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Sliding Mode Control Design for Single DOF System

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Abstract – Stability, rapid response, disturbance minimization and minimal error operation are crucial for the efficient functioning of Single Degree of Freedom (DOF) systems. To enhance these characteristics, numerous control methods have been proposed. In this study, the Sliding Mode Control (SMC) approach is introduced to achieve robust control of a Single DOF system. Initially, the transfer function of the system is derived and the system states are determined. Subsequently, an SMC controller is designed in accordance with these system states. Within this framework, an SMC sliding function is defined. Using this method, the system states are drawn to the boundary surface through the switching function *sign(s)*, enabling the desired dynamic behavior of the system. To ensure high performance and stable operation of the control system, the Lyapunov stability theory is incorporated into the controller design. A controller structure is developed based on the defined Lyapunov function. Consequently, an optimal control input is generated by leveraging the principles of the SMC method and Lyapunov stability theory. To demonstrate the effectiveness of the proposed method, results are obtained in the MATLAB/Simulink environment. The results are compared and analyzed for different gain values of *(K)*, specifically *(K = 15)*, *(K = 30)*, and *(K = 45)*. Among these, the optimal control input is achieved with (*K* = *45*). The proposed SMC controller (with *(K = 45)*) enables the system output to rapidly reach the desired reference input without oscillations, resulting in the design of a high-performance system.

Keywords – Lyapunov Stability, Matlab/Simulink, Robust Control, Single DOF, SMC.

I. INTRODUCTION

In modern control theory, ensuring the stable operation of a system, optimizing its dynamic behavior, and achieving high performance are of great importance. In this context, methods such as Lyapunov-based controller designs and the Sliding Mode Control (SMC) approach are frequently used to ensure the robust and effective operation of the system. In this study, these two methods were employed to investigate the control of a Single Degree of Freedom (DOF) system.

The primary objective of controller structures designed based on the Lyapunov stability criterion is to ensure system stability. By defining a Lyapunov function, optimal parameter values that keep the system within a stable region are computed and applied to the system. For instance, Lyapunov-based Model Predictive Control (MPC) has been utilized in the control of dual-arm robotic systems [1]. In an adaptive control study designed based on the Lyapunov approach [2], PID controller parameter values were determined using this method, enabling the control of a robotic arm. Apart from robotic applications, the Lyapunov approach has also been employed in studies aimed at minimizing vehicle path-tracking errors and computing optimal torque values [3-5]. Additionally, various methods such as Lyapunov-based adaptive PID controllers [6], artificial neural networks [7], and the Switched Systems Approach [8] have been used.

SMC, one of the control techniques, is well known for its effectiveness in nonlinear systems and its ability to provide high performance. Various systems have been controlled using SMC. One of the key reasons for the preference for this method in systems is its ability to maintain stability by keeping the sliding surface in the desired region. Besides systems controlled using SMC, hybrid structures that integrate SMC with other control methods have also been proposed. Examples of hybrid approaches include system control using fuzzy logic-based SMC [9,10], a control method integrating Kalman Filter for state estimation with an SMC-PI controller [11], and Single DOF control using SMC-based Optimum Brace Stiffness (OBS) [12].

In this study, the materials and methods section is presented first, providing general information on SMC as well as the proposed method design incorporating both SMC and Lyapunov approaches. Subsequently, Section 3 discusses the results obtained from Matlab/Simulink simulations. Finally, the conclusion section is presented.

II. MATERIALS AND METHOD

In this section, information regarding Sliding Mode Control (SMC), which is used in the controller design, is first provided. Subsequently, the proposed hybrid approach for system design is presented.

A. Sliding Mode Control

SMC is known as one of the robust control methods used to address uncertainties in systems and minimize disturbance effects. It ensures stable operation by maintaining the system on a predefined surface in the state space. This method has demonstrated effective results in electrical, robotic, and mechanical systems. When examining the working principle of this method, a sliding surface function (s) is first defined. The system states are maintained within this region, ensuring the desired dynamic behavior. The general sliding surface function of SMC is given in Equation 1.

$$S(x) = Cx + m \tag{1}$$

Here, *x* represents the system states, *C* is the gain value defined for the sliding surface and *m* is the parameter value designed for the system. The value of *m* may vary depending on different systems. In this study, instead of *m*, the derivative of the error value *e* is used. To generate the most suitable control input, Equation 2 is utilized. Here, the control input is given by the sum of u_{eq}, which ensures that the system remains on the sliding surface, and u_{sw}, which performs the switching function. Equation 3 represents the switching control signal.

$$u = u_{eq} + u_{sw} \tag{2}$$

$$u_{sw} = -Ksign(s) \tag{3}$$

B. Sliding Mode Control Design for Single DOF System

The transfer function of the Single DOF system is given in Equation 4. To obtain this function, the parameter values of the Single DOF system were referenced [13].

$$\theta(s) = \frac{100}{s^2 + 10s} u \tag{4}$$

$$\theta(s)\{s^2 + 10s\} = 100u \tag{5}$$

$$\ddot{\theta} + 10\dot{\theta} = 100u \tag{6}$$

In this equation, $\theta(s)$ represents the system's position, while u is defined as the control input applied to the controlled system. The system states and the canonical form equation are given below.

$$x_1 = \theta; \quad x_2 = \dot{\theta}; \quad \dot{x_2} = \ddot{\theta} = -10\dot{\theta} + 100u = -10x_2 + 100u \tag{7}$$

Here, x_1 and x_2 represent the system states. Using these equations, the system parameter values and the optimal SMC design have been achieved. In the SMC controller structure, the sliding surface function is first defined as given in Equation 8.

$$s = ce + \dot{e} \tag{8}$$

The expression e is known as the error value occurring between the system's position output and the reference input. The error and its derivative expressions are given in the equations below.

$$e = r - \theta \tag{9}$$

$$\dot{e} = \dot{r} - \dot{\theta} \tag{10}$$

$$\ddot{e} = \ddot{r} - \ddot{\theta} \tag{11}$$

Here, the reference input signal designated for the system is denoted as r. The Lyapunov stability criterion has been utilized to determine the optimal parameter values and ensure system stability. As is well known, in the Lyapunov stability criterion, the Lyapunov function is first defined as shown in Equation 12 [14].

$$V = \frac{1}{2}s^2 \tag{12}$$

For the system to operate stably, $s\dot{s} < 0$ must be satisfied. Since s is the sliding surface;

$$s = ce + \dot{e} \tag{13}$$

$$\dot{s} = c\dot{e} + \ddot{e} \tag{14}$$

After writing $\ddot{e} = \ddot{r} - \ddot{\theta}$ in Equation 14, and using the expression $\ddot{\theta} = -10\dot{\theta} + 100u$, the following equation is obtained.

$$\dot{s} = c\dot{e} + \ddot{r} - 10\dot{\theta} + 100u \tag{15}$$

In this case, the product of the sliding surface function and its derivative ($s\dot{s}$) can be expressed as shown in Equation 16.

$$s\dot{s} = s\{c\dot{e} + \ddot{r} - 10\dot{\theta} + 100u\} \tag{16}$$

In this equation, to satisfy the condition $s\dot{s} < 0$, the equation definition for the control signal is provided in Equation 17.

$$u = \frac{1}{100}\{c\dot{e} + \ddot{r} - 10\dot{\theta} + Ksign(s)\} \tag{17}$$

$$sign(s) = \begin{Bmatrix} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{Bmatrix} \qquad (s\dot{s} = Ksign(s) < 0) \tag{18}$$

In Equation 17, since the reference input signal is constant and the derivative of a constant signal is considered to be zero, the control signal (u) equation is obtained as shown below.

$$u = \frac{1}{100}\{-c\dot{\theta} - 10\dot{\theta} + Ksign(s)\} \tag{19}$$

The SMC controller structure in the Matlab/Simulink model is shown in Figure 1. The designed controller structure is implemented based on the control input expression provided in Equation 19. Upon examining the SMC structure, it can be observed that the system has two inputs and one output. The error and output values are defined as inputs, while the control input is defined as the output.

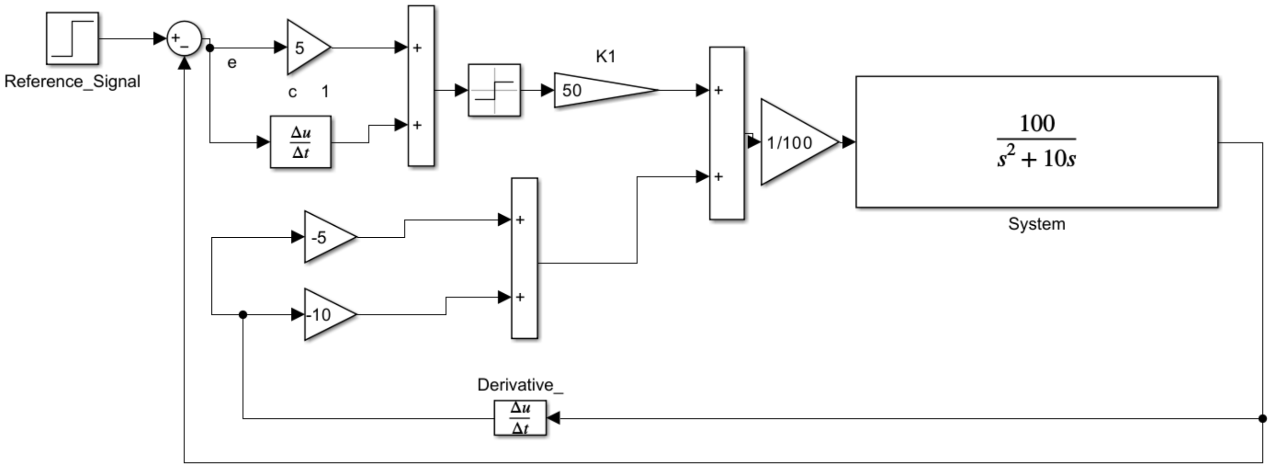

Fig. 1 Simulink Model Of Sliding Mode Controller

III. RESULTS

The results of the control system designed according to the SMC and Lyapunov stability theory have been obtained in the Matlab/Simulink environment. The sliding surface expression used in the SMC approach is defined as *s*. To ensure the stable operation of the system, the *Ksign(s)* function has been used. However, the gain value (*K*) in this function is unknown and is manually input by the user. In this study, the results obtained for different values of *K* are presented in Figure 2. The value *K* = *45* has been determined as the optimal value for maximum system performance. It can be observed that the system settles very quickly and reduces the steady-state error to zero within approximately 1 second.

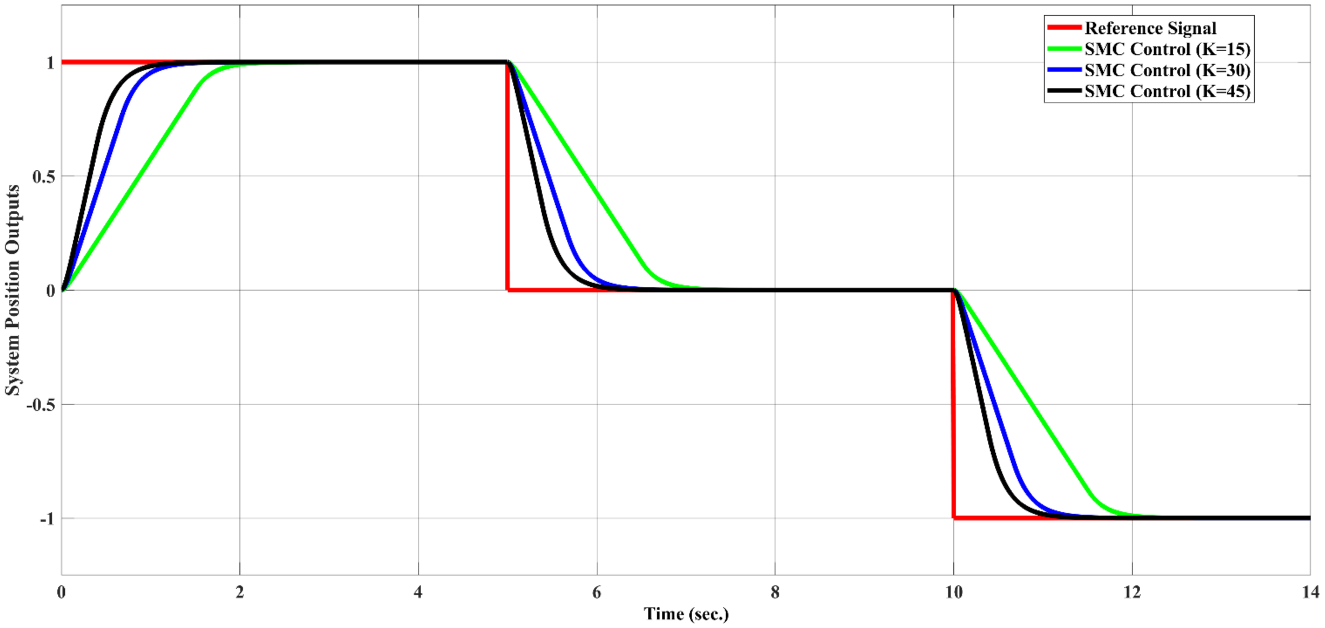

Fig. 2 Simulation Results of Sliding Mode Controller

In Figure 3, the control input applied to the system and the SMC sliding surface signals are provided. It can be observed that the sliding surface is maintained within the range of -0.5 to 0.5. Chattering is observed on the sliding surface. Although these oscillations are present in the simulation studies, they are undesirable in practical applications. The smaller the deviations on the sliding surface, the higher the system's performance.

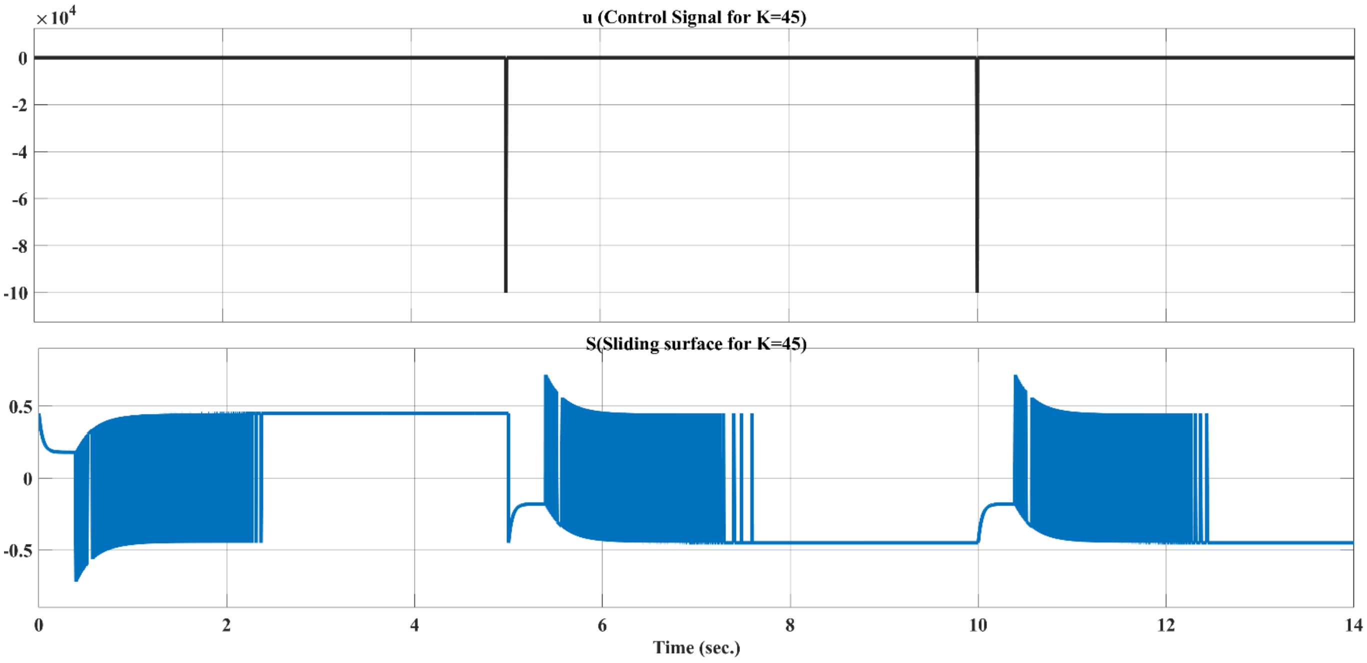

Fig. 3 Control Input Applied to a Single DOF System and SMC Sliding Surface

Furthermore, there is a parallel connection between the system's dynamic behavior and the sliding surface. Maintaining the sliding surface on a specific surface implies that the system's dynamic behavior is improved. In summary, it can be concluded that the system has reached a stable state and the controller structure is successful.

IV. CONCLUSION

In this study, an optimal control input was generated using a Lyapunov-based SMC structure and applied to a Single DOF system. The SMC sliding surface function was designed, ensuring the system's stability by keeping it on this surface. Additionally, Lyapunov stability theory was incorporated into the system, improving its performance. Experiments were conducted on the system using different *sign(s)* functions with varying *K* gain values, and the obtained results were compared in the Matlab/Simulink environment. According to the simulation results, it was observed that the SMC structure with $K = 45$ provided maximum performance. In future controller designs, Artificial Intelligence algorithms could be used for calculating the optimal *K* gain value. This controller structure can also be applied to various robotic applications, electrical, and mechanical circuits.

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