Managing Capacity Through Reward Programs

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Rewarding customers with own products or services has become an increasingly popular practice across a spectrum of industries such as airlines, hotels, and telecommunication. In these service industries, firms face demand uncertainty and strict short-term capacity constraint. When the market demand is low, firms hold excess capacities that would lead to intense price competition. In this paper we study the adoption and design of reward programs in the context of capacity management. We demonstrate that it is optimal for firms to offer capacity rewards when the market demand varies from one period to the other. By offering the reward programs, firms can effectively reduce available capacities when the market demand is low, and hence credibly show their unwillingness to undersell. Such a commitment can encourage their competitors to set their prices high. When firms provide reward programs, if a firm sets a higher price than the other and sells less today, in the future the firm can benefit from the other firm’s larger reduction in available capacity through rewards. Thus, reward programs also provide additional incentives for firms to set higher current prices. Finally, since reward programs can add flexibility in adjusting the available capacities to the market demand, firms increase the size of regular capacities with reward programs.

Key words: capacity management; competition; pricing under capacity constraints; reward programs

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1. Introduction
Promotional programs that reward loyal customers with the firms’ own products or services have become widespread in many markets, particularly in the service industries. Airlines, hotels, car rental, cruise lines, and telecommunication industries offer good examples where such reward programs have been adopted by virtually all the major competitors. The best-known example of loyalty programs is perhaps AAdvantage, the frequent-flier program offered by American Airlines since 1981. Over the years, most airlines have adopted variants of American Airlines’ frequent-flier program. Currently, more than 38 million members are, or have been, enrolled in these programs and billions of free miles have been awarded. According to the airlines, frequent-flier programs now have become the most important marketing tool for the industry (McCartney 1996). A similar pattern has also been observed in the hotel industry. Nearly all the major hotel chains have adopted frequent-guest programs where hotels reward their frequent guests with free room stays. Examples include Hyatt’s Gold Passport, Marriott’s Honored Guest, Sheraton’s Club International, Holiday Inn’s Priority Club, and Hilton’s HHonors.

In this paper, we study the adoption and optimal design of reward programs in the context of capacity management. Our model takes into consideration the following two important characteristics shared by such service industries as airlines, hotels, and telecommunications. First, these industries have strict capacity constraints. Once the capacities are established, the cost of making any adjustment—renting new gates and airplanes, building new hotels, or setting up additional communication networks—are quite high. Moreover, the unit of demand change (e.g., number of seats for airlines and number of rooms for hotels) is far smaller than the unit for capacity adjustment (e.g., number of airplanes for airlines and number of buildings for hotels). In other words, firms can add capacity only in significant lumps. Second, airlines and hotels face considerable demand uncertainty and seasonal variations. For instance, market demands typically are much higher during summer holidays and Christmas than during the rest of year. The high fixed cost and additional regulations, like
gates availability, prevent firms from fully tailoring capacities to demand variations. Firms are forced to develop capacities consistent with the peak demand, resulting in substantial excess capacity during periods of low demand. For example, during the first part of the nineties, the average occupancy rate was only about 63%–70% for hotels, and the average load factor rate was only about 60%–67% for airlines, including the capacity taken by redemption of reward offers (Proctor 1996). When a firm has excess capacity and low marginal cost, that firm can benefit from selling more to fill its capacity by undercutting the competitor’s price. As a result, the existence of excess capacities always leads to intense price competition.

We develop a game-theoretical model to analyze how reward programs may help alleviate this problem of excess capacities. Our analysis shows that reward programs provide firms with flexibility in capacity management and enable them to better adjust their available capacities in response to the actual level of market demand. When market demand is low, by handing out a part of their capacities as rewards, firms are able to credibly commit to smaller available capacities in the future, and thus reduce price competition. Since any excess capacity owned by a firm generates a threat of price cutting to the other firm, both firms prefer to adopt reward programs to reduce their excess capacities rather than free-riding on their competitors’ reward programs. We also find that reward programs may increase market prices by compensating a firm’s loss in current sales with a gain in future sales. To see the intuition behind this result, imagine that a firm charges a higher price and sells less than its competitor. In the future, the firm’s competitor will have to hand out more capacity in the form of rewards and retain a smaller capacity. The firm’s future profit increases due to the competitor’s larger capacity reduction through rewards. Such an incentive to set higher prices grows even stronger when the size of the reward amount increases. Finally, because firms can use the reward programs to effectively reduce excess capacities during the periods with low demand, they may set up larger initial capacities. In a model of demand seasonality, we find that firms set the initial capacities according to the state of peak demand and use the reward programs to eliminate the excess capacities during the low-demand period.

Our paper is related to three streams of research. First, we extend the literature on the relationship between capacities and price competition (e.g., Edgeworth 1925, Levitan and Shubik 1972, Kreps and Scheinkman 1983, and Davidson and Deneckere 1986.) Edgeworth (1925) shows that there may not exist any pure-strategy equilibrium in an industry with excess capacity. Levitan and Shubik (1972) characterize a mixed-strategy equilibrium for these types of games of price competition. More recently, Kreps and Scheinkman (1983) use a two-stage game, a stage of capacity decisions followed by a stage of capacity-constrained price competition, to show that firms do indeed commit to Cournot capacity when they have rational expectations of subsequent price competition. To examine the role of reward programs in capacity management, we extend the capacity-price framework already developed in the literature by adding the offering of reward programs. Moreover, as reward programs connect the current demand with future capacity, we develop a dynamic game with multiple periods of price competition. Our paper also examines the impact of reward programs on firms’ initial capacity decisions.

Second, our paper is related to studies on yield management, a method that has widespread acceptance in the airline and hotel industries (Kimes 1989, Weatherford and Bodily 1992). The goal of yield management is to maximize revenue yield per capacity unit (e.g., per seat for airlines) by employing the price-discrimination method. For example, airlines decide how many discount fares to sell while ensuring enough seats for late arrivals who are willing to pay full price. While research in yield management often studies complex pricing schemes but neglects competitive responses, our paper adopts simple pricing schemes and focuses on the strategic effect of excess capacity on price competition. Moreover, we introduce reward programs as a new element of capacity management in a competitive environment.

Finally, our paper contributes to the research on sales promotion, and reward programs in particular. Existing research on reward programs (e.g., Calkin 1990 and Kim et al. 2001) emphasizes the role of reward programs in creating consumers’ switching costs and therefore enhancing customer loyalty. According to Cairns and Galbraith (1990), firms may also use reward programs as an implicit device to encourage their employees to take business trips because these rewards are tax free. Unlike the existing research, we focus on reward programs that provide free capacity as incentive and, more importantly, examine the reward programs in the context of capacity management from a fresh perspective.

We organize the rest of this paper as follows. We propose an analytical model of reward programs and capacity-constrained price competition in §2. We analyze the model in §3 to demonstrate the strategic use of reward programs in capacity management. We consider several directions in which to extend our model and examine the implications to our key results in §4, and we conclude in §5 with a summary of main results.
2. A Model

Consider two identical firms (denoted as \( i = a, b \)) producing perfectly substitutable products or services in an industry with demand seasonality and strict capacity constraints. Such industry characteristics are common among airlines and hotels. To model the capacity constraint, we assume that each firm holds a capacity of \( k \) units. The marginal costs for both firms are constant up to their capacities, but become infinite for the production over \( k \) units. Essentially, we consider a market with strict capacity constraint studied by Edgeworth (1925). Without loss of generality, we further assume the marginal cost up to capacity to be zero. We assume that firms compete in a market of size \( \alpha \), where consumers’ reservation prices are uniformly distributed in a closed interval \([0, \alpha]\).

The firms compete in a three-period game. (We show the sequence of moves in Table 1.) In Period 0, the firms simultaneously announce their reward amount \( R_i \) (\( i = a, b \)). After the firms announce their reward decisions, they enter two periods of price competition. Our model setting that firms’ reward decisions are followed by repeated periods of price competition reflects the nature that reward programs are long-term decisions, while prices are short-term decisions. To model demand seasonality, we assume that the levels of demand are different between Period 1 and Period 2. More specifically, we let the demand be high in the first period (\( \alpha_1 = \alpha_{H} \)) and low in the second period (\( \alpha_2 = \alpha_{L} \)). When the firms make reward decisions, they know the demand states for both periods.

During each of the two periods, firms set their prices and the consumers make their purchase decisions. We let \( p_i \) and \( d_i \) denote the price and demand, respectively, for firm \( i \) (\( i = a, b \)) at period \( t \) (\( t = 1, 2 \)). Firms make their decisions independently and simultaneously. Consumers accumulate their reward points through purchases in the first period. In the second period, firms allocate a certain percentage of capacity to be distributed as rewards, and consumers redeem rewards before the firms decide the second-period prices.

Next we explain the reward programs and the market demand in more detail.

**Rewards and Redemption.** Given our interest in capacity management, we assume that firms offer capacity rewards, e.g., free round-trip tickets for airlines or free night stays for hotels. Firms face two reward decisions in our model: the amount of rewards offered to customers and the size of capacities allocated for reward redemption. First, at the beginning of the game, the firms decide the amount of points rewarded to their first-period customers. We let \( R_i \) denote the reward amount that firm \( i \) offers to each customer. In practice, the amount of rewards provided and actually redeemed depends on several decisions related to the design of programs, e.g., number of mileage or points rewarded for each unit of purchase, the minimum amount of mileage or points required to redeem a reward, and the expiration date. We interpret the reward variable \( R_i \) as a composite measure of these design factors. Since the firms make their reward decisions at the beginning of the game, they set their rewards to maximize the expected profits from the two subsequent periods. We assume that firms can correctly anticipate the impact of rewards on subsequent periods of price competition. We also assume that firms discount their future profits by a discount factor \( \delta \). Consumers accumulate their reward points through purchases in the first period. At the end of the first period, each firm knows the units sold by both firms, and hence the total accumulated reward points, denoted by \( \psi_i = R_i d_i \) for firm \( i \).

At the beginning of the second period, the firms decide the amount of capacity allocated for consumers to redeem their rewards. We define the size of capacities that firm \( i \) allocates for rewards by \( \phi_i = r_i d_i \), which is equal to \( r_i \) percentage of firm \( i \)’s first-period demand. Naturally, \( r_i \leq R_i \) and \( \phi_i \leq \psi_i \), \( i = a \) or \( b \). Firms can then calculate their capacities available in the second period, denoted by \( k_i = k - \phi_i \) (\( i = a, b \)). When a firm allocates an insufficient amount of capacity for redemption, i.e., \( r_i < R_i \), the firm faces an inventory of reward points (\( \psi_i - \phi_i \) for firm \( i \)). We recognize that offering an insufficient amount of capacity for rewards may result in dissatisfaction from those customers who are not able to redeem their rewards. Such customer dissatisfaction may lead to a firm’s future profit loss. To model such profit impact of the inventory of unredeemed reward points, we consider the following loss function to firm \( i \)’s profit, which is evaluated at Period 2.

\[
L(\psi_i - \phi_i) = \eta(\psi_i - \phi_i)^2, \quad (\eta > 0, \ i = a \ or \ b).
\]  

The parameter \( \eta \) of the above equation measures the extent of negative impact of such customer dissatisfactions on a firm’s future profit. Finally, when deciding the allocation of capacity for rewards, firms can

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Sequence of Moves</th>
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<tbody>
<tr>
<td>Period 0</td>
<td>Firms set the reward amount: ( R_a, R_b ).</td>
</tr>
<tr>
<td>Period 1</td>
<td>( (\alpha_1 = \alpha_{H}) ) Firms set the first-period prices: ( p_i^1, p_i^2 ).</td>
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<tr>
<td></td>
<td>(1.2) Consumers make the purchases, firms realize first-period demand: ( d_i^1, d_i^2 ).</td>
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<tr>
<td></td>
<td>Firms’ total accumulated reward points: ( \psi_i = R_i d_i ).</td>
</tr>
<tr>
<td>Period 2</td>
<td>( (\alpha_2 = \alpha_{L}) ) Firms allocate capacities to be handed out as rewards: ( \phi_i = \gamma d_i^1, \phi_b = \gamma b d_i^2 ).</td>
</tr>
<tr>
<td></td>
<td>(2.2) Firms set the second-period prices: ( p_i^2, p_i^2 ).</td>
</tr>
<tr>
<td></td>
<td>(2.3) Consumers make the purchases, firms realize second-period demand: ( d_i^1, d_i^2 ).</td>
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correctly anticipate the effect of reduced capacities on the subsequent price competition.

**Demand Functions.** Because competing firms offer identical products/services, demand for firm’s product/service at period $t$ only depends on the competing firms’ prices and capacities. More specifically, we formulate firm’s demand function at period $t$ as follows:

$$d_i^t(p_i^t, p_j^t | \alpha_i, k_i, k_j) = \begin{cases} \max[0, \min[k_i, \alpha_i - p_i^t]] & \text{if } p_i^t < p_j^t, \\ \max[0, \min[k_j, (\alpha_j - p_j^t)/2]] & \text{if } p_i^t = p_j^t, \\ \max[0, \min[k_j, \alpha_j - p_j^t - k_j]] & \text{if } p_i^t > p_j^t \end{cases} \quad (2)$$

where $i, j = a, b, i \neq j$, and $t = 1, 2$. Demand function (2) implies that if $p_i^t < p_j^t$, firm $i$ sells up to capacity $k_i$ or serves all those consumers whose reservation prices are larger than $p_i^t$. If $p_i^t = p_j^t$, then two firms equally share the market demand. If $p_i^t > p_j^t$, although consumers with reservation prices higher than $p_i^t$ are willing to purchase from firm $i$, consumers prefer to purchase from firm $j$, which offers a lower price. None of these consumers buy from firm $i$ if firm $j$’s capacity $k_j$ is large enough to cover the market demand. However, if firm $j$’s capacity $k_j$ is not large enough, those remaining consumers whose demands are not met by firm $j$ would buy from firm $i$ ($i, j = a, b, i \neq j$).

The firms’ initial capacities are identical and exogenously determined prior to the beginning of the game. In a game where two firms first set their capacities and then compete on prices with capacity constraints, Kreps and Scheinkman (1983) demonstrate that the Cournot capacities, denoted by $Q(\alpha, c)$, where $c$ is the cost of capacity, are indeed the equilibrium outcome. Considering a case of constant capacity cost ($c(k) = k c_0$) and the linear demand function proposed in (2), one can compute the Cournot capacity $Q(\alpha, c_0) = (\alpha - c_0) / 3$. Because market demand can be high or low in our model, we assume that the firms’ initial capacity $k$ between $Q(\alpha_t, c_0)$ and $Q(\alpha_H, c_0)$, where $Q(\alpha_t, c_0)$ and $Q(\alpha_H, c_0)$ denote the Cournot capacities for low-and high-demand scenarios, respectively. Following this assumption, the initial capacities should satisfy the condition $k \leq \alpha_t < 3k \leq \alpha_H$.\(^2\)

\(^1\)Here we implicitly assume that the lower-priced firm serves the consumers with the highest reservation prices first when it cannot satisfy the entire market. Such a rationing rule is called the efficient rationing rule, which has been widely adopted in the literature (e.g., Levitan and Shubik 1972, Kreps and Scheinkman 1983, and Staiger and Wolak 1992). A more detailed discussion of alternative rationing rules is available in Tirole (1988).

\(^2\)In the extreme case of low demand, $\alpha_t \leq k$, each firm has enough capacity to cover the entire market. In this case, since each firm has an incentive to undercut its price by a very small amount to capture the entire market, market price is driven down to the level of marginal cost, which is equal to zero here. In this case, each firm sells to $\alpha_t / 2$ consumers and holds an excess capacity of $k - \alpha_t / 2$. This case of low demand appears quite extreme and unrealistic. Moreover, results and intuitions on excess capacity resemble the case of $k \leq \alpha_t \leq 3k$. To keep our exposition concise, we will only consider $k \leq \alpha_t$ for the case of low demand in the rest of this paper.

### 3. Model Analysis

In this section, we analyze how firms may strategically use the reward programs in their capacity management when facing demand seasonality. In §3.1 we start our analysis with a benchmark case where firms do not offer rewards. In the benchmark case, the price competitions in two periods are not related. The results show that when market demand is low, the existence of excess capacities intensifies price competition. In §3.2 we analyze the model with reward programs. Our results show that firms can successfully mitigate price competition in the period with low demand by eliminating excess capacities through reward programs. In §3.3, we extend the model to study the firms’ decisions on the initial capacities.

We place the technical details associated with this section in Appendix 1, where we solve the equilibrium prices and rewards following the principle of backward induction. Specifically, we solve the problem by analyzing the second period first, and then backward induct to the first-period price competition. At Period 0, we study the decisions on reward amount. Throughout the paper, the goal of our analysis is to first search for the pure-strategy equilibrium. If there does not exist any pure-strategy equilibrium, we investigate the mixed-strategy equilibrium.

#### 3.1. Benchmark: Equilibrium Price Strategy Without Reward Programs

To develop a benchmark for comparison, we first describe the price strategies without reward programs. In this case, a firm’s price strategy is independent between two periods. We formulate the competing firms’ price-decision problems in (P1).

$$(P1) \quad (p_i^0) = \arg \max_{p_i} \pi_i = p_i^0 \cdot d_i^0(p_i^0, p_j^0 | \alpha_i, k, k),$$

where the demand function $d_i^0(\cdot)$ is given by Equation (2), $i = a$ or $b$. Without reward programs, firms have full capacities in both periods. Such a game of price competition with strict capacity constraint is already analyzed in the literature (e.g., Levitan and Shubik 1972 and Kreps and Scheinkman 1983). In general, the equilibrium prices depend on the size of market demand relative to the firms’ available capacities. Specifically, given that each firm has an initial capacity $k \in [\alpha_t / 3, \alpha_H / 3]$, when the market demand
is high in the first period \((\alpha_1 = \alpha_{1i})\), there exists a pure-strategy equilibrium where both firms sell their entire capacities. When the market demand is low in the second period \((\alpha_2 = \alpha_{1i})\), there exists only a mixed-strategy equilibrium. We next summarize the equilibrium results for each period. Again, these results of capacity-constrained equilibrium prices are due to Levitan and Shubik (1972) and Kreps and Scheinkman (1983). For details of derivation, readers may refer to these two papers.

**First Period** \((\alpha_1 = \alpha_{1i})\). In the first period, the demand is high as \(\alpha_i = \alpha_{1i}\). There exists a unique pure-strategy equilibrium where both firms sell up to their capacities. The equilibrium price is \((p|^{0}_i) = \alpha_{1i} - 2k\), and the equilibrium profit is \((\pi^{0}_i) = (\alpha_{1i} - 2k)k\) for firm \(i\), \(i = a, b\). At the equilibrium price, if a firm unilaterally reduces its price below \(\alpha_{1i} - 2k\), the firm’s profit decreases as the demand is constrained by capacity \(k\). If the firm unilaterally raises its price above \(\alpha_{1i} - 2k\), the firm’s profit becomes \(p(a_{1i} - k - p)\), which is lower than \((\alpha_{1i} - 2k)k\) because the marginal profit change \(\frac{\partial \pi_i }{\partial p_i} = 3k - \alpha_{1i} < 0\). In the equilibrium, both firms wish to sell more, but are constrained by their available capacities.

**Second Period** \((\alpha_2 = \alpha_{1i})\). In the second period, the demand is low as \(\alpha_i = \alpha_{1i}\). Unlike in the first period, setting the price at \(\alpha_{1i} - 2k\) to dump the entire capacity is no longer an equilibrium strategy. If one firm charges a price at \(\alpha_{1i} - 2k\), the other firm can be better off by increasing the price above \(\alpha_{1i} - 2k\) because the marginal profit change \(\frac{\partial \pi_i }{\partial p_i} = 3k - \alpha_{1i} > 0\). Consequently, there does not exist any pure-strategy equilibrium, and we have to resort to the mixed-strategy equilibrium. The mixed-pricing strategy is characterized by a distribution over a (common) price interval. One can characterize the distribution with the upper bound \(p\) and lower bound \((p)\) of the interval, and the cumulative probability distribution function \(\Phi_0(p)\).

\[
\Phi_0(p) = \frac{pk - (\alpha_i - k)^2/4}{p(2k - \alpha_i + p)}, \quad (3)
\]

for \(p \in [p, \bar{p}]\) where \(\bar{p} = \frac{\alpha_i - k}{2}\) and 
\[
p = \frac{(\alpha_i - k)^2}{4k}, \quad (4)
\]

\[
E\pi^*_2 = \frac{(\alpha_i - k)^2}{4}. \quad (5)
\]

Equation (5) shows that when market demand is low, each firm’s expected profit is identical to the Stackleberg follower’s profit given first-mover’s capacity at \(k\). We can infer from the above distribution that the expected price decreases with capacity \(k\). When a firm randomizes its price, it considers the risk of being undersold. Because a larger firm has greater incentive to set a lower price to sell more, a firm is exposed to greater risk of being undersold when its competitor holds a larger capacity. Consequently, on average, market prices decrease with industry capacities. The resulting equilibrium profits also decrease with the available capacities. Finally, at the beginning of the game, the firms anticipate a period of high demand followed by a period of low demand. Based on the above results, each firm’s total expected profits from the two periods is

\[
E\pi^*_i = (\alpha_{1i} - 2k)k + \delta \frac{(\alpha_i - k)^2}{4}. \quad (6)
\]

**Excess Capacities.** In the first period, the market demand is high and neither firm holds any excess capacity. In the second period, the market demand is low and firms follow the identical mixed strategy characterized by Equations (3) and (4). Given firm \(i\)’s price at \(p\) under a probability that is equal to \(1 - \Phi_0(p)\), firm \(j\) would price higher than \(p\). In this case, firm \(i\) would sell its entire capacity \(k\) and have no excess capacity. On the other hand, under a probability equal to \(\Phi_0(p)\), firm \(j\)’s price would be lower than \(p\). Then firm \(i\) would sell less than its capacity and hold an excess capacity \(k - (\alpha_i - k - p) = 2k - \alpha_i + p\). Therefore, when setting the price at \(p\), firm \(i\) expects an excess capacity that is equal to \((2k - \alpha_i + p)\Phi_0(p) = (p - p)k/p\). Because firm \(i\)'s price also follows the distribution \(\Phi_0(p)\), the firm’s expected amount of excess capacity should be

\[
\int_{p}^{\bar{p}} \frac{(p - p)k}{p} d\Phi_0(p).
\]

The existence of excess capacity is frequently observed in practice in the form of unoccupied seats for airlines, vacant rooms for hotels, or idle lines for telephone companies. Excess capacity means the underutilization of capacity investment. More importantly, as shown by the results above, the existence of excess capacities creates incentives for firms to undersell. Consequently, both firms would lower their prices to avoid being undersold, and the resulting equilibrium profits decrease with the size of excess capacities.

We now analyze the model where firms provide reward programs.

### 3.2. Equilibrium Results with Reward Programs

We will start with the discussion of the effect of reward programs on the second-period price competition, followed by the optimal allocation of capacities for reward redemption. We then move backwards to the first period and analyze the first-period price. Last, we present the equilibrium reward decisions.
3.2.1. Price Strategy in the Second Period. We formulate the competing firms’ second-period price-decision problems in (P2).

\[
\begin{align*}
(P2) \quad p^*_i(\alpha, k, k') &= \arg\max_{p_i} \pi^*_i(\alpha, k, k') \\
&= p^*_i \cdot d^*_i(p^*_i, p^*_j | \alpha, k, k'),
\end{align*}
\]

where the demand function \(d^*_i(\cdot)\) is given by Equation (2), \(i = a, b\). Through the reward program, firm \(i\) hands out \(\phi_i\) amount of its capacity in the second period to reward the first-period customers. As a result, firm \(i\)'s available capacity in the second period becomes \(k_i = k - \phi_i\) (\(i = a\) or \(b\)). In the benchmark case where no firm offers reward programs, there only exists a mixed-strategy equilibrium in the second period. With reward programs that hand out free products, there can exist a pure-strategy equilibrium if firms allocate sufficiently large amounts of capacities. Specifically, if \(\min_i \{2\phi_i + \phi_j\} \geq 3k - \alpha_L\), then in the equilibrium both firms sell up to their capacities in the second period. In this case, firm \(i\)'s second-period equilibrium price becomes \(p^*_i = \alpha_i - k_i^2 - k_j^2\), and the firm’s second-period equilibrium profit becomes \(\pi^*_i = (\alpha_i - k_i^2 - k_j^2)k_i^2\).

When \(\min_i \{2\phi_i + \phi_j\} < 3k - \alpha_L\), as in the benchmark case, there does not exist any pure-strategy equilibrium, and we resort to the mixed-strategy equilibrium. In the equilibrium, at least one of the firms will possess excess capacity. Both the price distributions and the resulting profits depend on the firms’ available capacities. Suppose that \(\phi_i < \phi_j\); then, \(k_j > k_i\) and the expected equilibrium profit for firm \(i\) will be \(\pi^*_i = (\alpha_i - k_i^2)^2/4\), and the profit for firm \(j\) would be \(\pi^*_j = (k_j^2/k_i)(\alpha_i - k_i^2)^2/4\). We can draw two conclusions from the firms’ expected profits. First, if a firm’s available capacity \(k_j^2 \geq \alpha_i/3\) (which is satisfied in the equilibrium, as we will show in §3.2.4), the firm’s profit would increase with the capacity reduction through rewards. Second, since \(\pi^*_j\) decreases with \(k_j^2\) and \(\pi^*_i\) decreases with \(k_i^2\), a firm’s profit increases when the other firm hands out more capacity rewards. We now summarize the above results in Proposition 1.

**Proposition 1.** In the second period, with the offering of capacity rewards through reward programs, firms increase the second-period prices and profits. A firm’s second-period profit increases with its competitor’s capacity reduction through reward programs.

Proposition 1 indicates that because the demand is low in the second period and firms have excess capacities, offering capacity rewards through reward programs can help reduce price competition and increase profits in the second period. The proposition also shows that a firm can benefit from its competing firm’s capacity reduction through reward programs.

3.2.2. Allocating Capacities for Reward Redemption. At the beginning of the second period, the firms decide the amount of capacities to be allocated for reward redemption. We formulate the firms’ capacity allocation decision problems in (P3).

\[
\begin{align*}
(P3) \quad r^*_i(\alpha, R_a, R_b) &= \arg\max_{r_i} \Psi^*_i(r_i | \alpha) \\
&= \pi^*_i(\alpha_i, k - r_id_i^*, k - r_id_j^*) - \eta(R_d^1 - r_i^2),
\end{align*}
\]

where \(\pi^*_i(\alpha_i, k - r_id_i^*, k - r_id_j^*)\) is firm \(i\)'s second-period profit function from consumer purchases solved by (P2). The loss function from the inventory of unredeemed reward points, \(\eta(R_d^1 - r_id_j^*)^2\), is defined in Equation (1). The loss function does not appear in Problem (P1) because when firms decide their second-period prices, the level of such inventory is already determined, and therefore becomes the sunk cost. However, when a firm decides the capacity allocation, the firm should not only consider the impact of available capacity on subsequent price competition, but also the profit impact of any unredeemed reward points due to insufficiently allocated capacity for rewards. In Proposition 2, we summarize the results from the analysis in Appendix 1.

**Proposition 2.** If the total reward points \(\psi_i \leq (3k - \alpha_i)/3\), then the optimal level of capacity allocated for rewards is \(\phi_i = \psi_i\). If \(\psi_i > (3k - \alpha_i)/3\), then \(\phi_i \in ((3k - \alpha_i)/3, \psi_i)\).

Proposition 2 shows that a firm should allocate an amount of capacity for reward redemption (\(\phi_i^*\)) close to \((3k - \alpha_i)/3\). Note that when \(\phi_i^* = (3k - \alpha_i)/3\), firm \(i\)’s remaining capacity that is available for sale is \(k_i^2 = k - (3k - \alpha_i)/3 = \alpha_i/3\). Therefore, with reward amount \(\phi_i^* = (3k - \alpha_i)/3\), firm \(i\) adjusts the available capacity back to the level without excess capacities in the equilibrium. Firms can thus perfectly resolve the problem of excess capacities. The result implies that when the accumulated amount of reward points \(\psi_i\) is not more than \((3k - \alpha_i)/3\), a firm should allocate a capacity for reward redemption \(\phi_i^*\) equal to \(\psi_i\), so that all reward points will be redeemed. However, when \(\psi_i > (3k - \alpha_i)/3\), the optimal \(\phi_i^*\) should be smaller than \(\psi_i\) because allocating \(\phi_i^*\) equal to \((3k - \alpha_i)/3\) is sufficient to cut the excess capacities. Any \(\phi_i\) that is larger than \((3k - \alpha_i)/3\) would have to cut the profitable capacities. However, the optimal \(\phi_i^*\) should be larger than \((3k - \alpha_i)/3\) because the firm incurs the future profit loss from any unredeemed reward points. In the equilibrium, firms have to balance the need to retain profitable capacities and the need to reduce customer dissatisfaction due to insufficient capacities for redemption.
In summary, because the market demand is low, the existence of excess capacities would intensify the market price competition. Unless the consumers have accumulated too many reward points, the firms would allocate sufficient capacities for the consumers to redeem all their reward points. The reduction of firms’ available capacities can effectively alleviate the price competition during the period of time with low market demand.

### 3.2.3. Price Strategy in the First Period

In the first period, both firms have full capacities; i.e., \( k_i^1 = k_j^1 = k \). We formulate the competing firms’ first-period price-decision problems in (P4).

(P4) \[
p^*_i(\alpha_{hi}) = \arg \max_{p_i} \pi_i(\alpha_{hi}) \]
\[
= p^*_i d^*_i(p^*_i, p^*_j | \alpha_{hi}) + \delta \Psi^*_2(\alpha_{hi}),
\]

where \( \Psi^*_2(\alpha_{hi}) \) is solved from Problem (P3). Problem (P4) indicates that each firm sets the first-period price to maximize total expected profits from both periods. When a firm sets its first-period price, the firm considers the effect of its first-period price on its first-period profit as well as the effect on the firm’s second-period profit. We summarize the main results from the analysis in Appendix 1 in Proposition 3 and discuss the intuitions afterwards.

**Proposition 3.** If \( R_i \leq (3k - \alpha_i)/(3k) \) and \( R_j \leq (3k - \alpha_j)/(3k) \), then \( p^*_i = \alpha_{hi} - 2k; \) otherwise, \( p^*_i \) can be larger than \( \alpha_{hi} - 2k \) when \( \alpha_{hi} - 3k \) is sufficiently small.

Proposition 3 shows that reward programs may increase firms’ first-period prices. First, when both \( R_i \) and \( R_j \) are lower than \( (3k - \alpha_i)/(3k) \) and both firms set the first-period prices equal to \( \alpha_{hi} - 2k \), each firm sells the full capacity \( k \) and has total reward points \( \psi_j \leq (3k - \alpha_i)/3 \). According to Proposition 2, in the second period the firms will allocate enough capacity to have all reward points redeemed. A firm will not sell lower than \( \alpha_{hi} - 2k \) in the first period because the firm’s demand will remain the same at \( k \), thus leading to a lower profit. If a firm sells higher than \( \alpha_{hi} - 2k \), the firm’s first-period demand will be lower than capacity \( k \), leading to a lower first-period profit. Moreover, the firm’s second-period profit decreases with a smaller amount of reward points \( \psi_j \) when \( \psi_j \leq (3k - \alpha_i)/3 \). Therefore, the equilibrium price should be \( \alpha_{hi} - 2k \).

However, when \( R_i > (3k - \alpha_i)/(3k) \), a firm may benefit from a first-period price that is higher than \( \alpha_{hi} - 2k \). According to Proposition 2, in the second period firm \( i \) will allocate an amount of capacity \( \phi^*_i \in ((3k - \alpha_i)/3, Rk) \) for reward redemption. Any unredeemed reward points will lead to the firm’s future profit loss. Clearly, by increasing the first-period price and reducing the first-period demand, the firm gains in the second period with a smaller amount of profit loss due to unfulfilled needs for free products. Interestingly, we find that when \( R_i > (3k - \alpha_i)/(3k) \), firm \( j \) may also have the incentive to price higher than \( \alpha_{hi} - 2k \) even though \( R_j < (3k - \alpha_i)/(3k) \). When firm \( j \) increases price, firm \( j \) decreases the first-period demand below capacity \( k \). As a result, firm \( j \) decreases the amount of rewards to be handed out and retains a larger capacity in the second period. The increased capacity \( k_j^2 \) will reduce firm \( i \)’s profit from price competition, and hence increases the other firm’s incentive to cut its capacity. Note that firm \( i \)’s profit depends on the profit from price competition as well as the loss from unredeemed rewards. A larger capacity retained by firm \( j \) decreases the relative importance of firm \( i \)’s profit from price competition, thus making it more important for firm \( i \) to reduce the inventory of unredeemed capacity. Consequently, in the equilibrium firm \( i \) would have a smaller capacity, which would benefit firm \( j \).

### 3.2.4. Decisions on Reward Amount

Firms decide reward amount \( (R_i) \) in the beginning of the game to maximize their own total (discounted) profits from the two subsequent periods. We let \( \pi_i(R_i, R_j) \) denote firm \( i \)’s total profits, including the profit impact of any reward inventories. We can formulate the firms’ decision problems on \( R_i \) as follows.

(P5) \[
R^*_i = \arg \max_{R_i} \pi_i(R_i, R_j) = \pi_i(R_i, R_j, \alpha_{hi}) + \delta \Psi^*_2(\alpha_{hi}'),
\]

where \( i = a \) or \( b \). When deciding the reward amount, firms can correctly anticipate the impact of rewards on subsequent periods of price competition. In Proposition 4, we summarize the main results from the analysis in Appendix 1.

**Proposition 4.** The equilibrium reward amount \( R^*_i = (3k - \alpha_i)/(3k) \). In the equilibrium, the first-period price \( p^*_i = \alpha_{hi} - 2k \) and profit \( \pi^*_i = k(\alpha_{hi} - 2k) \). In the second period, capacity allocated for rewards \( r^*_i = (3k - \alpha_i)/(3k) \), price \( p^*_2 = \alpha_{hi} - 2k \), and profit \( \pi^*_2 = \alpha_{hi} - 2k \).

To interpret the equilibrium reward amount, we need to understand the potential benefits and costs associated with the reward programs. Firms benefit from offering reward programs in the second period when market demand is low. As indicated by Proposition 1, firms increase their second-period profits by allocating their excess capacities for reward redemptions. The reduction of excess capacities mitigates price competition. If both firms offer reward amount \( R^*_i = (3k - \alpha_i)/(3k) \), then according to Proposition 2, in the second period firms will allocate enough capacity...
to meet the redemption of all reward points. The resulting second-period capacity will be $k_2 = \alpha_i/3$ for each firm, thus completely eliminating the excess capacities during the period of low demand. A firm will not offer more rewards than $(3k - \alpha_i)/(3k)$ because of the future profit loss due to unredeemed reward points. According to Proposition 2, when a firm offers more reward than $(3k - \alpha_i)/(3k)$, in the second period the firm will not provide enough capacity for redemption. The resulting inventory of unredeemed reward points would lead to customer dissatisfaction and future profit loss.

In the equilibrium, both firms sell up to their capacities in the first period when the market demand is high. Meanwhile, consumers accumulate their reward points through purchases. In our model, because firms know the seasonal market demand, the firms can correctly anticipate the amount of capacity reduction needed in the period of low demand. As a result, firms can set the right reward amount so that all accumulated reward points will be redeemed. In conclusion, firms can use reward programs to avoid a price war during a period of low demand, and obtain higher profits.

### 3.3. Size of Initial Capacities

So far we have treated the firms’ initial capacities $k_i$ as exogenously determined prior to the game. Since market demand can be either high or low over the time periods, we assumed that the level of initial capacity $k \in (Q(\alpha_{t1}, c_0), Q(\alpha_{t2}, c_0))$. In §3.2 we demonstrated that firms can use reward programs to reduce the excess capacities during the period of low demand. One may thus infer that the use of reward programs may affect the size of firms’ original capacities. Specifically, since the reward programs can mitigate the price wars resulting from the excess capacities, the offering of reward programs may increase the firms’ initial capacities to be closer to $Q(\alpha_{t1}, c_0)$. To demonstrate this result, we need to extend our model by adding a stage of decision on capacities ($k_i$ for firm $i$) before the decisions on rewards at the beginning of the game. We analyze the extended model in Appendix 1 and summarize the main results in the next proposition.

**Proposition 5.** (1) The equilibrium capacities are

$$k_2^* = k_1^* = \begin{cases} k_0 \in (Q(\alpha_{t1}, c_0), Q(\alpha_{t2}, c_0)) & \text{without reward programs;} \\ Q(\alpha_{t1}, c_0) & \text{with reward programs.} \end{cases}$$

(2) With reward programs, both firms provide the rewards equal to $1 - \alpha_i/(3Q(\alpha_{t2}, c_0))$ that reduce their excess capacities to zero during the period of low demand.

Proposition 5 confirms that without reward programs, the equilibrium capacities are between $Q(\alpha_{t1}, c_0)$ and $Q(\alpha_{t2}, c_0)$. More importantly, the proposition shows that each firm’s equilibrium capacity increases to $Q(\alpha_{t1}, c_0)$ when firms provide reward programs. While increasing their initial capacities, firms offer large enough rewards to ensure that no excess capacities exist during the period of low demand. Therefore, without reward programs we would observe insufficient capacities during the period of high demand, but excess capacities and intense price competition during the period of low demand. With reward programs, firms not only eliminate the excess capacities and mitigate price competition during the period of low demand, but also set sufficiently large capacities for the period of high demand.

### 4. Model Extensions

So far we have assumed that firms know the exact demand states of both periods. In reality, a firm may not be able to precisely predict the market demand. In §4.1, we generalize our model by incorporating the demand uncertainty in each period. In the rest of this paper, we will refer to the model that we analyzed in §3 as the basic model, and the model with demand uncertainty as the generalized model. Recall that the basic model also assumes that a firm’s first-period demand is not affected by the size of rewards. This assumption allows us to focus on the role of reward programs in capacity management. However, consumers may value the rewards and consider the reward value when they purchase in the first period (Kim et al. 2001). As a result, offering rewards may help a firm attract customers and increase demand in the first period. In §4.2, we discuss how a further model extension to incorporate such a demand effect may affect our main results. Finally, in the basic model we assume that handing out free products through the reward programs does not affect the firms’ primary demand in the second period. In reality, some of the primary market demand may overlap with the demand for free products. In §4.3, we extend the generalized model by allowing for such demand cannibalization in the second period and discuss whether firms’ incentives to provide reward programs would depend on the size of cannibalization.

#### 4.1. A Generalized Model with Demand Uncertainty

The generalized model extends the basic model by incorporating the firms’ uncertainty about future market demand. At the beginning of the game, when firms make reward decisions, the exact states of demand of either period are unknown. The firms set their rewards to maximize the expected profits from
the two subsequent periods. To model the demand uncertainty, we assume that the level of demand at time period $t$ can be either high ($\alpha_t = \alpha_H$) or low ($\alpha_t = \alpha_L$), $t = 1$ or 2. Before knowing the actual state of demand, firms share the same belief that the probability for $\alpha_t = \alpha_H$ is $\rho_H$, and the probability for $\alpha_t = \alpha_L$ is $(1 - \rho_L)$, $t = 1$ or 2. Note that the basic model of demand seasonality is a special case of the generalized model with $\rho_1 = 1$ and $\rho_2 = 0$.

Neither firm knows the actual state of demand in period $t$ ($\alpha_t$) until they enter the period $t$, $t = 1$ or 2. After observing the demand state in the first period, a firm sets its first-period price to maximize the firm’s total expected profits from both Period 1 and Period 2. In the second period, after observing $\alpha_T$, a firm decides the amount of capacity allocated for rewards before setting the second-period price. Compared with the sequence of moves of the basic model that is illustrated in Table 1, a similar table for the generalized model should add to the beginning of each period a stage in which firms observe the actual demand state of that period.

We next discuss the results from the generalized model. To avoid repetition with §3, we will limit our discussion to the new insights that arise from the feature of demand uncertainty. We solve the generalized model in Appendix 2, following the same approach as for the basic model. Due to the complexity of the model, we are not able to obtain closed-form solutions for reward decisions. Instead, we demonstrate the firms’ incentives to provide rewards by showing that a firm can gain profit from offering a reward program. We analyze the impact of such rewards on subsequent decisions, assuming that firms offer identical rewards.

**Price Strategy in the Second Period.** The competing firms’ second-period price-decision problems are similar to (P2). However, with demand uncertainty, the second-period demand can be high ($\alpha_2 = \alpha_H$) or low ($\alpha_2 = \alpha_L$). When demand is low, we have the same problem as in the basic model analyzed in §3.2.1. The main conclusion is that because market demand is low, firms can mitigate the price competition by handing out part of excess capacity in the form of rewards. Therefore, reward programs can increase both prices and profits.

When the demand is high, there exists a unique pure-strategy equilibrium where both firms sell up to their capacities. The equilibrium price is $p_l = \alpha_H - 2k + \phi_a + \phi_b$, $i = a, b$. Since both $\phi_a$ and $\phi_b$ are nonnegative, each firm’s second-period price goes up with capacity reduction through reward programs. However, because the market demand is high, firms would sell their full capacities even without the reward programs. Because firms can benefit from the additional capacities, the inventory of rewards becomes a burden to the firms.

**Allocating Capacities for Reward Redemption.** In the beginning of the second period, after realizing the demand state ($\alpha_2$), the firms decide the amount of capacities to be allocated for reward redemption. The decision problem can be formulated similar to (P3) in §3.2.2, for both the case of $\alpha_2 = \alpha_H$ and $\alpha_2 = \alpha_L$. When the demand is low ($\alpha_2 = \alpha_L$), Proposition 2 states that if $\psi_i \leq (3k - \alpha_L)/3$, then $\phi_i^* = \psi_i$; otherwise, $\phi_i^* \in ((3k - \alpha_L)/3, \psi_i)$. Essentially, a firm should allocate an amount of capacity for reward redemption ($\phi_i^*$) close to $(3k - \alpha_L)/3$. A firm has an inventory of unredeemed reward points if and only if rewarded points $\psi_i > (3k - \alpha_L)/3$.

For the case of high demand ($\alpha_2 = \alpha_H$), in Proposition 6 we summarize the results from analysis in Appendix 2.

**Proposition 6.** When $\alpha_2 = \alpha_H$, if $2\eta \psi_i \leq \alpha_H - 3k$ for both firms, then $\phi_i^* = 0$; otherwise,

$$
\phi_i^* = \frac{\eta}{3 + 2\eta} (\psi_i + \psi_j) + \frac{\eta}{1 + 2\eta} (\psi_i - \psi_j) - \frac{1}{3 + 2\eta} (\alpha_H - 3k), \quad (i, j = a, b, i \neq j). \tag{7}
$$

Proposition 6 states that when the second-period demand is high and $2\eta \psi_i \leq \alpha_H - 3k$, firms do not allocate any capacities for rewards. Because the demand is high, both firms desire additional capacities. The marginal profitability of the additional capacity units is measured by $\alpha_H - 3k$. Therefore, a larger $\alpha_H - 3k$ implies greater opportunity cost for a firm to assign the capacities for reward redemption. However, not allocating enough capacities for rewards will lead to customer dissatisfaction because not all the customers are able to redeem their reward points. If the negative profit impact because of such customer dissatisfaction is sufficiently small (a small $\eta$), then firms will not assign any capacities for rewards. A common practice among the airline and hotel reward programs is to contain “blackout dates” to ensure that no capacities are handed out as rewards during peak seasons or for busy routes.

However, if $\eta$ is sufficiently large that $2\eta \psi_i \geq \alpha_H - 3k$, firms assign part of their capacities for reward redemption even when the market demand is high. In recent years a number of reward programs have eliminated “blackout dates.” For example, after Starwood Hotels & Resorts offered no blackout dates for room redemptions in February 1999, Hyatt Hotels Corporation announced in May 2000 the elimination of award blackout dates for Gold Passport members. Given the firms’ capacities, our analysis indicates two possible reasons for the firms to eliminate “blackout dates”—a higher $\eta$ (growing customer dissatisfaction from the “blackout dates”) or a smaller
\(\alpha_{ij} - 3k\) (declined market demand during these peak dates).

Finally, we infer from Equation (7) that \(\phi^*_i\) (amount of capacity allocated for reward redemption) increases with \(\eta\) and \(\psi_i\), but decreases with \(\alpha_{ij} - 3k\) and \(\psi_i\). Both a larger \(\eta\) and a larger \(\psi_i\), lead to greater customer dissatisfaction as a result of the difficulty of redeeming rewards, suggesting more capacity to be allocated for rewards. On the other hand, a larger \(\alpha_{ij} - 3k\) implies greater market demand. A larger \(\psi_i\) implies smaller industrial capacity because firm \(j\) is expected to allocate more capacity for reward redemption and leave a smaller capacity for sale. Therefore, both a larger \(\alpha_{ij} - 3k\) and a larger \(\psi_i\) imply greater marginal profitability from existing capacities, calling for the allocation of less capacity for rewards and more capacity for sale.

In summary, the above results show that firms allocate the amount of capacity for reward redemption according to the realization of market demand. When the market demand is high, firms may use blackout policy to ensure that no capacities are given out as rewards, and retain the entire capacities for sale. However, the resulting customer dissatisfaction can hurt the firms’ future profits and, consequently, prevent the firms from not allocating any capacity for reward redemption. When the market demand is low, the existence of excess capacities would intensify the market price competition. In this case, unless the consumers have accumulated too many reward points, the firms would allocate sufficient capacities for the consumers to redeem all their reward points. The reduction of firms’ available capacities can effectively alleviate the price competition during a period of unfavorable market demand. Our results are consistent with casual observations of industry practices. For example, it is reported in trade publications that airlines preallocate a percentage of seats to the frequent fliers a couple of months prior to the departure date. The percentage varies across seasons: In peak seasons airlines only allocate 4% of capacity to free seats, while in idle seasons they typically allocate 10% of capacity to reward the frequent fliers. Similarly, percentage of seats allocated for rewards differs between the routes. The number of seats allocated to the frequent fliers on the busier routes is always much smaller than that on other routes.

**Price Strategy in the First Period.** We formulate the competing firms’ first-period price-decision problems in (P6).

\[
\begin{align*}
\text{(P6)} & \quad p^*_i(\alpha_i) = \arg \max_{p_i} \pi^*_i(\alpha_i) \\
& = p_i^*d_i^*(p_i^*, p_j^* | \alpha_i) \\
& \quad + \delta[\rho \psi^*_i(\alpha_{ij}) + (1 - \rho) \psi^*_j(\alpha_{ji})],
\end{align*}
\]

where \(\alpha_i = \alpha_{ij}\) or \(\alpha_{ji}\), \(\Psi^*_i(\alpha_{ij})\) and \(\Psi^*_j(\alpha_{ji})\) are the second-period profits for high- and low-demand scenarios, respectively. Unlike the basic model, here a firm does not know the demand state in the second period, and therefore sets the price to maximize total expected profits. We summarize in Proposition 7 the main results from the analysis in Appendix 2. We restrict our discussion to the case of symmetric equilibrium, where \(R_j = R_i = R^*\). Note that we will not impose such symmetry when we analyze firms’ reward decisions.

**Proposition 7.** (1) When \(\alpha_i = \alpha_{ij}\), if \(2\eta R^k \leq \alpha_{ij} - 3k\), then \(p^*_i = \alpha_{ij} - 2k\); otherwise, \(p^*_i\) can be higher than \(\alpha_{ij} - 2k\).

(2) When \(\alpha_i = \alpha_{ij}\), if \(2\eta R^k \leq \alpha_{ij} - 3k\), then the first-period mixed-strategy price equilibrium follows a price distribution that first-order stochastically dominates the price distribution without reward programs.

Proposition 7 shows that reward programs may increase firms’ first-period prices. When the demand is high in the first period and \(\eta\) is sufficiently large, as indicated by condition \(2\eta R^k > \alpha_{ij} - 3k\), the first-period price \(p_i^*\) can be higher than \(\alpha_{ij} - 2k\). Essentially, with a large \(\eta\), firms have to allocate capacities for reward redemption even if the second-period demand is high. Therefore, when \(\alpha_2 = \alpha_{ij}\) with a positive probability \(p_2\), a firm’s first-period sales will lead to a reduction in the firm’s second-period capacity, and hence a reduction in the firm’s second-period profit, thus creating an incentive for the firms to price high and sell less in the first period. Recall that in the basic model, the second-period demand is always low, and therefore a small amount of capacity rewards in the form of free products is always profitable. The result of Proposition 7 extends Proposition 3 by showing that in a generalized model where the second-period demand can be high, the first-period price \(p_i^*(\alpha_{ij})\) can be higher than \(\alpha_{ij} - 2k\) even when rewards \(R_i \leq (3k - \alpha_i)/3k\).

The second part of Proposition 7 shows an interesting effect of reward programs on firms’ dynamic mixed pricing strategies. When demand is low in the first period and firms do not offer reward programs, the equilibrium prices are characterized by the distribution \(\Phi_0(p)\) defined by Equations (3) and (4). As we show in Appendix 2, the equilibrium price distribution with reward programs \(\Phi_i(p)\) stochastically dominates the distribution without reward programs \(\Phi_0(p)\). Therefore, the average price in the first period increases with the offering of reward programs. To see the intuition behind this result, note that with reward programs, firms reward their customers an amount of capacity proportional to their first-period sales. If firm \(i’\)s price is higher than that of its competitor \(j\) in the first period, then firm \(i\) will sell less in the first
period. As firm \( j \) sells more in the first period, in the second period firm \( j \) has to allocate more capacity for reward redemption. Thus, firm \( i \) can benefit from firm \( j \)'s large capacity reduction in the second period. Such an incentive to price higher does not exist with the absence of reward programs.

**Decisions on Reward Amount.** We formulate the firms’ decisions on reward \( R_i \) in the generalized model as follows.

\[
R_i^* = \arg \max_{R_i} E\pi_i \quad \text{(P7)}
\]

\[
= \rho_1 \rho_2 E\pi_i(R_i | \alpha_{1t}, \alpha_{1l}) + \rho_1(1-\rho_2) E\pi_i(R_i | \alpha_{0t}, \alpha_{0l}) + (1-\rho_1)(1-\rho_2) E\pi_i(R_i | \alpha_{1t}, \alpha_{1l}),
\]

where \( E\pi_i(R_i | \alpha_{it}, \alpha_{it}) \) denote firm i’s total expected profits from two periods, conditional on actual demand state \( \alpha_i \) for the first and second periods, respectively, including the profit impact of any reward inventories, \( i = a \) or \( b \). We analyze (P7) in Appendix 2 and summarize the main results in Proposition 8.

**Proposition 8.** In the generalized model with demand uncertainty, both firms offer positive rewards in the equilibrium.

While the demand can be high or low in the second period, firms benefit from the reward programs only when the second-period demand is low. According to Proposition 1, when the second-period demand is low, firms increase the profits by allocating their excess capacities for reward redemptions. The reduction of excess capacities mitigates price competition. However, when the second-period demand is high, because firms desire more capacities to meet market demand and prefer not to allocate capacities for rewards, the accumulated reward points become a costly burden to the firms. However, not allocating enough capacities for reward redemption may lead to customer dissatisfaction and future profit loss. Therefore, a firm benefits from the reward program when the demand is low in the second period, but loses profit when the demand is high in the second period. In the generalized model, when the firms make reward decisions at the beginning of the game, the firms do not know the exact demand state of the second period, and hence cannot precisely predict the profitability of the reward programs.

Proposition 8 confirms that despite the uncertainty in the generalized model, the equilibrium reward is still positive. To see the intuition, consider that firm \( i \) provides a very small reward \( R_i \), while firm \( j \) does not offer any reward. We first look at the case of high demand in the second period (with probability \( \rho_2 \)). Since the number of accumulated reward points is very small, from Proposition 6 we know that firm \( i \) will allocate zero capacity for reward redemption. Since \( R_i \) is very close to zero, the marginal negative impact to the firm’s future profit due to unredeemed rewards \( R_i d_i^1 \) is also very close to zero. In other words, the potential loss of profit associated with reward \( R_i \) is very close to zero. We now look at the case of low demand in the second period (with probability \( 1 - \rho_2 \)). In this case, from Proposition 2 we know that each firm has a strong incentive to unilaterally reduce the available capacities closer to \( \alpha_i/3 \). Therefore, the gain of profit associated with the reward \( R_i \) is substantial. In conclusion, as long as the demand uncertainty exists and there is a positive probability of low demand realization (\( \rho_2 < 1 \)), firms will always provide some amount of rewards in the equilibrium. The results alleviate the concern that a firm may free-ride on the other firm’s capacity reduction through reward programs. If one firm reduces its capacity during the period of low demand, its competitor can benefit as well by doing nothing. Thus, there is a potential problem of free-riding that may prevent firms from cutting their excess capacities. Interestingly, while the opportunity to free-ride seems to undermine firms’ incentives on capacity reduction, we find that, indeed, it is optimal for both firms to provide positive reward amount. The reason behind this result is that the benefit in the low-demand scenario dominates the cost in the high-demand scenario.

**4.2. Positive Effect of Rewards on Demand Functions**

The demand function in Equation (2) assumes that a firm’s first-period demand is not affected by the size of rewards. We now discuss how our main results might be affected if we allow the consumers to consider the value of rewards when they purchase in the first period. Suppose that when firm \( i \) offers rewards \( R_i \), each consumer values the chance to obtain reward by \( v(R_i) \), which is increasing and concave with respect to \( R_i \). In the first period, a consumer now chooses the supplier with the smaller net payment \( p_i - v(R_i) \). Clearly, a firm’s first-period demand \( d_i' \), and hence the first-period profit \( \pi_i' \), increase with its reward \( R_i \). Since firms can now attract consumers through lower prices as well as high rewards, firms should prefer to provide more rewards. In the second period, the above demand effect does not exist because the consumers do not earn any more rewards from the purchases. However, the firms’ incentives to use rewards to manage their available capacities as discussed in Proposition 2 and Proposition 6 still exist.

By incorporating the positive demand effect, firms can use the accumulated reward points to manage
their capacity levels in the second period. Further, the firms can also use reward points to attract consumers in the first period. Compared with the generalized model where rewards do not affect demand, firms have greater incentives to offer rewards, and therefore the equilibrium reward amount should be higher. However, since larger rewards would reduce the consumers’ chances to redeem rewards in the second period, consumers’ marginal value from the additional rewards should be decreasing. Moreover, smaller opportunity to redeem reward points would lead to customer dissatisfaction, and hence reduce firms’ future profits at the end. We can therefore infer that while the positive demand effect would increase the firms’ reward offerings, the equilibrium amount of rewards offered would still be limited. Finally, it is important to point out that when consumers accumulate more reward points, reward programs can play more significant roles in firms’ capacity management.

4.3. Cannibalization with Primary Demand

Our analysis so far assumes that handing out free products through the reward programs does not affect the firms’ primary demand in the second period. In other words, we assume that the use of free products or services in the format of capacity rewards are either additional consumer demand or the part of primary demand associated with sufficiently low willingness to pay (the lower end of interval \([0, \alpha_L]\)). In reality, a free product or service offered as a reward may cannibalize the primary demand. Recall that we model consumers’ reservation prices as uniformly distributed in the interval \([0, \alpha_s]\), where \(s = H \) or \(L\). When the extent of cannibalization is small, the demand for free products is primarily distributed in the lower end of \([0, \alpha_L]\). On the other hand, a large cannibalization implies that the demand for free products is primarily distributed in the upper end of \([0, \alpha_L]\). To model the extent of cannibalization, we let the demand intercept

\[
\alpha_{c,s} = \alpha_s - \lambda_s (r_s d^l_s + r_s d^u_s), \quad (s = H \text{ or } L). \tag{8}
\]

Equation (8) assumes that \(\lambda_s\) percentage of satisfied demand for free products (or total redeemed reward points, which is equal to \(r_s d^l_s + r_s d^u_s\)) are distributed in the upper end of \([0, \alpha_s]\). The parameter \(\lambda_s\) represents the extent of overlapping between primary demand and demand for free products when the primary demand is \(\alpha_s\). In Appendix 2 we examine the firms’ incentives to provide reward programs by incorporating (8) into the generalized model. We summarize our main results in Proposition 9.

**Proposition 9.** When the demand cannibalization is modeled by (8), in the equilibrium the firms offer positive rewards if \(\lambda_L < 1 - ((\alpha_L - k)/2k)\rho_1\).

Proposition 9 states that the firms provide positive rewards when \(\lambda_L\) is sufficiently small. The degree of cannibalization can be small for several reasons. For instance, quite often business travelers collect free airline tickets or hotel stays through business trips, but use them for leisure consumption. Miller (1996) reports that more than 80% of the mileage earned from business trips had been used for leisure travels. Therefore, the part of demand that is cannibalized can be associated with the lower end of the reservation prices only. Moreover, consumers may use these free products or services as excess demand. For example, a consumer might never have taken a trip to the Caribbean without having received a free ticket.

Given the value of \(\lambda_L\), the condition \(\lambda_L < 1 - ((\alpha_L - k)/2k)\rho_1\) is more likely to be satisfied with a larger \(k\), smaller \(\alpha_L\), and smaller \(\rho_1\). Both a larger \(k\) and a smaller \(\alpha_L\) lead to more excess capacities, and hence greater benefits from reward programs when the demand is low. In Equation (8), the demand cannibalization increases with the size of first-period demand. Thus, a smaller \(\rho_1\) indicates a lower expected first-period demand, a smaller expected demand cannibalization, and hence an increased expected benefit from reward programs. Note that the condition does not depend on \(\lambda_H\) (the extent of cannibalization for the case of high demand). According to Proposition 6, when the market demand is high in the second period and the accumulated amount of reward points is small, firms do not allocate any capacities for reward redemption. Consequently, no cannibalization occurs, and hence \(\lambda_H\) does not affect firms’ incentives to provide marginal rewards. In general, we find that the potential demand cannibalization can diminish the profitability of reward programs by reducing the primary demand. Firms should therefore be cautious in using reward programs to manage capacities when the cannibalization is expected to be high.

5. Conclusions

In this paper, we use a dynamic game to demonstrate how firms can use reward programs to support their capacity management. Our results are particularly applicable to such industries as airlines and hotels, where firms often possess excess capacities due to the extremely high cost of adjusting their capacities to the fluctuation in market demand. When a firm holds perishable excess capacity, it has strong incentive to set a low price to sell more of its capacity. Its competitor, with the fear of being undersold, also tends to charge a lower price. Consequently, aggressive price competitions pervade these industries during the period of low demand. With reward programs, however, firms are able to commit to lower available capacities by giving out some of their capacities in
the form of rewards. Therefore, reward programs add flexibility for firms to adjust their capacities to market demand and avoid intense price competition during the period of low demand. With this additional flexibility in capacity management, we show that firms would set higher initial capacities. Our analysis also confirms that it is self-enforcing for each firm to offer reward programs to reduce the excess capacity. With reward programs, if a firm is undersold by its competitor at the current period, the firm can benefit from its competitor’s capacity reduction later on. This benefit can compensate the firm’s current loss due to its higher price, and consequently increase the market prices.

Our results on how firms can incorporate the reward programs in their capacity management provides further support for the popularity of reward programs in the hospitality industry. Other studies have also shown that firms may use reward programs to attract and retain customers. (For example, see Klemperer 1987 and Kim et al. 2001.) As we have shown in the paper, these alternative views may strengthen our results. Firms can use reward programs not only to attract consumers by rewarding customers with free products, but also use the accumulated reward points to reduce excess capacities during the period of low demand.

Appendix 1. Analyzing the Basic Model
The analysis follows the principle of backward induction.

Proof of Proposition 1 on Second-Period Price and Profits. If the capacity allocated for rewards $\phi_j < \phi_i$, then the second period available capacity $k'_j > k'_i$. Further, if $\max(2k'_j + k'_j, 2k'_i + k'_i) < k'_i$, there is a pure-strategy equilibrium where both firms sell up to their capacities. Otherwise, firms follow the mixed-strategy equilibrium. The resulting profit functions are derived in Kreps and Scheinkman (1983).

$$\pi^j_2 = \begin{cases} \frac{k - \phi_i (\alpha_L - k + \phi_i)^2}{k - \phi_j}, & \text{if } \phi_i > \phi_j \text{ and } 2\phi_i + \phi_j < 3k - \alpha_L; \\ \frac{(\alpha_L - k + \phi_i)^2}{4}, & \text{if } \phi_i \leq \phi_j \text{ and } 2\phi_i + \phi_j < 3k - \alpha_L; \\ \frac{(k - \phi_i)(\alpha_L - k - 2\phi_i + \phi_j)}{4}, & \text{if } \min(2\phi_i + \phi_j, \phi_i + 2\phi_j) > 3k - \alpha_L. \end{cases}$$

(A1.1)

In (A1.1), condition $\min(2\phi_i + \phi_j) > 3k - \alpha_L$ is equivalent to $\max(2k'_j + k'_j, 2k'_i + k'_i) < k'_i$. Proposition 1 follows the result that when demand is low, price and profits increase as the size of capacity decreases by handing out capacity rewards.

Proof of Proposition 2 on the Size of Capacity Allocated for Rewards. The capacity allocation problem is defined in (P3).

$$\phi^*_j = \arg\max_{\phi_i \leq \phi_j} \Psi^*_j = \pi^*_j - \eta(\psi_i - \phi_i)^2,$$

where $\pi^*_j$ is given by Equation (A1.1). Calculate the first-order derivative,

$$\frac{\partial \Psi^*_j}{\partial \phi_i} = \begin{cases} \frac{\alpha_L - k + \phi_i}{k - \phi_j} \frac{3k - \alpha_L - 3\phi_i}{4} + 2\eta(\psi_i - \phi_i), & \text{if } \phi_i > \phi_j \text{ and } \phi_i + 2\phi_i < 3k - \alpha_L; \\ 0 + 2\eta(\psi_i - \phi_i), & \text{if } \phi_i \leq \phi_j \text{ and } 2\phi_i + \phi_j < 3k - \alpha_L; \\ 3k - \alpha_L - (2\phi_i + \phi_j) + 2\eta(\psi_i - \phi_i), & \text{if } \min(2\phi_i + \phi_j, \phi_i + 2\phi_j) > 3k - \alpha_L. \end{cases}$$

(A1.2)

In (A1.2), first,

$$\frac{\alpha_L - k + \phi_i}{k - \phi_j} \frac{3k - \alpha_L - 3\phi_i}{4} > 0$$

when $\phi_i < (3k - \alpha_L)/3$, but becomes negative, and hence $\pi^*_j$ decreases with $\phi_i$ when $\phi_i \geq (3k - \alpha_L)/3$. Second, $2\eta(\psi_i - \phi_i) > 0$. Based on these two results, we can conclude with the following equilibrium allocations and resulting profits.

(i) If $\phi_i < (3k - \alpha_L)/3$, then $\phi^*_j = \psi_i$ and $\phi^*_j = \psi_i$. The equilibrium profits are:

$$\Psi^*_j^i = \frac{k - \phi_i^i (\alpha_L - k + \psi_i)^2}{k - \phi_j}, \quad \Psi^*_j^j = \frac{(\alpha_L - k + \psi_i)^2}{4}.$$

(A1.3)

(ii) If $\phi_i > (3k - \alpha_L)/3 > \psi_i$ and $\psi_i + 2\phi_j < 3k - \alpha_L$, then $\phi^*_j = \psi_i$ and $\phi^*_i$ is solved by

$$\frac{\partial \Psi^*_j}{\partial \phi_i} = \frac{\alpha_L - k + \phi_i^*}{k - \psi_i} \frac{3\psi_i^* - (3k - \alpha_L)}{4} + 2\eta(\psi_i - \phi_i^*) = 0.$$

(A1.4)

To see the relation between $\psi_i$ and $\phi^*_i$, we take the derivative against (A1.4) with respect to $\psi_i$ at both sides of (A1.4). We obtain the following:

$$\left[ \frac{3(\alpha_L - k + \phi_i^*) + 3\psi_i^* - (3k - \alpha_L)}{4(k - \psi_i)} + 2\eta \right] \frac{\partial \psi_i^*}{\partial \phi_i} = \frac{\alpha_L - k + \phi_i^*}{(k - \psi_i)^2} \frac{3\psi_i^* - (3k - \alpha_L)}{4} < 0.$$

(A1.5)

Since $\partial \psi_i^*/\partial \phi_i > 0$ at $\phi_i = (3k - \alpha_L)/3$ but $\partial \psi_i^*/\partial \phi_i < 0$ at $\phi_i = \psi_i$, $\phi^*_i \in ((3k - \alpha_L)/3, \psi_i)$. The profit

$$\Psi^*_j^i = \frac{k - \phi_i^* (\alpha_L - k + \phi_i^*)^2}{k - \psi_i} - \eta(\psi_i - \phi_i^*)^2, \quad \Psi^*_j^j = \frac{(\alpha_L - k + \phi_i^*)^2}{4}.$$

(A1.6)

(iii) If $\psi_i > (3k - \alpha_L)/3$ and $\psi_i \geq (3k - \alpha_L)/3$, or $\psi_i > (3k - \alpha_L)/3 > \psi_i$ but $\psi_i + 2\phi_j$ is sufficiently large, then the equilibrium allocations are solved by

$$\frac{\partial \Psi^*_j}{\partial \phi_i} = 3k - \alpha_L - (2\phi_i + \phi_j^*) + 2\eta(\psi_i - \phi_i^*) = 0.$$

(A1.7)

From Equation (A1.7) for both firms, we can solve the equilibrium allocation

$$\phi^*_i = \frac{\eta}{3 + 2\eta} (\psi_i + \phi_i) + \frac{\eta}{1 + 2\eta} (\psi_i - \phi_i) + \frac{1}{3 + 2\eta} (3k - \alpha_L).$$

(A1.8)
Similar to (A1.5), we can obtain from (A1.8) the relation between $\psi_i$ and $\phi_i^*$:

$$\frac{d\psi_i^*}{d\psi_i} = \frac{\eta}{3+2\eta} - \frac{\eta}{1+2\eta} = -\frac{2}{3+2\eta} \eta < 0. \quad (A1.9)$$

Finally, the profit $\Psi^*_2 = (k - \phi_i)(\alpha_i - 2k +\phi_i^* + \gamma) - \eta(\psi_i - \phi_i^*)$.

**Proof of Proposition 3 on First-Period Price.**

(P4) $p_i^* (\alpha_i) = \arg \max \pi^i(\alpha_i) = p_i^0d_i^1(p_i^1, p_i^1 | \alpha_i) + \delta \Psi^*_2(\alpha_i),$ $i \neq i$

where $d_i^1(p_i^1, p_i^1 | \alpha_i)$ is given by Equation (2), $\Psi^*_2(\alpha)$ are solved from (P3).

We first show that $p_i^1 (\alpha_i) \geq \alpha_i - 2k$. If firm $i$ lowers $p_i$ below $\alpha_i - 2k$, $d_i^1$ remains at $k$ due to capacity constraint. As a result, $k_i$ and $\Psi_i$ remain the same, but $\pi_i^1(\alpha_i)$ is lower with a smaller $p_i^1$. Thus, firm $i$'s total profit ($\pi^i$) would decrease with the price decrease.

Assuming that $p_i^1 = \alpha_i - 2k$, we consider a small increase of $p_i$ from $\alpha_i - 2k$. Then $d_i^1 = \alpha_i - k - p_i^1 < k$. We first calculate the impact on the firm's first-period profit

$$\frac{d\pi_i^1}{dp_i^1}(p_i^1 = \alpha_i - 2k) = (\alpha_i - k - 2p_i^1 | p_i^1 = \alpha_i - 2k) = 3k - \alpha_i < 0. \quad (A1.10)$$

We now calculate the impact of price increase on second-period profit $\Psi^*_2$, following (A1.3)–(A1.9) for different values of $\psi_i$ and $\phi_i$. Since $d_i^1 = d_i^1 = k$ and the reward amount $\phi_i = R_k$, the condition $\psi_i \leq (3k - \alpha_i)/3$ is equivalent to $R_i \leq (3k - \alpha_i)/3$.

(i) If $R_j \leq R_i \leq (3k - \alpha_i)/(3k)$, from (A1.3) we have $\Psi^*_2 = (\alpha_i - k + R_i, k)^2/4$.

Then

$$\frac{\partial \Psi^*_2}{\partial p_i^1} = \frac{\partial \psi_i^*}{\partial p_i^1} - \frac{3k - \alpha_i - 3kR_k \alpha_i - k + k}{4} = R_i < 0.$$

If $R_j \leq R_i \leq (3k - \alpha_i)/(3k)$, from (A1.3) we have $\Psi^*_2 = (\alpha_i - k + R_i, k)^2/4$. Then

$$\frac{\partial \Psi^*_2}{\partial p_i^1} = \frac{\partial \psi_i^*}{\partial p_i^1} = R_i \frac{\partial d_i^1}{\partial p_i^1} = 0.$$

Therefore, if $R_j \leq (3k - \alpha_i)/(3k)$ and $R_i \leq (3k - \alpha_i)/(3k)$, we have $\Psi^*_2 / \partial p_i^1 \leq 0$. Together with (A1.10),

$$\frac{\partial \psi_i^*}{\partial p_i^1} = 3k - \alpha_i - 3kR_k \alpha_i - k + k \frac{(R_i, k < 0).}{4}$$

Therefore, $p_i^* \geq \alpha_i - 2k$. (ii) If $R_j \geq (3k - \alpha_i)/(3k) > R_i$ and $R_i + 2R_j < (3k - \alpha_i)/k$, then $\Psi^*_2$ is given by (A1.6). Applying the Envelope Theory,

$$\frac{\partial \Psi^*_2}{\partial p_i^1} = \frac{\partial \psi_i^*}{\partial p_i^1} + \frac{R_i d_i^1}{\partial p_i^1} = 2\eta R_i (\psi_i - \phi_i^*) = 2\eta R_i (R_i, k - \phi_i^*) > 0.$$

The impact of the increase of $p_i^1$ on $i$'s total profit

$$\frac{\partial \pi_i}{dp_i^1} = \frac{\partial \psi_i^*}{dp_i^1} + \frac{\partial \psi_i^*}{dp_i^1} = (3k - \alpha_i) + 2\eta R_i (R_i, k - \phi_i^*). \quad (A1.12)$$

If $R_j > (3k - \alpha_i)/(3k) > R_i$ and $2R_i + R_j < (3k - \alpha_i)/k$, then

$$\Psi^*_2 = (\alpha_i - k + \phi_i^*)^2/4$$

depends on $\psi_i$ only through $\phi_i$. The impact of the increase of $p_i^1$ on firm $i$'s total profit

$$\frac{\partial \pi_i}{dp_i^1} = \frac{\partial \psi_i^*}{dp_i^1} + \frac{\partial \psi_i^*}{dp_i^1} = -(\alpha_i - 2k) + \frac{\partial R_i (\alpha_i, k + \phi_i^*)}{2 \partial \phi_i^*}.$$

From (A1.5), we know that $\partial \psi_i^*/\partial \phi_i^* < 0$. Therefore, in both (A1.12) and (A1.13), when the market demand $\alpha_i$ is close to $3k$ and rewards are large, an increase of $p_i$ above $\alpha_i - 2k$ could lead to an increase of total profits.

(iii) $R_i > R_j \geq (3k - \alpha_i)/(3k)$, or $R_i > R_j \geq (3k - \alpha_i)/(3k)$. In this case,

$$\frac{\partial \pi_i}{dp_i^1} = \frac{\partial \psi_i^*}{dp_i^1} + \frac{\partial \psi_i^*}{dp_i^1} = -(\alpha_i - 3k) - \delta R_i (\alpha_i, R_i, k - \phi_i^*). \quad (A1.14)$$

From Profit Function (A1.9), it is clear that $\partial \pi^*_2 / \partial \psi_i < 0$. Therefore, as in (A1.12) and (A1.13), $\partial \pi^* / \partial p_i$ can be positive when rewards are large and $\psi_i$ is close to $3k$.

**Proof of Proposition 4 on Reward Decisions.** Our proof for the result $R_i^* = (3k - \alpha_i)/(3k)$ proceeds in two steps. First, we show that, within the range $R_i \leq (3k - \alpha_i)/(3k)$ and $R_j \leq (3k - \alpha_i)/(3k)$, the equilibrium reward amount is $R_i^* = R_j^* = (3k - \alpha_i)/(3k)$. Second, we show that no firm can do better by unilaterally increasing rewards from $(3k - \alpha_i)/(3k)$.

We begin with the rewards $R_i \leq (3k - \alpha_i)/(3k)$ and $R_j \leq (3k - \alpha_i)/(3k)$. From Equation (A1.11), we know that in the first period, $p_i^1 = p_i^1 = \alpha_i - 2k$ and $k_i^1 = k_i^1 = k$. From (A1.3), we know that in the second period, $\phi_i^* = \psi_i$ and $\phi_i^* = \psi_i$. We can then write firms' reward-decision problem as follows:

(PA1.1) $R_i^* = \arg \max_{R_i \leq (3k - \alpha_i)/(3k)} \psi_i$,

$$\begin{cases}
\frac{k(\alpha_i - 2k)}{k - r} + \frac{R_j (\alpha_i - k + R_j, k)^2}{4}, & \text{if } R_i > R_j;
\frac{k(\alpha_i - 2k)}{4} + \frac{(\alpha_i - k + R_j, k)^2}{4}, & \text{if } R_j \leq R_i.
\end{cases}$$

Since $\pi^*_2 = [(k - R_j, k)/(k - R_j)]((\alpha_i - k + R_i, k)^2/4)$ reaches maximum at $R_i = (3k - \alpha_i)/(3k)$, we can conclude that, given $R_i \leq (3k - \alpha_i)/(3k)$ and $R_j \leq (3k - \alpha_i)/(3k)$, the equilibrium rewards $R_i^* = R_j^* = (3k - \alpha_i)/(3k)$. Under this reward amount, in the second period the allocated capacity for rewards $R_i^* = R_j^* = (3k - \alpha_i)/(3k)$. For both firms, $k_i^1 = \alpha_i/3$, $\Psi^*_2 = \alpha_i^2/9$, and $\pi^* = \pi^* = (\alpha_i - 2k) + \alpha_i^2/9$.

Now we show that neither firm can be better off by unilaterally increasing rewards. To demonstrate, we let $R_i = (3k - \alpha_i)/(3k)$ fixed and consider a small increase of $R_i$ above $(3k - \alpha_i)/(3k)$. First, as we have shown earlier, in the second period the capacity allocated for rewards will be $\phi_i^* > (3k - \alpha_i)/(3k) \geq \phi_i$. The available capacity in the second period will be $k_i^1 \leq \alpha_i/3 \leq k_i^1$ and hence the second-period
profit $\Psi_2^* < \alpha_i^2 / 9$. Second, based on Equation (A1.14), when $\eta$ is sufficiently small, firm $i$'s profit $\pi_i^* = k(\alpha_i - 2k)$. Otherwise, $\pi_i^* < k(\alpha_i - 2k)$. Overall, $\pi_i^* \leq k(\alpha_i - 2k)$. Combining the above discussion on firm $i$'s profit in both periods, we can conclude that $\pi^* < k(\alpha_i - 2k) + \alpha_i^2 / 9$ when firm $i$ unilaterally increases $R_i$ above $(3k - \alpha_i)/(3k)$. Therefore, no firm will deviate from reward amount $(3k - \alpha_i)/(3k)$. In the equilibrium,

$$R_i^* = R_j^* = \frac{3k - \alpha_i}{3k}, \quad r_i^* = r_j^* = \frac{3k - \alpha_k}{3k}, \quad \pi^* = k(\alpha_i - 2k) + \frac{\alpha_i^2}{9},$$

(A1.15)

**Proof of Proposition 5 on Initial Capacity Size.** We now extend the model and formulate the firms’ decisions on the initial capacity $(k_i)$.

(PA1.2) $k_i^* = \arg \max \pi_i = \pi_i^*(k_i, k_j, \alpha_i + \delta \Psi_2^*(\alpha_i)).$

**Capacity Without Reward Programs.** Without reward programs, $\pi_i^*(k_i, k_j, \alpha_i)$ and $\Psi_2^*(\alpha_i)$ are independent. Since firms decide the equilibrium capacity to maximize $\pi_i^*(k_i, k_j, \alpha_i) + \delta \pi_i^*(k_i, k_j, \alpha_i)$, the equilibrium capacity $k_i^*$ should be within the interval $(Q(\alpha_i, c_i))$, where $Q(\alpha_i, c_i)$ is the equilibrium capacity for $\pi_i^*(k_i, k_j, \alpha_i)$ only, and $Q(\alpha_i, c_i)$ for $\pi_i^*(k_i, k_j, \alpha_i)$ only. Capacity $k_i^*$ should be closer to $Q(\alpha_i, c_i)$ with smaller $\delta$.

**Capacity with Reward Programs.** Following the same argument that leads to (A1.15), we have

$$r_i^* = R_i^* = \frac{3k - \alpha_i}{3k} \quad \text{and} \quad \Psi_2^* = \frac{\alpha_i^2}{9}, \quad (i = a \text{ or } b).$$

(A1.16)

Substituting (A1.16) into (PA1.2), we find that firm $i$ decides $k_i$ to maximize $\pi_i^*(k_i, k_j, \alpha_i)$ only. Following Kreps and Scheinkman (1983), the capacity should be at the Cournot level $Q(\alpha_i, c_i)$.

$$k_i^* = \frac{Q(\alpha_i, c_i)}{3k} = Q(\alpha_i, c_i).$$

(A1.17)

Substituting (A1.17) into (A1.16), we have reward amount

$$r_i^* = R_i^* = 1 - \frac{\alpha_i}{3Q(\alpha_i, c_i)}.$$ 

(A1.18)

**Appendix 2. Analyzing the Generalized Model**

In this appendix, we analyze the generalized model discussed in §4. Consistent with the results obtained in the basic model, we assume $R_i \leq (3k - \alpha_i)/(3k)$ and $R_j \leq (3k - \alpha_j)/(3k)$.

**Proof of Proposition 6 on Allocating Capacity for Rewards when $\alpha_i = \alpha_j$.** The problem formulation (PA2.1) is similar to (P3).

(PA2.1) $\phi_i^*(\alpha_i) = \arg \max \psi_i = \pi_i = [\alpha_i - (k - \phi_i) - (k - \phi_j)] \cdot (k - \phi_i) - \eta(\psi_i - \phi_i)^2.$

The first-order derivative is

$$\frac{\partial \psi_i^*}{\partial \phi_i} = -[(\alpha_i - 3k) + (2\phi_i + \phi_j)] + 2\eta(\psi_i - \phi_i), \quad (i = a, b).$$

The optimal $\phi_i$ can be either zero or positive.

(i) If $(\partial \psi^*_i / \partial \phi_i)(\phi_i = 0) = -2(\alpha_i - 3k) / 2\eta < 0$, i.e.,

$$2\eta\phi_i < \alpha_i - 3k, \quad (i = a, b),$$

(A2.2) then $\phi_i^*(\alpha_j = \alpha_i) = 0$ for both firms. Substituting $\phi_i^*(\alpha_j = \alpha_i) = 0$ into (PA2.1), we have

$$\psi_i^* = (\alpha_i - 2k)k - \eta(R_i d_i)^2.$$ 

(A2.3)

(ii) If (A2.2) does not hold, we solve the first-order condition of (A2.1), $\partial \psi_i^* / \partial \phi_i = 0$.

$$2 + \eta(\phi_i^*_1 < \alpha_i - 3k), \quad (i, j = a, b, i \neq j).$$

(A2.4)

Solving (A2.4) simultaneously for both firms, we can obtain

$$\phi_i^* = \frac{\eta}{3 + 2\eta} (\psi_i + \phi_i) + \frac{\eta}{1 + 2\eta} (\psi_i - \phi_i) - \frac{1}{3 + 2\eta} (\alpha_i - 3k),$$

(i, j = a, b, i \neq j). (A2.5)

We can obtain the equilibrium profits by substituting (A2.5) into the firms’ profit functions.

**Proof of Proposition 7 on First-Period Price Competition.**

(P6) $p_i^*(\alpha_i) = \arg \max \pi_i^*(\alpha_i)$

$$= p_i^1(\alpha_i) = \frac{p_i^1 | \alpha_i}{\bar{\eta} + \partial \psi_i^* / \partial \alpha_i} + \delta \psi_i^*(\alpha_i) \cdot (1 - \rho_2 \psi_i^*(\alpha_i)).$$

(A6.1)

When $\alpha_i = \alpha_j$, and $R_i = R_j = R \leq (3k - \alpha_i)/(3k)$, according to Equation (A1.3), we have $\psi_i^*(\alpha_i, R) = (\alpha_i - k + Rk)^2 / 4$. The impact of an increase of $p_i^1$ on $\psi_i^*$, $\partial \psi_i^* / \partial p_i^1 = 0$.

When $\alpha_i = \alpha_j$, we showed in (A2.1)–(A2.5) that $\phi_i^*$ can be zero or positive.

(i) If $2\eta R k - \alpha_i < 3k$ as in (A2.2), then $\phi_i^* = 0$. In this case,

$$\frac{\partial \psi_i^*(\alpha_i)}{\partial p_i^1} = -[2\eta Rk] = 2\eta R^2 k.$$ 

(A7.1)

The impact of an increase of $p_i^1$ on firm $i$’s total profit

$$\frac{\partial \pi_i^*(\alpha_i)}{\partial p_i^1} = \frac{\partial \psi_i^*(\alpha_i)}{\partial p_i^1} + \delta \psi_i^*(\alpha_i) \cdot \frac{\partial \psi_i^*(\alpha_i)}{\partial p_i^1}$$

$$= (3k - \alpha_i) + \delta (2\rho_2 \eta Rk^2 + (1 - \rho_2) x) \times 0 < 3k - \alpha_i + \delta \rho_2 (R_i - 3k) < 0.$$ 

(A2.8)

Therefore, $p_i^1(\alpha_i) = \alpha_i - 2k$. 

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(ii) If \( 2\eta Rk > \alpha_H - 3k \), then \( \phi^*_i \) is given by (A2.5). Substituting \( \psi = \psi_j = Rk \) into Equation (A2.5), we have \( \phi^*_i = (2\eta Rk - (\alpha_H - 3k))/(3 + 2\eta) \) and \( \phi_i/\phi_j \) for any price \( p \),

\[
\begin{align*}
\partial \Psi_1(\alpha_N) &= \left[ \frac{\partial \psi_1(\alpha_H)}{\partial \phi_i} + \frac{\partial \psi_1(\alpha_H)}{\partial \phi_j} \right] \partial \phi_i \\
&= - \left[ \frac{\partial \psi_1(\alpha_H)}{\partial \phi_i} + \frac{\partial \psi_1(\alpha_H)}{\partial \phi_j} \right] \\
&= 2\eta(Rk - \phi_i)R + (k - \phi_i)/(3 + 2\eta)(1 + 2\eta).
\end{align*}
\]

Now we can calculate the overall profit effect of the price increase:

\[
\begin{align*}
\partial \psi_1(\alpha_H) &= \partial \psi_1(\alpha_H) + \partial p_i \partial \psi_1(\alpha_H) \frac{\partial \psi_1(\alpha_H)}{\partial p_i} + \partial p_j \partial \psi_1(\alpha_H) \frac{\partial \psi_1(\alpha_H)}{\partial p_j} \\
&= -(\alpha_H - 3k) + \rho_2 \frac{2\eta Rk - \phi_i)R}{(3 + 2\eta)(1 + 2\eta)} + 0 \\
&= -(\alpha_H - 3k) \left[ 1 - \rho_2 \frac{2\eta Rk - \phi_i)R}{(3 + 2\eta)(1 + 2\eta)} \right] \\
&\quad + \rho_2 \frac{2\eta Rk}{3 + 2\eta} \left[ \frac{3R + (3 + 2\eta - 2\eta R - \phi_i)R}{(3 + 2\eta)(1 + 2\eta)} \right].
\end{align*}
\]

The above expression (A2.10) can be positive when \( \alpha_H - 3k \) is small.

Proof of Proposition 7(2). Low Demand in the First Period. We first characterize the mixed-strategy equilibrium with rewards, \( \Phi_{1,\delta}(p) \). We then compare the price distribution with that without rewards. When \( 2\eta Rk < \alpha_H - 3k \), \( r^*_i(\alpha_H) = 0 \), and according to (A2.2), \( \Psi_2(\alpha_H) = (\alpha_H - 2k)k - \eta(Rd_i)^2 \).

(i) The upper bound is \( \hat{p}_{1,k} \). If a firm charges \( \hat{p}_{1,k} \), the firm will be undersold by its competitor with probability equal to one. The firm’s expected profit at the upper bound is

\[
E(\pi(\hat{p}_{1,k})) = \hat{p}_{1,k}(\alpha_L - \hat{p}_{1,k} - k) \\
+ \delta \rho_2 \left[ k(\alpha_H - 2k) - \eta R^2(\alpha_L - \hat{p}_{1,k} - k)^2 \right] \\
+ \delta(1 - \rho_2) \frac{\alpha_L - k + Rk}{4}.
\]

(ii) The lower bound is \( p_{1,k} \). By charging \( p_{1,k} \), the firm will always undersell its competitor.

\[
E(\pi(p_{1,k})) = p_{1,k} + \delta(1 - \rho_2)E_{\hat{p}}(p_{1,k}) \frac{(\alpha_L - k + Rk)^2}{4} \\
+ \delta \rho_2 k(\alpha_H - 2k) - \delta \rho_2 \eta(Rk)^2,
\]

(A2.14)

where

\[
E_{\hat{p}}(p) = \frac{k - R}{R (\alpha_k - \hat{p} - k)} \left[ \hat{p} > \hat{p}, \right. \quad i \neq j,
\]

(A2.15)

denotes the mean value of the capacity ratio computed according to the price distribution \( \Phi_{1,\delta}(p) \) over the price range \([\hat{p}, \pi_{1,k}]\). We can then solve \( p_{1,k} \) from \( E(\pi(p_{1,k})) = E(\pi(\alpha_L)) \).

(iii) To derive the price distribution, consider firm \( i \)'s profit when it sets the price at \( p \).

\[
\begin{align*}
E(\pi(p)) = \Phi_{1,\delta}(p) \left[ p(\alpha_L - p - k) + \delta(1 - \rho_2) \frac{(\alpha_L - k + Rk)^2}{4} \right. \\
+ \delta \rho_2 k(\alpha_H - 2k) - \delta \rho_2 \eta R^2(\alpha_L - p - k)^2 \right] \\
+ \frac{1}{(1 - \rho_2)} \left[ p + \delta \rho_2 k(\alpha_H - 2k) - \delta \rho_2 \eta R^2 \right] \\
+ \delta(1 - \rho_2) \frac{\alpha_L - k + Rk}{2} E_{\hat{p}}(p).
\end{align*}
\]

We solve the price distribution by equating the above equation to the firm’s expected profit, \( E(\pi(p)) = E(\pi(\alpha_L)) \), given by (A2.13).

\[
\Phi_{1,\delta}(p) = \left[ \frac{p - (\alpha_L - k)^2}{4} \frac{1}{1 + \delta \eta R^2} - \delta \rho_2 \eta R^2 \right] \\
\cdot \left[ p(p + 2k - \alpha_L) - \delta \rho_2 \eta R^2 \right] \\
\cdot (k^2 - (\alpha_L - p)^2) - H\right]^{-1},
\]

(A2.16)

where \( p \in [p_{1,k}, \pi_{1,k}] \), \( H = (1 - \rho_2)(\alpha_L - k + Rk)^2/4(1 - \rho_2) \).

To prove that price increases with the reward program, we next show that the price distribution function \( \Phi_{1,\delta}(p) < \Phi_0(p) \) for any price \( p < \bar{p} = (\alpha_L - k)/2 \), where \( \bar{p} = (\alpha_L - k)/2 \) is the upper bound of the distribution \( \Phi_0(p) \) given by Equation (3) of the benchmark case. Since \( H > 0 \), and in the numerator of Equation (A2.16),

\[
\begin{align*}
\frac{(\alpha_L - k)^2}{4} \frac{1}{1 + \delta \eta R^2} - \delta \rho_2 \eta R^2 k^2 \\
= - \frac{(\alpha_L - k)^2}{4} \frac{1}{1 + \delta \eta R^2} + \frac{(\alpha_L - k)^2}{4} \frac{\delta \rho_2 \eta R^2}{1 + \delta \eta R^2} \\
= - \delta \rho_2 \eta R^2 \left[ k^2 - \left( \alpha_L - \alpha_L - k \frac{\alpha_L - k}{2} \right)^2 \right] - \frac{(\alpha_L - k)^2}{4} \delta \rho_2 \eta R^2 \\
= - \frac{(\alpha_L - k)^2}{4} \frac{\delta \rho_2 \eta R^2}{1 + \delta \eta R^2} \\
= - \frac{(\alpha_L - k)^2}{4} \frac{\delta \rho_2 \eta R^2}{1 + \delta \eta R^2} \\
< - \frac{(\alpha_L - k)^2}{4} \frac{\delta \rho_2 \eta R^2}{1 + \delta \eta R^2},
\end{align*}
\]

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we have $\Phi_{ij}(p) < \Phi_{ij}(p^*)$ for any price $p \leq (\alpha_i - k)/2$. Thus, the price distribution with reward programs first-order stochastically dominates the distribution without reward programs.

Proof of Proposition 8 on Reward Amount. At the beginning of game, firms decide the reward amount to maximize the expected profits.

\[(P7)\quad R^*_i = \arg \max_{R_i} E\pi_{i} \]

\[= \rho_i \rho_j \pi_{i}(R_i | \alpha_i, \alpha_j) + \rho_i (1 - \rho_j) \pi_{i}(R_i | \alpha_i, \alpha_j)
+ (1 - \rho_i) \rho_j \pi_{i}(R_i | \alpha_i, \alpha_j)
+ (1 - \rho_i) (1 - \rho_j) \pi_{i}(R_i | \alpha_i, \alpha_j),\]

where $\pi_{i}(R_i | \alpha_i, \alpha_j) = E\pi_{i}(R_i | \alpha_i, \alpha_j) + \delta E\pi_{i}(R_i | \alpha_i, \alpha_j)$. To prove that $R^*_i > 0$, we consider a unilateral positive deviation for $R_i$ at $R_i = R_j = 0$, and show that $(\delta E\pi_{i}/\delta R_i)(R_i = 0, R_j = 0) > 0$.

Second-Period Profit with Positive $R_i$ and $R_j = 0$.\n
(i) $\alpha_2 = \alpha_L$. Since $R_i$ is small and Condition (A2.2) is satisfied, $r^*_i(\alpha_j) = 0$. From (A2.18),

\[
\Psi_i^*(\alpha_i) = (\alpha_i - 2k)k - \eta(R_i d_i^*)^2 \quad \text{and}
\Psi_i^*(\alpha_i) = (\alpha_i - 2k)k.
\]

From (A2.17), we can obtain

\[
\frac{\partial \Psi_i^*(\alpha_i)}{\partial R_i} = -\eta(R_i^2(2k) - 1) = -2\eta d_i^* R_i^2 \approx 0.
\]

(ii) $\alpha_2 = \alpha_L$. Since $R_i$ is small, we have $r^*_i = R_i > r^*_j = R_j = 0$ and $k^*_i < k^*_j = k$. As a result,

\[
\Psi_i^*(\alpha_i) = \left(\frac{k - R_i d_i^*}{k} \right) \left(\alpha_i - k + R_i d_i^* \right)^2 \quad \text{and}
\Psi_i^*(\alpha_i) = \left(\frac{k - R_i d_i^*}{k} \right) \left(\alpha_i - k + R_i d_i^* \right)^2.
\]

From (A2.19), we can obtain

\[
\frac{\partial \Psi_i^*(\alpha_i)}{\partial R_i} = \frac{R_i (\alpha_i - k + R_i d_i^*) (\alpha_i - k + 3k + 3R_i d_i^*)}{k} \approx 0.
\]

First-Period Profit and Total Profits with Positive $R_i$ and $R_j = 0$.\n
(i) $\alpha_1 = \alpha_H$. First we establish that $p_i^*(\alpha_H) = \alpha_H - 2k$. Following the same argument as in Appendix 1, we can claim that firm $i$’s total profit ($\pi^*$) should decrease with a unilateral price decrease below $\alpha_H - 2k$. Consider a small increase of $d_i^*$ above $\alpha_H - 2k$. Since $R_i$ is very small and $R_j = 0$, the results (A2.6)–(A2.8) are applicable. Therefore, $p_i^*(\alpha_H) = \alpha_H - 2k$. The firm’s demand $d_i^* = k$ and profit $\pi^*_i(\alpha_H) = (\alpha_H - 2k)k$.

Substituting $d_i^* = k$ into (A2.17) and (A2.19), we can obtain $\partial \Psi_i^*(\alpha_h)/\partial R_i \approx 0$, and $\partial \Psi_i^*(\alpha_h)/\partial R_i \approx (\alpha_h - k) \cdot (3k - \alpha_h)/4$, respectively. Then, the marginal impact of reward $R_i$ on firm $i$’s profit when $\alpha_1 = \alpha_H$ is

\[
\frac{\partial \pi^*_i(\alpha_H)}{\partial R_i} (R_i = R_j = 0) = \delta (1 - \rho_2) \frac{(\alpha_i - k)(3k - \alpha_i)}{4}.
\]

(ii) $\alpha_1 = \alpha_L$. In this case, there does not exist any pure-strategy equilibrium. (Formal proofs are available upon request.) To characterize the price distribution, we let $p_i^*, p_i^*$ be the lower and upper bound of firm $i$’s price distribution, and $p_j^*, p_j^*$ be the lower and upper bound of firm $j$’s price distribution. Since two firms have identical first-period profit functions, and according to (A2.17) and (A2.19), $\pi^*_i$ decreases with $p_i^*$, overall firm $j$ has more incentive to lower price than firm $i$. Therefore, $p_i^* \leq p_j^*$, and, given the nonexistence of a pure-strategy equilibrium, $p_j^*$ should not be a mass point in firm $i$’s price distribution. Considering

\[
\pi^*_i(p_j^* | \alpha_i) = (\alpha_i - k - p_j^*) + \delta \rho_2 (\alpha_H - 2k)k
+ \delta (1 - \rho_2) \frac{(\alpha_i - k + R_j k)^2}{4}.
\]

We can compute $p_j^* = (\alpha_i - k)/2$ and

\[
\pi^*_i(\alpha_i) = \frac{(\alpha_i - k)^2}{4} + \delta \rho_2 (\alpha_H - 2k)k
+ \delta (1 - \rho_2) \frac{(\alpha_i - k + R_j k)^2}{4}.
\]

We now show that $p_i^*, p_j^*, p_j^*$, and neither is a mass point in the distribution. If $p_i^* > p_j^*$, then firm $i$ can increase profit by moving density in $(\pi^*_i, \pi^*_j)$ to a point slightly below $p_j^*$. If $p_i^*$ is a mass point, then firm $i$ can increase profit by naming a mass point slightly below $p_j^*$. An implication of the above result is that when a firm’s price is at $p_j^*$, the firm’s demand will be $k$ units. Further, the nature of mixed-strategy equilibrium implies that $\pi^*_i(\alpha_i) = p_i^* k = p_j^* k = \pi^*_i(\alpha_1)$. Therefore, from (A2.23),

\[
\pi^*_i(\alpha_1) (R_i = R_j = 0) = \delta (1 - \rho_2) \frac{k(\alpha_i - k)}{2}.
\]

Combining (A2.21) and (A2.24), we can conclude that

\[
\frac{\partial \pi^*_i}{\partial R_i} (R_i = R_j = 0) = \rho_1 \frac{\partial E\pi_{i+1}(\alpha_i - \alpha_H)}{\partial R_i} + (1 - \rho_1) \frac{\partial E\pi_{i+1}(\alpha_i - \alpha_L)}{\partial R_i}
= \rho_1 \delta (1 - \rho_2) \frac{(\alpha_i - k)(3k - \alpha_i)}{4}
+ (1 - \rho_1) \delta (1 - \rho_2) \frac{(\alpha_i - k)}{2} > 0.
\]

Therefore, the equilibrium reward amount $R^*_i > 0$.

Proof of Proposition 9 on the Impact of Demand Cannibalization. To model the cannibalization effect of rewards $R_i d_i^*$, we let

\[
a_{C_i} = a_i - \lambda_i(r_i d_i^* + r_j d_j^*), \quad (s = H \text{ or } L),
\]

where $\lambda_i$ represents the extent of overlap with primary demand when $\alpha_2 = \alpha_s$. To show the firms’ incentives in offering reward programs, we let $R_i = R_j = 0$ and study firm $i$’s profit change from a small increase of $R_j$ by (A2.25).
Combining (A2.29) and (A2.31), we can conclude that $0$ can derive the expected profit as:

$$
\Psi_2^r (\alpha_l) = \frac{k - R_i d_i^r (\alpha_l - \lambda_i R_i d_i^r + k + R_i d_i^r)^2}{k}.
$$

Then the impact of a positive $R_i$ should be revised accordingly.

$$
\partial \Psi_2^r (\alpha_l) = (\alpha_l - \lambda_i R_i d_i^r - k + R_i d_i^r)^2.
$$

(A2.27)

With a very small $R_i$,

$$
\frac{\partial \Psi_2^r (\alpha_l)}{\partial R_i^r} = \frac{R_i (\alpha_l - k + (1 - \lambda_i) R_i d_i^r)}{4}.
$$

(A2.28)

First-Period Profit and Total Profits with Positive $R_i$ and $R_j = 0$.

(i) $\alpha_2 = \alpha_{H_i}$. Similar to the case of no cannibalization, we have $p_i^*(\alpha_2) = \alpha_{H_i} - 2k$. Similar to (A2.21), we can compute the effect of a positive $R_i$ on firm $i$'s profit when $\alpha_i = \alpha_{H_i}$.

$$
\frac{\partial \pi^t (\alpha_{H_i})}{\partial R_i^r} (R_i = 0) = 0 + 0 + \delta (1 - \rho_2) (\alpha_{H_i} - k) (3k - \alpha_{L_i} - 2 \lambda_i k).
$$

(A2.29)

(ii) $\alpha_1 = \alpha_L$. Similar to the case of no cannibalization, we can derive the expected profit as:

$$
\pi^t (\alpha_L) = \left( \frac{\alpha_L - k}{2} \right)^2 + \delta \rho_2 (\alpha_{H_i} - 2k) k + \delta (1 - \rho_2) (\alpha_L - \lambda_i R_i k - k + R_i k)^2.
$$

(A2.30)

Then the impact of a positive $R_i$ on firm $i$'s profit when $\alpha_i = \alpha_L$ is

$$
\frac{\partial \pi^t (\alpha_L)}{\partial R_i^r} (R_i = 0) = \delta (1 - \rho_2) 2k (1 - \lambda_i) (\alpha_L - k) > 0.
$$

(A2.31)

Combining (A2.29) and (A2.31), we can conclude that

$$
\frac{\partial E\pi_i}{\partial R_i^r} (R_i = 0) = \rho_1 \delta (1 - \rho_2) (\alpha_L - k) (3k - \alpha_L - 2 \lambda_i k) + (1 - \rho_1) \delta (1 - \rho_2) 2k (1 - \lambda_i) (\alpha_L - k) + \delta (1 - \rho_2) (\alpha_L - k) (2k (1 - \lambda_i) - \rho_1 (\alpha_L - k)).
$$

(A2.32)

For (A2.32) to be positive, we need

$$
\lambda_i < 1 - \frac{\alpha_L - k}{2k} \rho_1.
$$

(A2.33)

References


