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Early Age Tests to Predict 28 Days Compressive Strength of Concrete

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The compressive strength of concrete is the most common performance measure used by the engineer in designing buildings and other structures. Usually two specimen concrete cylinders are cast for each day's representative compressive strength test. The 3-days or 7-days tests are done to assess the early gain of concrete strength and make a crude estimate of the 28 day strength at site. However, 28-days tests are mandatory as per design/construction code requirements. This paper is an attempt to develop a simple mathematical model based on concrete's nature of strength gain to predict the compressive strength of concrete at 28th day from early age results. The model is a simple equation (a rational polynomial) that consists of only two constants and one variable which is the age of concrete in days. The proposed model has a good potential to predict concrete strength at different age with high accuracy.

Key word: cylinder crushing strength, concrete, compressive strength, strength prediction.

1. INTRODUCTION

Designing a concrete structure requires the concrete compressive strength to be used. The design strength of the concrete normally represents its 28thday strength. In construction works 28 days is a considerable time to wait for the test results of concrete strength, while it also represents the quality control process of concrete mixing, placing, compaction, proper curing etc. Concrete mix design is a process based on code recommendation and requires some previous experience. If due to some error in mix design or mix preparation at site the test results fail to achieve the designed strength, then repetition of the entire process becomes mandatory, which can be costly and time consuming. For every failure, it is necessary to wait at least 28 days, thus the need for an easy and reliable method for estimating the final strength at an early age of concrete is a long felt matter. Hence, a rapid and suitable concrete strength prediction would be of great significance [Kheder et al, 2003].

Researchers are very keen to explore the concrete behaviour and for this reason prediction of the concrete strength is being marked as an active area of research. Many studies are being carried out in this area [Zain et al., 2010]. Different approaches using regression functions have been proposed for predicting the concrete strength [Oluokun et al., 1990; Popovics, 1998]. Traditional modelling approaches are established based on empirical relation and experimental data which are improving day by day. Some smart modelling system utilizing artificial neural network [Nath et al., 2011] and support vector machines [Gupta, 2007] are developed for predicting compressive strength of concrete.

Objective of all studies that have been carried out was to make the concrete strength predictable and increase the efficiency of the prediction. In this paper, an attempt is made to develop a relation between concrete strength and its age and finally express this relationship with a simple mathematical equation.

2. COMPRESSIVE STRENGTH OF CONCRETE

Concrete is a product of two major components, one is the cement paste and the other is the bulk inert mass. In order to form the cementing medium, cement would mix with water. Coarse aggregates and fine aggregates are the part of inert mass. In properly mixed concrete, these materials are completely surrounded and coated by cement paste filling all the void space between the particles. With time, the setting process of the concrete starts and it begins to gain its strength.

Strength is the design property of the concrete. Characteristics like, durability, permeability, volume stability may be important in some cases of designing concrete structures but strength is the most important one. An overall picture of concrete quality is reflected by the concrete strength. The process of strength growth is called 'hardening'. There are many factors which control compressive strength of concrete. Mix proportioning, aggregate quality, aggregate gradation, type of cement, mixing and placing method, curing of concrete, curing temperature and the water cement ratio are some of the significant parameters. Water cement ratio (w/c) alone has the most prominent impact on concrete strength. A minimum amount of water is necessary for proper chemical reaction in the concrete and additional amount of water increases the workability but reduces strength.

3. MIX-DESIGN DATA

Total 56 sets of test data (called Group-1 here) had been used for developing the mathematical model. These are taken from a previous study by Garg [2007] and the validation of the model is done using the experimental data (called Group-2) from a recent work [Hasan, 2012]. These tests are carried out recently in the Concrete laboratory of civil engineering department of Bangladesh University of Engineering and Technology (BUET). Primarily, the Group-1 data are used to model the behaviour of normal stone aggregate concrete with time (age). Ordinary Portland cement and mix design method of ACI Committee [ACI 211.1-91, 1991] had been used for mix design. For testing the cylinders ASTM Standard [ASTM C39, 2011] recommendation had been followed. Randomly selected 10 sample data sets of Group-1 are listed in Table 1. All the 23 data sets of Group-2 are used for validation (Table-7). However, only 10 sample data from Group-2 is shown in Table 2 to indicate the mix proportions used.

Ranges of material properties and concrete strengths achieved for Group-1 and Group-2 data sets are summarized in Table 3. Concrete are prepared using Cement (C), Coarse-Aggregate (CA), Fine-Aggregate (FA) and Water (W). No admixtures or additives are used in either case. Different mix proportions of the ingredients and different water cement (w/c) ratio were used to study the variation in strength. All the specimens were immersed in water for curing. Variation of temperature was negligible and hence its effect is ignored.

4. DERIVATION OF THE MODEL

In order to understand the strength gaining pattern of the concrete with age, strength versus day curve was plotted for every single set of model test data (Group-1). It was observed that every curve follows a typical pattern. Fig 1 is a representative figure showing the strength gaining pattern with age of concrete for three sets. MATLAB curve fitting tool [MATLAB 2010a] was used to plot these data and also for the analysis purposes. From the plotted data the best fit curve for each set was drawn. The plotted best fit curves show a good correlation and the average value of the square of the correlation coefficient is 0.997. The value of correlation coefficients of the three representative data sets of Fig 1 is given in Table 3.

Table 1: Concrete Mix Proportion of Group-1 Samples

Sl. No.	Concrete strength (MPa)			FM of sand	W/C ratio	Mix proportion of concrete (kg/m^3)				CA size ratio (10mm:20mm)
	7day	14day	28day			Water	Cement	FA	CA	
1	13.84	17.8	25.6	2.4	0.52	185	356	797	1057	2:1
2	14.08	17.91	19.53	2.4	0.5	185	370	781	1055	2:1
3	15.71	18.57	25.57	2.4	0.48	190	396	744	1057	2:1
4	16.91	20.13	25.97	2.4	0.52	190	365	775	1056	2:1
5	14.44	19.06	23.06	2.4	0.48	185	385	767	1056	1:1
6	15.08	20.26	24.84	2.4	0.52	185	356	797	1057	1:1
7	16.11	21.77	26.84	2.4	0.48	190	396	744	1057	1:1
8	17.82	23.20	25.00	2.4	0.52	190	365	775	1056	1:1
9	19.98	26.75	29.32	2.4	0.46	190	413	727	1057	2:1
10	20.00	21.53	25.97	2.4	0.44	185	420	732	1055	2:1

Table 2: Concrete Mix Proportion of Group-2 Samples

Number	Concrete strength (MPa)			FM of sand	W/C ratio	Mix proportion of concrete (kg/m3)				CA size ratio (10mm:20mm)
	3day	7day	28day			Water	Cement	FA	CA	
1	14.49	23.33	32.34	2.56	0.40	178	448	650	1124	2:1
2	18.89	23.38	30.90	2.56	0.48	213	443	643	1113	2:1
3	12.01	22.22	28.58	2.56	0.53	234	439	637	1102	2:1
4	10.09	14.53	20.01	2.56	0.57	255	435	630	1091	2:1
5	21.58	28.58	38.50	2.56	0.47	191	404	707	1111	2:1
6	10.53	17.85	23.34	2.56	0.53	213	400	700	1100	2:1
7	16.97	24.58	35.34	2.56	0.59	234	396	692	1089	2:1
8	23.62	31.86	39.06	2.56	0.65	255	392	686	1078	2:1
9	8.09	12.41	26.82	2.56	0.51	190	370	740	1111	2:1
10	17.01	22.38	29.54	2.56	0.52	191	370	740	1111	2:1

Table 3: Property Ranges of Group-1 and Group-2 Tests

Name	Unit	Range (Group-1)	Range (Group-2)
Coarse aggregate (CA)	(kg/m^3)	985-1078	1042-1124
Fine aggregate (FA)	(kg/m^3)	665-826	630-826
Cement (C)	(kg/m^3)	356-475	312-448
Water (W)	(kg/m^3)	185,190	177-255
Fineness modulus (FM)of sand		2.4, 2.6	2.56
W/C ratio		0.4-0.52	0.4-0.76
CA size ratio (10mm:20mm)		1:1, 2:1	2:1
3rd day test strength		-	5.29-23.61
7th day test strength	MPa	13.84-27.82	8.61-31.86
14th day test strength	MPa	17.8-37.6	-
28th day test strength	MPa	19.53-39.37	12.37-39.06

The next step of the study was to determine a general equation for these curves being plotted from the model test data. Investigation shows that all the curves maintain a good correlation with the following simple equation:

$$f'_{c,D} = \frac{D}{D+q}\ p \qquad (1)$$

where, $f'_{c,D}$ = Strength of the concrete at Dth day.(D = 1,2,3,…..); D= Number of days; p and q are constants for each curve but different for different data sets (curves). It may be mentioned that this equation (Eq. 1) is similar to the equation (Eq. 2) proposed by ACI committee [ACI 209-71,1971] for predicting compressive strength at any day based on 28 days strength.

$$(f_c{'})_t = \frac{t}{a+b.t}.(f'_c)_{28d} \qquad (2)$$

Here, a and b are constants, $(f'_c)_{28d}$ = 28-day strength and t is time. This equation (Eq. 2) can be recast to similar form of Eq. 1.

To utilize the derived equation (Eq. 1), just value of two constants (p and q) are to be determined. Note that the constant q has the unit of day and p has the stress unit.

Table 4 shows the values of p and q for three arbitrary data sets. These are obtained from the best fit curves for each set of data. The values of p

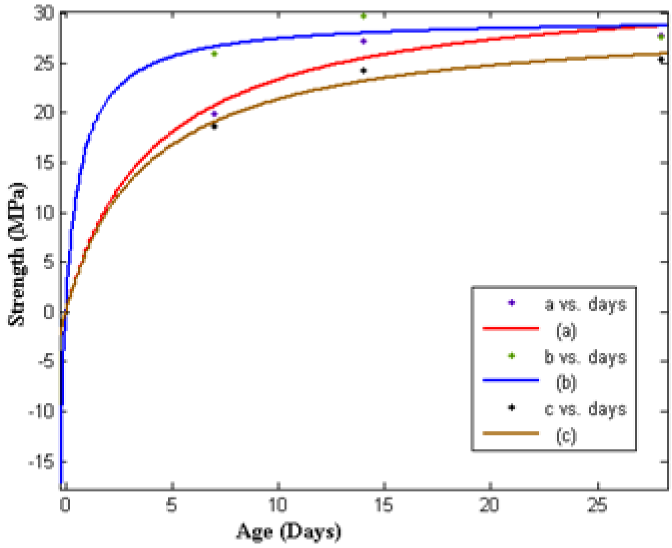

Fig. 1: Strength gaining curve for representative sets

Table 4: Representative Sample Sets Correlation

Set	Compressive strength test results (MPa)			Square of coefficient of correlation	Value of p	Value of q
	7 days	14 days	28 days			
(a)	20.40	22.73	32.55	0.991	32.94	4.091
(b)	18.62	24.24	25.33	0.992	29.53	0.757
(c)	23.48	26.42	28.97	0.995	29.35	3.730

and q can also be determined by putting strength test results in Equation 1 for any two days and solving it, but for this at least two test results for two different days are required. In this study, an attempt has been made to determine these values from only one day test result. An empirical relation is developed for this particular case (particular type of ingredients of concrete) to solve the problem. It is observed that, all values of p, q and strength of a particular day $f'_{c,D}$ for each set maintain a correlation of polynomial surface. In other words, values of p can be expressed as the function of q and $f'_{c,D}$ [which represent a polynomial surface]. This correlation is established using MATLAB surface fitting tool. The equation of the correlation is given below in Eq. 3:

$$p = a + b.q + c.f'_{c.D} + d.q.f'_{c.D} + e.\{f'_{c.D}\}^2 \qquad (3)$$

Where, $f'_{c,D}$ = Strength of the concrete at Dth day; (D = 1, 2, 3 …) and a, b, c, d and e are the coefficients of different terms. This polynomial relation of p, q and $f'_{c,D}$ is valid for different day test results of concrete strength, just the coefficients [a, b, c, d, e] of Eq. 3 will be different for different D-values. As the correlation for 7th day test result of concrete [D=7] is built up, the values of the coefficients becomes, a = - 6.26, b = 0.7898, c = 1.478, d = 0.0994 and e = - 0.0074. Putting these values in Eq. 3 equation (Eq. 4) is obtained. Figure 2 shows the polynomial surface corresponding to Equation 4.

Using 14th day strength results [D=14] the coefficients of Eq. 3 becomes, a = - 4.527, b = - 1.041, c = 1.373, d = 0.1406 and e = - 0.0125. Substituting these values in Eq. 3 ,Eq. 5 is derived:

Now, if the 7 day strength value is put in Eq. 4, it becomes a linear equation in p and q. Thus, solving two linear Equations 1 and 4, values of p and q are obtained for each case. Finally, after finding the values of p and q, the unique equation for a particular case can be formed. Using this procedure, all 56 different concrete data sets are analysed and the model predicted values of p and

$$p = -6.26 + 0.7898q + 1.478f'_{c.7} + 0.0994q.f'_{c.7} - 0.0074\{f'_{c.7}\}^2 \qquad (4)$$

$$p = -4.527 - 1.041q + 1.373f'_{c.14} + 0.1406q.f'_{c.14} - 0.0125\{f'_{c.14}\}^2 \quad (5)$$

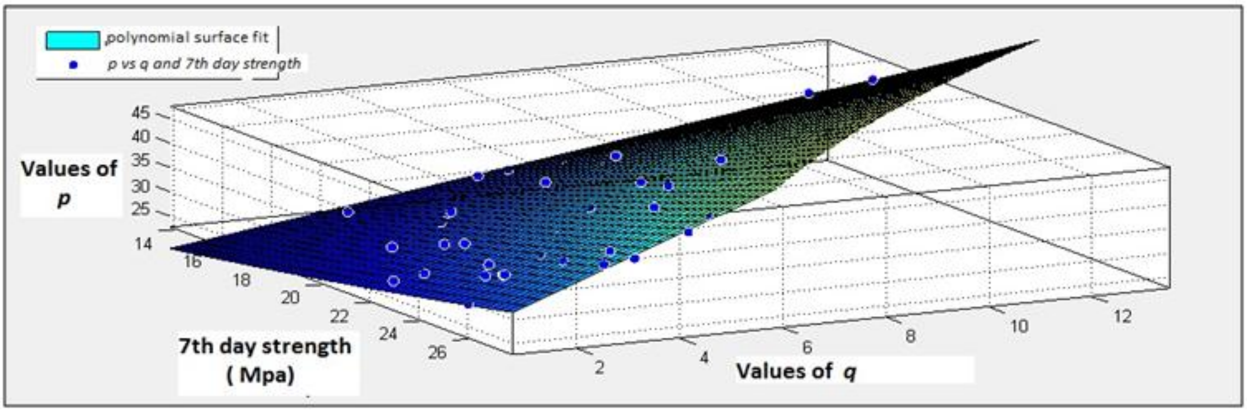

Fig. 2: Polynomial surface representing Equation 4

q, the unique equation for a particular case can be formed. Using this procedure, all 56 different concrete data sets are analysed and the model predicted values of compressive strength for 28th day are compared with experimental values. Same procedure is repeated when Eq. 5 (that relates p, q with 14 day strength) is used with Eq. 1 to evaluate the values of p and q. To validate the performance of the proposed model, Group-2 data obtained from tests in a different laboratory of a different country are used.

5. PERFORMANCE AND VALIDATION

The performance of the proposed equations are evaluated by three statistical parameters, mean absolute error (MAE), root mean square error (RMSE) and normal efficiency (EF); their expressions are given below

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}(|P_i - A_i|) \quad (6)$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(P_i - A_i)^2} \quad (7)$$

.

$$\text{EF} = \left(1 - \frac{1}{n}\sum_{i=1}^{n}\frac{(|Pi - A_i|)}{A_i}\right) \times 100\ \% \quad (8)$$

Here, A_i = Actual value; P_i = Predicted value; n = number of data (1, 2, 3 …).

Prediction of 28th day strength of concrete for some data sets of Group-1 is listed in Table 5. These predictions are made based on results of either 7 day or 14 day cylinder strength and are compared separately. The fineness modulus (FM) of fine aggregate (FA) was 2.4 for the first six sample data sets selected here and 2.6 for the last four. The ratio of the coarse aggregate sizes (10mm: 20mm) for first four sets was 2:1 and for the rest it was 1:1. The effectiveness of the proposed model is summarized below in Table 6 considering all the 56 test data of Group-1.

Group-2 test data are from completely different source. These are used to check the validity and versatility of the above mentioned model. Prediction of 28th and 3rd day strength are made from solving the Equation 1 & 4 [D=7] i.e. using 7 day test result. Table 7 shows the predicted values of the 3 days and 28 days strengths. Predictions efficiency of the model for Group-2 data are summarised in Table 8. All the samples are made with coarse aggregate having the maximum size ratio (10mm: 20mm) of 2:1.

Table 5: Prediction of 28 days Strength of Concrete for Group-1

Number	Concrete strength results (MPa), Ai			Concrete Mix Ratio (C:CA:FA)	W/C Ratio	Water (kg/m3)	Solving Equation 1 & 4 [D=7]		Solving Equation 1 & 5 [D=14]	
							Predicted Concrete strength (MPa), Pi	Pi/Ai	Predicted Concrete strength (MPa), Pi	Pi/Ai
	7day	14day	28day				28day		28day	
1	13.84	17.8	25.6	1:2.24:2.97	0.52	185	20.83	0.81	22.43	0.88
2	14.08	17.91	19.53	1:2.11:2.85	0.50	185	21.16	1.08	22.45	1.15
3	15.71	18.57	25.57	1:1.88:2.67	0.48	190	23.41	0.92	22.68	0.89
4	16.91	20.13	25.97	1:2.12:2.89	0.52	190	25.25	0.97	23.69	0.91
5	14.44	19.06	23.06	1:1.99:2.74	0.48	185	21.65	0.94	22.94	0.99
6	17.55	21.95	24.77	1:1.74:2.51	0.44	185	26.73	1.08	25.29	1.02
7	19.22	25.55	28.55	1:1.94:2.45	0.46	185	26.41	0.93	29.03	1.02
8	21.60	21.82	24.88	1:2.20:2.76	0.50	185	29.72	1.19	25.16	1.01
9	23.48	26.42	28.97	1:1.84:2.48	0.46	190	31.81	1.10	30.00	1.04
10	27.68	32.71	38.40	1:1.56:2.21	0.40	185	35.93	0.94	37.44	0.97

Table 6: Effectiveness of the Model in predicting 28 days Strength based on 7 and 14 days tests for Group-1

	Using 7th day strength result	Using 14th day strength result
Root Mean Square Error [RMSE]	3.07	3.02
Mean Absolute Error [MAE]	2.57	2.51
Efficiency [EF (%)]	91.0	91.4
Avg. Pi/Ai (min-max)	1.01 (0.80-1.24)	1.02 (0.81-1.20)

Table 7: Prediction of 3 and 28 days Strength of Concrete for Group-2

Sl. No.	Concrete strength results (MPa), A_i			Concrete Mix Ratio (C:CA:FA)	W/C Ratio	Water (kg/m^3)	Solving Equation 1 & 4 [D=7]		Solving Equation 1 & 4 [D=7]	
							Predicted Concrete strength for 3rd day (MPa), P_i	Pi/Ai	Predicted Concrete strength for 28th day (MPa), Pi	P_i/A_i
	3day	7day	28day				3days		28days	
1	14.49	23.34	32.34	1:1.45:2.51	0.41	185	15.91	1.10	31.65	0.98
2	18.89	23.38	30.90	1:1.45:2.51	0.50	220	15.94	0.84	31.70	1.03
3	12.01	22.22	28.58	1:1.45:2.51	0.55	241	15.01	1.25	30.42	1.06
4	10.09	14.53	20.01	1:1.45:2.51	0.61	262	9.131	0.9	21.78	1.09
5	21.57	28.58	38.50	1:1.75:2.75	0.50	199	20.49	0.95	36.73	0.95
6	10.53	17.85	23.33	1:1.75:2.75	0.56	220	10.76	1.02	28.36	1.22
7	16.97	24.58	35.34	1:1.75:2.75	0.61	241	16.93	1.00	32.95	0.93
8	23.61	31.86	39.06	1:1.75:2.75	0.67	262	23.78	1.01	39.39	1.01
9	8.09	12.41	20.63	1:2:3	0.54	198	7.72	0.95	18.87	0.91
10	17.01	22.36	29.54	1:2:3	0.54	199	15.14	0.89	30.60	1.04
11	11.57	15.17	27.42	1:2:3	0.61	220	9.56	0.83	22.66	0.83
12	7.69	10.77	15.17	1:2:3	0.67	241	6.64	0.86	16.58	1.09
13	18.45	26.58	36.22	1:2.5:3	0.50	177	18.66	1.01	34.91	0.96
14	14.01	17.09	28.10	1:2.5:3	0.56	199	11.75	0.83	25.59	0.91
15	5.29	9.93	14.41	1:2.5:3	0.63	220	6.09	1.15	15.38	1.07
16	5.73	9.17	12.61	1:2.5:3	0.70	241	5.60	0.98	14.28	1.13
17	20.41	30.50	37.90	1:2.5:3.25	0.58	190	22.37	1.10	38.33	1.01
18	12.13	18.41	25.78	1:2.5:3.25	0.68	220	14.12	1.17	22.21	0.86
19	9.52	17.05	20.57	1:2.5:3.25	0.75	241	10.73	1.13	25.51	1.24
20	7.29	11.51	13.46	1:2.5:3.25	0.80	254	7.27	1.00	16.61	1.23

Sl. No.	Concrete strength results (MPa), A_i			Concrete Mix Ratio (C:CA:FA)	W/C Ratio	Water (kg/m^3)	Solving Equation 1 & 4 [D=7]		Solving Equation 1 & 4 [D=7]	
							Predicted Concrete strength for 3rd day (MPa), P_i	Pi/Ai	Predicted Concrete strength for 28th day (MPa), Pi	P_i/A_i
	3day	7day	28day				3days		28days	
21	9.09	16.05	20.89	1:2.5:3.5	0.62	195	10.14	1.12	23.90	1.14
22	7.33	12.25	16.01	1:2.5:3.5	0.70	217	7.609	1.04	18.65	1.16
23	5.57	8.61	12.37	1:2.5:3.5	0.77	239	5.246	0.94	13.47	1.09

Table 8: Prediction Efficiency of the Proposed Model using Group-2 data

	Predicting 3rd day strength result	Predicting 28th day strength result
Root Mean Square Error [RMSE]	1.58	2.53
Mean Absolute Error [MAE]	1.19	2.12
Efficiency [EF (%)]	93.4	90.2
Avg. Pi/Ai (min-max)	1.00 (0.83-1.25)	1.04 (0.83-1.24)

6. CONCLUSION

A simple mathematical model is proposed to predict the compressive strength of concrete from the early age test results [any single day test]. Performance of the model is quite satisfactory. The concrete strength gain characteristic with age is modelled by a simple mathematical equation (rational polynomial). A polynomial surface equation is developed in terms of the two constants and concrete strength of a particular day. Early age test data have been used to get values of the two constants required for the prediction. The proposed equations have the potential to predict concrete strength for any age. There are scopes for further study to evaluate the values of these constants without any early age test if the two constants can be estimated from sufficient number of previous test results on typical concrete samples. The model is for ordinary Portland cement concrete. Similar model for other cement types or concrete with admixture can be developed. A simple and practical approach has been described for prediction of 28-day compressive strength of ordinary concrete and the proposed technique can be used as a reliable tool for assessing the design strength of concrete from quite early age test results.

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