**ROBUST ACTUATOR FORCE ANALYSIS OF A HEAVY-DUTY MANIPULATOR USING GMM/GMR**

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Abstract

Uncertainty is inevitable in modelling of physical world, including sensor noise, control errors and inaccurate models of the environment. In robotic grasping force analysis (GFA), the uncertainties mainly come from the location of contact points and the coefficient of friction. Previous researches of uncertainty analysis are mainly focusing on of robotic grasp synthesis and planning, while little work has been done from the aspect of grasping force. In this paper, a point contact model with uncertainty is proposed to investigate the effect of uncertainty in GFA, which was solved using Monte Carlo simulation. Moreover, to predict the minimal actuator force in an industrial heavy-duty manipulator, a Gaussian mixture model (GMM) is proposed to describe the relationship between the object configuration and the minimal actuator force.

Key Words

Grasping force optimization, uncertainty, Gaussian mixture model, Gaussian mixture regression

1. Introduction

A variety of uncertainties exist due to the incomplete information when modelling the physical world. As for robotic grasping and manipulation, the uncertainty mainly comes from the discrepancy between the real physical world and the simplified theoretic model: the motion accuracy, the sensor noise and resolution, the mechanical structure deformation, and all of the relevant parameters, such as the coefficient of friction, which is not easy to obtain a precise value. To cope with uncertainties, the robotic system can either employs an efficient control method using sensory feedback [1], [2], which can response quickly to uncertainty, or a robust model, including compliant mechanical structure and robust control method that can passively adapt to unexpected perturbation from the environment [3], [4].

In robotic grasping and manipulation, one of the fundamental problems is to compute the contact (grasping) forces between the grasped or manipulated object and the fingers of the robotic hand [5]. In order to accomplish some manipulation tasks, the right grasping forces should be exerted exactly on the object to move the object to desired position. However, due the multi-contacts between objects and hands, the grasping force analysis (GFA) problem is usually indeterminate

1. Usually, by choosing an objective function according to the manipulation task, the GFA can be transformed into a constrained optimization, which can find the optimal2 grasping forces among all the possible solutions. This problem has been solved as a linear programming problem by linearizing the friction constraints [6] and a conic optimization problem by formulating the friction constraints as a second-order cone [23]. More recently, it is observed that the friction cone constraint can be expressed as a semi-positive matrix and then the grasping optimization problem can be solved efficiently using gradient flow on a special Riemannian manifold [7]. Based on this significant observation, the problem is further formulated using linear matrix inequality (LMI), which can be solved easily by barrier method [8]. However, one of the main assumptions of these researches is that the contact location and the coefficient of friction should be precisely known. While this assumption is valid for most of the precise grasps where the fingertips only have a small area to contact with object, in some industrial applications, as the heavy-duty manipulator in this paper, this assumption is no longer valid due to the large contact areas between gripper and object. Also, even for the dexterous robotic hand, the precise contact point is difficult to obtain because of the deformation of the object or soft fingertips.

The uncertainty in robotic grasping has been considered in some research areas, such as the effect of un-
certainty on the searching of force-closure grasps [9], the contact representation [10]. To the best of our knowledge, few works have been done in the area of GFA when taking the uncertainty of contact location and coefficient of friction into account. However, as shown in this paper, these two kinds of uncertainties have a significant influence on the final optimal grasping forces. In this paper, to model the contact location uncertainty, an uncertain point contact model (UPCM) has been proposed. Based on this model, the grasping force optimization problem can be solved using Monte Carlo simulation [11]. Moreover, after obtaining a large number of optimal grasping forces for different object configurations, the relationship between the optimal grasping forces and the object configurations can be learnt by Gaussian mixture model (GMM) [12], which can be further used to predict the optimal grasping forces for new object configuration without solving the constrained optimization problem. Though GMM has been used in robot learning before, the applications are mainly focusing on robot motion generation [13].

The contribution of this paper lies in two aspects. First, a UPCM is proposed to deal with the uncertainty of contact location in robotic grasping, which can lead to more realistic optimal grasping forces. Second, a GMM is learnt from simulation results to predict the relationship between grasping forces and object configurations. The rest of this paper is organized as follows: the GFA problem and UPCM are presented in next section. In Section 3, the grasping force optimization problem is solved using Monte Carlo simulation. A GMM is learnt in Section 4 to predict the optimal grasping forces and the object configurations of contact location in robotic grasping, which can lead to more realistic optimal grasping forces. Second, a GMM is learnt by Gaussian mixture model (GMM) [12], which can be further used to predict the relationship between grasping forces and object configurations. The rest of this paper is organized as follows: the GFA problem and UPCM are presented in next section. In Section 3, the grasping force optimization problem is solved using Monte Carlo simulation. A GMM is learnt in Section 4 to predict the optimal actuator force of a heavy-duty manipulator in forging industry, followed by a conclusion in Section 5.

2. Uncertainty in GFA

In this section, the grasping force optimization problem will be formulated first with focus on the constraints on the grasping forces. Then, considering the uncertainty of the contact location, a UPCM is proposed to describe this uncertainty.

2.1 Grasping Force Analysis

Considering a multifingered robotic hand grasp an object with \( N \) contact points, the contact models of which are point contact with friction\(^3\) (PCwF) [14]. In this object-hand grasping system, there are several constraints due to the object force equilibrium, the finger joint equilibrium and the friction at each contact points. In the following section, these different constraints will be introduced.

First, from the object force equilibrium of all the external forces exerted on the object we have:

\[
Gt = -w_o \tag{1}
\]

The grasp map can be expressed as:

\[
G = [G_1, \ldots, G_N] \in \mathbb{R}^{6 \times 3N}
\]

and \( G_i \in \mathbb{R}^{6 \times 3}, i = 1, \ldots, N \) is the grasp mapping at contact point \( i \), which can map the contact forces expressed in the local contact framework to the object framework [15]. \( w_o \in \mathbb{R}^6 \) is the external force (including gravity of the object) exerting on the object and represented in the object framework. \( t = [t_{11}, t_{12}, \ldots, t_{1n}, t_{i1}, t_{i2}, \ldots, t_{Nn}, t_{N1}, t_{N2}]^T \in \mathbb{R}^{3N} \) stands for the contact forces at all the contact points and \( t_{1n}, t_{i1}, t_{i2} \), are the normal contact force, the two tangential contact forces at contact point \( i \) respectively, which are represented in the local contact framework.

Next from the force equilibrium of all the finger joints, we have:

\[
\tau = J^T t \tag{2}
\]

\( J \) is the Jacobian matrix that mapping the contact forces to joint torques \( \tau \in \mathbb{R}^M \) [16].

The friction cone constraint can be represented as:

\[
u_it_in = \sqrt{t_{ii1}^2 + t_{ii2}^2} \tag{3}
\]

\( u_i \) is the static coefficient of friction at the \( i \)th contact point. This constraint can be further formulated as:

\[
f_{ci} = \left\{ t_i \in \mathbb{R}^{3 \times 1} | g_i(t) = u_it_in - \sqrt{t_{ii1}^2 + t_{ii2}^2} \geq 0 \right\} \tag{4}
\]

Using the Cartesian product of \( f_{ci} \), we have the friction constraints for all the contact points as follows:

\[
FC = f_{c1} \times \cdots \times f_{cn} = \{(t_1, \ldots, t_n) | t_i \in f_{ci}, \ i = 1, \ldots, N \} \tag{5}
\]

The GFA problem is indeterminate as there are many solutions satisfying the constraints (1)–(5). As discussed earlier, by choosing a proper objective function \( F_{task}(t) \) according to the task [16], [17], then the GFA problem can be described as a constrained optimization as follows:

\[
\begin{align*}
\min & : & F_{task}(t) \\
\text{s.t.} & : & Gt = -w_o \\
& : & \tau = J^T t \\
& : & t \in FC
\end{align*}
\]

2.2 Uncertain Point Contact Model

It is easy to find that the location of contact points, orientation of contact frameworks (defined mainly by the contact normal direction) and the coefficient of friction are related to the GFA problem, but their impact on the optimal grasping forces is still not clear now. What will happen if these parameters are not precisely known in a real application? Here we try to reveal this impact and in this paper we will focus on the uncertainty of contact point location and the coefficient of friction between the object and the robotic fingers, as in the rest of this section, we will introduce the UPCM.

In the research of robotic grasping, the contact types are usually labelled as: frictionless point contact (FPC), PCwF, soft finger contact (SFC) [14]. In fact, all of these different contact models are based on a fundamental principle in contact mechanics with some reasonable
Figure 1. The equivalent contact force and moment applying on a contact area.

assumptions [18]: the equivalent contact force transmitted from one contact area to another through a point of contact can be resolved into a normal force, a tangential force and the moment, Fig. 1.

In a point contact model with friction, the spin moment $M_z$, arising from tangential friction forces within the contact area, is usually ignored. Also, within the contact area, there always exists a point that:

\[ M_x = 0 \quad M_y = 0 \quad (7) \]

This point is called zero moment point (ZMP) [19]. In fact, the ZMP is not always located on the centre of the contact area, which is determined by the actual distribution of contact forces. That is to say, the real equivalent contact point cannot be precisely known. For a dexterous grasp, the contact area is so small that we can assume the contact point is just on the fingertip. However, for some cases, such as the heavy-duty manipulator, the contact area is relatively very large that makes the assumption no longer valid.

Here, an UPCM was proposed to describe the uncertain characteristic of the equivalent contact point, i.e., ZMP. In this model, the contact area between the fingertip and the grasped object is approximately represented as:

\[ a_i \in \varepsilon_i = \{ \overline{a_i} + P_i u | ||u||_2 \leq 1 \} \quad (8) \]

where $P_i \in \mathbb{R}^{3 \times 3}$ is called the uncertain matrix and $\overline{a_i}$ denotes the theoretic location of equivalent contact point.

(1) When $\text{rank}(P_i) = 3$, the ZMP locates in an ellipsoid;

(2) When $\text{rank}(P_i) = 2$, the ZMP locates in an ellipse;

(3) When $\text{rank}(P_i) = 1$, the ZMP locates on a line;

(4) When $P_i = 0$, the ZMP is determinate and locates on the theoretic position.

When considering the uncertainty of contact point location, the grasping force optimization (6) can be formulated as following.

\[
\begin{align*}
\text{min} & : & F_{\text{task}}(t) \\
\text{s.t.} & : & G(a)t = -w_o \\
& & \tau = J(a)^T t \\
& & t \in FC \\
& & a = \{(a_1, \ldots, a_N)|a_i \in \varepsilon_i, \quad i = 1, \ldots, N}\end{align*}
\]

Here, $a = \{(a_1, \ldots, a_N)|a_i \in \varepsilon_i, \quad i = 1, \ldots, N\}$ represents the actual location of the contact points. When the actual location of the contact points $a$ is given, the optimization (9) can be transformed equally to the primal problem (6). When $a$ is not determined, that is, at least one of the uncertain matrix satisfies: $P_i \neq 0$.

The problem (9) turns out to be a robust optimization problem that requires the constraints be satisfied for all of the possible values of the uncertain parameters $a = \{(a_1, \ldots, a_N)|a_i \in \varepsilon_i, \quad i = 1, \ldots, N\}$. Though we do not know the actual precise location of contact points, we know the approximate contact area where the contact point must locate in. The robust optimization requires that wherever the real locations of equivalent contact points in the contact area, the constraints must be satisfied.

3. Robust Grasping Force Optimization

When the problem data is given, i.e., given the value of $a$, the problem (9) can be transformed into an unconstrained nonlinear programming problem (10), where $r$ stands for a positive penalty parameter and $h_i(t), g_j(t)$ describes all of the equality and inequality constraints in (9), respectively. When the parameter $r$ is large enough, the solution of optimization (10) is equal to that of (9) [20].

\[
\begin{align*}
\text{min} : & \quad F_{\text{task}}^T F_{\text{task}} + r \left[ \sum_i (h_i(t))^2 + \sum_j (g_j(t))^2 \right] \\
\end{align*}
\]

Furthermore, the optimization (10) can be further transformed into a least-square optimization problem as follows:

\[
\begin{align*}
\text{min} : & \quad \phi = \psi(t)^T \psi(t) \\
\psi(x) = & \begin{bmatrix} F_{\text{task}} \\ \sqrt{r} h(t) \end{bmatrix} \\
\end{align*}
\]

Then, the optimal solution of (11) can be easily obtained using unconstrained optimization algorithms like Newton method or L-M method.

We should note that this simplification is based on the given value of $a$, whose real value is hard to obtain. In the rest of this section, a Monte Carlo method will be used to simulate the actual location of contact points in UPCM.

Here, we assume that the contact area is an ellipse, i.e., $\text{rank}(P_i) = 2$, which can be expressed as:

\[
\varepsilon_i = \left\{ \overline{a_i} + \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} r \cos \theta \\ r \sin \theta \end{bmatrix} | \theta \in [0, 2\pi], |r| \leq 1 \right\} \quad (12)
\]

$m$ and $n$ are the two axes of the ellipse, which can be estimated according to size of fingertips. We use the Monte Carlo method [11] to represent the uncertainty of the contact location. The parameters $r, \theta$ in (12) are the
random variables with given Gaussian distribution, which stand for the distance and direction from the theoretic contact location to the actual contact location, respectively, Fig. 2. The optimization process is given in Fig. 3. First, we randomly initialize the values of \( r, \theta \), then we can run the optimization algorithm (11) and obtain an optimal value of \( F_{\text{task}} \). Repeat this process for \( Nm \) times and we obtain \( Nm \) optimal solutions \( F_{\text{task}} \), we choose the biggest optimal value of \( F_{\text{task}} \) as the final result, i.e., the worst-case as the final result.

4. Minimal Actuator Force Prediction via GMM/GMR

In last section, the GFA problem with uncertain contact locations can be solved by combining optimization and Monte Carlo simulation, Fig. 3. In this section, we will apply this framework to compute the minimal actuator force of a heavy-duty manipulator in forging industry [21]. Also, in order to predict the minimal actuator force for different weights and configurations of the workpieces, a GMM is learnt using the data from the simulation.

4.1 Minimal Actuator Force

The manipulator is used for forging large shafts that usually are tens of tons, Fig. 4. The workpiece can be rotated by the manipulator along the long axis. It is very crucial to know the minimal actuator force for a given workpiece as we need to know how large the force should be applied by the actuator in order to manipulate the workpiece, which is usually estimated by the experience. When given the weight and the rotation angle of the workpiece, a robust minimal actuator force can be obtained using the robust optimization process as in Fig. 3. The details of the structure of manipulator and the computation process are given in [21].

When \( Nm = 10 \), the minimal actuator force changes with the number of simulations, as shown in Fig. 5. For the same 20T (ton) workpiece, the value of the minimal actuator force can have a fluctuation up to 25\% [4]. As the coefficient of friction can not be exactly determined, we change the value of it from 0.3 to 0.6. Here, we assume all the coefficients at all the contact points are the same, which is often the case in our heavy-duty manipulator. For each value of the coefficient of friction, the final minimal actuator force is obtained by 5 simulations, i.e., \( Nm = 5 \). It is easy to note when the coefficient of friction increases, the minimal actuator force will decrease Fig. 6. This observation also coincides with our intuition that the rougher an object is, the easier we can grasp it. The change of the minimal actuator force with the rotation angle of workpiece is shown in Fig. 7, which indicates that grasp configuration also has a significant influence on the minimal actuator force. For each value of the rotation angle, the final optimal actuator force is also obtained by setting \( Nm = 5 \).

4.2 Gaussian Mixture Model

Until now, the minimal actuator force can be obtained when the weight and configuration of the workpiece is given. However, during the manipulation of the workpiece, the configurations may change from time to time and also there are many different types of workpieces, from 20T to 80T. To predict the minimal actuator force when the weights and configurations are changing, we use the GMM [12] to build the joint distribution, i.e., \( P(F, \theta, W) \) of the weight \( W \), the rotation angle \( \theta \) and the minimal actuator force \( F \). The choice of using GMM has the advantages of

\[ \text{As shown in Fig. 5, the maximal value is 165T and the minimal value is 132T.} \]
For a GMM, the joint probability distribution of all variables is represented as a sum of $K$ Gaussian distributions,

$$P(F, \theta, W) = \sum_{k=1}^{K} \pi_k N(F, \theta, W | \mu_k, \Sigma_k)$$  \hspace{1cm} (13)$$

where $\pi_k$ is the prior of the $k$th multidimensional Gaussian component and $\mu_k, \Sigma_k$ are respectively its mean and covariance, such that:

$$\mu_k = \begin{bmatrix} \mu_{F,k} \\ \mu_{\theta,k} \\ \mu_{W,k} \end{bmatrix}, \quad \Sigma_k = \begin{bmatrix} \Sigma_{FF,k} & \Sigma_{F\theta,k} & \Sigma_{FW,k} \\ \Sigma_{F\theta,k} & \Sigma_{\theta\theta,k} & \Sigma_{\theta W,k} \\ \Sigma_{FW,k} & \Sigma_{W\theta,k} & \Sigma_{WW,k} \end{bmatrix}$$  \hspace{1cm} (14)$$

An expectation-maximization algorithm is used [12] to learn the parameters in the GMM model, including the prior $\pi_k$, mean $\mu_k$ and covariance $\Sigma_k$.

In this paper, we randomly choose 300 $\theta \in [0, 180]$ and $W \in [20, 80] \times 10^4 N$, and from the optimization process, we can obtain 300 minimal actuator forces $F$. That is to say, we have 300 training data points $(F, \theta, W)$. The number of the Gaussian distribution set using Bayesian information criterion (BIC) [13], here, $K = 4$.

### 4.3 Gaussian Mixture Regression

GMM can describe the joint distribution of all the relevant variables; however, more often, we want to predict some output variables given the input variables. Here, we want to know the minimal actuator force when the weight and configuration of the workpiece are given. To this end, we can compute the conditional probability $P(F|\theta, W)$ by means of Gaussian mixture regression (GMR) [22]:

$$P(F|\theta, W) \sim N(\hat{F}, \hat{\Sigma}_{FF})$$  \hspace{1cm} (15)$$

The expectation is:

$$\hat{F} = \sum_{k=1}^{K} h_k \left[ \mu_{F,k} + \left( \sum_{F\theta,k} \sum_{FW,k} \right) \left( \sum_{\theta\theta,k} \sum_{W\theta,k} \right)^{-1} \left( \begin{bmatrix} \theta \\ W \end{bmatrix} - \begin{bmatrix} \mu_{\theta,k} \\ \mu_{W,k} \end{bmatrix} \right) \right]$$  \hspace{1cm} (16)$$

The covariance is:

$$\hat{\Sigma}_{FF} = \sum_{k=1}^{K} h_k^2 \left[ \sum_{F\theta,k} \sum_{FW,k} - \left( \sum_{F\theta,k} \sum_{FW,k} \right) \left( \sum_{\theta\theta,k} \sum_{W\theta,k} \right)^{-1} \left( \sum_{\theta F,k} \sum_{W F,k} \right) \right]$$  \hspace{1cm} (17)$$

$$h_k = \frac{\pi_k N(\begin{bmatrix} \theta \\ W \end{bmatrix}; \begin{bmatrix} \mu_{\theta,k} \\ \mu_{W,k} \end{bmatrix}, \begin{bmatrix} \sum_{\theta\theta,k} \sum_{W\theta,k} \sum_{\theta W,k} \sum_{WW,k} \end{bmatrix})}{\sum_{i=1}^{K} \pi_i N(\begin{bmatrix} \theta \\ W \end{bmatrix}; \begin{bmatrix} \mu_{\theta,i} \\ \mu_{W,i} \end{bmatrix}, \begin{bmatrix} \sum_{\theta\theta,i} \sum_{W\theta,i} \sum_{\theta W,i} \sum_{WW,i} \end{bmatrix})}$$
Using the 300 data points, first we obtain the relationship between the weight of the workpiece and the minimal actuator force of the manipulator, Fig. 8. We find that the minimal actuator force increases linearly with the weight of the object. Denote $r = \frac{F}{W}$, the problem becomes computing the conditional probability $P(r|\theta)$, which can be easily solved as (15). The result is plotted in Fig. 9. The top figure in Fig. 9 is the original training data points and there is a strong coupling between the ratio $r$ and the workpiece configuration $\theta$. In the middle figure, a GMM with four Gaussian distributions is learnt to model the joint distribution of the training data points. The lowest figure is the prediction given by GMR and the solid line is the expectation of the minimal actuator force for different object configurations. In addition, the band above and below the solid line shows the 95% confidence interval of the prediction. The smaller the band is, the more confident the prediction is. In our application, due to the safety consideration, we usually choose the biggest value of the predictions which are shown as black points in the lowest figure. All these predictions for different types of workpieces are examined in the factory, which conform well to the empirical results.

5. Conclusion

In this paper, in order to predict the minimal actuator force of a heavy-duty manipulator in forging industry, we divide this problem into two steps. First, when the weight and configuration of the workpiece is given, the minimal actuator force is formulated as a GFA problem. To model the contact location uncertainty in this problem, an UPCM is proposed. This GFA problem with UPCM is solved using Monte Carlo simulation. Second, when the weight and configuration of the workpiece are changing, a GMM is learned using the simulation results from the first step, which can describe the nonlinear relationship among the object weight, object configuration and the minimal actuator force.

The framework of learning from the optimization results has several advantages. First, we do not need to obtain the data from experiments, which may be expensive
or impossible. Instead, we use the optimization method to formulate the problem and obtain the training data. As for the heavy-duty manipulator, it is impossible to obtain a large number of data of the minimal actuator force. Second, the uncertainty can be considered in the optimization process and further encapsulated in the GMM. Third, the GMR provides a close form to compute any conditional probability of the GMM, which allows us to predict the actuator force given the current rotation angle and weight of workpiece in a real time.

We are now planning to extend this work to more general manipulation system. In this paper, there is only one variable of the object configuration, i.e., the rotation angle. Usually, there are six variables to describe the configuration of an object in three-dimensional space. Also, there is only one variable of the actuator force, for more dexterous manipulation system, how to predict the joint torques with respect to the object configuration will be much more difficult\(^5\), which is one of the promising directions for our future work.

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**References**


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\(^5\) The number of train data will increase exponentially with the number of the variables, which is called “curse of dimensionality” [22].

**Biographies**

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