A Convolution Model for Heart Rate Prediction in Physical Exercise

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Abstract: During exercise, heart rate has proven to be a good measure in planning workouts. It is not only simple to measure but also well understood and has been used for many years for workout planning. To use heart rate to control physical exercise, a model which predicts future heart rate dependent on a given strain can be utilized. In this paper, we present a mathematical model based on convolution for predicting the heart rate response to strain with four physiologically explainable parameters. This model is based on the general idea of the Fitness-Fatigue model for performance analysis, but is revised here for heart rate analysis. Comparisons show that the Convolution model can compete with other known heart rate models. Furthermore, this new model can be improved by reducing the number of parameters. The remaining parameter seems to be a promising indicator of the actual subject's fitness.

1 INTRODUCTION

Exercising has a proven therapeutic effect on the cardiovascular system. To avoid overstrain, determining an optimal training dose is crucial. In general, heart rate prediction based on physical activity can be a useful tool in properly controlling and monitoring the strain that a smart training device imposes on a subject during exercise (Achten and Jeukendrup, 2003). Hence, accurately predicting heart rate from work load information is an essential part in models used for training control since too much and wrong exercising can do more harm than good.

If an accurate prediction shows a heart rate too high or an unexpected increase or decrease of the heart rate, workload can be reduced or improved in adequate time. Ignoring the limits of the physical capabilities will risks overtraining and will not only nullify the effect of the exercise but also reduce the subject's motivation (Lehmann et al., 1993; Smith, 2003). Any physical mobilization and training activity for a human subject must therefore be highly sensitive to the subject's physical capabilities and actual physical condition in order to be effective. This means that a trainer or therapist that plans the workout must be able to understand and predict with reasonable accuracy how the subject's cardiovascular system will respond to a certain exercise strain, e.g., by measuring and monitoring the subject's heart rate (Borresen and Lambert, 2008). Reliable prediction requires a model that establishes a functional relation between the strain to which the subject is exposed over time and the response of the cardiovascular system, as illustrated in Figure 1.

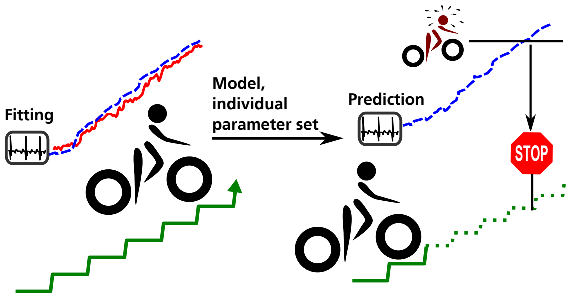

Figure 1: Overview of the fitting and prediction process. On the left-hand side, a heart rate model is fitted according to the measured heart rate of a subject and performed strain. With these individualized parameters, the model can then be used to predict heart rate for a given workload before the work commences and prevent exhaustion (right-hand side).

Suitable models depends on a preliminary fitting process where model specific parameters are adapted to the subject in order to fit a simulated heart rate to the measured heart rate based on some performed strain. After the fitting, the model can then be used to predict heart rate for a whole training session. This

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prediction could be helpful in planning the training beforehand since any crossing of the personal performance limit can be predetermined.

However, these models are often mathematical models with a number of parameters that can rarely be explained physiologically. Furthermore, a large number of parameters can lead to problems with computing time, error handling, and prediction instability. The paper presents a mathematical heart rate model where all four parameters have a physiological meaning. During the experiments, the number of parameters could be reduced down to one degree of freedom, leading to much more stability and a very fast computation.

The structure of the paper is as follows: In Section 2, the process of heart rate prediction is explained in general, followed by a brief overview of usual heart rate prediction models. In this context, the new Convolution model is presented in Section 3. In Section 4, data material and executed experiments are explained, followed by the presentation, evaluation and discussion of results. The paper is completed by a conclusion and an outline for future experiments in Section 5.

2 HEART RATE PREDICTION DURING EXERCISE

Usually, the human body does not adapt to strain immediately. The reaction is delayed, so heart rate increases after a certain time of physical activity, and regeneration in relaxation is also delayed which results in hysteresis. The adaption rate of these processes depends greatly on the specific person, and modeling requires individual adaptation. Each suitable model should therefore have at least one parameter that can account for this individual component.

In general, many heart rate models $\mathcal{M}$ can be considered as functions mapping all parameters $\vec{\alpha}$ required by the model, and a strain curve u to a prediction of a heart rate curve y:

$$\mathcal{M} : \mathcal{P} \times \mathbb{R}^* \longrightarrow \mathbb{R}^*,$$

where $\mathcal{P}$ is the parameter set, and both input (i.e., strain curve) as well as output (i.e., heart rate curve) are real time series, denoted by $\mathbb{R}^* := \bigcup_{n \in \mathbb{N}} \mathbb{R}^n$. The data can be assumed as an equidistant, discrete time series. The estimated heart rate at point of time t is labeled by $y(t) = \mathcal{M}(\vec{\alpha}, u)$, where $\vec{\alpha} \in \mathcal{P}$ is the parameter setting and $u = u_1, ..., u_t \in (\mathbb{R}^+)^*$ serves as the model input. In the conducted experiments, u is defined as a sequence of positive values and given by the considered workload. An additional constraint in computing $y(t)$ is added: only elements $u(s)$ with $s \leq t$ are allowed to enable real-time applications. In some models, the measured heart rate up to the actual point in time serves as an additional model input.

Within the last ten years, a variety of models for heart rate prediction have been discussed. Some typical mathematical concepts are systems of differential equations or variants of a Hammerstein model. While (Cheng et al., 2007) introduce a nonlinear state-space model to predict the heart rate behavior of a subject based on the running velocity on a treadmill, (Paradiso et al., 2013) use the same model to regulate the heart rate using a cyclic ergometer. Both models include nonlinear components to simulate changes in the organism due to long term exercise. The fuzzy Takagi-Sugeno model by (Mohammad et al., 2011) deals with 12 parameters and is commonly used for optimizing physical activity for elderly non-trained people. During cycling exercises it is used to control the power system which can regulate the amount of strain and hereby control the heart rate. Further model-based systems exist for both, running (Su et al., 2007; Su et al., 2010; Koenig et al., 2009) and cycling (Leitner et al., 2014; Le et al., 2008) on different training devices.

A linear time invariant (LTI) model from (Baig et al., 2010) can be used for this topic as well, but it is presented in literature explicitly for a *single-step* prediction. This means, only the next heart beat is predicted using preceding measurements of its specific workload. To use it for a whole session prediction, the heart rate must be estimated iteratively and a beforehand predicted heart rate must be used. The original model and its adjustment have four parameters to scale previously measured or predicted values of heart rate and strain.

Even fitness trackers or smartphone apps support their users with heart rate information and are usually able to inform the user about, e.g., an increasing heart rate. Exemplary, (Sumida et al., 2013) presents a method to estimate heart rate with a smartphone based on walking speed and acceleration. Here as well, heart rate is simulated on demand during the exercise.

So the most common applications for these models are automatic control systems, especially for treadmills or cycle ergometers. During this kind of exercise, a quick system response to actual heart rate is necessary but in this case it is not necessary to simulate or predict a whole training session in advance. Usually, only some seconds up to a few minutes are predicted.

Nevertheless, planning a training as a whole in advance might be important when doing outdoor activities where the subject has to deal with the actual environmental setting. In this case, knowing the limit beforehand is crucial if overtraining is to be avoided. The workload can then be optimized for a planned route, similar to the prediction of a velocity protocol in running as presented in (Brzostowski et al., 2013). The Convolution model presented here is able to predict a whole training session beforehand.

3 THE CONVOLUTION MODEL APPROACH

For the related task of predicting a measure for fitness in general, the Fitness-Fatigue model (Calvert et al., 1976) has been widely used since its first description in the early seventies. This model works with convolution to compute the actual prediction by using not only the last input value but also all previous input values in decreasing intensity. This method means that the shorter the time span between an input value and the current point in time, the stronger its influence on the computation of the currently computed output value. Its great advantage is therefore a weighted consideration of past strain with a slight effect on actual physiological response, performance in general or heart rate in particular. Because of the delayed reaction of human body to any strain, a model based on convolution seems to be promising for predicting the heart rate response to strain as well.

Here, elements from the estimated heart rate sequence y at time t follows:

$$y(t) = a_2 \cdot \left[\frac{1}{a_1} (u * e^{-\bullet/a_1})(t) \right]^{a_4} + a_3.$$

In contrast to the original Fitness-Fatigue model, the proposed model uses four parameters to improve adapting a strain value to a predicted heart rate. As our experiments show, these parameters allow this approach to successfully predict a heart rate curve from a strain curve. This parametrization is not simply a mathematical trick; each of these parameters has a direct physiological origin and meaning:

a_1 : *memory parameter* used for convolution. This parameter describes the effect of former strain on actual heart rate, i.e., how much influence does previous strain have.

a_2 : *impact parameter* used as a multiplicative factor. This parameter explains the impact of rising strain on heart rate (e.g., proportional or disproportional), i.e., it illustrates how strong the reaction to strain becomes and how steeply heart rate increases over time.

a_3 : *level parameter* used as an additive constant to lift the predicted heart rate up to a suitable level. Every subject has a specific resting heart rate, from which heart rate under strain ascends.

a_4 : *slope parameter* used as exponent. This allows a non-linear reaction of the heart rate to increasing strain near the personal performance limit. Hence this parameter can be used to refine the conceptually related impact parameter.

Our experiments show that the number of parameters can be reduced. A linkage was found between the memory and the impact parameter using a polynomial of the second degree, $a_1 = x_1 + x_2 \cdot a_2 + x_3 \cdot a_2^2$, with suitable values for x_i. Additionally, level and slope parameter a_3 and a_4 could be predefined, so that the arising model has only one degree of freedom left by use of a_2.

4 EXPERIMENTS

In this paper, the terms *fitting* and *prediction* (instead of *training set* and *test set* following machine learning phrases) are used. Nevertheless, fitting describes the direct fit of parameters to given data, while prediction makes use of these identified parameters without any changes and applies them to different given data of the same subject. A *training* in this context always refers to physical exercise, and a *test* or *protocol test* refers to standardized protocol exercise tests realized by a cycle ergometer.

The data used was obtained by two male volunteers doing a standardized test every two to four weeks during an approximate seven month period — a third one started later, so his total period was two months. The sports of the three volunteers is cycling and their tests were performed on the cycle ergometer "Cyclus 2" (RBM elektronik-automation GmbH, Germany). The protocols followed a step-size protocol: starting with 50 W, increased by 25 W every 3 minutes. The test protocol were examined until termination by exhaustion. 17 tests were collected altogether.

In general, heart rate models are fitted to a varying number of training sessions for one and the same person using Levenberg-Marquardt as suggested by (Busso et al., 1997). The individualized model can then be used to predict further sessions.

Session prediction is used for predicting a whole time series, i.e., to predict the heart rate curve for a given strain over a certain time, which usually is a whole training session. Especially for planning such a training session, it is important to assess the behavior of the heart rate to a given workload at a given time (Ludwig et al., 2015).

For measuring the quality and accuracy, the *root-mean-square error* (RMSE) is considered.

To prove the competitiveness of the Convolution model in relation to existing heart rate models, a special type of a cross-validation, namely *past only cross-validation*, is invented and performed:

In real usage, only past training sessions will be available for fitting. Unlike the common leave-one-out cross-validation (Refaeilzadeh et al., 2009), only training series up to one point of the past are used for fitting and heart rate is predicted for all training sessions in the future compared to the specified point in time. We call this a *past only cross-validation*, which is conducted for model comparison.

The Convolution model is evaluated here in comparison to the Takagi-Sugeno model (without feedback model control) and the adjusted LTI model, which have showed best results in previous studies on analytical non-machine learning models only (Ludwig et al., 2015). Additionally, a simple polynomial model is used as baseline: As described in (Ludwig et al., 2015) and (Füller et al., 2015), a polynomial model is suitable as a baseline scaling function for mapping any kind of input data (such as workload) to any kind of output data (such as heart rate). We use this baseline function to determine the fitting-quality without any physiological modeling.

In earlier experiments, the Convolution model was computed for some other data sets, e.g., in running, where it could compete with published models. But these data sets are not comparable for this investigation and therefore not considered in this paper.

While data sets are available for three persons with 4, 5, and 8 training sessions and a fitting was computed over at least two training sessions, the number of possible experiments results in 30, with respect to the time line. Exemplary for the subject with 5 training sessions, the fitting on the first two training sessions results in 3 data sets for prediction, fitting on the first three training sessions results in 2 prediction sets and fitting on all but the last training session leads to another prediction experiment.

To analyze possible dependencies and restrictions, several experiments are conducted for reducing the amount of parameters in the Convolution model step by step and dependent on the results.

4.1 Results

Competitiveness in General. To prove that the Convolution model can compete with other analytical models, a past only cross-validation was performed with 30 experiments as stated before. As a result, median RMSE and standard deviation are illustrated in Figure 2 for two variations of the Convolution model, and the remaining three literature models, Takagi-Sugeno model, LTI and Polynomial model. Outliers are marked with crosses. The Convolution model with one parameter (Conv. (1p)) is explained explicitly in the next paragraph. This comparison shows how the Convolution model gains the smallest median error and smallest deviation. Furthermore, it achieves some of the smallest errors overall. Beyond the Convolution model, Takagi-Sugeno yields better results than LTI and the baseline Polynomial model, which confirms results by (Füller et al., 2015). Nevertheless, it is conspicuous that the Takagi-Sugeno model produces some large outliers with errors of above 25 bpm.

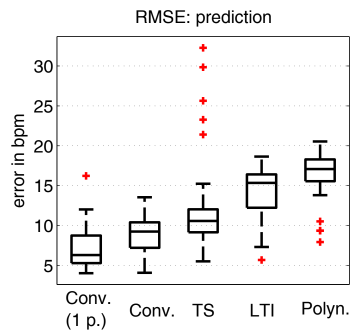

Figure 2: Median RMSE and standard deviation for heart rate prediction of two variations of the Convolution model, Takagi-Sugeno model, LTI and Polynomial model. Outliers are marked by crosses.

Parameter Reduction and Further Competitiveness. Multiple experiments were performed to reduce the number of necessary parameters within the Convolution model. First of all, each single parameter was set to an appropriate value while the remaining three parameters were left arbitrary. Since some fitting data appears to show that the current effect of using bygone strain seems to correlate with the impact of the actual effects of strain, the slope parameter and the impact parameter are bound together using a polynomial – once with a degree of two, once with a degree of five. The parameters for both were computed using a fit curve to data MATLAB function. Additionally, the setting of different experiments is combined. Table 1 shows median, standard deviation and mean value over all 30 experiments for the following nine different settings. Here, experiment number 0 serves as a baseline, where the past only cross-validation is executed with all 4 parameters. Experiments $1-8$ are described hereafter and lead to the following findings:

1. Exponential slope parameter is fixed to $a_4 = 0.9$:

Table 1: Median, standard deviation and mean in different experiments for reducing the parameters of the Convolution model.

No. exp.	Median	STD	Mean
0	9.25	2.29	8.95
1	9.24	2.28	9.96
2	6.12	2.65	6.82
3	9.34	2.50	9.11
4	9.26	2.29	8.95
5	9.75	2.36	9.09
6	9.26	2.26	9.03
7	6.91	2.58	7.31
8	6.31	2.56	7.08

The exponential parameter can be set to 0.9 for all subjects without increasing the median error. Compared to the baseline experiment, this degree of freedom does not appear to be necessary and a_4 can be fixed without any substantial loss of accuracy, except for some cases, as the higher mean error implies.

2. The level parameter a_3 is set to a precalculated resting heart rate for each subject individually: The prediction seems to be much more stable if the resting heart rate is fixed. This experiment yields the smallest errors over all performed experiments.

3. The memory parameter a_1 has been shown to range between 1.8 and 2.2, so it is set to $a_1 = 2$: Since the error is increased compared to the baseline, this setting has to be improved by combination with other experiment settings or some different approach.

4. The impact parameter a_2 has been shown to alternate in an area around 0.002 and is therefore set to this value, so $a_2 = 0.002$: The fixed impact parameter yields similar results compared to the baseline experiment. It seems to be that this degree of freedom is not a necessity.

5. Since a further look at a_1 and a_2 indicates a dependency, a linkage with a polynomial of the second degree is examined: This linking results in slightly higher errors compared to the baseline experiment.

6. A dependency of memory parameter a_1 and impact parameter a_2 with a polynomial of the fifth degree is examined: Likewise, linking leads to slightly higher errors compared to the baseline experiment.

7. Settings of experiments 1 and 2 were combined, so a_3 and a_4 are predefined as stated before: This fixation results in errors not quite as small as in experiment 2, but it might be reasonably comparable and is much better than the baseline experiment.

8. Settings of experiments 1, 2 and 5 combined, i.e., a_3 and a_4 are fixed and the polynomial of the second degree is applied additionally: Compared to experiment 7, an improvement with smaller errors can be assessed. Except for fixing only the level parameter, this experiment gains the smallest errors.

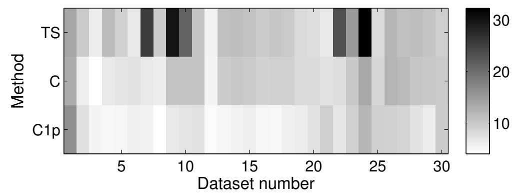

Figure 3: Heat map for the Takagi-Sugeno model (TS), the Convolution model (C), and the Convolution model with one parameter (C1p) for all 30 training session experiments. Lower RMSE values are colored lighter, higher errors are colored black.

Since the 8$^{\text{th}}$ experiment yields the best combination of small errors and few parameters, an enhanced Convolution model is built that has only one degree of freedom using the impact parameter a_2, denoted by α. Here, the additive level parameter is fixed to the individual resting heart rate for each person, the exponential slope parameter is set to 0.9, and the corresponding polynomial of the second degree is given as $a_1 = 2.06 + 158.8 \cdot a_2 - 36750 \cdot a_2^2$.

As stated before, Figure 2 illustrates prediction accuracies for the considered literature models and the two Convolution model approaches. The model labeled “Conv. (1p)” is the Convolution model from experiment number 8. It can be seen that the median value for this one parameter Convolution model is lowest, and even the best outliers with smallest errors could be reached using this model approach. As stated before, the four parameter Convolution model can easily compete with Takagi-Sugeno model, as shown by its lower median error and its lower error regarding outliers.

Figure 3 compares errors for the three models for every single training session. The color bar visualizes the RMSE while white is used for very small errors and a black coloring is used for RMSE values of 25 bpm and above. In most of the 30 experiments, smaller errors are generated by one of the Convolution model approaches than by the Takagi-Sugeno model. There are only few distinct identifiable counterexamples, such as the predicted data sets with number 21 and 25. In contrast, some data set predictions show huge errors using the Takagi-Sugeno model, while both Convolution model approaches can

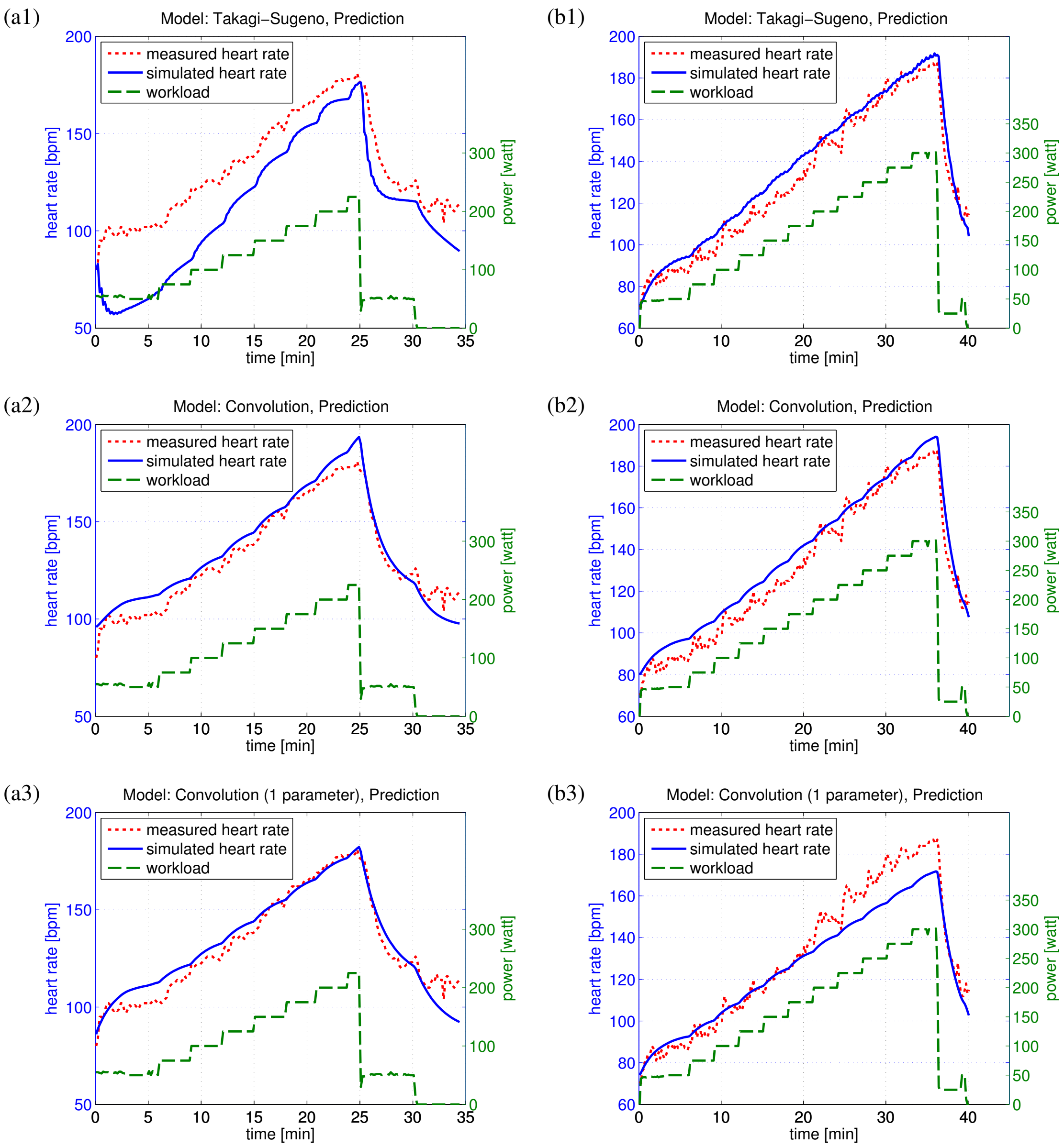

Figure 4: Three typical heart rate prediction examples for label **(1)** Takagi-Sugeno model, **(2)** Convolution model, **(3)** Convolution model with one parameter. Sets **a** and **b** illustrated two different experiments, but the same predicted training session each for all three models.

deal with the same prediction setting, with numbers 7, 9, 10, 22, 23 and 24 leading the way.

As an example, prediction 21 and 22 are visualized in Figure 4, where strain is given in watt and measured heart rate is plotted against predicted heart rate for better comparison of model accuracies. Here, each column shows figures computed with these three models but using the same data set. The figures in the first row are predictions executed from Takagi-Sugeno, the second row illustrates predictions using the four parameter Convolution model, and the last row presents prediction results from the one parameter Convolution model approach. While set *a* illustrates an example (no. 22) where the error of Takagi-Sugeno model is huge, in comparison to Convolution model approaches, which can both deal with this setting, set *b* illustrates an example (no. 21) where the error of all three models is in a similar range, but the one parameter Convolution model performs a bit worse than the others.

4.2 Evaluation and Discussion

Comparison of the Convolution model against known analytical models has shown that the Convolution model yields a comparable or even better accuracy. The Convolution model with one parameter seems to bring further enhancement. The experiments show that restrictions to the parameter area can actually improve prediction accuracy. At least, it seems to be reasonable to set the level parameter a_3 to an individual resting heart rate value. Since the error in experiment number 8 is only slightly higher than the error in experiment number 2, the advantages of one parameter instead of three should be considered: computation is much faster, fitting is more stable and risks of local minima are reduced because of the reduced complexity. Since the linkage of a_1 and a_2 using a polynomial of the fifth degree yields slightly smaller errors compared to using the presented polynomial of the second degree, this linkage combined with the other two parameter specifications might be a valid option, too. But since the average deviation is comparably small, we decided to uphold the model as simple as reasonable possible. Therefore we preferred this one parameter model against the other possibilities.

Given, however, that the training zone for aerobic and anaerobic training are approximately 15 to 20 bpm wide (10% of the maximum heart rate), such a prediction accuracy would not be sufficient. For a detailed training plan, an accuracy of 5% of the maximum heart rate is desired—which can be achieved using the Convolution model, which yields an error of around 9 bpm (approach with four parameters) or 7 bpm (approach with one parameter).

In some cases, especially the first minutes of training show huge deviation between the measured and the predicted heart rate values. By neglecting first minutes of a prediction, the accuracy can be improved. But since this is the case for all models, we ignored this potential improvement and took these first stages into account without exception.

The Convolution model with only one parameter not only yields good prediction results but also allowed the changes in this remaining parameter for each subject over time to be observed. In doing so, the remaining parameter seems to correspond to the fitness process itself. Figure 5 shows that—apart from some peaks in the beginning for two subjects—the value of this parameter decreases while the subject's training program continues. Since the subjects reported that they had trained regularly during the experiment period, the measured behavior might explain their increased fitness. This assumption is based on a few data only, hence the results will need to be validated using more and larger data sets. At the moment, the remaining parameter as fitness indicator can only be taken as a conceptual idea that needs further investigation.

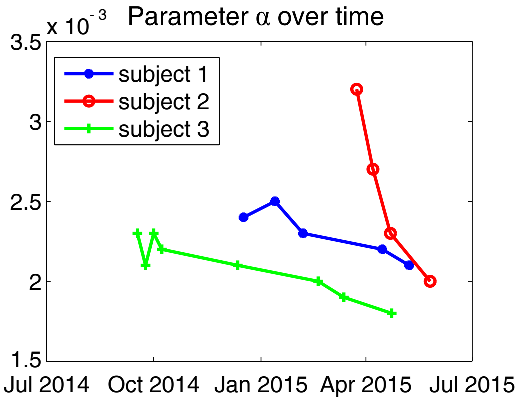

Figure 5: The remaining parameter over time for all three subjects.

Regarding other than cycling sports, first experiments in running were conducted: Using data of one male athlete, six exercises performed on a treadmill were performed. Predicting the heart rate in these running data results in a median RMSE of 10.68 ± 6.37 bpm using the Takagi-Sugeno model and in a median RMSE of 6.49 ± 2.35 bpm using the Convolution model with four parameters, indicating that the Convolution model is applicable to other sports than cycling as well.

5 CONCLUSION AND FUTURE WORK

This work presented a new model for heart rate prediction. First of all, a comparison to other heart rate models was performed. It has been shown that the Convolution model can fairly compete with other analytical models for predicting the heart rate a whole training session in advance. Moreover, reducing the number of arbitrary parameters leads to even smaller errors and more stability. First experiments on the remaining parameter lead us to the assumption that this parameter might indicate the actual fitness condition of a subject.

An important next step will be to analyze the usefulness of this model in simulating outdoor workouts. Many cyclists have their well-known training routes or plan their ride in advance. Therefore, a previous simulation based on strain according to GPS profiles might be beneficial in training planning.

Additionally, the usefulness of the Convolution model should be investigated for other sports comprehensively, such as cycling on other protocols, cycling without any protocol, running with or without protocol, and others.

Furthermore, experiments using a larger data set with more subjects have to show if a correlation between changes in fitness and in the remaining Convolution model parameter α can be consistently observed.

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