longstaff and Schwartz (1995) has a threshold that is exogenous, known *ex ante*, and that is an independent variable with support [0,100]. Draws represent a percentage of the solvency-threshold value of assets. By construction, firm-level recovery rates are unrelated to bank debt share. Like our model, this alternative implies a negative relationship between loan coupon rates and recovery (but the

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measure and of the specification (e.g. linear, spline with various knots, quadratic, various measurement dates), the coefficient on volatility was generally statistically insignificant and its sign was not robust. The presence or absence of volatility in the specification did not affect other results.

reason is different: firms with low exogenous threshold values are more likely to have low loan recoveries, which should be priced *ex ante*). Unlike our model, this alternative has no implication for the impact of the presence of covenants on recovery rates. In a simulated version of this alternative model, both loan and bond recovery rates are negatively correlated with bank debt share because of the influence of the share of senior loans on the frequency of full-recovery and zero-recovery observations. If such observations are excluded, instrument-level recovery rates are uncorrelated with bank debt share. Only about half of loans have full recoveries (due to seniority), and covenants do not matter.[14]

Deadweight costs are related to bank debt share: Deadweight costs of bankruptcy drive a wedge between recoveries and threshold values. Thus, if bank debt share is negatively correlated with deadweight costs, a postive empirical relationship between firm-level recovery rates and bank debt share might be observed even if bank debt share is unrelated to threshold values of assets. For example, banks might reduce bargaining frictions more if their share of the firm's debt is larger, and bargaining frictions might be related to deadweight costs. Moreover, if deadweight costs are priced at loan issuance, a negative empirical relationship between bank debt share and loan coupon rates might be observed.[15] We do not associate this deadweight-cost story with any single fully fleshed-out model because it has the potential to fit with several. Our primary approach to testing the story is to include in the empirical specification common proxies for bargaining frictions, which we assume are related to deadweight costs. We are interested in whether inclusion or exclusion of such proxies has a material effect on the estimated coefficient on bank debt share, which we would expect if it is standing in for deadweight costs.[16]

Leland's model: In Leland and Toft (1996), firms with shorter-maturity debt have larger required debt-service payments because the amount of principal coming due in each period is larger. In this model and others in the spirit of Leland (1994), all debt service payments must be financed by outside equity. The default threshold is the firm value which equates the value of equity's option to the required debt-service payment. Thresholds, and thus firm-level recovery rates, are higher the higher the coupon interest rate (the opposite of our model's prediction) and are higher the shorter the maturity of debt (our model is silent about maturity effects). Thus, if bank debt maturities are shorter than bond maturities on average, oa positive empirical relationship between total recovery rates and bank debt share might be due to maturity effects in the absence of controls for maturity. Other models might also suggest a role for maturity. We include loan coupon rate and measures of maturity in some specifications.

The alternatives are all in the spirit of structural models of default. Such models abstract from many aspects of bargaining and contracting. We believe these are important, but in this

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[14] Simulated thresholds were distributed uniformly on [0,1], as were simulated bank debt shares. If draws of thresholds (bank debt share) are from the empirical distribution of total recovery rates (bank debt share) as represented by our data, the fraction of bank debt with very high recovery rates is higher but other relationships are similar.

[15] If we add deadweight costs that are sensitive to bank debt share to the random-threshold simulation mentioned above, a positive correlation between firm-level recovery rates and bank debt share arises by constructioni. The relationship for loan recovery rates again depends on whether full-recovery and no-recovery loans are excluded. The sign of the relationship for bond recovery rates is indeterminate, depending on the strength of the relationship between deadweight costs and bank debt share. Again only about half of loans are simulated to have full recoveries, and covenants do not matter.

[16] Any mechanism that causes χ to be negatively correlated with λ would be relevant. We focus on deadweight costs as the most intuitive specific alternative.

paper we do not address them due to the difficulty of adequately covering the diverse literature.[17] However, our second alternative implicitly sheds light on models in which bargaining frictions influence recovery.[18]

Table 1 summarizes the implications of our model and the alternatives. Blank cells in a column mean no testable implication of the alternative.

2.4 Data

For bankrupt firms, we measure recovery rates, the debt structure of firms, and firm and debt characteristics. Recovery rates and debt structure are from Standard and Poor's LossStats database, which tracks debt structure and ultimate recovery for each debt instrument outstanding at default for each firm in the database.[19] For example, suppose a firm defaulted and declared bankruptcy on 1 June 1998, that it emerged from bankruptcy exactly one year later, and that the firm's debt on the bankruptcy date consisted of a single bank loan and a single bond issue. Suppose that at emergence, the holders of the loan and bond received a mixture of cash and debt obligations of the emerging firm in compensation for their claims. LossStats records:

- The market value of such compensation at the time of emergence, separately for each pre-bankruptcy debt instrument.[20]
- The identity and some characteristics of the firm and of its experience in bankruptcy, such as the court which handled the case.
- Some characteristics of each debt instrument, such as original-issue amount, amount of principal outstanding at default, coupon interest rate, whether the instrument is subordinated or secured, and the priority class to which the instrument is assigned by the bankruptcy court.

We use only data for bankruptcies, not for distressed restructurings.[21] Almost all the firms are U.S. firms. Most had publicly issued debt or equity outstanding at default.

The LossStats release that we use ends in late 2006, but bankruptcies appear in the database only after they are resolved (because only then can ultimate recovery be determined). This raises the possibility of bias: Firms that take a long time to emerge from bankruptcy may be more likely to be omitted from our analysis. A common supposition is that the debt of such firms tends to have smaller recovery. Rather than complicating estimation by including corrections for censoring,

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[17] Particularly notable are models that focus on pre-bankruptcy default and renegotiation, such as Fan and Sundaresan (2000), in which bankruptcy is only one potential outcome of bargaining and asset value is only one factor considered by agents.

[18] For our purposes, it is reasonable to ignore bargaining by equityholders after bankruptcy is declared because, in the U.S., equityholders of bankrupt firms in effect lose control of the firm. Their claim is deeply subordinated by the court, so their threat against debtholders is weak once bankruptcy is declared.

[19] Most available recovery databases do not support measurement of debt structure at bankruptcy or of recoveries to the firm's debt taken as a whole because they have data for only a subset of debt instruments. S&P obtains LossStats data primarily by analyzing SEC filings and bankruptcy court documents. Values of compensation received at emergence are gathered from a variety of sources.

[20] LossStats has information for the complete debt structure of each firm, but not about equity or preferred stock claims and their recoveries, nor about accounts-payable or other liabilities (discussed further below).

[21] In pre-bankruptcy bargaining and contracting, agents' expectations about bankruptcy outcomes and how it will work are likely to influence what they do. We focus on bankruptcy as a step toward better understanding of contracting and bargaining.

Table 1. Summary of predictions of alternatives

Predictions in ( ) denote cases where the variable may be standing in for another variable. For example, in the deadweight costs alternative, λ is expected to be positive because, in the absence of controls for variation in deadweight costs across firms, bank debt share might stand in for such controls.

Alternative	Variable or testable implication								
	Sign on λ	Sign on c	Proxies for Frictions	Loan maturity	Covenants matter	High loan recovery	Instrument level loan on λ	bond on λ	Interactions
Our model	+	-			yes	yes	weak -	+	λ and c
Random threshold	- or 0	-					-	-	
Random threshold w/deadweight costs	(+)	-	affect λ emt				-	indet.	
Leland et al	(+)	+		- & affects λ emt			(+)	-	

we demonstrate robustness of results to dropping from the sample those firms with bankruptcy dates in later years. Mean (median) time in bankruptcy is 14 (12) months, regardless of how many trailing bankruptcies are dropped, and the longest bankruptcy took a bit less than six years to resolve. In obtaining most results, we drop the ten bankruptcies declared in 2005 and 2006 that had appeared on LossStats as of the release date, leaving 644 usable bankruptcies. We check robustness by dropping all bankruptcies after 1997 and find similar results.

We matched LossStats observations to entries in Compustat (to obtain financial statement variables and ratings) and to Loan Pricing Corporation's Dealscan database (primarily to obtain information about loan covenants). In creating and cleaning variables, we used a variety of sources to learn about details of bankruptcies or debt structure, especially SEC filings and Moody's Bond Record. The date of Compustat balance-sheet and income-statement variables is the latest fiscal year-end date that precedes the bankruptcy date. Where the available fiscal year-end data is more than 1.1 years before the bankruptcy date, we eliminate the firm from the Compustat-matched subsample.

2.5 Recovery measures

We normalize recovery cash flows by amount-owed in order to work with recovery rates. Our instrument-level measure of recovery is R_i/D_i, where R_i is the total dollar amount of the recovery received by holders of debt instrument i and D_i is the total amount owed according to the terms of the debt contract and the rules of bankruptcy. Our total, firm-level recovery measure is R/D, where R is the sum of dollar recoveries on all of the firm's debt instruments and D is the sum of amounts owed. Thus, firm-level recovery rates are weighted averages of the recovery rates on the firm's individual obligations. For some of our examinations of recoveries on loans and bonds, we compute similar weighted averages of all individual bank debt (bond) recoveries so that we have a single representative bank debt (bond) recovery for the firm.

We examine firm-level recovery (and representative loan and bond recoveries for each firm) partly because our model suggests it and partly because it seems likely to be a cleaner measure than individual debt instrument recoveries for examining how characteristics of the firm and the economic environment affect bankruptcy thresholds. Some firms have many bonds or loans for small amounts and some have a few large ones, so pools of individual instruments implicitly weight the experiences of firms differently.

Figure 6 illustrates the different properties of instrument- and firm-level recovery. The hump-shaped line plots the kernel-smoothed empirical distribution of firm-level recovery at emergence for our data. The U-shaped line plots the empirical distribution of recovery for individual debt instruments (not combined into a single representative loan and bond for each firm). The instruments resemble collar options so it is unsurprising that their distribution is bimodal with peaks at or near out-of-the-money (zero recovery) and full recovery.[22] As noted previously, the firm-level data in

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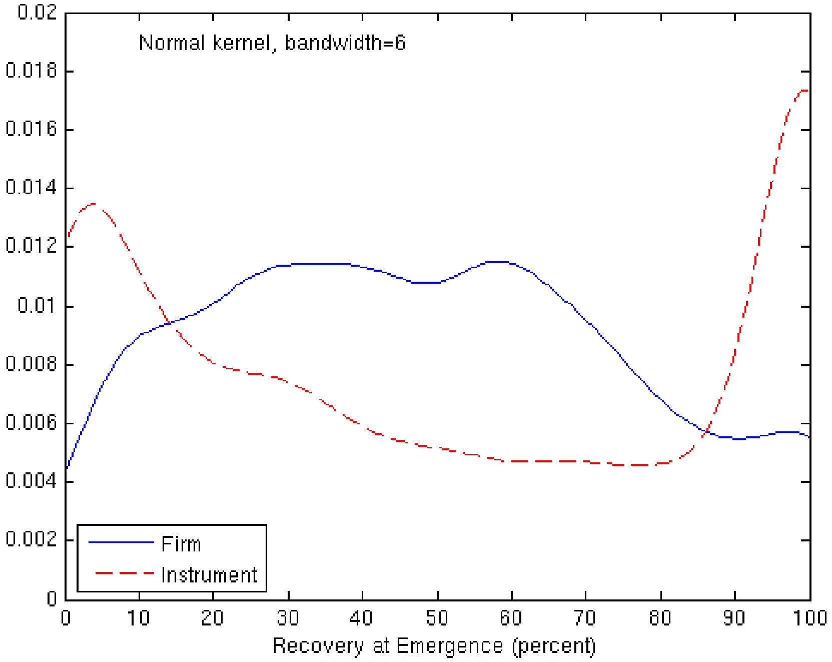
[22] Using pools of individual instrument-level recoveries would require adequate controls for the non-linear payoff properties of instruments, which are difficult to develop. Individual debt instruments of a bankrupt firm, like any corporate debt, are contingent claims on the value of the firm at emergence from bankruptcy. However, payoff properties of the claims are specified by bankruptcy law rather than the pre-bankruptcy contractual terms of the instrument. The court uses the rules of absolute priority to rank-order debt instruments into classes. Assets are allocated to each class in order of priority until assets are exhausted. Thus, a debt instrument of a bankrupt firm is similar to a collar option written on the value of the firm: It can be out-of-the-money, receive part of its claim, or all of its claim, depending on court's determination of the value of the firm. A number of practical problems in measuring the strike prices or attachment points for each instrument make satisfactory specification of the relationship between

the figure were constructed from the instrument-level data, so differences in sample selection do not drive shapes of the curves.

The small fraction of firms with high total recovery rates (illustrated by the rather low level of the hump-shaped line at recovery rates about 80 percent) is striking, as is the wide spread of recovery rates. Such a distribution either implies that most firms are deeply insolvent on the bankruptcy filing date or that deadweights costs (or other jumps in firm value) are much larger and far more variable in the cross-section than previous studies have found. Deep insolvency is to be expected in our model, but is difficult for many structural models of default to account for.

Figure 6: Distribution of Recovery at Firm-level and Instrument-level



Kernel estimate of density of recovery at emergence. The “firm” curve is for total recovery on all debt, whereas the “instrument” curve is for all individual debt instruments observed in our data.

We examine ultimate recovery, defined as recovery received at emergence from bankruptcy, because data limitations are such that it is the best available measure of the payoff on debt of a bankrupt firm. Much of the empirical literature has examined recovery-at-default, proxied by the secondary market trading price of defaulted debt instruments approximately 30 days after default. One problem is that post-default price data are not available for many instruments, making it impossible to construct firm-level measures of recovery-at-default for most bankruptcies. Another problem is that the trading price soon after default embeds estimates of the seniority class to which the instrument will be assigned by the court as well as market estimates of the present discounted value of the firm at emergence.[23],

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instrument-level recovery and firm characteristics a subject for future research.

[23] Our model allows for variation in firm value during the time between bankruptcy and emergence but assumes a

2.6 Bank debt share

"Loans" are any debt that LossStats' broad debt type variable describes as a "Line of Credit," "Revolving Credit," or "Term Loan" (examples of other common classifications are "Subordinated Bonds," "Senior Unsecured Bonds," etc.). We examined LossStats' more detailed description of each instrument (and other sources as necessary) and removed from the loan category any instrument that does not appear to be arms-length debt owed to bank-like lenders (for example, loans from suppliers or parents) (results are robust to leaving such instruments as loans).

In our model, loans without financial covenants are not "bank debt" and should not be included in measures of bank debt share. However, in some models, it is the nature of the lender that matters. Thus, identification of loans with covenants is potentially useful. We matched firms and loans with LPC Dealscan and consulted other sources in order to identify loans with financial covenants. Of 509 sample firms with any loan amount outstanding at bankruptcy, we found positive evidence of financial covenants in loans for 428 and only 13 firms with loans having no financial covenants. For the remaining 68 firms, insufficient information is available.

For most of our empirical exercises, we include in measures of bank debt share all loans except those at the 13 firms identified as having no covenants. In some exercises, we employ separate measures of the shares of debt with covenants, without covenants, and with unknown covenant status.

2.7 Non-debt claims

At emergence from bankruptcy, firm value is allocated not only to holders of pre-bankruptcy debt claims, but also to pay administrative costs of the bankruptcy, to pay taxes, and to other claims such as accounts payable.[24] As noted, our data report only recoveries on debt, so our firm-level recoveries represent a lower-bound estimate of the value of the firm's assets at emergence. For example, accounts payable are usually treated as "general unsecured claims." Other things equal, a larger share of accounts payable in total liabilities should reduce the dollar amount of our measure of firm-level recovery. We check robustness by using the shares of different types of non-debt and non-equity claims in total liabilities as predictors and find that only accounts payable predicts our measure of recovery, as discussed further below.[25]

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constant time in bankruptcy. To examine robustness to the effect of variations in time in bankruptcy purely due to the time value of money, we discounted values at emergence back to the bankruptcy date using a variety of assumptions about discount rates. Our results are robust to use of any of the discounted measures. The cross-sectional variation of discount factors due to time in bankruptcy is quite small relative to the cross-sectional variation in recovery rates at emergence.

[24] Many bankrupt firms obtain superpriority debtor-in-possession (DIP) loan commitments, usually from banks, which almost always are repaid in full if any balances are outstanding at emergence. However, funds are generally not drawn under such facilities. The facilities help the firm to continue operations by assuring trade creditors that the firm will not experience a liquidity problem while in bankruptcy. Only a few DIP loans appear in our data (which includes only loan commitments for which balances were outstanding at bankruptcy or emergence), and in only one case were new balances added after the bankruptcy was filed. Thus, our results are not affected by the superpriority status of DIP loans.

[25] Deviations from absolute priority that involve transfers from holders of one debt instrument in our sample to another are immaterial to our analysis of firm-level recovery. However, deviations that involve payoffs to equityholders will reduce firm-level debt recovery. Bharath et al. (2007) offer evidence that such deviations are usually small during our sample period.

2.8 Summary statistics

Table 2 presents mean, median, minimum and maximum values for many of the variables that appear in the analysis below, for the full sample (644 usable observations) and for the Compustat-matched subsample (389 observations). Control variables are described below as results are discussed. Average firm-level recovery is about 50 percent, but individual-firm recoveries range widely, with the best outcome being a gain of 63 percent of the amount of the claim and the worst being a total loss.[26] The total amount of debt claims is $290 million for the median firm. The median firm in the Compustat-matched subsample had approximately a zero net worth at the fiscal year-end before filing (BookLeverageRatio$\approx$ 1, computed as liabilities/assets) and had four debt instruments outstanding.

On average, bank debt represents about one-third of all firm debt, and ranges from none to all. 24 percent of sample firms had no bank debt as of the filing date. Although our model admits such firms, they may be unusual, so we include dummy variables in regressions for such firms and also for firms with all bank debt.

About 62 percent of Compustat subsample firms had an S&P rating at the fiscal year-end before filing, and only about 10 percent of such firms were rated BB or better (very few were rated investment-grade).

3 Empirical Results

Table 3 reports estimates from simple ordinary least-squares models of firm-level recovery. Parameter estimates and statistical significance are similar when produced by Tobit estimation or when all observations with recovery rates above 100 percent are dropped (not tabulated)). p-values are computed from standard OLS variance-covariance matrices; White's (1980) specification test does not reject homoskedasticity for any of the regressions we run. Cases where use of robust standard errors is material to statistical significance are mentioned below.

Somewhat arbitrarily, we regard column 1 as the base specification because it is estimable for a relatively large sample.

3.1 Bank debt share

Bank debt share is an economically and statistically significant predictor of firm-level recovery rates, with a 1 percentage point increase in share associated with about a one-quarter percentage point increase in recovery rate, other things equal (bank debt share is measured as a fraction, so the estimated coefficient is the change in percentage points of recovery for a change in share from 0 to 1, or 27.00). If all debt structure variables are dropped from the regression, the adjusted R-square drops by more than half, roughly implying that debt structure is a more important predictor of recovery than all the other variables taken together. These results are consistent with the alternatives we consider only if bank debt share is proxying for something else.

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[26] The dollar value received by debtholders at emergence sometimes exceeds the amount of debtholders' claims. Some such cases may arise because our measure of claims is imperfect, but many occur because time elapses between filing of the firm's plan of reorganization and emergence from bankruptcy. If the value of the firm increases sharply during this interval, or if the court's estimate of value as embodied in the plan of reorganization is too small, debtholders may receive some value that would have gone to equityholders (or other deeply subordinated claimants) in a world of instantaneous action and perfect information. Our results are robust to dropping all observations with firm level recovery greater than 100 percent, which comprise less than 5 percent of observations in our sample.

Table 2. Sample summary statistics

Data are for all bankruptcies in the LossStat database with bankruptcy filing dates before 2005. Debt claim amounts and assets-of-firm are in millions of dollars. Compustat data are as of the most recent fiscal year-end date preceding the bankruptcy filing date, except that data for year-end dates more than 1.1 years prior to the filing date are eliminated. Number of debt instruments is the number of separate debt obligations of the firm at the time bankruptcy is filed, whereas number of priority classes is the number of different class labels assigned by the court that are shown for debt instruments in LossStats.

	Full Sample				Compustat Subsample			
Variable	Mean	Median	Min	Max	Mean	Median	Min	Max
NumberOfBankruptcies			644				389	
RecoveryRateInPct	50	48	0	163	51	49	0	138
AmountOfClaims$Mil	680	290	12	32869	792	355	12	32869
TotalBookAssets$Mil					1856	445	2	103914
LoanIntRateSpread	2.81	2.75	0.32	10.50	2.75	2.75	0.32	7.00
Debt Structure								
BankDebtShareAsFrac	0.34	0.28	0.00	1.00	0.33	0.28	0.00	1.00
NoBankDebtDummy	0.24	0.00	0.00	1.00	0.24	0.00	0.00	1.00
AllBankDebtDummy	0.08	0.00	0.00	1.00	0.06	0.00	0.00	1.00
SecuredDebtShareInPct	0.44	0.41	0.00	1.00	0.42	0.39	0.00	1.00
AllSubDebtDummy	0.05	0.00	0.00	1.00	0.06	0.00	0.00	1.00
NoSubDebtDummy	0.38	0.00	0.00	1.00	0.35	0.00	0.00	1.00
SubDebtShareInPct	0.31	0.25	0.00	1.00	0.32	0.25	0.00	1.00
Frictions								
YearsInBankruptcy	1.23	1.01	0.05	5.79	1.23	0.95	0.09	5.79
YearsFromPlanToEmerge	0.48	0.33	0.01	3.82	0.46	0.33	0.03	3.82
YearsInDefaultPreFile	0.31	0.08	0.00	3.15	0.28	0.06	0.00	3.06
PrePackagedBRDummy	0.26	0.00	0.00	1.00	0.26	0.00	0.00	1.00
NumberDebtInstruments	4.47	4.00	1.00	55.00	4.91	4.00	1.00	55.00
NumberPriorityClasses	2.21	2.00	1.00	16.00	2.34	2.00	1.00	16.00
Bad Actors								
FraudDummy	0.05	0.00	0.00	1.00	0.06	0.00	0.00	1.00
FiledAgainWithin5YrsDumm	0.04	0.00	0.00	1.00	0.06	0.00	0.00	1.00
Presiding Court Dummies								
Court_CA	0.06	0.00	0.00	1.00	0.07	0.00	0.00	1.00
Court_NY	0.19	0.00	0.00	1.00	0.20	0.00	0.00	1.00
Court_DE	0.35	0.00	0.00	1.00	0.37	0.00	0.00	1.00
Court_IL	0.03	0.00	0.00	1.00	0.02	0.00	0.00	1.00
Court_TX	0.08	0.00	0.00	1.00	0.08	0.00	0.00	1.00
Compustat Variables								
NonIntangAssetToTotal					0.87	0.97	0.20	1.00
BookLeverageRatio					1.19	1.00	0.25	5.50
OpIncomeToAssets					0.03	0.05	-0.25	0.25
AcctsPayableToTotLiabs					0.09	0.07	0.00	0.30
PPE_ToAssets					0.38	0.37	0.00	0.96
RatedBBOrBetter					0.10	0.00	0.00	1.00
RatedSingleB					0.30	0.00	0.00	1.00
RatedCCC					0.14	0.00	0.00	1.00
RatedWorseThanCCC					0.08	0.00	0.00	1.00
Most Industies Not Shown								
BubbleFirmDummy	0.08	0.00	0.00	1.00	0.09	0.00	0.00	1.00
UtilityDummy	0.02	0.00	0.00	1.00	0.02	0.00	0.00	1.00

Table 3. Main firm-level recovery rate regressions

The dependent variable in OLS regressions, with p-values based on conventional standard errors, is the firm-level recovery rate at emergence. The shares of bank, subordinated and secured debt are the fractions of each type of debt outstanding at default. The utility dummy indicates regulated public utilities, such as natural gas delivery companies. Court dummies identify the location of the court that supervised the bankruptcy. The omitted court is "all others." Industry dummies are based on a judgmental collapsing of industry codes provided by S&P into sixteen categories, all of which are included in regressions reported in all but column 3, but only utility, telecom, computer, and airline are shown to save space. Others are statistically insignificant.

	(1)		(2)		(3)		(4)		(5)	
Independent	Base case		Add coupon		No deadweight		Add maturity		Separate covs	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Intercept	56.82	<.0001	78.57	<.0001	58.64	<.0001	53.66	0.0001	56.75	<.0001
Share bank debt	**27.00**	**<.0001**	26.55	0.0045	24.69	0.0002	**41.79**	**<.0001**		
Share bank debt with cov									**26.62**	**0.0001**
Share bank debt no cov									**6.48**	**0.7978**
Share bank debt no info yet									**28.24**	**0.0015**
Loan maturity (years left)							**-2.74**	**0.0644**		
Bond maturity (years left)							**-3.26**	**0.1135**		
Loan coupon spread			**-4.51**	**0.0030**						
No bank debt dummy	-2.27	0.5881			-4.15	0.2599			-2.53	0.5567
All bank debt dummy	9.48	0.0998	5.56	0.4120	9.22	0.1082			9.59	0.0980
Share secured debt	-2.44	0.5510	-3.59	0.5902	-1.87	0.6450	-10.61	0.1065	-2.30	0.5770
All sub debt dummy	-10.62	0.0945			-10.56	0.0954			-10.39	0.1048
No sub debt dummy	2.28	0.5604	3.77	0.5031	1.48	0.6884	8.66	0.1237	2.26	0.5663
Share sub debt	-9.74	0.1434	-7.89	0.4400	-10.06	0.1140	-0.22	0.9812	-9.66	0.1480
Bankruptcy year dummies:										
1987-88	-1.42	0.8740	-8.66	0.5187	-2.41	0.7838	0.00	.	-1.47	0.8704
1989	-13.15	0.1252	17.34	0.4270	-14.34	0.0905	10.89	0.6884	-13.29	0.1249
1990	-12.03	0.0822	-14.53	0.3290	-13.33	0.0507	-30.61	0.0119	-12.08	0.0827
1991	-2.46	0.6789	-12.53	0.2567	-2.97	0.6144	3.33	0.7524	-2.45	0.6806
1992	0.43	0.9457	-15.89	0.1865	0.72	0.9082	-19.72	0.1359	0.44	0.9439
1994	-6.02	0.4300	-0.59	0.9674	-6.31	0.4019	-12.57	0.3047	-5.94	0.4371
1995	0.45	0.9493	0.49	0.9678	0.57	0.9355	6.90	0.6020	0.59	0.9332
1996	-2.16	0.7683	-1.15	0.9204	-3.35	0.6463	-2.90	0.8137	-2.24	0.7605
1997	-9.03	0.2576	-8.40	0.4791	-10.78	0.1750	-9.67	0.3940	-8.98	0.2615
1998	-13.36	0.0787	-23.82	0.0669	-14.77	0.0493	-19.15	0.1047	-13.22	0.0826
1999	-7.18	0.2417	-18.94	0.0653	-8.18	0.1743	-13.96	0.1447	-7.29	0.2361
2000	-15.34	0.0111	-20.91	0.0363	-15.91	0.0072	-18.78	0.0400	-15.21	0.0121
2001	-16.44	0.0039	-23.04	0.0176	-17.09	0.0021	-21.25	0.0188	-16.27	0.0045
2002	-15.45	0.0070	-18.23	0.0706	-15.14	0.0074	-18.67	0.0412	-15.30	0.0079
2003	-1.68	0.7831	-1.47	0.8849	-1.11	0.8534	-3.67	0.6941	-1.65	0.7870
2004	-2.82	0.6831	-1.08	0.9235	-1.67	0.8075	-8.88	0.3730	-2.69	0.6979
Time in bankruptcy	0.63	0.6738	0.68	0.7476			1.22	0.5717	0.65	0.6633
Time from plan to emerge	-5.07	0.0579	-11.82	0.0107			-8.29	0.0340	-5.06	0.0587
Time in default pre-filing	-0.10	0.9657	1.90	0.6665			2.96	0.4151	-0.11	0.9614
Prepackaged bankruptcy	6.38	0.0265	1.50	0.7133			4.90	0.1977	6.41	0.0263
Number debt instruments	0.15	0.6177	0.52	0.1545			0.70	0.0889	0.14	0.6411
Number priority classes	0.28	0.7932	-0.74	0.5650			0.29	0.8690	0.27	0.8036
Fraud dummy	0.18	0.9712	-2.35	0.7353			-0.30	0.9647	0.21	0.9674
Filed again within 5 yrs dum	-6.01	0.2667	-5.54	0.4273			-11.47	0.1279	-5.83	0.2844
Court dummies:										
California	-1.84	0.7078	14.84	0.0778	-2.19	0.6549	9.77	0.1853	-1.98	0.6878
New York	-5.51	0.0825	-5.02	0.2561	-4.76	0.1282	-1.52	0.7235	-5.56	0.0806
Delaware	-5.55	0.0536	-6.15	0.1203	-4.50	0.1143	-5.03	0.1817	-5.56	0.0535
Illinois	-3.26	0.6214	-6.83	0.4828	-2.70	0.6816	6.34	0.5138	-3.33	0.6150
Texas	-2.18	0.6125	-0.04	0.9947	-2.90	0.4970	-1.26	0.8252	-2.16	0.6165
Selected industry dummies:										
Bubble-firm dummy	-16.06	0.0081	-25.36	0.0039	-15.69	0.0097	-22.53	0.0080	-16.19	0.0078
Utilities	24.80	0.0013	10.74	0.2860	23.19	0.0015	11.76	0.3000	24.85	0.0014
Telecom	-5.39	0.3647	-4.21	0.6076	-4.46	0.4490	-0.75	0.9255	-5.19	0.3858
Computer	-5.34	0.2440	-6.62	0.3187	-4.63	0.3063	-8.15	0.1745	-5.29	0.2498
Airline	-7.04	0.4314	-13.04	0.4863	-5.13	0.5604	23.94	0.1195	-6.88	0.4431
Number observations	644		339		644		356		644	
Adjusted R-squared	**0.28**		0.27		0.28		**0.29**		**0.28**	

3.2 Loan coupon interest rate

We measure loan interest rates as the mean spread over LIBOR (in percent) paid by the borrower on loans outstanding at the time of bankruptcy, as reported in LossStats (LIBOR is our proxy for the risk-free rate; spreads are missing for some loans in LossStats, so the sample is smaller for this exercise). When the spread is added to the base specification, as in column 2 of Table 3, the coefficient is negative, statistically significantly different from zero, and with a magnitude roughly consistent with our model. The result is consistent with our model and with the aforementioned alternatives except for Leland (1994) and related models. However, the point estimate is not statistically significantly different from zero if robust errors are used.

3.3 Bargaining frictions

In most specifications, we include several proxies for bargaining frictions among creditors, shown in Table 3 in the rows immediately following the coefficients for year-of-bankruptcy dummies. These include:

- The time in bankruptcy (measured in years from filing and emergence).
- The time between the filing of the first plan of reorganization to emergence. Median time is 4 months. Much longer times are likely due to the first plan being voted down and thus with substantial bargaining problems among creditors.
- The time between the borrower's first default on a debt payment and the bankruptcy filing date.
- A dummy for pre-packaged bankruptcies, in which the firm has negotiated a tentative plan of reorganization with creditors before filing, implying lesser bargaining frictions.
- The number of debt instruments outstanding at filing.
- The number of priority classes into which the court aggregated the debt instruments.
- A dummy for bankruptcies involving pre-bankruptcy fraud problems at the borrower.
- A dummy for firms that filed for bankruptcy again within five years of emergence.[27]

Only two of the proxies are statistically significant predictors of recovery rates. Longer time from plan filing to emergence is associated with smaller recovery and prepackaged bankruptcies are associated with larger recovery. These results are consistent with modest variation in deadweight costs that is influenced by bargaining frictions.[28]

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[27] Wang (2007) offers evidence that recovery rates are lower for bankruptcies precipitated at least in part by fraud, and for bankruptcies managed by certain courts. We construct a fraud variable in a manner similar to Wang (2007) (by examining Lynn Lopucki's Bankruptcy Research Database, supplemented by some additional frauds we noticed while cleaning the data). We also used similar sources to identify firms that experienced more than one bankruptcy within five years of that recorded in any given observation (often called "Chapter 22" bankruptcies).

[28] Prepackaged bankruptcies are more likely to be those in which the equityholders' bankruptcy threshold is above that of the bank's, and such bankruptcies are likely to feature a higher recovery (the firm may not have time to negotiate a prepack if its value has fallen below the bank's threshold). Thus, it is not clear that prepackaged bankruptcies are characterized by lesser bargaining frictions.

Importantly, when we omit all such proxy variables from the specification, the coefficient on bank debt share remains statistically significant and does not change much, as shown in column 3 of Table 3. [29] This is evidence against the alternative in which bank debt share is merely serving as a proxy for the impact of banks on deadweight losses.

A number of authors, such as Lopucki (2005), have suggested that the efficiency of bankruptcy courts varies and that firms that are deeply insolvent may be more likely to file in some venues than others. We include dummy variables for each bankruptcy court that handles a substantial volume of bankruptcies in our sample (the omitted category is all other courts). Although coefficient values are negative for the court dummies, none are very large and, looking across the columns of Table 3, none are robustly statistically significant. Moreover, other results are not materially affected if we omit the court dummies (not tabulated).

3.4 Debt maturity

The appropriate definition of empirical debt maturity measures for use in recovery regressions varies across alternative models. Overall average maturity (pooling loans and bonds) seems most appropriate in the case of Leland and Toft (1996). When such a measure is included in the specification, its estimated coefficient is near -6 and is statistically significant (with OLS errors, but not with robust errors) (not tabulated). This is consistent with Leland and Toft (1996), but the estimated coefficient on bank debt share does not change much, implying that debt structure variables are not proxying for the firm's debt maturity profile. We experimented with other measures of maturity, and in no case is the effect on the estimated coefficient on bank debt share material. One example is displayed in column 4 of Table 3, which includes mean time to maturity separately for bank debt and bonds.[30] In the particular specification shown, both coefficients are negative. Time to average bond maturity is statistically significant and time to loan maturity is not, but signs and statistical significance are not very robust to variations in the definitions of measures nor to use of robust errors. It is worth noting that loan maturities are not very short: Mean remaining loan time to maturity at bankruptcy is 2.3 years, whereas for bonds the mean time is 5.9 years. Typical term to maturity at origination for loans is 3 to 5 years; for bonds it is 7 to 10 years.[31]

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[29] The finding that total time in bankruptcy does not matter is a bit of a surprise. It is conventional wisdom that deadweight costs of bankruptcy increase with duration of the bankruptcy. However, as noted previously, average time in bankruptcy in our sample is relatively short. It is possible that the conventional wisdom comes from experience in the 1970s or early 1980s and that bankruptcy practice has changed. Moreover, most empirical studies to date have examined recovery at the individual debt-instrument level, which implicitly overweights bankruptcies of firms with many instruments. We ran regressions similar to those in Table 3 using instrument-level data and found that results for variables such as time in bankruptcy and court dummies are sensitive to details of the specification. We view this as a potential subject for future research.

[30] Sample size is smaller because maturity is missing for some instruments in LossStats. The increase in the magnitude of the coefficient on bank debt share from 27 to 35 is a feature of the smaller sample, not the addition of maturity variables.

[31] In the specification shown, remaining loan maturity is measured as the log of the mean number of years remaining to maturity on the bankruptcy date for all outstanding loans of the borrower, and similarly for bond maturity. If time is measured in levels rather than log of levels, significance patterns remain similar and the estimated coefficients are near -1, implying economically modest effects of maturity on recovery rates. If time is measured as minimum time to maturity statistical significance is marginal at best and coefficients are economically small.

3.5 Role of covenants

In column 5 of Table 3, we split bank debt share into three parts: the ratio of bank debt to total debt only for the 428 firms having loans with covenants (zero for other observations), the ratio of bank debt to total debt only for the 13 firms with loans without covenants, and the ratio only for the 68 firms for which we are unable to identify covenant status (for each observation, the three variables sum to the value of our normal bank debt share variable). Consistent with our model, the coefficient on the share of bank debt without covenants not statistically significantly different from zero. We expect that most of the unidentified-status firms had loans with covenants, so it is unsurprising that the estimated coefficient for that variable is similar to that for the variable for firms with covenants and that both are statistically significantly different from zero.

We are reluctant to push this result too hard because of the small number of firms with no covenants. But the very small number of loans without covenants supports our assumption that such covenants are common, especially for distressed firms.[32]

3.6 Loan recovery rates

Our model implies that bank debt should usually receive a high recovery rate (because banks choose the threshold to protect themselves), although some lower recoveries are to be expected due to asset volatility during the bankruptcy period. The median and mean loan recovery rate are 99.5 and 85 percent, respectively. Figure 7 displays the distribution of loan recovery rates for all loans (the left bar of each group of three bars).[33] The recovery rate on bank debt (senior, secured, first-lien loans with covenants) is 90 percent or more for about 70 percent of sample firms. For the remaining firms, bank debt recovery rates are roughly uniformly distributed between 0 and 90 percent.

Some of S&P's recovery measurement methods are noisier than others. When we exclude observations measured by the noisiest of S&P's three methods (the middle bar at each decile in the Figure), bank debt creditors receive a full recovery (90 percent or more) at 81 percent of bankrupt firms. When we include only bank debt for bankruptcies filed in good years (roughly, non-recession years), 85 percent receive a full recovery and there is some bunching of other observations in the high deciles.

We inspected some of the bankruptcies with very low bank debt recovery rates and found that some had collateral that turned out to be of little value and some bank debt ended up being classified as equal in priority to the firm's bonds). Some are cases where the asset value of the firm may have experienced a big negative jump (such as some telecom and tech firms during 2001-2003). It appears that banks sometimes make idiosyncratic mistakes. Such mistakes are outside the scope of our model but are not very inconsistent with it.

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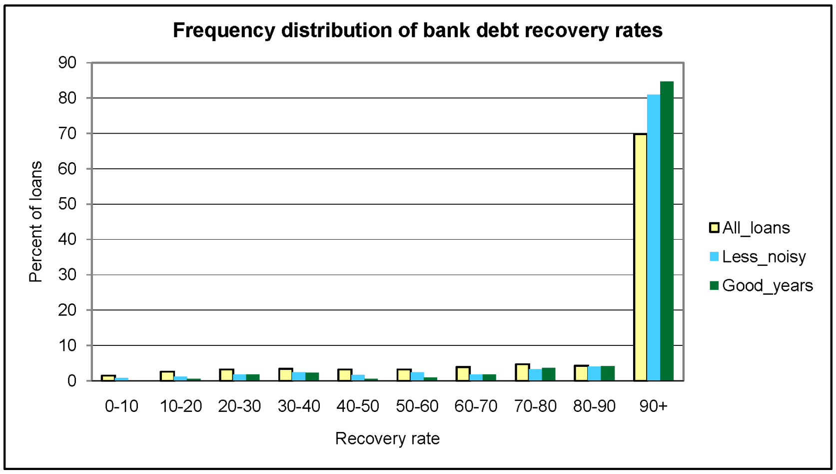
[32] Many borrowers had multiple loans outstanding at bankruptcy. We measure variables as if all a borrower's loans have covenants if any do. This is reasonable because the loans are usually similar in bankruptcy priority. In our model, the lender with covenants should take into account similar-priority loans in making decisions about foreclosure because recovery will be shared with holders of such loans.

[33] Individual loans were aggregated to one representative loan per bankruptcy; results are similar for the distribution of individual loans.

Figure 7: Frequency distribution of bank debt recovery rates



3.7 Instrument-level regressions

Returning to the top panels of Figure 2, we expect the empirical relationship between recovery rates on loans alone (bonds alone) and bank debt share to be negative (positive) conditional on the coupon spread, with a negative shift as the coupon spread increases that is roughly proportional to the square of bank debt share. Table 4 reports results for specifications which include both bank debt share and the product of the coupon spread and the square of bank debt share.[34] Point estimates of coefficients on the variables of interest have the signs predicted by our model. Coefficients on bank debt share are not statistically significant but those on the interaction of coupon rate and bank debt share are weakly significant. Parameter estimates may be less precise than in firm-level regressions partly because, as noted previously, empirical relationships are conditional on other parameters of the model. We lack controls for some parameters, and the resulting noise is likely to be more material for instrument-level regressions than firm-level regressions. Overall, these results could be consistent with both our model and the alternatives, though the alternatives are silent about the impact of an interaction between loan coupon interest rate and the square of bank debt share.

3.8 Non-linear specifications, and more about coupon rate

For realistic parameter values and for values of bank debt share well below 1, our model implies a nearly one-for-one relationship between firm-level recovery rates and bank debt share. For higher values of bank debt share, the slope is flatter (see Figure 2). Moreover, the relationship depends on the loan coupon spread. We chose a linear base specification for our regressionsbecause the non-linearities are not huge and because the coupon spread is available only for a subset of our data. However, the non-linearities are of some interest because it is not obvious they are implied by the alternative models.

The first column of Table 5 reports results when we spline the bank debt share variable with a single knot at 0.3, which is between the mean and median values of bank debt share. We find evidence of the flatter slope shown in the bottom panel of Figure 2 for high values of bank debt share: The coefficient on the low-share segment is 78, whereas the coefficient on the high-share segment is 30. The coefficients are statistically significantly different from zero and from each other. However, significance of the difference in estimated coefficients is not robust to the value of bank debt share at which the knot is located.

The second column of Table 5 reports results when we interact bank debt share with the coupon spread. As shown in the bottom panel of Figure 2, our model implies the negative impact of coupon spread rises with bank debt share (not with its square as in the loan and bond recovery regressions). We find evidence of this non-linearity: The coefficient on bank debt share is larger at about 53 than in the base specification (perhaps due to less attenuation from mis-specification), and the coefficient on the interaction term is negative and statistically significant.[35]

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[34] We limit the sample to bankruptcies of firms with both loans and bonds outstanding and with coupon spread data available, so dummies for all-bank-debt, no-bank-debt, etc. are not included in regressions. For each such bankruptcy, we aggregated all loans (bonds) into a single representative loan (bond) with the recovery rate being the rate for all the loans (bonds) taken together (in essence, a claim-weighted average, like our firm-level recovery measure). Such aggregation allows us to ignore differences in priority across individual instruments, which are very difficult to measure well. Our model's implications are for representative loans and bonds.

[35] The larger coefficient on bank debt share is not a feature of the smaller sample: In the subsample with coupon spread data but no interaction term, the coefficient on bank debt share is similar to base-case results.

Table 4. Debt instrument-level recovery rate regressions

The interest rate spread on bank debt is the spread over LIBOR as recorded in the LossStat database. Where a firm has multiple bank loans outstanding, the mean spread is used. Other variables are as in previous tables. The second column reports base-case regression results for the subsample for which spreads are available to support comparisons.

Independent Variable	(1) Loans		(2) Bonds	
	Coeff.	p-value	Coeff.	p-value
Intercept	115.60	<.0001	71.90	<.0001
Share bank debt	**-17.49**	**0.3106**	**19.92**	**0.2854**
Share-bank-debt^2 * coupon	**-9.52**	**0.1017**	**-12.51**	**0.0471**
Share sub debt	-0.44	0.9513	-19.27	0.0138
Bankruptcy year dummies:				
1987-88	-11.25	0.4855	-23.23	0.1834
1989	9.51	0.6798	9.16	0.7132
1990	4.28	0.8010	-18.32	0.3191
1991	-6.67	0.5795	-21.88	0.0935
1992	-20.90	0.1059	-19.67	0.1590
1994	6.62	0.6659	-16.99	0.3058
1995	4.49	0.7540	-0.29	0.9851
1996	1.56	0.9030	-7.66	0.5801
1997	-2.93	0.8347	-30.71	0.0437
1998	-26.97	0.0715	-46.01	0.0047
1999	-6.11	0.5899	-34.28	0.0055
2000	-13.08	0.2315	-33.97	0.0043
2001	-5.25	0.6203	-38.59	0.0009
2002	-4.96	0.6525	-32.33	0.0071
2003	-3.47	0.7553	-7.95	0.5092
2004	-0.34	0.9776	-11.70	0.3724
Time in bankruptcy	1.59	0.5144	-2.09	0.4295
Time from plan to emerge	-2.80	0.6223	-9.10	0.1391
Time in default pre-filing	3.82	0.4288	2.40	0.6454
Prepackaged bankruptcy	2.15	0.6418	-0.80	0.8730
Number debt instruments	-0.61	0.1570	1.05	0.0235
Number priority classes	-1.42	0.2897	-3.10	0.0334
Fraud dummy	-10.37	0.2446	15.16	0.1159
Filed again within 5 yrs dum	-0.11	0.9889	-5.41	0.5093
Court dummies:				
California	13.74	0.1993	20.24	0.0808
New York	-4.39	0.3965	-1.50	0.7889
Delaware	-3.87	0.3974	-4.44	0.3694
Illinois	-9.02	0.3795	-2.00	0.8571
Texas	-3.96	0.5636	-0.23	0.9755
Selected industry dummies:				
Bubble-firm dummy	-31.99	0.0022	-7.05	0.5281
Utilities	-0.71	0.9541	14.63	0.2733
Telecom	1.00	0.9137	-12.95	0.1967
Computer	-14.51	0.0917	4.40	0.6354
Airline	-10.86	0.5757	-6.84	0.7445
Number observations	279		279	
Adjusted R-squared	**0.18**		**0.23**	

Table 5. Alternative specifications of firm-level recovery rate regressions

All details are the same as in Table 3 with the following exceptions. In Column 1, bank debt share has a spline representation with a single knot at 0.3. The "dummy for 0.3+ segment" coefficient allows the intercept for the second segment of the spline to differ. In column 2, bank debt share and the loan coupon interest rate (measured as the mean spread over LIBOR on bank debt of the given borrower). The last two columns show results when the sample is restricted to bankruptcies filed in the years 1987-97 and 1998-2004, respectively.

	(1)		(2)		(3)		(4)		(5)	
Independent	Spline λ		Interact λ, c		Only λ		1989-97		1998-2004	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Intercept	50.64	<.0001	64.63	<.0001	47.65	<.0001	53.22	<.0001	54.98	<.0001
Share bank debt			**53.32**	**<.0001**	**37.61**	**<.0001**	**32.09**	**0.0044**	**27.03**	**0.0030**
Share bank debt range 0-0.3	**78.37**	**0.0013**								
Share bank debt range 0.3+	**30.09**	**0.0071**								
Intercept for 0.3+ segment	**4.00**	**0.5827**								
Share bank debt * coupon			**-8.93**	**0.0017**						
No bank debt dummy	4.63	0.3911	4.42	0.6746			-3.17	0.6484	0.25	0.9628
All bank debt dummy	8.77	0.1804	4.67	0.4911			0.34	0.9753	9.86	0.1549
Share secured debt	-1.74	0.6714	-2.55	0.7033			-1.00	0.8780	-5.65	0.3135
All sub debt dummy	-10.27	0.1066	0.00	.			-4.71	0.5730	2.74	0.8185
No sub debt dummy	1.82	0.6437	4.22	0.4529			-3.07	0.6547	7.30	0.1460
Share sub debt	-10.18	0.1294	-5.98	0.5586			-24.18	0.0190	-0.21	0.9811
Bankruptcy year dummies:										
1987-88	-1.07	0.9048	-7.94	0.5531	-3.99	0.6590	-0.80	0.9326		
1989	-13.49	0.1150	17.62	0.4186	-15.92	0.0665	-10.43	0.2578		
1990	-11.84	0.0862	-14.80	0.3192	-14.74	0.0349	-11.76	0.1030		
1991	-2.50	0.6729	-13.30	0.2280	-2.87	0.6339	0.15	0.9807		
1992	0.06	0.9917	-14.70	0.2203	-0.85	0.8930	0.88	0.8898		
1994	-5.86	0.4417	-0.85	0.9529	-3.56	0.6443	-2.97	0.7072		
1995	0.68	0.9236	-0.38	0.9751	5.23	0.4587	4.20	0.5774		
1996	-2.38	0.7442	-1.84	0.8733	2.12	0.7721	-1.58	0.8444		
1997	-8.77	0.2698	-8.89	0.4531	-6.06	0.4473	-9.39	0.2834		
1998	-13.17	0.0821	-26.73	0.0398	-10.26	0.1799			-11.78	0.1331
1999	-7.11	0.2448	-19.08	0.0628	-4.36	0.4793			-3.26	0.6205
2000	-15.14	0.0121	-21.66	0.0297	-12.90	0.0328			-11.24	0.0776
2001	-16.10	0.0046	-24.55	0.0112	-14.46	0.0109			-13.74	0.0194
2002	-14.89	0.0092	-18.83	0.0606	-11.54	0.0425			-10.40	0.0728
2003	-2.39	0.6948	-3.37	0.7367	2.33	0.6998			4.00	0.5091
2004	-2.18	0.7522	-2.95	0.7891	1.10	0.8724				
Time in bankruptcy	0.40	0.7878	0.29	0.8912	0.92	0.5396	1.57	0.4861	-0.32	0.8760
Time from plan to emerge	-4.87	0.0677	-11.28	0.0146	-4.51	0.0957	-5.73	0.1475	-3.42	0.3663
Time in default pre-filing	-0.15	0.9480	1.75	0.6915	-0.63	0.7915	-1.69	0.5951	-0.98	0.7949
Prepackaged bankruptcy	6.07	0.0346	0.96	0.8134	6.77	0.0199	6.93	0.1770	4.54	0.2086
Number debt instruments	0.15	0.6312	0.62	0.0876	0.26	0.3921	-0.42	0.5881	0.40	0.2462
Number priority classes	0.29	0.7832	-0.88	0.4943	0.08	0.9319	2.70	0.1609	-1.32	0.3136
Fraud dummy	0.11	0.9828	-1.84	0.7905	-2.06	0.6856	-5.88	0.6079	0.89	0.8770
Filed again within 5 yrs dum	-5.28	0.3291	-6.38	0.3603	-7.28	0.1805	-0.29	0.9706	-8.19	0.2878
Court dummies:										
California	-1.39	0.7771	14.98	0.0747	-2.87	0.5616	-3.62	0.5960	-2.71	0.7101
New York	-5.52	0.0814	-6.18	0.1641	-7.01	0.0279	-7.01	0.1569	-6.70	0.1172
Delaware	-5.51	0.0546	-6.61	0.0941	-6.51	0.0240	-5.45	0.2831	-7.04	0.0499
Illinois	-3.50	0.5945	-7.07	0.4667	-4.48	0.5000	-5.64	0.6181	-4.52	0.5828
Texas	-2.37	0.5813	-1.78	0.7725	-2.10	0.6288	-2.63	0.6735	-2.42	0.7001
Selected industry dummies:										
Bubble-firm dummy	-15.60	0.0099	-25.68	0.0035	-15.20	0.0129			-17.77	0.0043
Utilities	24.09	0.0018	12.27	0.2201	25.84	0.0010	39.54	0.0003	10.94	0.3201
Telecom	-6.49	0.2760	-2.68	0.7445	-3.40	0.5678	8.14	0.6915	-6.78	0.3200
Computer	-5.69	0.2138	-4.90	0.4614	-7.35	0.1132	1.05	0.8928	-6.88	0.2446
Airline	-7.50	0.4008	-8.46	0.6492	-5.24	0.5611	2.10	0.8524	-12.96	0.4093
Number observations	644		339		644		256		388	
Adjusted R-squared	**0.29**		**0.27**		**0.26**		**0.29**		**0.31**	

We view these exercises as material evidence in favor of our model. Of course, it might be possible to make a model having similar predictions of non-linearities, but we are not aware of one.

3.9 Other debt structure variables

Firms with no bank debt are not inconsistent with our model but, intuitively, it seems likely that their bankruptcy thresholds are determined by mechanisms outside our model. To account for this and for the possibility of non-linearities at corners and for unmodeled aspects of debt structure, most regression specifications include dummies for firms with no bank debt at filing and firms with all bank debt, for the share of debt that is secured, for the share that is contractually subordinated, and dummies for firms with all subordinated debt and no subordinated debt. The all-bank-debt and all-subordinated-debt dummy coefficients are statistically significantly different from zero at or near the 10 percent level in the base specification, but the magnitude and significance of such coefficient varies a lot across specifications show in Table Table 3. When we split the sample in columns 4 and 5 of Table 5, the (continuous) share of subordinated debt is a material predictor only for the early years and the all-bank-debt dummy is material (though not statistically significant) only for the later years, while bank debt share is an economically and statistically significant predictor in both subsamples. When we omit the additional debt structure variables entirely, as in column 3 of Table 5, the estimated coefficient on bank debt share is substantially larger than in the base specification. (If we omit the bank debt share variable, the adjusted R-square drops by about half.) Taken together, these results make us reluctant to speculate about the economic mechanisms that might cause some additional debt structure variables to be material. [36]

3.10 Other auxiliary and control variables

3.10.1 Year and industry

In addition to the variables already mentioned, we include in all specifications dummies for year-of-bankruptcy-filing and for the industry of the borrower. Some are statistically and economically significant predictors of firm-level recovery rates. These variables are not of particular interest for the purposes of this paper, but may be of independent interest to some readers due to attention given them in previous literature. We discuss them in Appendix G.

3.10.2 Financial statement variables and credit ratings

Table 6 reports results of regressions for a subsample of firms for which we were able to find usable data in Compustat. Compustat's balance sheet, income statement, and debt-rating variables are as of the firm's fiscal year-end data prior to the bankruptcy date (firms for which usable fiscal year-end data is more than 1.1 years prior to the bankruptcy date are dropped).

The Compustat subsample affords an opportunity to examine whether other characteristics of the firm are associated with recovery, such as the nature of its assets, its size, or its operating cash flow not long before filing. Estimates imply that most such characteristics are not predictive of firm-level recovery, whether debt structure variables are included or not. The borrower's S&P

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[36] That the secured debt share variable is not significant is not particularly surprising given that collateral merely gives one class of claimants priority over other classes. However, liens might protect assets from dissipation by the firm prior to bankruptcy. Perhaps such protection is not very material to recovery rates because banks take into account the degree of such protection on a case-by-case basis in setting the foreclosure threshold.

Table 6. Firm-level recovery rate regressions, Compustat-matched subsample

The dependent variable in OLS regressions, with p-values based on conventional standard errors, is the firm-level recovery rate at emergence. All other variables are as in Table 3, except that balance sheet and income statement variables, as well as rating dummies, are from Compustat and are dated as of the firm's last fiscal year-end date before filing bankruptcy for which data are available.

	(1)		(2)		(3)		(4)	
Independent	Base case		Add firm vars		No debt struc		Ratings	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Intercept	65.72	<.0001	67.67	<.0001	66.59	<.0001	70.48	<.0001
Share bank debt	**20.11**	**0.0225**	**29.43**	**0.0011**			**23.77**	**0.0077**
No bank debt dummy	-3.49	0.5056	1.99	0.7195			-3.88	0.4652
All bank debt dummy	14.00	0.0647	13.55	0.0715			14.08	0.0705
Share secured debt	-4.76	0.4024	-6.62	0.2365			-6.13	0.2896
All sub debt dummy	-17.94	0.0209	-18.71	0.0171			-18.48	0.0187
No sub debt dummy	2.02	0.6697	4.23	0.3691			2.31	0.6259
Share sub debt	-13.23	0.0967	-8.29	0.3008			-11.86	0.1389
Firm characteristics								
Log Total Assets			0.68	0.6591	1.14	0.4950	-32.75	0.0844
Non-intang. assets/assets			0.76	0.9332	8.43	0.4010		
Book liabs./assets			**-5.86**	0.0198	**-8.04**	0.0032		
Operating income/assets			15.59	0.2410	21.40	0.1402		
Accts payable/tot liabilities			**-51.40**	0.0150	-28.95	0.2033	1.03	0.8380
PPE/assets			-6.95	0.3723	-9.47	0.2702		
Ratings								
BB or safer							-2.78	0.4252
B							4.49	0.3272
CCC							3.01	0.5702
CC or worse							1.38	0.7959
Time in bankruptcy	-0.56	0.7561	-0.83	0.6528	-1.86	0.3595	-0.95	0.6016
Time from plan to emerge	-0.50	0.8884	0.45	0.8987	-0.34	0.9307	0.35	0.9209
Time in default pre-filing	-0.24	0.9401	2.35	0.4848	1.43	0.7000	-1.76	0.6093
Prepackaged bankruptcy	8.72	0.0160	7.52	0.0372	2.38	0.5497	7.95	0.0311
Number debt instruments	0.09	0.7913	0.00	0.9973	0.04	0.9258	0.05	0.8752
Number priority classes	-0.07	0.9532	1.14	0.4440	0.87	0.5058	0.25	0.8304
Fraud dummy	2.30	0.6841	-5.75	0.3384	-6.80	0.2957	1.62	0.7769
Filed again within 5 yrs dum	-6.40	0.2786	-7.67	0.1857	-6.97	0.2744	-6.67	0.2590
Court dummies:								
California	4.09	0.4762	4.45	0.4311	-0.24	0.9688	5.29	0.3580
New York	-1.98	0.6211	-1.65	0.6831	-3.03	0.4978	-0.31	0.9402
Delaware	-2.19	0.5478	-2.55	0.4795	-4.85	0.2219	-0.86	0.8138
Illinois	2.61	0.7891	0.73	0.9408	-4.81	0.6569	3.88	0.6911
Texas	-1.23	0.8158	0.44	0.9319	-1.74	0.7620	-0.85	0.8719
Selected industry dummies								
Bubble-firm dummy	-13.42	0.0571	-8.94	0.2076	-13.83	0.0760	-10.88	0.1286
Utilities	26.97	0.0122	22.71	0.0352	21.20	0.0747	24.38	0.0255
Telecom	-9.05	0.1866	-12.50	0.0790	-12.12	0.1190	-14.70	0.0404
Computer	-9.00	0.1021	-11.49	0.0424	-19.06	0.0023	-11.37	0.0430
Airline	-18.67	0.0654	-19.85	0.0558	-26.69	0.0195	-20.22	0.0458
Number observations	388		376		376		383	
Adjusted R-squared	**0.32**		0.36		0.20		0.33	

rating at the fiscal year-end before filing also is not significant. Moreover, the significance of the bank debt share variable is maintained in the smaller Compustat subsample, regardless of what other variables are included.

The variable of most interest for purposes of checking robustness is the share of total liabilities that is accounts payable. This category includes trade credit extended to the firm, which is likely to be treated by the court as a senior unsecured claim and, especially in the case of small accounts payable, is likely to be paid in full during Chapter 11 bankruptcies in order to reduce the number of creditors and to permit the firm to continue operating with normal trade relationships. Because accounts payable are not measured in LossStats, a marginal additional dollar of payables represents an additional dollar of claims. Payments to such claims reduce our measured firm-level recovery rate by reducing assets available to pay debtholders. The estimated coefficient on the accounts payable variable in column 2 of Table 6, at -51, implies a reduction of about half a percentage point of our measured recovery rate for each additional percentage point of total liabilities that are accounts payable, which is a sensible magnitude. Inclusion of the variable does not materially affect the estimated coefficient on bank debt share.

Another exception is leverage. Measured as the ratio of book total liabilities to total assets, the coefficient estimate implies a moderate reduction in firm level recovery rate of about five percentage points if book leverage increases from its median ratio of 1 to a value of 2.

In the spirit of our model, perhaps it is unsurprising that most observable firm characteristics are not strongly associated with recovery. They might be in a world with an exogenous default boundary, but banks can observe such variables and thus can be expected to take such variables into account in setting the default boundary in a manner likely to erase any correlation.[37]

3.11 Summing up the evidence

We regard the totality of the empirical evidence as providing strong support for our model. Though some of its predictions are not strongly supported, none of our model's predictions is resoundingly rejected, and some of its predictions that are supported are novel. We find a consistent and enormously robust role of bank debt share in predicting firm-level recovery that is difficult to pass off as being due to maturity effects or deadweight costs of bankruptcy. Debt structure variables contribute more than half of the explanatory power of the base case regression, which along with the large size of coefficients implies debt structure is economically important. Bank debt with covenants matters but not bank debt without covenants, implying that it is the control rights granted by the covenants that matters, not bank status. Loan coupon rates predict firm-level recovery in a manner predicted by our model, and we find predicted non-linearities and interaction effects of coupon rates and bank debt share that are not obvious implications of alternatives. Results are robust to inclusion or exclusion of a wide array of auxiliary and control variables and to use of early-years versus late-years subsamples. The great majority of loan recovery rates are near 100 percent as predicted.

We specify and discuss alternatives in order to make a case that the empirical relationships we find are not simply due to alternative mechanisms. We do not recommend that the reader interpret the evidence as rejecting the alternatives. For example, we do find a negative relationship between recovery rates and maturity in some specifications as predicted by Leland and Toft (1996), and our evidence that the relationship with coupon rates is the opposite of that predicted by Leland and

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[37] We are grateful to Richard Cantor for this point.

Toft (1996) is sensitive to the choice of estimator of the covariance matrix. We believe it is likely that bankruptcies are generated by a diversity of mechanisms. This is conceptually consistent with a first-passage model: If different mechanisms generate bankruptcy filings at different asset values for any given firm, the mechanism associated with the highest threshold will describe the decision for that firm. The evidence strongly supports our model as capturing the determinants of many bankruptcies, but it does not imply our model describes all bankruptcies. Given the large number of extant models, sorting out which models describe which bankrutpcies is a subject for future research.

Discussion

This paper offers a model and evidence supportive of a hypothesis that private debtholders play an important role in determining the value of assets at which firms declare bankruptcy. In order to protect the recovery they receive, and using the control rights granted by loan covenants, private lenders set a threshold that is higher the larger is their share of the firm's debt. Because asset value at bankruptcy strongly influences the value distributed to claimants at emergence, a higher private debt share is associated with higher ultimate firm-level recovery rates. Our model also sheds light on the long-standing puzzle of relatively low average recoveries on defaulted corporate bonds.

We do not claim that banks always set the default boundary — casual inspection of the news reveals obvious cases of strategic default by equityholders — but banks' role appears to be of substantial empirical importance. Nor do we claim that our empirical evidence applies throughout the world — an implication of our paper is that recovery rates are likely to be quite sensitive to the legal and practical feasibility of the conditional control rights that covenants give creditors and to details of bankruptcy law and practice.

In closing, we offer some suggestions for future research. First, our results suggest that literature on the capital structure decision might be enriched by analysis of the choice of the private debt share of total debt. We assume the share is exogenous, which is reasonable for firms near the bankruptcy threshold, but it is clearly a choice variable for very solvent firms. Given that debt structure influences the states of the world in which bankruptcy occurs, the debt composition decision may interact with the leverage decision. More research is needed to reveal the nature and relevance of such interactions.

Second, modeling of recoveries on individual debt instruments, which has been the focus of most empirical work on recovery to date, might be revisited. Combining a model of firm-level recovery with non-linear modeling of the impact of debt instrument seniority might provide more insight than models suggested to date, which are usually linear and ignore debt composition.

Third, our examination of firm-level measures of ultimate recovery differs from almost all prior studies. Most have examined samples of recoveries to individual debt instruments and some have interpreted results as revealing information about the relationship between firm characteristics and recovery rates. To the extent that our results differ for similar variables, more research may be needed because we expect that firm-level explanations would be revealed most clearly in firm-level regressions. Our purpose in this appendix is not to criticize previous work, merely to point out in passing some auxiliary results that may be interesting.

Fourth, in analyzing the extent to which recovery rate risk is systematic, which is important to debt pricing and risk management, we speculate that interpretation of results, and robustness, may be cleaner by doing analysis at the firm level. Moreover, controlling for debt composition may be important because the aggregate distribution of debt composition may vary over the business cycle. For example, if average bank debt share is lowest as cyclical peaks are approached, our results imply one would expect lower average recoveries during recessions. The nature of cycles in debt composition is an open empirical question.

Finally, our paper may help point the way toward resolution of some puzzles implicit in existing literature. Davydenko (2005) finds that while fixed boundary models of default do reasonably well in predicting default rates on average, such models do not perform so well in the cross section. Cross sectional variation in the absence of controls for debt structure is natural in our framework because the boundary differs with bank debt share. Faulkender and Petersen (2006) find that firms with bonds outstanding are considerably more leveraged on average than firms with only private debt in their capital structure. We do not examine capital structure decisions, but in our model, a firm that wished to increase leverage while holding its bankruptcy probability fixed could do so by issuing more bonds and no more loans.

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A Optimal foreclosure when recovery is deterministic

We require the following intermediate result:

Lemma 2
The function $\Xi(\kappa)$ is decreasing in σ and is bounded from above by

$$\lim_{\sigma\to 0} \Xi(\kappa) = \frac{r}{\mathcal{C} - \kappa(r-\rho)}$$

for all $\kappa < \mathcal{C}/(r-\rho)$.

The limit as $\sigma \to 0$ can be derived from the asymptotic formula for the ${}_1F_1$ function in FWC 07.20.06.0008.01. To see that $\Xi(\kappa)$ must be decreasing in σ, observe that

$$\Xi(\kappa) = \frac{d}{d\kappa}\left(\frac{\psi(V;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)}\right)\Bigg|_{V=\kappa}$$

and so

$$\frac{\partial}{\partial\sigma}\Xi(\kappa) = \frac{\partial}{\partial\kappa}\frac{\partial}{\partial\sigma}\left(\frac{\psi(V;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)}\right)\Bigg|_{V=\kappa}$$

The ratio $\psi(V)/\psi(\kappa)$ can be interpreted as the present value of receiving \$1 contingent on future bankruptcy (see Leland, 1994, p. 1219 for the corresponding expression in the strategic default model). As the value of this option must be increasing in σ whenever the option is out of the money, we have

$$\frac{\partial}{\partial\sigma}\left(\frac{\psi(V;\alpha,\beta,\zeta)}{\psi(\kappa;\alpha,\beta,\zeta)}\right) > 0$$

for all $\kappa < V$. At $V = \kappa$, the option is worth exactly \$1, regardless of σ, so $\partial/\partial\sigma(\psi(V)/\psi(\kappa))$ must be decreasing in κ for V in the neighborhood of κ. This implies that $\partial\Xi(\kappa)/\partial\sigma$ must be negative.

When recovery is deterministic ($\tau = 0$ and/or $\tilde{\sigma} = 0$), recovery is given by $B(\kappa) = \min\{\lambda, \kappa\}$. For all $\kappa > \lambda$, $B(\kappa) = \lambda$ and $B'(\kappa) = 0$, so

$$\mathcal{F}(\kappa) = -\left(\lambda\frac{c}{r} - \lambda\right)\Xi(\kappa) < 0.$$

For all $\kappa < \lambda$, $B(\kappa) = \kappa$ and $B'(\kappa) = 1$, so

$$\begin{aligned}\mathcal{F}(\kappa) &= 1 - \left(\lambda\frac{c}{r} - \kappa\right)\Xi(\kappa)\\ &> 1 - \left(\lambda\frac{c}{r} - \kappa\right)\frac{r}{\mathcal{C} - \kappa(r-\rho)} = \frac{\gamma(1-\lambda) + \delta + \kappa\rho}{\mathcal{C} - \kappa(r-\rho)} \geq 0\end{aligned}$$

Note that the application of Lemma 2 is valid because $\kappa < \lambda \leq 1 \leq \mathcal{C}/(r-\rho)$. It follows that the cusp point at $\kappa = \lambda$ is the optimal foreclosure threshold.

B Boundary values of the first order condition

We prove these results under the extended model of Section 1.3. For $\kappa \to 0$, we make use of the limit

Result 1

$$\lim_{V\to 0}\Xi(V;\alpha,\beta,\zeta) = \alpha(\beta-1)\zeta = r/\mathcal{C}.$$

This follows from the asymptotic limit (FWC 07.20.06.0009.01)

$$\lim_{z\to\infty} z^a {}_1F_1(a, a+b, z) = \frac{\Gamma(a+b)}{\Gamma(b)}$$

and by noting that $\alpha(\beta-1) = 2r/\sigma^2$. It is easily verified that $M(0, D, s^2) = 0$ and $M_1(0, D, s^2) = 1$, which implies that $B'(0) = \exp(\chi + \eta^2/2)$ and $B(0) = 0$.

To prove Proposition 3, observe that $\kappa B'(\kappa) \to 0$ for large κ, but $\kappa\Xi(\kappa) = \alpha + o(\kappa^{-1})$ As $\lambda\frac{c}{r} - B(\kappa)$ is positive and bounded, we have $\kappa\mathcal{F}(\kappa)$ asymptotically negative and bounded.

C Proof of Lemma 1

Define the function $\tilde{\Xi}(V,\zeta) \equiv \zeta \cdot \Xi(V;\alpha,\beta\,\zeta)$ for α,β fixed. It is easily checked that $\tilde{\Xi}$ depends only on the product ζV and not on V and ζ individually, which implies

$$V \cdot \frac{\partial}{\partial V}\tilde{\Xi}(V,\zeta) = \zeta \cdot \frac{\partial}{\partial \zeta}\tilde{\Xi}(V,\zeta).$$

We substitute back to get

$$\zeta \cdot \frac{\partial}{\partial \zeta}\Xi(V;\alpha,\beta\,\zeta) = \Xi(V;\alpha,\beta\,\zeta) + V \cdot \Xi'(V;\alpha,\beta\,\zeta).$$

To sign the right hand side, observe that

$$\begin{aligned}\Xi'(V) &= \Xi(V)^2 - \frac{\alpha\zeta\psi'(V;\alpha+1,\beta-1,\zeta)}{\psi(V;\alpha,\beta,\zeta)}\\ &= \Xi(V)\left(\frac{\alpha}{V}\frac{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))} - \frac{\alpha+1}{V}\frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right).\end{aligned}$$

This implies

$$\begin{aligned}\zeta\cdot\frac{\partial}{\partial\zeta}\Xi(V;\alpha,\beta\,\zeta) = \Xi(V)\cdot&\left(1+\alpha\frac{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))} - (\alpha+1)\frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right)\\ &= \Xi(V)\cdot\left(1-\frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right)\\ &+\alpha\Xi(V)\cdot\left(\frac{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))} - \frac{{}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V))}{{}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))}\right). \quad (15)\end{aligned}$$

The ${}_1F_1$ function is decreasing in its first parameter when the argument is negative, so the first term in the last line of equation 15 is positive for all finite $V > 0$. The main theorem in Barnard et al. (2009) guarantees that

$${}_1F_1(\alpha+1,\alpha+\beta,-1/(\zeta V))^2 > {}_1F_1(\alpha+2,\alpha+\beta,-1/(\zeta V)) \cdot {}_1F_1(\alpha,\alpha+\beta,-1/(\zeta V))$$

for all finite V, so the second term in equation 15 is positive as well.

D Asymptotic analysis for large expected bankruptcy shocks

This appendix shows that $\kappa^* \to 0$ whenever χ is very large in magnitude. When $\chi \to -\infty$, we have

$$B(\kappa) = \mathrm{E}\left[\min\{\lambda, \exp(\chi+\eta^2/2)V_{t+\tau}\}|V_t = \kappa\right] \to 0$$

and

$$B'(\kappa) = \exp(\chi+\eta^2/2)M_1(\exp(\chi+\eta^2/2)\kappa, \lambda, \sqrt{\tau\sigma^2+\eta^2}) \to 0$$

for any fixed κ. Therefore, for χ sufficiently large and negative, $\mathcal{F}(\kappa)$ is dominated by the term $-\lambda\frac{c}{r}\Xi(\kappa)$ which is negative. This pushes us to the corner solution $\kappa^* = 0$.

When $\chi \to \infty$, we have

$$B(\kappa) = \mathrm{E}\left[\min\{\lambda, \exp(\chi+\eta^2/2)V_{t+\tau}\}|V_t = \kappa\right] \to \lambda$$

for any $\kappa \to 0$, so again $B'(\kappa) \to 0$. Therefore, for χ sufficiently large and positive, $\mathcal{F}(\kappa)$ is dominated by the term $-(\lambda\frac{c}{r} - \lambda)\Xi(\kappa) < 0$. This pushes us towards $\kappa^* = 0$, though the corner solution will not be reached for any finite χ.

E Covenant boundary and waiver fees

In this Appendix, we introduce a finite covenant boundary ν. Whenever $V_t \leq \nu$, the borrower is considered to be in violation of covenants and the bank has an option to foreclose at will. Whenever $V > \nu$, covenants are satisfied and the bank cannot foreclose. Loan contracts may specify a fee to be paid to the bank when a covenant violation is waived, and in other cases something similar might be achieved by renegotiation at the time of covenant violation. For simplicity, we assume that a waiver penalty of w is added to the coupon rate c whenever $\kappa < V \leq \nu$.

To maintain clarity in notation, we mark with a check any parameter that pertains under $V > \nu$, and mark with a hat any parameter that pertains under $V \leq \nu$. Thus, $\check{c}$ is the normal coupon rate, and $\hat{c} = \check{c} + w$ is the penalty coupon rate. (Think "smile" for the normal state and "frown" for the violation state.) We allow for the possibility that the contract requires lower dividend payments to equityholders when $V \leq \nu$, so similarly distinguish $\hat{\delta} \leq \check{\delta}$ and $\hat{\rho} \leq \check{\rho}$. All other fundamental parameters are fixed across the two regimes, but derived parameters such as α, β and ζ vary and so are marked with checks and hats. For fixed κ, the loan price is

$$F(V) = \begin{cases} \hat{F}(V) & \text{if } V \leq \nu, \\ \check{F}(V) & \text{if } V > \nu. \end{cases} \tag{16}$$

where $\hat{F}(V)$ and $\check{F}(V)$ are solutions to equation (4) under the two parameter regimes. It is important to recognize here that the $\hat{F}$ and $\check{F}$ functions differ from equation (7) because the relevant boundary conditions are not the same.

For the moment, take default boundary κ as fixed. The lower boundary value for $\hat{F}$ is $\hat{F}(\kappa) = B(\kappa)$. The upper boundary value for $\check{F}$ is $\check{F}(\infty) = \lambda\check{c}/r$. Two additional boundary restrictions are required to provide the upper boundary of $\hat{F}(V)$ and lower boundary of $\check{F}(V)$ where they join at covenant threshold $V = \nu$. These are given by the smooth pasting conditions, $\hat{F}(\nu) = \check{F}(\nu)$ and $\hat{F}'(\nu) = \check{F}'(\nu)$. As V is driven by a diffusion, passage across the threshold at ν is an accessible event, which implies that F must be continuous at $V = \nu$. Dixit (1993, §3.8) provides a no-arbitrage argument for continuity in the first derivatives.

Let f_ν be the value of the loan at ν. Solution to $\check{F}(V)$ proceeds exactly as for the baseline model, except that the lower boundary is $\check{F}(\nu) = f_\nu$. This implies

$$\check{A}_1 = \left(\lambda\frac{\check{c}}{r} - f_\nu\right)\frac{1}{\psi(\nu; \check{\alpha}, \check{\beta}, \check{\zeta})} = \left(\lambda\frac{\check{c}}{r} - f_\nu\right)\frac{1}{\check{\psi}_1(\nu)}$$

where for convenience we define

$$\check{\psi}_1(y) = \psi(y; \check{\alpha}, \check{\beta}, \check{\zeta}).$$

We similarly define for the violation state

$$\begin{aligned} \hat{\psi}_1(y) &= \psi(y; \hat{\alpha}, \hat{\beta}, \hat{\zeta}) \\ \hat{\psi}_2(y) &= \psi(y; 1 - \hat{\beta}, 1 - \hat{\alpha}, \hat{\zeta}) \end{aligned}$$

The boundary conditions for $\hat{F}(V)$ lead to simultaneous linear equations

$$\begin{aligned} \hat{A}_1 \cdot \hat{\psi}_1(\kappa) + \hat{A}_2 \cdot \hat{\psi}_2(\kappa) &= \lambda\frac{\hat{c}}{r} - B(\kappa) \\ \hat{A}_1 \cdot \hat{\psi}_1(\nu) + \hat{A}_2 \cdot \hat{\psi}_2(\nu) &= \lambda\frac{\hat{c}}{r} - f_\nu. \end{aligned}$$

which has solution

$$\begin{aligned}
\hat{A}_1 &= \frac{1}{\hat{\Delta}}\left(\hat{\psi}_2(\nu)\left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right) - \hat{\psi}_2(\kappa)\left(\lambda\frac{\hat{c}}{r} - f_\nu\right)\right) \\
\hat{A}_2 &= \frac{1}{\hat{\Delta}}\left(-\hat{\psi}_1(\nu)\left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right) + \hat{\psi}_1(\kappa)\left(\lambda\frac{\hat{c}}{r} - f_\nu\right)\right)
\end{aligned}$$

where $\hat{\Delta}$ is the determinant

$$\hat{\Delta} \equiv \hat{\psi}_1(\kappa)\hat{\psi}_2(\nu) - \hat{\psi}_1(\nu)\hat{\psi}_2(\kappa)$$

Finally, we impose $\hat{F}'(\nu) = \check{F}'(\nu)$ to pin down f_ν as

$$f_\nu = \frac{\lambda\frac{\check{c}}{r}\check{\Xi}(\nu) + \lambda\frac{\hat{c}}{r}\hat{\Xi}(\kappa,\nu) - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right)\hat{\Xi}(\nu,\nu)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} \tag{17}$$

where

$$\hat{\Xi}(a,b) \equiv \frac{1}{\hat{\Delta}}\left(\hat{\psi}_1(a)\hat{\psi}_2'(b) - \hat{\psi}_2(a)\hat{\psi}_1'(b)\right).$$

The two-variable Ξ function extends the one-variable function in the sense that

$$\lim_{\nu\to\infty}\Xi(\nu,\kappa) = \Xi(\kappa).$$

Two examples are shown in Figure 8. The solid curve is $F(V)$. The points $(\kappa^*, B(\kappa^*))$ and $(\nu, F(\nu))$ are marked with circles. Observe that F need not be monotonic in V. If the waiver fee is high enough, then the loan is most valuable when covenants are in violation while V is still not too close to the default boundary. In this case, F peaks between κ and ν, and $\check{F}$ converges to its asymptotic value from above rather than from below.

The dashed curves are lower and upper bounds derived from the baseline model. The value of the loan must be no less than the value of a loan in which parameters are held fixed at $c = \check{c}$, $\delta = \check{\delta}$, and $\rho = \check{\rho}$, and where the initial condition is a value of $B(\kappa)$ at $V = \kappa$. Similarly, $F(V)$ can be no greater than the value of a loan in which parameters are held fixed at $c = \hat{c}$, $\delta = \hat{\delta}$, and $\rho = \hat{\rho}$, for the same initial condition. Therefore,

$$F^{lower}(V) \leq F(V) \leq F^{upper}(V)$$

where

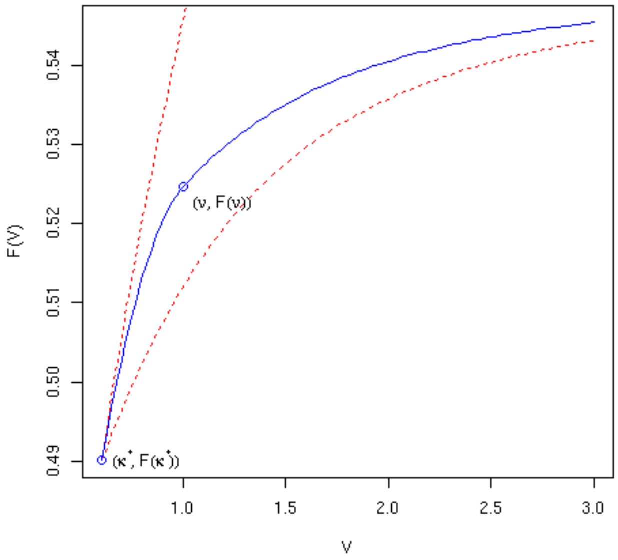
$$\begin{aligned}
F^{lower}(V) &= \lambda\frac{\check{c}}{r} - \left(\lambda\frac{\check{c}}{r} - B(\kappa)\right)\cdot\frac{\check{\psi}_1(V)}{\check{\psi}_1(\kappa)} \\
F^{upper}(V) &= \lambda\frac{\hat{c}}{r} - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right)\cdot\frac{\hat{\psi}_1(V)}{\hat{\psi}_1(\kappa)}
\end{aligned}$$

Observe that F clings to its upper bound at very low V (where the violation state parameters are the dominant influence), and converges to its lower bound as V tends to infinity (where the normal state parameters dominate).

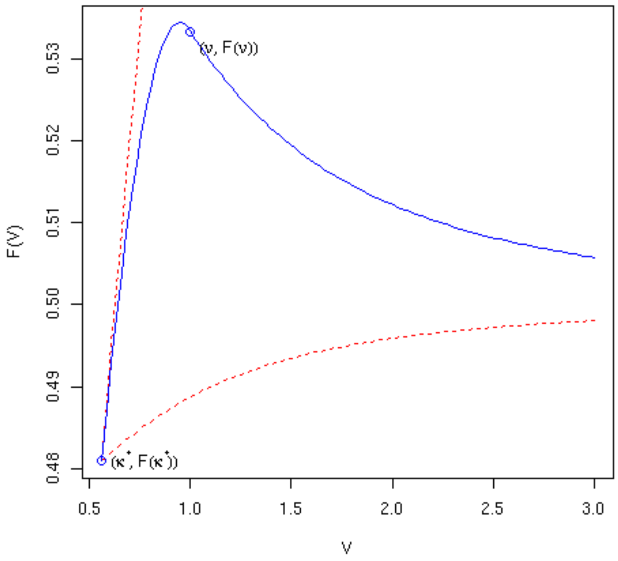
To complete the solution of our model, we solve for the optimal κ^* using the first order condition (9), and find

$$\mathcal{F}(\kappa) = B'(\kappa) - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right)\hat{\Xi}(\nu,\kappa) + \left(\lambda\frac{\hat{c}}{r} - f_\nu(\kappa)\right)\hat{\Xi}(\kappa,\kappa) \tag{18}$$

Figure 8: Loan value and bounding functions



(a) Small waiver fee ($\check{c} = r + 0.005, \hat{c} = \check{c} + 0.01$)



(b) Large waiver fee ($\check{c} = r, \hat{c} = \check{c} + 0.03$)

Solid blue line is $F(V)$, dashed red lines are upper and lower bounds from baseline model. Parameters: $\nu = 1$, $r = 0.05$, $\sigma = \tilde{\sigma} = 0.2$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\chi = \eta = 0$, $\tau = 1$.

where we have written $f_\nu(\kappa)$ to emphasize the dependence of f_ν on κ. As κ^* is constrained to the interval $[0, \nu]$, corner solutions must be checked. Otherwise, numerical solution for κ^* is straightforward.

We can rearrange equation (18) to emphasize its relationship to the FOC for the baseline model. We substitute in equation (17) to arrive at

$$\mathcal{F}(\kappa) = B'(\kappa) - \left(\lambda\frac{\hat{c}}{r} - B(\kappa)\right)\left(\hat{\Xi}(\nu,\kappa) - \frac{\hat{\Xi}(\nu,\nu)\hat{\Xi}(\kappa,\kappa)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)}\right) + \lambda\frac{w}{r}\frac{\check{\Xi}(\nu)\hat{\Xi}(\kappa,\kappa)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} \tag{19}$$

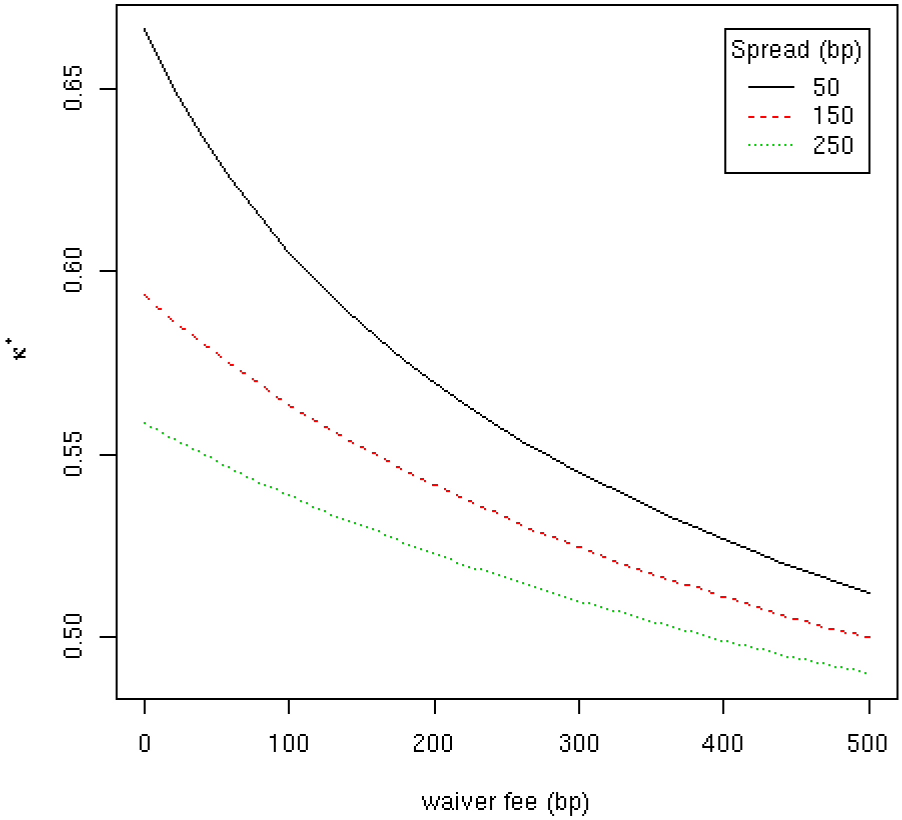
When the penalty state does not alter contractual parameters (i.e., $w = 0$, $\hat{\delta} = \check{\delta}$, and $\hat{\rho} = \check{\rho}$), then $\check{\Xi}(\nu) = \hat{\Xi}(\nu)$. Some tedious algebra can verify that

$$\hat{\Xi}(\nu,\kappa) - \frac{\hat{\Xi}(\nu,\nu)\hat{\Xi}(\kappa,\kappa)}{\hat{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} = \hat{\Xi}(\kappa)$$

in which case equation (19) reduces to equation (10).

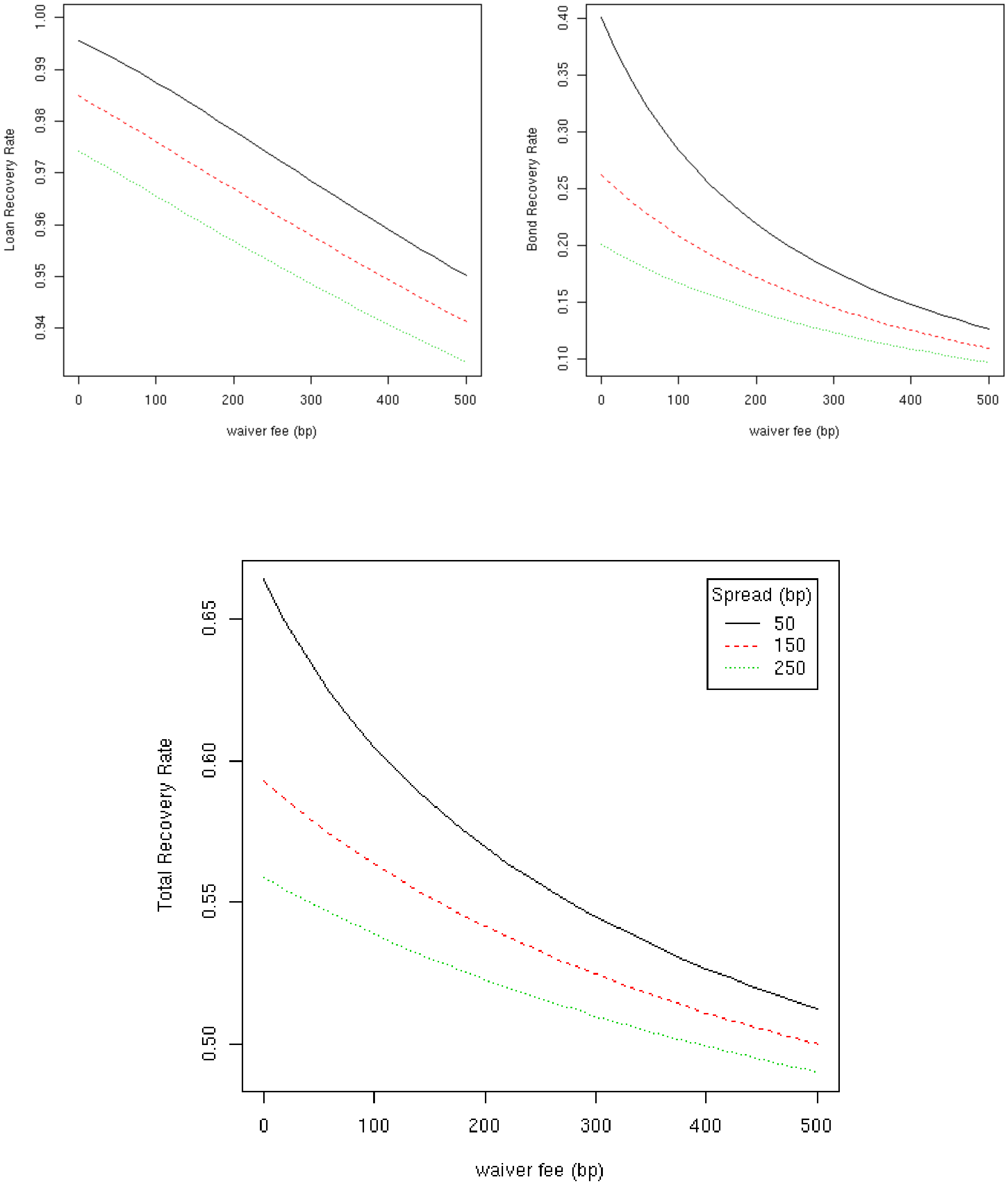
Figure 9 explores the dependence of the optimal foreclosure boundary on the waiver fee $w = \hat{c} - \check{c}$ and the normal state spread $\check{c} - r$. We find that κ^* decreases with w over this range of parameters. As the waiver fee is received by the bank only until foreclosure (or a return to the "normal" state $V > \nu$), an increase in the waiver fee increases the bank's incentive to forbear. Finally, in Figure 10, we explore the effect on recovery. As we would expect, the higher is w, the lower is the recovery rate for both debt classes. The effect shown on the loan's loss given default can be quite large on a relative basis, even if not terribly large on an absolute basis.

Figure 9: Effect of waiver fee on foreclosure threshold



The "spread" in the legend is $\check{c} - r$, expressed in basis points. Parameters: $r = 0.05$, $\sigma = \tilde{\sigma} = 0.2$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\chi = \eta = 0$, $\tau = 1$, $\nu = 1$.

Figure 10: Effect of waiver fee on recovery



Recovery rates at emergence. The "spread" in the legend is $\check{c} - r$, expressed in basis points. Parameters: $r = 0.05$, $\sigma = \tilde{\sigma} = 0.2$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\chi = \eta = 0$, $\tau = 1$, $\nu = 1$.

F Miscellaneous details about the data

F.1 Recovery measures

Placeholder. [38]

F.2 Structural subordination

In the U.S., most subordination is contractual.[39] Structural subordination refers to cases where debt is a claim on a holding company and the debt is not guaranteed by subsidiary operating companies. Holding company debtholders are not legal claimants in the operating company bankruptcies and will receive a recovery only if the holding company's equity interest in the subs is worth something at emergence (or if the holding company has other assets). Thus, structurally subordinated debtholders often lose everything or almost everything. Because we are interested in recovery to the firm as a whole, without regard to the structure of the firm, we have identified cases of related-company bankruptcies in LossStats and have combined each set of related entities into a single simulated entity. There are six such cases. Results are robust to use of uncombined data.

G Discussion of measurement and results for year and industry dummies

Returning to Table 3, industry effects on recovery rates are weak at best, with the exception of Utilities. All regressions include a full set of industry dummies that we created by boiling down S&P's more than 100 industry designations appearing in LossStats to 17 categories (retail is the omitted category in regressions). Coefficients on most of these dummies are never statistically significant and are omitted from tables. Only those industries that have signficant coefficients in some specifications are tabulated. Chief among these are utilities, which are associated with firm-level recovery rates about 30 percent higher than the average of about 50 percent. Like prior researchers, we speculate that the regulated nature of utilities in the United States is responsible.

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[38] The recovery measures computed by S&P embody some practices and assumptions that seemed potentially problematic to us. We used raw cash flow amount, type, and date information in LossStats to produce measures that are suitable for our purposes. Our measures are highly correlated with those of S&P (Pearson correlations are between 0.97 and 0.99), and our results are robust to use of measures based on a wide variety of assumptions. Detailed information about variable construction and data cleaning is in an appendix available from the authors.

[39] At issuance, the indenture for a subordinated debt instrument specifies the existing debt instruments to which the new debt is subordinated. At emergence from bankruptcy, holders of the subordinated debt promise to make side-payments of their recovery to holders of the debt to which theirs is subordinated, up to the point at which the recipients' bankruptcy claims are fully satisfied. Leaving aside the subordination agreement, subordinated debt is just another general unsecured claim, that is, it is "senior unsecured debt." The subordination agreement is a private contract that is typically enforced and implemented by the bankruptcy court as part of the agreed-upon plan of reorganization, but if the bankruptcy court does not enforce it, separate lawsuits for enforcement of the agreement must be litigated in other courts.

Often accounts payable and other general unsecured claims are not included in the list of debt to which the instrument is subordinated. Thus, in some cases, if the gross recovery received by subordinated debtholders is not exhausted by the contractual side-payments, subordinated debtholders may have positive recoveries even if some senior claimants do not have a full recovery. This is not a violation of absolute priority. None of these details of contractual subordination are material for our firm-level recovery estimates, but they are material for instrument-level analysis.

Regulators may play a role in forcing utilities into bankruptcy "early," as they probably have a preference that firms they regulate not become deeply insolvent.

We created a dummy variable for "bubble" firms, which are defined as firms in the telecom, internet, or energy trading sectors that filed for bankruptcy in the year 2000 or later. We classified bubble firms by inspection, as S&P's industry classifications are not always indicative. Coefficient estimates imply that such firms have economically and statistically significantly smaller recovery rates than other firms. We regard this result as consistent with the finding of Acharya et al. (2007) that recoveries are lower for firms whose industry is deeply distressed when bankruptcy is filed. If we omit this variable from specifications, coefficients on the Telecom and Computer industry dummies are often economically and statististically significant (not tabulated), but in the presence of the bubble dummy they usually are not.

Dummies for the year in which bankrutpcy was declared also appear in most regression specifications (1993 is the omitted year; 1987 and 1988 are combined because the number of observations for those years is small). The dummies are intended as controls for cyclical and trend effects. Most coefficients are not significantly different from zero and no trend is evident, but point estimates hint of the possibility of important cyclical effects, with lower average recoveries during recessions.