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SYMMETRY IN ARCHITECTURE: TOWARD AN OVERDUE REASSESSMENT

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Abstract: *The mathematical concept of symmetry, in its fullest sense, figured large in architectural history up to the early twentieth century. However, for the better part of a century, architecture and related disciplines have marginalized the consideration of symmetry in favour of a "functionalist" conception of design. More recently, dramatic developments in mathematics, physics, biology, neuroscience, environmental psychology, and other fields have given new dynamism to the ancient topic of symmetry. These findings carry implications for architecture and other environmental design professions that have, until now, been poorly understood, where they have been considered at all. This paper examines the new findings and what they reveal about current design orthodoxy as well as shedding new light on historic precedents. It concludes that there is an urgent need for a reassessment, toward a new agenda of research and practice.*

Keywords: symmetry, architecture, neuroscience, Christopher Alexander.

1 INTRODUCTION

Mathematics has an ancient relationship to architecture, and the topic of symmetry has long been a core concern of both (Salvadori, 2015). In the last century, the concept of symmetry has received renewed interest in many fields, including physics, mathematics, chemistry, information sciences, and cognitive psychology. This trend can be readily observed with the Google Books Ngram Viewer, which plots the frequency of terms within published books from 1800 to the present. From a steady plateau of .004%, usage of the term "symmetry" surges to a peak of about .012% in 1990, and is now about .007% (Fig. 1).

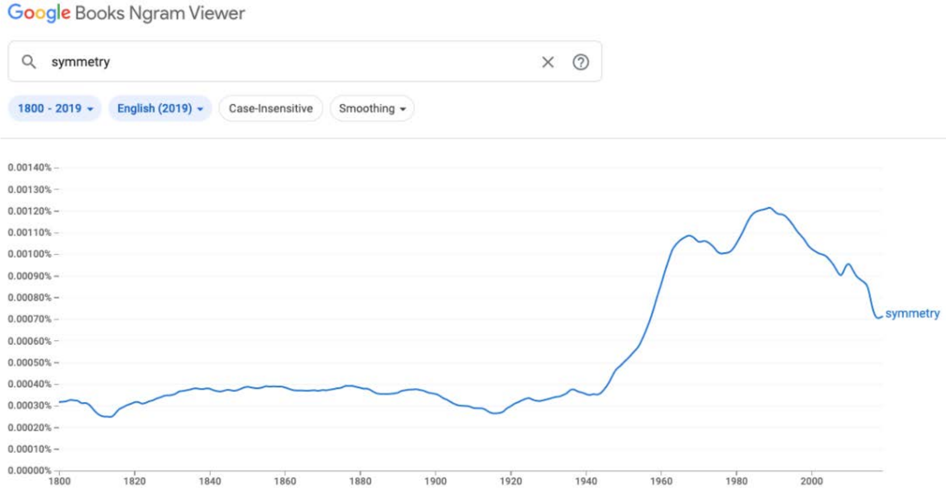
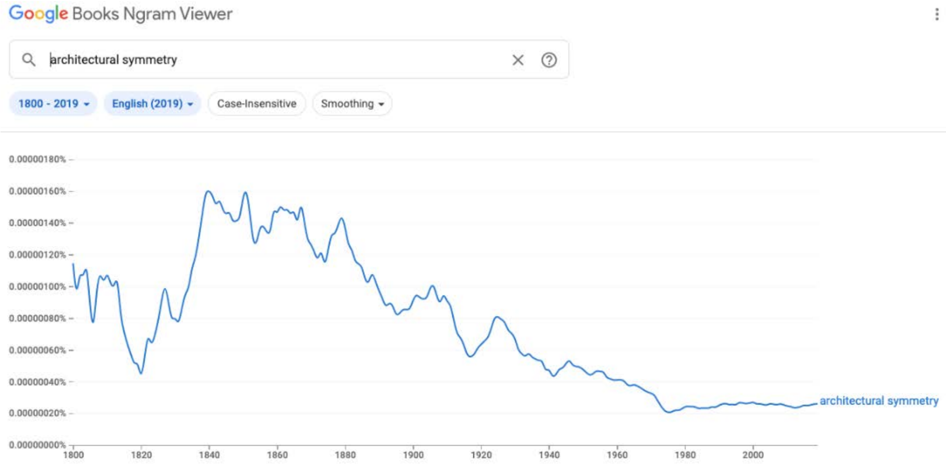


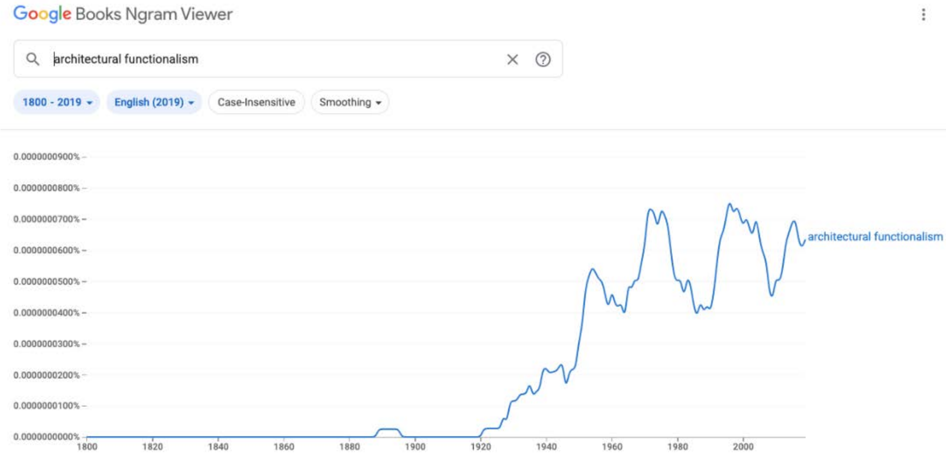
Figure 1: Frequency of the term "symmetry" in books since 1800, according to Google Books Ngram Viewer (Google, 2021a).

Something very different—and intriguing—occurs in the field of architecture, however. Rather than paralleling the rise in usage of the term "symmetry" in other fields, the term "architectural symmetry" follows a marked decline in the 20th century according to Google Books Ngram Viewer, particularly after about 1920 (Fig. 2). This was a period when new theories of architecture became prevalent, notably the vague but catchy concept of functionalism as outlined by Adolf Loos in his essay, "Ornament and Crime", written in 1910 (Loos, 1998), Le Corbusier in his book Towards a New Architecture (1923), the Congrès Internationaux d'Architecture Moderne in its Charter of Athens (as documented by Le Corbusier, 1943), as well as by other leaders of the modernist movement in architecture. For Loos, even the concept of beauty is contingent on this elusive functionalism: "It is, therefore, out of the question that something not satisfactorily performing its intended function can be beautiful" (Loos, 1998, p. 63). For Le Corbusier and his colleagues, not only buildings but the entire city must be reconceived as an assemblage of so-called functional units, zoned into their proper position: “Zoning is an operation carried out on the city map with the object of assigning every function and every individual to its rightful place” (Le Corbusier, 1943, para. 15). In that document, the term “function” appears no less than 37 times.



architectural symmetry

Figure 2: Frequency of the term “architectural symmetry” in books since 1800, according to Google Books Ngram Viewer (Google, 2021b).



architectural functionalism

Figure 3: Frequency of the term “architectural functionalism” in books since 1800, according to Google Books Ngram Viewer (Google, 2021c).

Indeed, when one searches on the term “architectural functionalism”, the graph of its frequency is almost an exact inverse of the term “architectural symmetry” (Fig. 3). From a state of almost nonexistence prior to 1920, the term surges into prominence after 1920, reaching peaks in the 1950s and beyond. This evidence strongly suggests, then, that the concept of architectural functionalism—whose precise meaning will be analysed later in this paper—has filled a gap (or perhaps created it) where the term "architectural symmetry" once existed in the literature.

What, then, was the cause of this decline in writing about (and implementing) architectural symmetry, in favour of the term "architectural functionalism"? Why did architects consider symmetry an obsolete topic to be cast aside, while in other disciplines it remained a dynamic and productive topic? Finally, what does that say about new opportunities for the fields of environmental design, and what they might learn from more recent scientific advances?

2 SYMMETRY BEYOND SIMPLE REFLECTION

To many lay people, understanding of the concept of "symmetry" may be quite limited to simplistic geometric ideas, often connoting only the most familiar example of two halves of an image reflecting each other as in a mirror. As trained mathematicians know, however, the concept is far broader. In addition to reflectional symmetry in geometry, there is rotational symmetry, translational symmetry, scaling symmetry, and many compound forms of these (Lockwood and MacMillan, 1978). Moreover, there are also varieties of information symmetry, and the symmetries of group theory (which include geometric symmetries but also other kinds) (McWeeny, 2002). There are also processes of symmetry-breaking, which are important in understanding many biological and physical processes (Marijuán, 1996). Some physicists have even advanced the concept that symmetry is a fundamental property of the universe (Bekenstein, 2004).

Most readers will be aware that the word "symmetry" comes from the Greek *syn* (together) + *metron* (measure), thus referring to a correspondence between different structures with similar measurements or correspondences. This concept can therefore apply broadly to many fields (as evidenced by this journal, dedicated to its exploration across domains). In the specific case of geometry, the "symmetrical" form in question is said to be invariant under a given transformation, which may include reflection, rotation, translation, scale changes, or other mappings. One can describe the results as "symmetrical" when the transformed portion of a structure is compared to the original portion, and it demonstrates a correspondence. In the human environment, this could take the form of visible patterns that express a common origin, or a relationship of some kind to other structures, or even to the viewer (as in cognitive symmetries, which are discussed further below).

Figure 4: Common forms of symmetry in the natural world (top), and in human architectures (bottom). From left to right: reflectional (tiger and classical façade with columns), rotational (sun during eclipse and stained-glass roof), translational (duck family and Islamic brick-and-tile wall), and scaling (fern leaf and Islamic Muqarnas pattern). Image credits: Top row, left: S Taheri via Wikimedia Commons; centre left: public domain (via Pixabay); centre right, public domain (via Maxpixel); right, public domain (via Free Nature Stock). Bottom row, left: Ryan Kaldari via Wikimedia Commons; centre left, Thomas Ledi via Wikimedia Commons; centre right, public domain (via Pixabay); right, public domain (via Pixabay).

3 SYMMETRY IN ARCHITECTURE UP TO 1900

As in other fields, the topic of symmetry was for many centuries a dynamic one in architecture. Indeed, an interest in symmetry in human environments and structures can be found in antiquity, and even further into prehistory (Hodgson, 2011). As Salvadori (2015) documented, there is evidence of many complex kinds of symmetry in ancient American pre-Columbian, Indian, and Chinese architectures. The symmetric geometries of ancient Egyptian structures have been noted (Rossi, 2004). Fractal scaling symmetries are obvious in sub-Saharan African architecture (Eglash, 1999). Centuries of Islamic Architecture offer us a pictorial textbook of applied plane symmetries. Pythagoras was perhaps the earliest to codify concepts of symmetry in Western mathematics (Vamvacas, 2009). In turn, he had a profound influence on Plato and others to follow, in their thinking about the properties of symmetry in nature and in human constructions (Kahn, 2001).

Following Plato, the Roman theorist Marcus Vitruvius Pollio (known more commonly as Vitruvius) applied the concept of symmetry more directly to the field of architecture.

His conception of “symmetry” was clearly less focused upon reflectional or rotational kinds of symmetry, as is often naïvely assumed, and more focused upon a deeper conception of correspondences under transformation, which included many other kinds of symmetry including translational, scaling, and information symmetries. His definition was:

> “… a proper agreement between the members of the work itself, and relation between the different parts and the whole general scheme, in accordance with a certain part selected as standard. Thus, in the human body there is a kind of symmetrical harmony […] and so it is with perfect buildings.” (Pollio, 2001, Book 2, Chapter 2)

Under this definition, a “certain part selected as standard” may be transformed via a change in size (scaling symmetry) or repeated position (translational symmetry), or through formation of proportional groups (informational symmetry). There is also the clear implication of combinations or compound symmetries. Vitruvius’ linking of symmetry to the concept of harmony also suggests a grouping of related symmetries.

The seminal Renaissance architect and theorist Alberti (1404–1472) also had a much more comprehensive understanding of symmetry, well beyond the simplistic reflectional or mirror symmetry that is often assumed. The Latin phrase used by Alberti was *ita ut mutuo ad speciem correspondeant*, or “so as to correspond to one another as a species,” that is, an interrelation of parts and wholes, or what he termed *concinnitas* or harmony (Alberti, 1988). Thus, Alberti took forward and deepened Vitruvius’ and Pythagoras’ already broad concepts of symmetry.

Alberti’s writings on this subject reflect a surprisingly contemporary view of the complex interrelation of systemic or part-whole relations:

> “Beauty is a kind of concord and mutual interplay of the parts of a thing. This concord is realized in a particular number, proportion, and arrangement demanded by harmony, which is the fundamental principle of nature […] There are three basic things which contain everything that we seek: number, what I have called proportion, and arrangement (*numerus*, *finitio*, *collocatio*). But besides these there is something else which originates from the linking and mutual relationship of these things, and which makes the surface of beauty

> glisten with a marvelous brilliance; this thing we call harmony (*concinnitas*).” (Alberti, 1988, Book 9, Section 5, pp. 337–340)

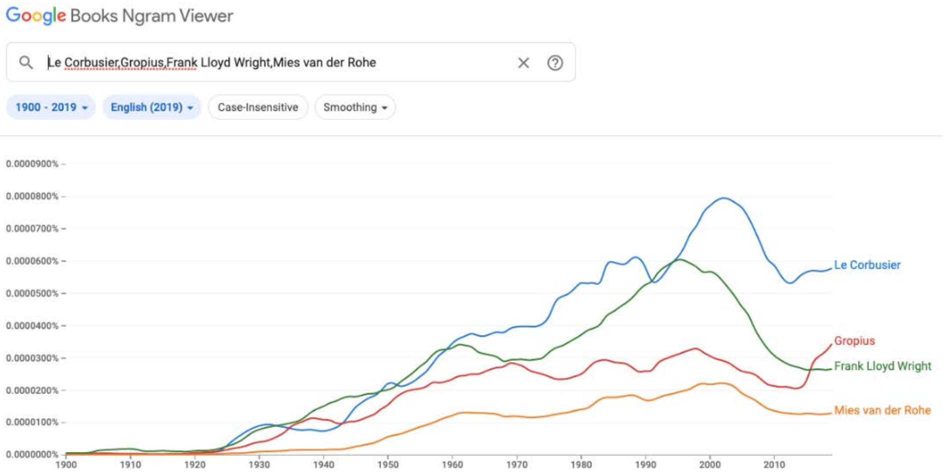
By “linking and mutual relationship” one might think of modern systems theory or group theory, and moreover tied to even more recent findings in neuroscience about aesthetic perceptions, about which more will be said below. What can be noted for the moment is that the forms of symmetry in buildings prior to 1900 were remarkably complex, and went far beyond simple reflectional or “mirror” symmetries.

Figure 5: Examples of buildings from before 1900 that exhibited many forms of symmetry, including reflectional, rotational, translational, scaling, and compound forms. Left, Santa Caterina del Sasso Monastery, Lake Maggiore, Italy; Center, Central Market, Valencia, Spain; right, Taktsang Monastery in Bhutan. Image credits: left, public domain (via Pixabay); center, @nosoylasonia via Photohere; right, Douglas J. McLaughlin via Wikimedia Commons.

4 THE “MODERN” ERA: SYMMETRY MISPLACED

There is little doubt that the three most influential architects of the early twentieth century were Le Corbusier, Walter Gropius, and Ludwig Mies van der Rohe. We focus on this trio of European architects because of the enormous impact they have had in displacing traditional world architecture after World War II. Once again, one can see from Google Books’ Ngram Viewer that references to Le Corbusier and Gropius in particular eclipsed those of even the most famous American architect, Frank Lloyd Wright (Fig. 6). Le Corbusier’s influence in particular is felt keenly even today, with an all-time peak in book references as late as the year 2000.

None of these three architects had a mathematical education, or even, evidently, formal education in mathematics beyond grade school. Indeed, each of them had remarkably little formal education as such. Le Corbusier had no university education, having trained at a municipal art school in his hometown of La-Chaux-de-Fonds, Switzerland (Brooks, 1999). Ludwig Mies van der Rohe, the son of a stone carver, left school at a young age to become an apprentice in his father’s shop, and later learned as an apprentice to designers and architects (Schultze and Windhorst, 2012). Walter Gropius alone had formal architectural education—though not directly in mathematics or the sciences—first at the Technical School in Munich and then the Konigliche Technische Hochschule in Berlin (Isaacs, 1991). He interrupted his formal education early, however, when he received a large inheritance from a great aunt. At that time and forever after, Gropius’ technical skills were known to be greatly limited. In fact he famously could not draw, relying upon associates to carry out this basic component of professional practice throughout his life (Crohn, 2019; Curl, 2018).


Le Corbusier,Gropius,Frank Lloyd Wright,Mies van der Rohe

1900 - 2019 · English (2019) · Case-Insensitive · Smoothing

Figure 6: Google Books Ngram Viewer of four of the most influential architects of the 20th century, Le Corbusier, Walter Gropius, Ludwig Mies van der Rohe and Frank Lloyd Wright (Google, 2021d).

There is no indication in the research literature that either Le Corbusier, Mies van der Rohe, or Gropius had more than a superficial understanding of the mathematical concept of symmetry. Like most lay people, they seem to have assumed that “symmetry” was limited to simple mirror relations, or proportional relations of the sides of a rectangle at best. Evidence for this simplification can be seen most visibly in Le Corbusier’s concept of the “Modulor”, a proportioning system that was purported to be based upon the human body, but was the subject of intense criticism at the time (Benelli, 2015) and up to the present day, by both architectural scholars and mathematicians

(Millais, 2020; Rozhkovskaya, 2020; Salingaros, 2018). Yet as far as the literature today indicates, this was the only time that any of the three leading architects expressed mathematical ideas of any kind, let alone concepts of symmetry.

What they did express were myriad opinions about technology and society, and the imperatives those supposedly created for a "new architecture". This was most visible in Le Corbusier's 1923 treatise, *Towards a New Architecture*, which rapturously proclaimed a new dawn for architecture:

> "A great epoch has begun. There exists a new spirit. Industry, overwhelming us like a flood which rolls on toward its destined ends, has furnished us with new tools adapted to this new epoch, animated by the new spirit." (Le Corbusier, 1923, p. 6).

It was postulated that this new architecture would learn from the more mechanically inclined engineering disciplines, and gleefully discard the "expiring" architecture of the past:

> "The American engineers overwhelm with their calculations our expiring architecture" (Le Corbusier, 1923, p. 31).

Le Corbusier particularly admired the engineers' emphasis on function, and their aim to solve well-defined technical problems—for the most part not involving people directly—and not to invent forms:

> "The modern engineer, then, pursues function first and form second…" (Le Corbusier, 1923, p. viii).

But a more careful examination of Le Corbusier shows that many of the "functional forms" that he adopted were irrelevant to their actual functions, e.g., creating habitable dwellings, or assuring human comfort. Instead, they were highly derivative images aimed only at capturing the allure of industrial marvels of the age, including airplanes, cruise ships, and grain silos. His *Towards a New Architecture* includes examples of the tall towers of grain elevators and the portholes and pipe railings of ships, as well as other industrial (and non-architectural) structures (Fig. 7). One may note, however, that human beings are not grains of wheat to be elevated for storage, nor are their buildings likely to go crashing through the waves or hurtling through the air. That is not their function, and the idea that these are elements of a "functionalist" design is risible.

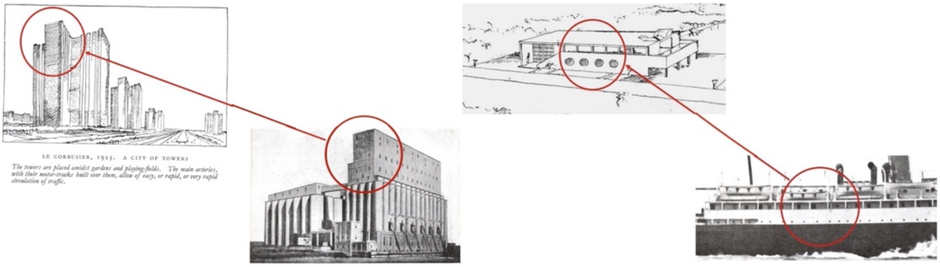


Figure 7: Le Corbusier's "functionalism" seems purely metaphoric when applying the towering forms of grain elevators, or the portholes and pipe railings of cruise ships (Le Corbusier, 1923).

That these forms were now possible for buildings also thanks to new technologies and new industrial processes is not in dispute. That they are the *necessary* forms of a new functionalist architecture is a tendentious *ex cathedra* claim, lacking in anything like scientific evidence. If Le Corbusier wishes to express himself through these new forms, and if he imagines that this is the fitting expression of a new age, it is one thing to do so as an individual designer; but this is hardly a basis on which to command that the entire profession shall be re-structured, following a rejection of all that came before. Yet this fanatical intolerance comes through in Le Corbusier's draft of the Athens Charter—which he claims was transcribed from the actual 1933 meeting of the *Congrès Internationaux d'Architecture Moderne* (although the evidence shows he himself wrote it much later) (Gold, 1998).

> "70 The practice of using styles of the past on aesthetic pretexts for new structures erected in historic areas has harmful consequences. Neither the continuation of such practices nor the introduction of such initiatives will be tolerated in any form. Such methods are contrary to the great lesson of history. Never has a return to the past been recorded, never has man retraced his own steps." (Le Corbusier, 1943, para. 70)

Aside from the intolerant fanaticism, this is a preposterous claim on its face. "Never" did the Renaissance return to the past treasures of Greece and Rome? "Never" did Palladio recapitulate Vitruvius, Jefferson, and Wren recapitulate Palladio, or for that matter, Rome recapitulate Greece? On the contrary, form languages evolve with time, typologies and patterns come and go, picked up again in revivals by later generations, not unlike the way biological evolution builds on (and sometimes recapitulates) previous patterns to generate more adaptive and more complex forms. The history of architecture (in Western culture, as in others) is an evolutionary fugue, and far from a straight line.

Alexander (2003) focuses on the evolutionary mechanism governing natural form languages in architecture, and opposes this to the imposition of simplistic styling, which leads to uniformisation. Alexander's work on the biological analogy of how forms evolve builds on the ground-breaking work of D'Arcy Wentworth Thompson (1952).

Le Corbusier's edict in the Athens Charter is, then, a most curious, even illiterate, reading of history, and as discussed previously, it is also a rather ignorant consideration of the role of symmetry. Where did Le Corbusier (and to some extent his colleagues Gropius and Mies) get these wrong ideas?

5 BRANDING (AND MARKETING) INDUSTRIAL MODERNITY

Insight into the formative influences on Le Corbusier's thinking, and that of his two close colleagues, can be found in a remarkable convergence of people and ideas around the year 1910. In that year, three young protégés were working in Berlin for Peter Behrens, the corporate architect for the industrial giant AEG (Anderson, 2002). Behrens has since become known as the father of corporate branding, for he designed not only buildings but logos, stationery, signage, and even some products. There was an important shift in emphasis in Behrens' work, away from building design *per se* and toward establishing a marketing narrative about the company and its products. The emphasis—and it is a strictly visual one—would be on industrial power, allure, excitement, and the promise of the future. For architecture, this was an especially important shift toward relying upon the psychological manipulation of the advertising world. No longer would buildings be oriented to their place, but to their time—and especially, to their role in heralding a glorious industrial future (Journel, 2015).

The three young protégés of Behrens are already familiar to this paper's readers. They are none other than Le Corbusier, Walter Gropius, and Ludwig Mies van der Rohe. They saw the power (and the profits, to be sure) offered by the new industrial regime. They surely observed the excitement among many members of the public about the promise of the new technology: its power and allure, its capacity to overcome human frailties, its promise to conquer disease and deprivation. Above all, they saw the opportunity to market and brand this new vision of the future, with logos, product design, and of course building design, as Behrens had demonstrated.

Le Corbusier in particular took up the promotional opportunities with zeal. Moving to Paris in 1917, he co-founded the advertising magazine *L'Esprit Nouveau* in 1920 (Colomina, 1996). Many of his later publications, including *Towards a New Architecture,* were drawn from his articles in the magazine, which featured breathless promotions of myriad products including aircraft, cars, and many other kinds of machines. As Colomina writes:

> "Purist culture […] is a 'reflection,' in both the specular and intellectual sense of the word, on the culture of the new means of communication, the world of advertising and mass media… The very idea of the 'machine age', we can see now, served the period as a symbolic concept, doubtless to say largely induced by the advertising industry. Retrospectively speaking, from the point of view of criticism, the concept of the 'machine age' had served the purpose of sustaining the myth of the 'modern movement' as an autonomous artistic practice and of the architect as 'interpreter' of the new industrial reality." (Colomina, 1996, p. 67)

In this newly imagined regime, the three architectural leaders (and others, including their Austrian contemporary Adolf Loos) assumed that there was no longer a place for ornament, pattern, or symmetry. As Loos wrote in his seminal 1910 treatise "Ornament and Crime" (Loos, 1998), while modern Western people might lament the loss of ornament, pattern and symmetry in the machine age, on the contrary, they should celebrate the loss as a mark of their (Western) cultural supremacy:

> "Are we alone […] supposed to be unable to do what any Negro, all the races before us have been unable to do? [...] Then I said: weep not. Therein lies the greatness of our age, that it is incapable of producing a new ornament. We have outgrown ornament; we have fought our way to freedom from ornament […]

> I am preaching to the aristocrat, I mean the person who stands at the pinnacle of mankind…" (Loos, 1998, pp. 20–24)

Le Corbusier expressed a similar sentiment a few years later:

> "Decoration is of a sensorial and elementary order, as is colour, and is suited to simple races, peasants, and savages… The peasant loves ornament and decorates his walls." (Le Corbusier, 1923, p. 123)

Therefore, both Loos and Le Corbusier declared modern (Western) civilization at the beginning of the 20th Century to be "the pinnacle of mankind," where ornament, pattern and symmetry were unnecessary, being suitable only for "primitive" people. Worse, ornament constituted a "crime"—just as "the modern man who tattoos himself [like a Papuan] is a criminal or a degenerate." Instead, all forms should be stripped down and made to conform to a rigorous program of "functionalism" (although with dominant image-based aspects irrelevant of actual function, as discussed previously). This proposed functionalism was understood to be machine-like, in the manner of early 20th century machines—that is to say, primitive ones by today's standards. Elementary parts needed to be mass produced, stripped to their simplest and most economical designs, and then recombined to form machines—the fundamental constituents of the new world. This was a totalizing regime that must also include architecture. Just as automobiles were machines for moving, so too, houses were machines for living.

The impact of these three venerated individuals (Le Corbusier, Gropius, and Mies) and their program of so-called functionalist design can hardly be overstated. The rapturous vision of a sanitized, powerful, purist future was experienced by millions at the 1939 World's Fair, and in endless advertisements of the period. A public eager for technological progress accepted those utopian promises because they *looked* so convincing. After the devastation of World War II, many civic leaders were convinced that the future lay with this exciting new stripped down, functionalist regime (Curl, 2018). Over the next decades, as the fame and influence of the three architects grew (Fig. 6), the human environment was also radically transformed.

6 FORM FOLLOWS FUNCTION?

It is not our purpose here to make *ad hominem* attacks on the pioneers of industrial modernism for their racist views, which were not uncommon in their day. Rather, we seek to understand the attitudes and beliefs that were in fact an integral part of a profoundly influential new theoretical framework—one that remains unquestioned orthodoxy for many practitioners and academics even today.

Our point is this: the racist, culturally supremacist attitudes they held were not peripheral to their agenda of a totalizing new international architecture, but at its very core. Among other fateful consequences, this new agenda resulted in the near-extinction of endless varieties of mathematical symmetry in architecture, expressed through rich cultural differentiations. These differentiations had existed in many forms around the globe, but they had also occurred within European and American cultures as well. These too were ruthlessly extinguished under the new totalizing regime.

As discussed further below, what was on offer, then, was a momentous change in the ways that human beings had structured their environments up to that time—incorporating free expression of innate notions of symmetry—with important impacts that we must reckon with today.

Compounding this problem, there was a remarkable fallacy in the very idea of "functionalism". For aesthetic experiences are *also* one "function" of architecture—as the image-based designs of Le Corbusier actually (but unintentionally) reveal. The "function" of the images in his case is none other than to create an aesthetic experience—one that accorded with a marketable set of images and theories of an invented modernity. This pseudo-functionalism is evident in the iconic Seagram Building designed by Ludwig Mies van der Rohe, often celebrated by architects as a masterful exemplar of functionalism (Curl, 2018). Yet its exposed I-beams are in fact purely ornamental, without any structural function (Schulze and Windhorst, 2012). This practice was of course diametrically opposed to the doctrines of functionalism and "ornament as crime"—yet it remained an unacknowledged contradiction, once again revealing that the supposed "function" of functionalism was mostly about images. Mies, Le Corbusier, and their friend Gropius were better at marketing than at authentic design philosophy.

It is now understood that "function" within a technological system is not only a matter of discrete parts operating within elementary machinery to produce designed outcomes. Far more commonly, functions are the outcomes of vastly complex adaptive systems put to human use. These complex systems generate, and are embedded within, symmetric structures. Thus, function is ultimately inseparable from symmetric structure.

This is a much deeper sense of “design”, not merely as the generation of visual forms, but as the transformation of complex adaptive systems. Herbert Simon famously observed that “everyone designs who devises courses of action aimed at changing existing situations into preferred ones” (Simon, 1969). Those existing situations may indeed be vastly complex—and so may the preferred ones. But the “function” of the design is none other than to achieve these “preferred” states, however irreducibly complex they may be, and however deeply embedded within a context of symmetric structures.

An important illustration comes from the famous phrase “form follows function”, an ideological rallying-call for the modernist design movement (Michl, 1995). But what does “form follows function” mean in practice? It is in fact a coded slogan applied to eliminate complex symmetries. By insisting on superficial appearances, the slogan promotes a stripped-down machine aesthetic associated with an imaginary functional efficiency. Visual purity dominates as an image, and more often than not, compromises or entirely displaces the actual function. As noted earlier, this function is inseparably embedded within the symmetrical structures of the environment, both natural and human.

In nature, form and function are indivisible, and this is also true in general for traditional and vernacular architectures. Since our neurophysiology prepares us for life in an environment containing multiple symmetries, our brain parallels that complexity. Human creations are consequently shaped by the innate structure of our neural system. Humans evolved to process a specific set of natural symmetries, and those automatically appear whenever we create something, resulting in both utilitarian and ornamental characteristics—which turn out to be related. Indeed, the intellectual distinction drawn at the start of the 20th Century between these two concepts is entirely alien to human physiology. Unselfconscious human creations follow the example of nature in not distinguishing between form and function (Alexander, 2003).

The modernist slogan “form follows function” actually represents a radical separation between the function and symmetry that are found together in all adapted structures, both man-made and natural. Frank Lloyd Wright, who understood the complexity of natural forms, wrote scathingly about this:

> "Form follows function? [...] the level of mere dogma […] the already trite cliché […] the new slogan […] became another aestheticism […] easy to use for prefabricated teaching." (Wright, 1949)

Christopher Alexander made a similar point in his 2003 book *The Nature of Order*:

> "In nature there is essentially nothing that can be identified as a pure ornament without function. Conversely, in nature there is essentially no system that can be identified as functional which is not also beautiful in an ornamental sense. In nature there is simply no division between ornament and function." (Alexander, 2003, Book 1, p. 404).

But "form follows function" took its place as unquestioned dogma, alongside that equally damaging modernist motto "ornament is crime". Together, these two seductive memes made it seem that complex symmetries (and the related ornamental structures) could be eliminated from the architecture of the 20th century without harm, and indeed, with benefits.

What happened had a long-term monumental impact on society, as it replaced the deeper symmetric structures that had existed up to that time with a new set of preferred images linked to progress. Henceforth, those distinctive "images of modernity" would propagate independently of any function, in what is known as memetic transmission (Salingaros, 2013, Chapters 10 and 12). A specific set of non-functional images entered the collective unconscious and fed the false impression that a building became functional in practice simply by copying those images. (This is a classic case of "reverse-causation fallacy", when people mistakenly confuse cause with effect in a process).

7 ADVANCES IN OTHER FIELDS

Even as architecture was eschewing the topic of symmetry through the mid-20th century and into the 21st, other fields saw robust development. In mathematics, the basic categories of symmetry—reflectional, rotational, translational and scaling—were enriched with many other categories, including the "fractal" forms of scaling symmetry, the "glide reflection" symmetries that produce many complex forms of tiling and other patterns, complex forms of information symmetry, and processes of "symmetry-breaking" in physics (Mehaffy, 2020; Salingaros, 2013; 2020).

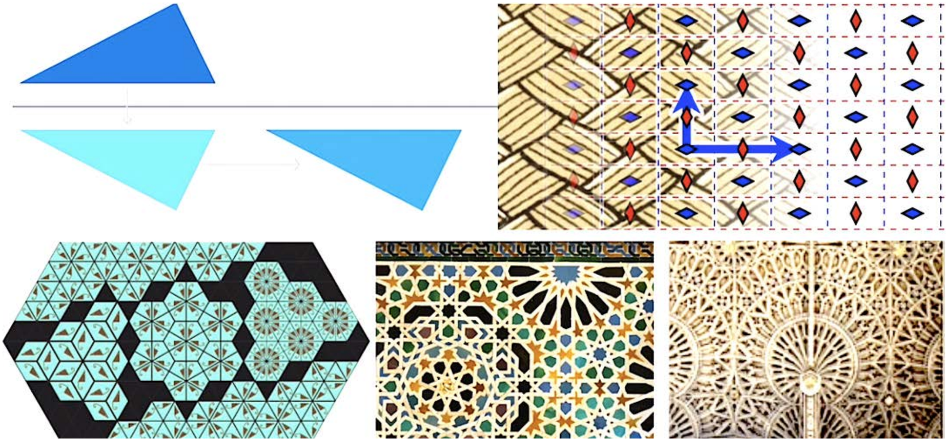


Figure 8: The "glide reflection," a form of compound symmetry, generates many rich tiling and interlocking patterns, including those of the Alhambra (bottom centre) and Westminster Abbey (bottom right). Images: Top left, Kelvinsong via Wikimedia Commons; top right, Martin von Gagern via Wikimedia Commons. Bottom left, Mathemalchemy group & Susan Goldstein; others in public domain.

As discussed earlier, while some architects continue to associate symmetry only with the much more limited class of mirror symmetries—for example, buildings whose left half is the mirror image of their right half—the concept is far broader, as readers of this journal are likely aware. The word "symmetry" refers to any transformation among structures or forms that preserves recognizable qualities. When a correspondence is found between the original form and the transformed one, that relation is described as a symmetry (Lockwood and MacMillan, 1978).

Another major subject area that has developed important applications of symmetry is known as group theory (McWeeny, 2002). A "symmetry group" is the group of all transformations within a geometric object, under which the object is invariant. This theory sheds important light on the structures of crystals and other complex geometric structures.

What now constitutes modern physics is largely the result of discovering group symmetries, starting in about the 1920s. The families of elementary particles and their interactions are classified according to symmetry groups (astonishing results that were awarded a series of Nobel prizes). All discussions on the fundamental structure of matter today employ the language and tools of symmetry groups. It is worth noting, however, that architectural culture has managed to keep itself isolated from all these pivotal scientific advances for over a century. We find this epistemological disconnect—in a profession that shapes the earth's surface, at a critical moment in human history—nothing short of alarming.

The concept of symmetry is also important in other branches of mathematics, where non-geometric structural invariances through transformation are also described as symmetries (Petitjean, 2003). Of course, an "equation" is at its heart nothing other than an expression of a symmetry between the two sides being "equated", through the transformations expressed in their formulae.

Symmetry plays a key role in information theory (Darvas, 2005; Jakulin, 2005; Jaynes, 1957). One might consider "information" as merely a useful form of symmetrical relationship between any two structures in the world, which often involves the use of one structure to model the other through their symmetries. For example, one may model the pattern of a sound wave through transformation into its digital representation, the measurements of which preserves a suitable level of symmetrical relationship to the original wave (Collier, 1996). Information need not be a human invention, however, as one can recognize similar phenomena in nature, e.g. the "information" contained in DNA (Marijuán, 1996). Once again, the symmetrical structure is what is invariant through transformation; in the case of DNA, the pattern of proteins expressed by the transformations of a given sequence of DNA are invariant, repeated (with of course occasional mutations) in the symmetrical copies in other cells of the same DNA sequence.

It has even been proposed (e.g., by Bekenstein, 2004) that "information" (hence symmetry, in this sense) underlies the fundamental structure of the universe. One can observe that the structures of the universe transform and differentiate from one another as a fundamental structural process, preserving some symmetries while others are broken. This "symmetry-breaking" transformation may be key to the differentiation of structures, and the evolution of their complexity. Without such a process, the universe might well remain an undifferentiated homogeneous field. This indeed may have been its state before the initial “symmetry-breaking event” of the Big Bang, and subsequent differentiation processes (Zurek, 1996).

In the biological sciences, the topic of symmetry has seen rich development. In biology and medicine, symmetry is a core topic of embryology and morphogenesis. This is most evident in the reflectional, rotational, translational, and scaling symmetries that are visible in beautiful flower and plant patterns. Recent evidence suggests that the symmetrical patterns of flowers may not be only a result of their morphogenetic process, but may also serve as important attraction signals for pollinator insects, and further, may serve to guide the behaviour of insect pollinators such as landing (Dafni, Lehrer, and Keyan, 1997; Ushimaru and Hyodo, 2005). In all these processes, one can see an essential relationship between process and structure, including the transmission of information. In many cases (such as the insect signalling) this information conveys the potential biological benefits of the structure in question.

Related findings indicate that symmetry functions as a form of biological signalling, which may convey the signaller’s general health and genetic suitability (Watson and Thornhill, 1994). The work of Enquist and Arak (1994) suggests that the ability to perceive both unbroken and broken symmetries may be a key to object recognition.

A group-theoretical treatment of how design elements combine (not undertaken here) has to blend mathematics with neuroscience. Human evolution employed sophisticated information-processing schemes to guarantee our survival. We continuously sift through enormous amounts of environmental information, seeking out meaningful clues to phenomena that will determine our actions (Peterson, 2013; Salingaros, 2020). As bilaterally-symmetric animals whose eyes scan horizontally, we have preferences that severely cut down on the types of combinations that lead to understandable form. As a result, human cognition ignores empty regions, and is made anxious by random forms and shapes. What is easiest for humans to comprehend is a high degree of complexity that is organized via the symmetries that we describe in this paper.

8 THE ROLE OF SYMMETRY IN HUMAN BIOLOGY AND COGNITION

Humans of course are not exempt from the dynamics of symmetry in biology, and evidence has grown that the role of symmetrical structures is crucial for life. For example, Grammer and Thornhill (1994) found that potential mates judge one another to be more sexually desirable in the presence of higher degrees of facial reflectional and rotational symmetry. Similar symmetrical features are also associated with a greater suitability of a potential life partner (Rhodes, Proffitt, Grady and Sumich, 1998).

Enquist and Arak (1994) noted that humans utilize the ability to perceive symmetry as an aid in structural legibility and environmental cognition. They suggest that this legibility produces a pleasurable experience, which may be a key component of aesthetic pleasure, or what is more commonly called “beauty”. Evidence from functional MRI studies supports this view (Grinde and Patil, 2009).

Single neurons in the brain can recognize complex symmetric patterns. Such cells preferentially fire when presented with concentric circles, crosses with an outline, stars of various complexity, and other concentrically-organized symmetrical figures (Salingaros, 2013, Chapter 4; Zigmond, 1999). Clusters of these specialized neurons then coordinate to trigger facial recognition through a face’s characteristic symmetries (Chang and Tsao, 2017). This built-in ability is crucial for the baby to recognize its mother and other animals of its own species—an innate cognitive mechanism that continues to be employed throughout adult life. Human social evolution depended upon decoding others’ emotions and feelings through facial expressions. Evolution, in fact, links symmetric pattern recognition to higher human intelligence. Information detection neurons that respond to more complex shapes are situated in increasingly advanced regions in the hierarchical organization of the visual cortex. These symmetry-seeking neurons are used to unconsciously interpret architecture (Ruggles, 2018; Sussman and Hollander, 2021).

These findings strongly suggest that symmetry, in its many forms, is an essential biological attribute for sensing and seeking the conditions of health and well-being, in humans as much as in other species. It seems that the ability to perceive symmetries may convey otherwise hidden information to an organism about both the morphogenesis and the morphology—that is, the process and product—of the structures it encounters. Whether it be choosing a mate, finding food, or settling in a salubrious environment—the information conveyed through symmetries appears invaluable.

There is even evidence that the presence of natural forms and geometries, including their various symmetrical properties, have positive effects on human health and well-being in their own right. In a landmark study, Ulrich (1984) showed a positive relationship between the ability to view a natural environment and the speed and completion of recovery from surgery, including shortened hospitalization time and reduced morbidity. Similar properties were subsequently found by other investigators, since referred to as “biophilia” (Kellert and Wilson, 1993). Later work found that in addition to natural environments, built environments also have significant impacts upon human well-being and can confer important benefits through biophilic geometries and related characteristics (Kellert, Heerwagen and Mador, 2008). It appears that there is a close correlation between the preference for certain environments and their capacity to promote “restoration”, or improved health and well-being (van den Berk, Koole and van der Wulp, 2003). These findings are part of a growing body of literature demonstrating the significant impact of aesthetics on well-being and health, both positively and negatively (Cold, 1998).

Research is discovering the specific geometric characteristics of optimal human environments, including their symmetrical properties. Hagerhall, Purcell and Taylor (2004) investigated human preferences for the fractal dimension of landscapes, and found a correlation between high scaling symmetry and strong preferences. It seems that our brain favours hierarchical scaling (Salingaros, 2013, Chapter 7).

Summarizing findings from his own team’s and others’ neurological research, Zeki (2019) concludes that “mathematical principles of symmetry, harmony and proportion […] are part of the cognitive apparatus of all brains.” He also asserts that these principles “have to be respected” in order to understand how humans perceive structures as beautiful (Zeki, 2019, p. 19).

Moreover, Zeki says, the experience of beauty is not a luxury, but a neurological requirement for healthy brains: “In our daily activity, we search for and seek to satisfy that quality; in simpler terms, we seek the beautiful to nourish the emotional brain since, from a neurobiological point of view, all areas of the brain must be continually nourished in a way that corresponds to their specific function […] Hence, whatever other demands go into architectural design, beauty must be a central element. Its experience adds to the health of its individuals and thus to society’s wellbeing. It is not a luxury, but an essential ingredient in nourishing the emotional brain” (Zeki, 2019, p. 19).

Beauty, then, is an essential experience of a healthy human life, and in turn it is closely linked to symmetry. Both concepts appear essential to the human need for understanding the natural and built worlds, and the need to find healthy conditions within them. One may go so far as to say that the experience of mathematical properties like symmetry is essential to our ability to find meaning within our human environments (Salingaros, 2020). As investigators now turn to using "deep learning" to analyse beauty (Cetinic *et al.*, 2019; Seresinhe *et al.*, 2017), the present discussion offers one basic piece of advice: scaling symmetries need to act together with the other mathematical symmetries for an attractive result.

9 EVIDENCE FOR NEGATIVE IMPACTS OF A "SYMMETRY DEFICIT" IN THE HUMAN ENVIRONMENT

There is notable evidence that a deficit in the experience of natural environments and their forms can be harmful, especially to children. The journalist Richard Louv (2008), citing this evidence, coined the term "nature deficit disorder" to describe the adverse impact upon children who are deprived of regular contact with natural environments. Grinde and Patil (2009) also found negative impacts on health and quality of life from environments devoid of nature, while Kellert (2002) surveyed the literature and found evidence that "children's emotional, intellectual, and values-related development, especially during middle childhood and early adolescence, is greatly enhanced by varied, recurrent, and ongoing contact with relatively familiar natural settings and processes" (Kellert, 2002, p. 146).

One may object here and cite those who claim that beauty is entirely a subjective phenomenon and "in the eye of the beholder". However, the evidence from neuroscience, and its connection to objective properties of symmetries, makes a strong rebuttal of this claim, and demonstrates large areas of overlap between people (if not complete unity) in the experience of beauty (Alexander, 2003; Kawabata and Zeki, 2004). Zeki and Chén (2019) and Zeki, Chén and Romaya (2018) do make a distinction between "artifactual" and "biological" perceptions of beauty. Their research distinguishes between two broad classes of priors on which the brain's inferential systems operate: artifactual priors such as man-made objects, versus biological priors such as colour categories and faces. While the former may be highly variable between people dependent upon different life experiences, e.g., the specialized training of professionals, the latter are largely a consequence of inherited responses to invariant geometric relationships, and these responses are widely shared among divergent populations. The impacts of these relationships result from the innate biological systems for experiencing the world, which are widely shared among humans. Zeki and Chén (2019) suggest that there is greater unanimity in architectural beauty than previously thought, exactly because “there is a heavy dose of biological beauty, dependent upon inherited brain concepts.”

Thus, particular expressions may be manipulated as highly variable constructs within the arts or other creative domains, and this variation accounts for the differentiation of perceptions and tastes. At their core, however, is a shared biological response—what Mehaffy (2020) refers to as *structura naturalis,* in contrast to the *structura mentis* of cognitive invention. In this conception, the constructed and the innate are not opposites, but poles of an overlapping continuum from the biological to the artifactual—the world of life, and the world of art. As Jane Jacobs observed (1961), when there is a confusion between the two domains, “the results impoverish life instead of enriching it” (Jacobs, 1961, p. 373).

With regard to biological response, the variability of preference for built environments does depend, at least to some extent, on their actual structural properties. For example Stamps (1994) observed that, although natural environments are generally preferred over built ones, it is also the case that within built environments, old buildings are more frequently preferred over new ones. It is important to note that these older structures, and the urban environments they comprise, often possess the same geometric characteristics as natural environments, sometimes including literal plant forms, but more often the symmetric properties of natural structures, including nested reflectional, translational, and scaling symmetries (Salingaros, 2010; 2013).

There is some evidence that environmental symmetry plays a key role in cognition, cognitive development and learning, notably that of children (Pornstein and Krinsky, 1985). This work builds on the seminal theories of Jean Piaget, who recognized that perception and learning in children requires the integration of elements of a pattern with symmetrical components (Piaget and Lambercier, 1943). More recent evidence has suggested that cognition is partly “embodied” in the physical world, in the form of a network of “external sensory data [linked] with internal cognitive programs” (Ballard, Hayhoe, Pook and Rao, 1997). These findings provide preliminary evidence (with more research needed) that a symmetry deficit in the human environment could have a real impact on children's cognitive development, and in the continued cognitive health of adults.

10 FRONTIERS OF SYMMETRY IN ARCHITECTURE: OUTLINING A NEW AGENDA FOR RESEARCH AND PRACTICE

A number of these developments in mathematics and other fields offer tantalizing leads for further development in architecture and the built environment. For example, it may be possible to create reasonably accurate predictive measurements of the human benefits of aesthetic characteristics of buildings, from the point of view of health and well-being. This approach could build on earlier work by the mathematician George Birkhoff (1933) who developed a mathematical model for the predicted aesthetic impact of an art object, expressed as $M = f(O/C)$, where O stands for order and C for complexity. For Birkhoff, order "O" was intimately related to symmetry, as when an object "is characterized by a certain harmony, symmetry, or order (O), more or less concealed, which seems necessary to aesthetic effect" (Birkhoff, 1933, pp. 3–4; Douchova, 2016). Several years later, Eysenck (1942) proposed a major modification to Birkhoff's theory, arguing that O/C (order divided by complexity) should be replaced by $O \times C$ (order times complexity). In other words, the complexity of a structure magnifies its perceived order. This revised formula better reflects the observation that compound symmetries, or combinations of symmetries, play a key role in the experience of beauty and its salubrious benefits.

More recently, Salingaros (1997) proposed a "thermodynamic analogy" for the aesthetic properties of buildings, as discussed by his colleague Christopher Alexander, and using an index similar to Eysenck's. An attempt was made to classify two distinct types of complexity into "disorganized" versus "organized", and to show how mathematical ordering transforms the former into the latter. In turn, Alexander proposed a kind of phenomenological model of the "15 Fundamental Properties" of preferable environmental geometries, which suggested different combinations of symmetry (Mehaffy, 2020). A body of work links information theory (referred to previously) to entropy and thermodynamics, which in turn link to complexity theory. Some of the most exciting recent results apply this work to understand the morphological properties of organisms, hence to how our brain is programmed to interpret its environment (Peterson, 2013).

The insights of Piaget and others suggest that a general mathematical-visual mechanism by which humans live and perceive the world is to group adjoining geometrical elements via symmetries into larger wholes (Pornstein and Krinsky, 1985; Salingaros, 2010). For example: (i) two juxtaposed mirror images are joined to make a symmetric whole; (ii) aligned repetitions of the same element are added to define a larger whole having translation symmetry; (iii) juxtaposed elements related via rotation can be grouped into a larger round whole. This is the simplest mathematical mechanism for ordering human environments. Symmetric relations also work at a distance, although their strength decreases. One can also combine simple symmetries, such as translation with reflection, into what is known as a ‘glide symmetry’ (Fig. 8).

The work of Piaget and others also suggests that humans build up larger-scale complex wholes using repeating individual units (Salingaros, 2010). The process can be observed in cultural artefacts and traditional design methods. One simply varies the repeating units sufficiently to distinguish them from each other, but not enough so as to remove their basic similarity. That is, one makes small changes among units of a larger symmetry that are the same on a particular scale. One uses the convenience of repeating the same unit, but then distinguishing each unit (or at least some of the units) through added variations on different scales. This is called “breaking the largest-scale symmetry”, since an otherwise perfect symmetry (translational, rotational, reflectional, etc.) is no longer strictly valid, although it obviously still dominates.

This interpretation of symmetry breaking comes from physics, where important phenomena (such as the appearance of electrical charge and mass) occur because an exact global symmetry is relaxed slightly—and is not at all destroyed. Yet that slight approximation enables the generation of new forces and sub-symmetries that would otherwise not act in a situation of perfect global symmetry.

Limited variation on a smaller or larger scale (“symmetry breaking”) also prevents informational collapse. If one breaks a repetition symmetry ever so slightly, then additional information is required to specify the whole structure, more than was necessary when perfect symmetry was present. Yet it is still much more economical to build up complex structures through combinations and ordering, rather than to have an entirely random structure, since even approximate symmetry saves an enormous amount of information specifying the overall complexity (Collier, 1996).

Traditional artefacts from all over the world such as pottery, carvings, textiles, and oriental carpets show, upon closer inspection, that every repeating unit has been made slightly different with the result that it prevents informational collapse and monotony. Vernacular architectures are replete with approximate symmetries and more formal architectures often exhibit requisite variety: e.g., individualized Byzantine and Romanesque capitals; imperfectly symmetrical Gothic façades; architectural elements repeated with variations, and so on (Alexander, 2003).

Symmetry breaking also makes sense from an informational point of view. One intuitively expects that a large-scale complex whole will contain much more information than a single unit used to generate it. The measure of complexity ought to be proportional to the size of the system, something that fails with monotonous repetition (in which there is no increase of complexity as the system scales up in size). But the key to adaptive symmetry is that the amount of complexity increases logarithmically and not exponentially with the size of the system. This quality makes the system comprehensible. By breaking the symmetry through the injection of additional information on smaller scales, there is indeed more information contained in the large-scale system than was present in an isolated unit.

The role of symmetry is investigated by Alexander (2003), considering “the art of building and the nature of the universe” (the subtitle of his major treatise *The Nature of Order*). In that work, he considered the geometric properties that characterize buildings and other human environments that are most conducive to well-being, and that, as he put it, “have life.” He found that he could identify just fifteen fundamental geometric properties. Each of the fifteen properties was a variety of symmetry, or was closely related to symmetry: e.g., “local symmetries”, “levels of scale”, “alternating repetition”, “echoes”, and so on. Alexander’s earlier work produced a remarkably fruitful series of spinoffs, including the design patterns movement in software, wiki and Wikipedia, Agile, Scrum and other innovations (Cunningham and Mehaffy, 2014). It seems very likely that his more recent work also offers fruitful avenues for further development.

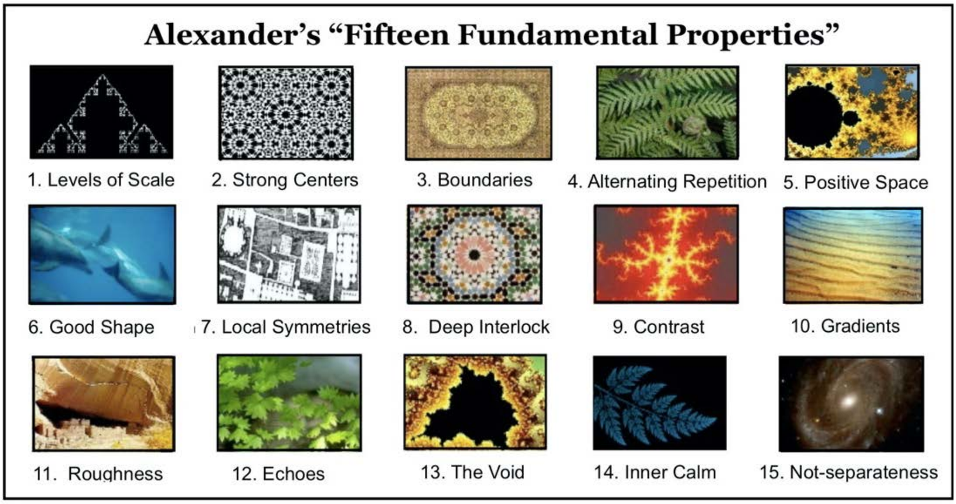


Figure 9: Alexander's "Fifteen Fundamental Properties" as described in his treatise, *The Nature of Order* (2003). The properties of symmetry are evident in all of the properties, to varying degrees.

All of these insights point to tantalizing opportunities (and needs) for further research into the role of symmetry in human experience, learning, culture, health and well-being. They point specifically to the need for greater understanding of the role of symmetry in the human environment, its impacts, and its capacity to improve critical aspects of human experience, resilience and sustainability.

11 CONCLUSION

It is clear that the topic of symmetry continues to be a dynamic and fruitful one in many fields—but architecture is not one of them. Indeed, as we have seen, architecture as a profession continues to be dominated by a school of thought that is charitably described as an obsolete relic of an earlier and more elementary industrial age. A more critical view is that architectural leadership has become wilfully illiterate in the fields whose findings might complicate their comfortable sinecures as art supplies to an unsustainable technocracy.

Were this situation to change—and there is mounting evidence that it must, not only for the quality of the human environment, but for the well-being and even the sustainability of human culture—a great many sacred cows would have to be slaughtered: (i) the idea that novelty and disruption are paramount; (ii) the idea that the products of machinery, any machinery, will make perfectly good human environments; (iii) the idea that only artistic Philistines will challenge that idea; and (iv) perhaps most important, the idea that the art of building lies not in illuminating and enriching the forms and patterns of life and nature, but in supplanting them with artistic abstractions. As the urbanist Jane Jacobs famously observed, this confusion of the roles of art and life is bad for life—and likely bad for art too (Jacobs, 1961, p. 373).

There is indeed a growing body of research into this unacceptable situation, too, and the cognitive biases and delusions that fuel it. They include research on the great divergence between what architects see and value, versus what almost all non-architects do (Gifford, Hine, Muller-Clemm and Shaw, 2002) and the "architectural myopia" that seems to account for it (Salingaros and Mehaffy, 2011); the "geometrical fundamentalism" that encourages architects to see and value more stripped-down, "pure" forms (Mehaffy and Salingaros, 2001); and the "construal level theory" that explains how psychological distance can result in the imposition of "construals" by professionals, in place of the actual factors that might satisfy their clients' and the public's needs (Trope and Liberman, 2011).

This problem is hardly a parochial concern of architects. At this moment in history, the world is urbanizing at an unprecedented rate, with profound impacts upon resources, human health and well-being, and the sustainability of civilization itself (UN-Habitat, 2013). Architects, as putative leaders of the built environment professions, will play a central role in this drama. Yet the leadership is currently promulgating a wave of building types that can only be described as perverse. Notable in this category is the all-glass skyscraper, currently a fetish of the construction industry, but (in addition to their resource gluttony) they prevent any complex symmetry, even if the architect wanted to design such. This is now a universal building typology, and universally popular with global finance, which through its radically fungible nature discourages the incorporation of anything other than the most banal and simplistic mirror symmetry. Here is a trillion-dollar global activity that only further erodes the type of symmetry that is (as the research herein describes) good for our mental health. It also likely plays a key role in the care and sustenance we give our buildings over time, in contrast to an unsustainable world of disordered, throwaway buildings.

Architects, therefore, need to provide ethical leadership as well as other forms of leadership. In that quest, the first outstanding task is to establish that the profession does indeed need realignment and reform, and moreover, needs greater literacy in the fields that point to its deficiencies. From there, a research agenda might be established that integrates recent knowledge emerging from the other disciplines with actual architectural practice. In particular, science has already developed a fruitful union between artificial intelligence and medicine, and this framework can easily transfer to creating complex symmetry for healing environments.

Such a broad agenda should include the investigation of new ideas for generating form and pattern, and for assuring that their properties are rich and evocative, and not merely banal reflections of other “accepted” works. (For an agenda of symmetry-based research certainly does not imply the mere imposition of pale simulacra.) At the same time, these new forms must address real human need and perception, and not the mere prerogatives of the form-giving artist, as too much of design does now. Such an agenda will seek to understand how particular patterns and relationships are recurrent, and should be so because those provide optimal human environments, but also how new patterns, new forms, and new complexities might be incorporated in pursuing health and well-being.

Finally, such an agenda might dare to learn from the ten thousand years of architectural evolution, with its many still-valuable treasures, no less than from the four billion years of natural evolution, and its vast richness of pattern, complexity, order, and—yes—symmetry.

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