Dynamic Model and Parameter Measurement of Interior Permanent Magnet Synchronous Motor

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ABSTRACT

To properly design and optimize a control system of an interior permanent magnet (IPM) synchronous motor, the machine model and its accurate parameters must be known. The IPM synchronous motor characteristics and its model are presented in this paper. Simple methods to measure IPM synchronous motor parameters such as armature resistance, flux linkage (λ_f), d-axis inductance (L_d), q-axis inductance (L_q) have been discussed. *The measurement techniques for L_d and L_q with and without neutral connection available have also been discussed. These methods are simple and do not require any complex theory or special equipment. Its effectiveness is demonstrated by experimental result.*

1. INTRODUCTION

Recently, Permanent magnet synchronous motors (PMSM) have become very popular in a wide variety of industrial, household electrical appliances (such as washing machine, refrigerator, vacuum cleaner etc) and electric vehicle applications, by providing high power density and high efficiency compared to other types of motors [1,2]. They offer the highest torque/volume of all motors by exploiting their rotor saliency. Rotor excitation, using the recently developed rare-earth magnet materials such as the Neodymium-Iron-Boron material also makes such a motor the world's most efficient and compact. The interior permanent magnet (IPM) synchronous motor also offers the possibility of wider speed range, compared to their surface magnet version, due to the fact that the magnets are securely embedded inside the rotor shaft.

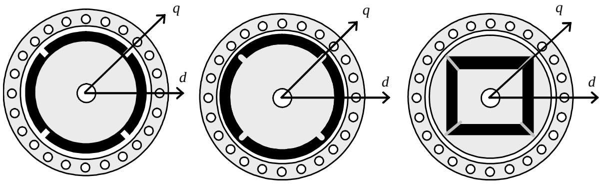
Accurate motor parameters is a mandatory to achieve high performance sensorless vector or direct torque control of IPM synchronous motor drive [2-5]. The IPM synchronous motor characteristics and its model are presented in this paper. Then the parameter measurement techniques for IPM synchronous motor are discussed. The measurement techniques for L_d and L_q with and without neutral connection available have also been discussed. A method to obtain two-phase circuit parameters from physically measured data has been established. Finally, the accuracy of the measured motor parameter validated, by using them in the sensorless direct torque control (DTC) of IPM synchronous motor drive. The measured motor parameters are used to estimate the speed of the IPM synchronous motor.

2. MODELING OF IPM SYNCHRONOUS MOTOR

The permanent magnet Synchronous motor has numerous advantages over other machines that are conventionally used for ac servo drives. The stator current of induction motor contains magnetizing as well as torque producing components. The use of permanent magnet in the rotor of the PMSM makes it unnecessary to supply magnetizing current through the stator for constant air-gap flux; the stator current need only to be torque producing. Hence for the same output, the PMSM will operate at a higher power factor because of the absence of the magnetizing current and will be more efficient than the induction motor. The conventional wound-rotor synchronous machine (SM), on the other hand, must have dc excitation on the motor, which is often supplied by brushes and slip rings. This implies rotor losses and regular brush maintenance, which implies downtime. The key reason for the development of the PMSM was to remove the foregoing disadvantages of the SM by replacing its field coil, dc power supply, and slip rings with a permanent magnet [6]. The PMSM, therefore, has a sinusoidal induced electromagnetic force (EMF) and requires sinusoidal currents to produce constant torque just like the SM.

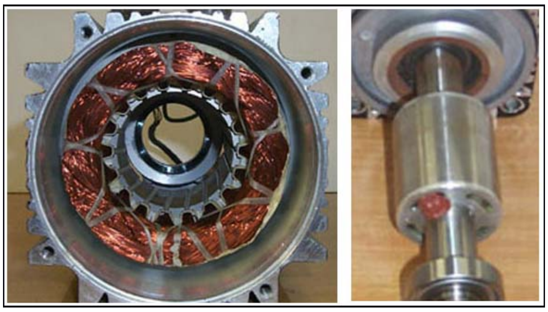
There are various rotor configurations of PMSM, which also influence the current control algorithms. PMSM can have magnets mounted on the surface of the rotor or buried inside the rotor. They are known as surface PMSM and buried or IPM synchronous motor, respectively. The most commonly used rotor configurations reported for high-performance drives are shown in figure 1, where the solid areas represent permanent magnets and the grey areas represent the iron. Since the permeability of ferrite and rare-earth magnets is close to that of air, the surface PM synchronous motor, shown in figure 1(a), usually has a large effective air gap and therefore has small stator inductances. A further consequence of large air gap is that the stator (or armature) reaction is negligible and therefore the surface PM synchronous motor is only good for constant-torque operation. The inset PM and IPM synchronous motors, shown in figures 1 (b) and (c), normally have small air gap and therefore the effects of armature reaction are significant. This makes IPM synchronous motor good for constant torque operation as well as field weakening operation. Furthermore, since the permanent magnets are buried inside the rotor, the IPM synchronous motor is mechanically robust and allows high-speed applications when compared to the surface PM synchronous motor.

The picture of stator and rotor of an IPM synchronous motor is shown in figure 2.



(a) Surface PM motor (b) Inset PM motor (c) IPM motor

Figure 1. Typical rotor cross-sections of PMSMs.



(a) Stator (b) Rotor

Figure 2. Stator and rotor of an IPM synchronous motor.

The stator of a PMSM and a wound-rotor SM are similar. The permanent magnet used in the PMSM is of modern rare-earth magnet materials with high resistivity, so induced currents in the rotor are negligible. In addition, there is no difference between the back EMF produced by a permanent magnet and that produced by an excited coil. Hence the mathematical model of a PMSM is similar to that of the wound rotor SM. The well-known motor model in *dq*-reference frame, which is synchronously rotating with the rotor, where *d*-axis is aligned with the magnet axis and *q*-axis is orthogonal to *d*-axis, is usually used for analyzing the PM synchronous motor. The motor model is shown as in equations (1) – (7) [7].

$$v_q = R\, i_q + p\, \lambda_q + \omega\, \lambda_d \qquad (1)$$

$$v_d = R\, i_d + p\, \lambda_d - \omega\, \lambda_q \qquad (2)$$

where λ_d and λ_q are the *d*- and *q*- axes stator flux linkages and are given by the following equations.

$$\lambda_d = L_d i_d + \lambda_f \qquad (3)$$

$$\lambda_q = L_q\, i_q \qquad (4)$$

Substituting equations (3) and (4) into the general torque equation (5) in *dq*-axis reference frame, gives the torque expression for a PM synchronous motor in equation (6).

$$T = \frac{3}{2} P\, (\lambda_d\, i_q - \lambda_q\, i_d\,) \qquad (5)$$

$$T = \frac{3}{2} P [\lambda_f\, i_q + (L_d - L_q) i_d\, i_q\,] \qquad (6)$$

The equation for the motor dynamics is

$$T = T_L + B\omega_r + Jp\omega_r \qquad (7)$$

In equations (1) – (6), v_d, v_q, i_d, i_q, L_d and L_q are the *d* and *q* axes stator voltages, currents and inductances respectively. *R* is the stator resistance and λ_f is the amplitude of the magnet flux linkage. ω is the electrical angular velocity in rad/sec, ω_r is the rotor speed, *B* is the damping coefficient, *J* is the moment of inertia, and *P* is the number of pole pairs. *p* is the operator *d/dt*.

The torque equation (6) consists of two terms. The first term represents the excitation torque, which is produced by the interaction of permanent magnet flux and i_q and is independent of i_d. The second term is the reluctance torque, which is proportional to the product of i_d and i_q and to the difference of L_d and L_q. For the surface PM synchronous motor, the reluctance torque is zero since $L_d = L_q$, while for the IPM synchronous motor, higher torque can be produced for the same i_d and i_q if (L_d - L_q) is larger. This is one of the advantages of IPM synchronous motor over surface PMSM. In addition, to weaken the magnetic flux produced by the permanent magnets, small negative i_d is needed as shown in equation (3), since L_d is relatively large for an IPM synchronous motor. A consequence of this is that i_q does not need to be reduced too much for keeping the stator current within its limitation. When applying negative i_d, the reluctance torque is positive, since $L_q > L_d$ in IPM synchronous motors, which compensates the torque drop in field weakening operation, and therefore the speed range can be extended. The relative amplitudes of the excitation and reluctance torque are determined during the motor design stage by selecting suitable values of parameters, λ_f, L_d and L_q.

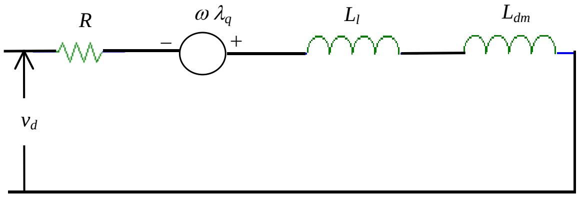
3. PARAMETER MEASUREMENT OF IPM SYNCHRONOUS MOTOR

The most common parameters used in the implementation of control algorithms are the simplified model parameters. These are:-

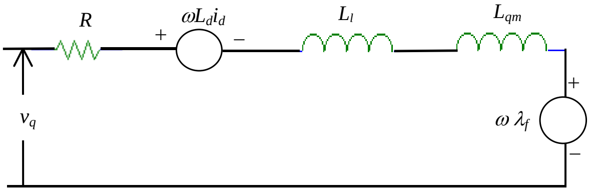
- R_a – Armature resistance,
- L_d - the direct axis self inductance,
- L_q - the quadrature axis self-inductance, and
- λ_f - the permanent magnet Flux linkage

These parameters are used to calculate control rules such as voltage limit ellipses and maximum torque-per-ampere trajectories in vector or direct torque control [2,5,8].

The picture of the motor used in this paper is shown in figure 2. The rotor structure of the other IPM synchronous motor used in this paper is as shown in figure 1(c). The stator of this IPM synchronous motor is not like a standard induction motor. The slots are skewed and thus cogging torque is very small. The equivalent circuit of the PM synchronous motors in *dq*-axis synchronously rotating reference frame is shown in figure 3. Since the PM synchronous motors do not have any damping windings in the rotor, the rotor circuit is open and is not shown in this figure. The *dq*-axis synchronous inductances have been splited into a stator leakage inductance L_l and a magnetizing inductance L_m.



(a) *d*-axis model.



(b) *q*-axis model.

Figure 3. The equivalent circuit of IPM synchronous motor in *dq*-reference frame.

3.1 Armature Resistance (R_a) Measurement

The stator resistance of the motor can be easily measured from the standard DC measurement. Since R_a is the resistance between line-to-neutral, R_a would be one half of the measured line-to-line resistance. For most small to medium size PM synchronous motors, skin effect is not significant and can be safely neglected. Winding resistance value is highly temperature-dependent. When winding resistance R_o is measured, temperature T_0 (°C) of the winding must be recorded and the resistance R_t at another temperature *T* should be calculated using the following formula.

$$R_t = R_0(K+T)/(K+T_0) \tag{8}$$

where *K* is the constant determined by the material ($K = 234.5$ for copper). Resistance value at 25°C is often used for published data.

3.2 Determination of Back EMF Constant λ_f

The back emf constant of a PM synchronous motor is determined from the open circuit phase voltage when it is drive by other motor. If the back emf is assumed to be sinusoidal, it is related to rotor speed and phase voltage as follows.

$$V = \omega\lambda_f \tag{9}$$

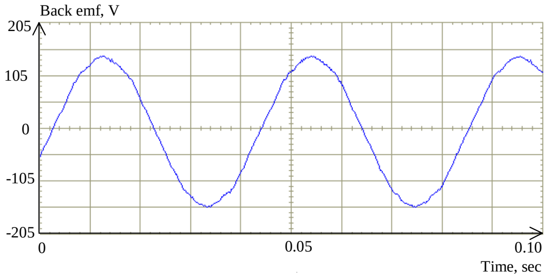


Figure 4. Back emf of the IPM at 734 rpm.

When the IPM motor rotates at 734 rpm, the open-circuit line to line voltage is as shown in figure 4. The back emf of this motor is almost sinusoidal. This waveform can be approximated by the following expression:

$$V = 142 \quad \sin\omega t \tag{10}$$

The rms value of the phase voltage is 57.96 volts. The calculated back emf constant is 0.377 Wb.

3.3 Determination of Synchronous Inductances

3.3.1 Stator self inductances

In a magnetically linear synchronous motor, the self-inductance of a phase winding is always positive and has a second harmonic variation with rotor position (θ) due to the different permeance values associated with the *q*- and *d*-axes. The self inductance L_{aa}, L_{bb}, L_{cc} are given by [9]:

$$L_{aa} = L_0 + L_l + L_2\cos(2\theta) \tag{11}$$

$$L_{bb} = L_0 + L_l + L_2\cos 2(\theta - 2\pi/3) \tag{12}$$

$$L_{cc} = L_0 + L_l + L_2\cos 2(\theta + 2\pi/3) \tag{13}$$

Where L_0 is the average component of self inductance due to self flux-linkage crossing the airgap, L_l is the phase leakage inductance and is due to self flux-linkage which does not cross the airgap (slot leakage and end winding), and L_2 is the magnitude of the second harmonic component.

3.3.2 Stator mutual inductances

The mutual inductance between stator phases also exhibits a second harmonic variation with θ due to the rotor geometry. The mutual inductance between phase A and phase B is evaluated by considering the airgap flux linking phase A when only phase B is excited. The resulting equations are [9]:

$$M_{ab} = M_0 + M_2\cos 2(\theta - \pi/3) \tag{14}$$

$$M_{bc} = M_0 + M_2\cos 2(\theta + \pi/3) \tag{15}$$

$$M_{ca} = M_0 + M_2\cos 2(\theta + \pi) \tag{16}$$

3.3.3 Synchronous Inductances

The IPM synchronous motor can be characterized by its *d*-axis inductance L_d and *q*-axis inductance L_q. As is seen from the torque equation (6) the reluctance torque per ampere is proportional to (L_q - L_d) while the saliency ration (L_q/L_d) determines many of the motor's operation characteristics such as field weakening range and power factor. These synchronous inductances are independent of the permanent magnets and can be therefore measured before or after the magnets are put into the rotor. The *dq*-axis synchronous inductances are given by:

$$L_d = L_{dm} + L_l \tag{17}$$

$$L_q = L_{qm} + L_l \tag{18}$$

where L_l is the stator leakage inductance and L_{dm} and L_{qm} are the magnetizing inductances

It has been shown in [10] that for sinusoidally distributed windings

$$L_{ab0} = -L_0 / 2 \quad (19)$$

$$L_2 = L_{ab2} \quad (20)$$

From *dq* axis theory [9], the *dq*-axis inductances are therefore obtained from the following equations.

$$L_d = \frac{3}{2}(L_0 + L_2) + L_l \quad (21)$$

$$L_q = \frac{3}{2}(L_0 - L_2) + L_l \quad (22)$$

Rearranging in order to obtaining L_0, L_2:

$$L_0 = \frac{L_d + L_q}{3} + \frac{2}{3} L_l \quad (23)$$

$$L_2 = \frac{L_d - L_q}{3} \quad (24)$$

For non-sinusoidally distributed windings, the relationship in (19) and (20) is no longer valid [11]. The more general definition is as follows.

$$L_d = (L_0 - Lac_0) - (L_2/2 + Lac_2) + L_l \quad (25)$$

$$L_q = (L_0 - Lac_0) + (L_2/2 + Lac_2) + L_l \quad (26)$$

3.3.4. Measurement of the Synchronous Inductances

There are various test methods for determining the synchronous inductances, L_d and L_q, and these methods are usually classified into running and standstill test methods. Running test methods were mainly developed for testing line-start motors with a squirrel cage and include the no-load test, slip test, zero power-factor test and so on. The tests are generally performed from a fixed-voltage fixed-frequency supply and are normally applicable to the synchronous reluctance motors and the PM synchronous motors before the permanent magnets are put into the rotor. Standstill test methods have the advantage that they can usually be applied to motors containing permanent magnets and the tests are easier to perform than running tests. The main types of standstill tests are the AC standstill test, the DC bridge test, the instantaneous flux-linkage test and the standstill torque test. In this study, the AC standstill test was adopted to measure the synchronous inductances of the IPM motor.
The AC standstill test is the easiest to perform and is widely applicable to inverter-fed PM synchronous motors. The effect of saturation is underestimated due to the nature of this test and is therefore mainly useful for measuring the unsaturated inductances. In other words, it is not applicable if the inductances show much saturation.

3.3.4.1 Synchronous Inductance Measurement for IPM motor with neutral connection

The circuit connection for synchronous inductance measurement with available neutral connection is shown as in figure 5.

The self-inductance and mutual inductance are obtained as:

$$L_{aa}(\theta) = \frac{\sqrt{(V_A / I)^2 - R^2}}{\omega} \quad (27)$$

$$L_{ac}(\theta) = \frac{V_C}{\omega} \quad (28)$$

where θ is the electrical rotor angle and ω is the frequency of the supplied AC voltage.

The measured self-inductance and mutual inductance waveforms of the prototype IPM motor are shown in figure 6. They can be approximated as;

$$L_{aa} = 0.332 + 0.037 \sin 2\theta - 0.007 \sin 4\theta \quad (29)$$

$$L_{ac} = -0.1 + 0.025 \sin 2\theta - 0.005 \sin 4\theta \quad (30)$$

Comparing with equations (11) and (16), one can obtain

$$L_0 + L_l = 0.332$$
$$L_2 = 0.037$$
$$Lac_0 = -0.1$$
$$Lac_2 = 0.02$$

Substituting them into with equations (25) and (26), the synchronous inductance of the prototype motor are calculated as:

$$L_d = 0.3855\ H$$

$$L_d = 0.4755\ H$$

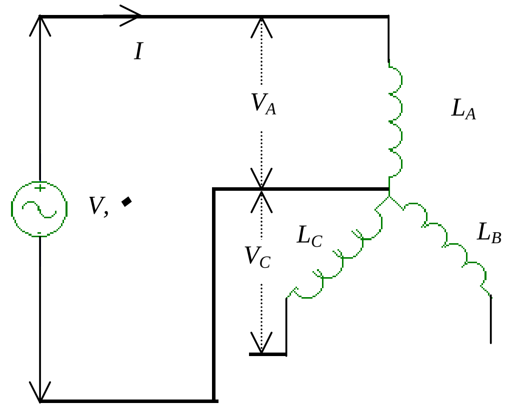
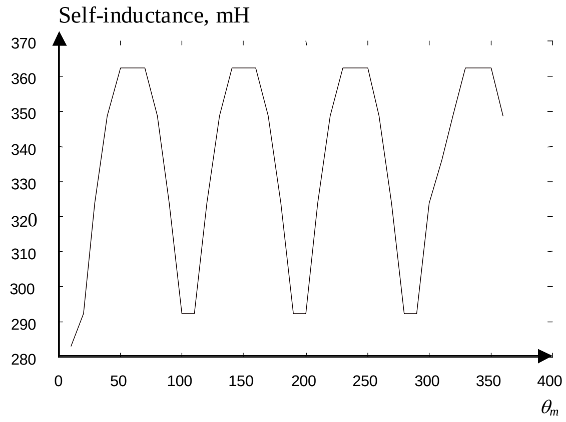
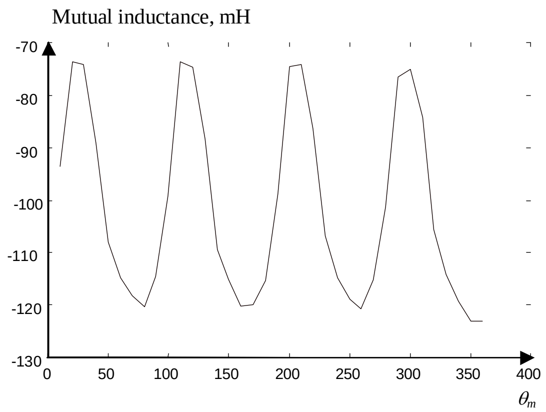


Figure 5. Circuit connection for inductance measurement of IPM motor with neutral connection.



(a)

Self inductance



(b) Mutual inductance

Figure 6. Measured self-inductance and mutual inductance.

TABLE I
Parameters of the IPM Synchronous Motor
(Prototype IPM motor developed at UNSW)

Number of pole pairs, P	2
Stator resistance R	18.6 Ω
Magnet flux linkage λ_f	0.447 Wb
d-axis inductance L_d	0.3885 H
q-axis inductance L_q	0.4755 H
Phase voltage V	240 V
Phase current I	1.4 A
Base speed ω_b	1500 rpm
Crossover speed ω_c	2400 rpm
Rated torque T_b	1.95 Nm
Rated power P_r	1 kW

3.3.4.2 Synchronous Inductance Measurement for IPM motor without neutral connection

The circuit connection for synchronous inductance measurement with no neutral connection available is shown in figure 7. The self-inductance can be obtained as,

$$L_{aa}(\theta) = \frac{2}{3}\frac{\sqrt{(V_A/I)^2 - (1.5R)^2}}{\omega} \tag{31}$$

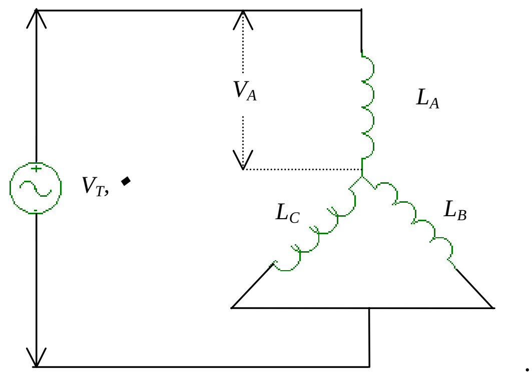


Figure 7. Circuit connection for inductance measurement of IPM motor without neutral connection.

The measured self-inductance waveforms of the IPM motor with no neutral connection are shown in figure 8. This can be approximated as,

$$L = 0.07365 + 0.02905\sin 2\theta \tag{32}$$

From the maximum and minimum valuses of the inductance, the calculated d-axis and q- axis inductances are:

$L_d = 0.0446$ H and $L_q = 0.1027$ H

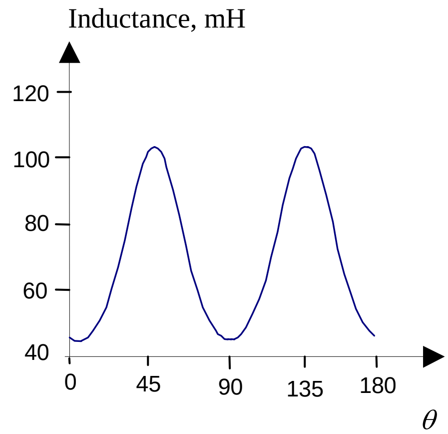


Figure 8. Measured self-inductance of IPM motor.

TABLE II
Parameters of the IPM Synchronous Motor
(Kollmorgen motor model: B-402A)

Number of pole pairs, P	2
Stator resistance R	5.8 Ω
Magnet flux linkage λ_f	0.377 Wb
d-axis inductance L_d	0.0448 H
q-axis inductance L_q	0.1024 H
Phase voltage V	132 V
Phase current I	3 A
Base speed ω_b	1260 rpm
Crossover speed ω_c	1460 rpm
Rated torque T_b	3.7 Nm
Rated power P_r	1 kW

4. SENSORLESS DTC DRIVE USING THE MEASURED MOTOR PARAMETER

Figure 9 shows the block diagram of a sensorless direct torque controlled IPM motor drive. The accuracy of the measured motor parameter validated, by using them in the sensorless direct torque control (DTC) of IPM synchronous motor drive. The measured motor parameters are used to estimate the speed of the IPM synchronous motor. The modeling and experimental results (figure 10) show that the speed estimator works very well with the measured parameters. The details of the speed estimator can be found in reference [4].

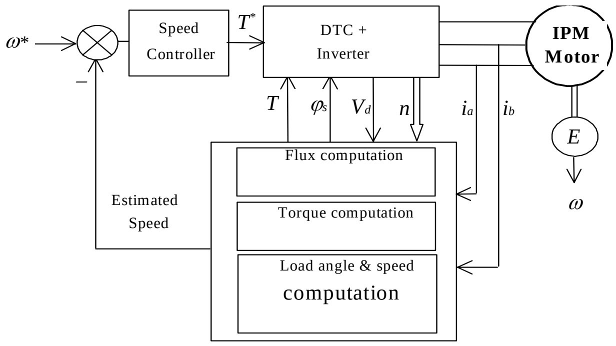
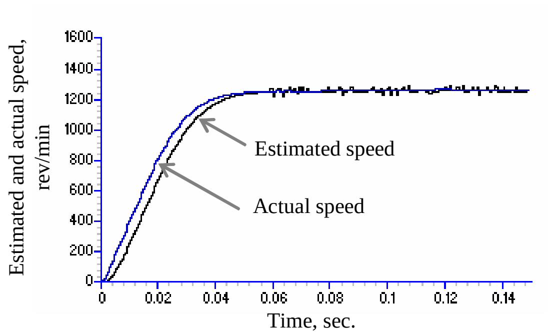
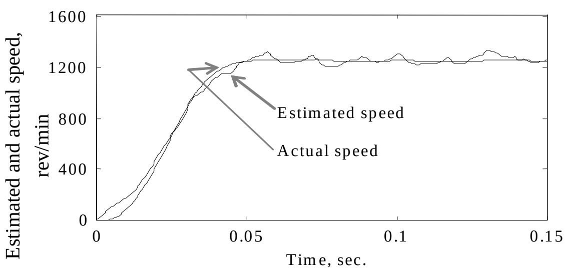


Figure 9. Sensorless Direct torque control of IPM synchronous motor drive.



(a) Simulation result.



(b) Experimental results.

Figure 10. Estimated and actual speed (measured using speed sensor).

5. CONCLUSIONS

The IPM synchronous motor characteristics and its model are discussed in this paper. Then the parameter measurement techniques for IPM synchronous motor are presented. The measurement techniques for L_d and L_q with and without neutral connection available have also been discussed. A method to obtain two-phase circuit parameters from physically measured data has been established. Finally, the accuracy of the measured motor parameter validated, by using them in the sensorless direct torque control (DTC) of IPM synchronous motor drive. The measured motor parameters are used to estimate the speed of the IPM synchronous motor. The modeling and experimental results show that the speed estimator works very well with the measured parameters.

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