UNSTEADY MHD FREE CONVECTION AND CHEMICALLY REACTIVE FLOW PAST AN INFINITE VERTICAL POROUS PLATE

By

M.C. RAJU * S.V.K VARMA ** R. RAMAKOTESWARA RAO ***

** Department of Humanities and Sciences, Annamacharya Institute of Technology and sciences (Autonomous), Rajampet, Andhra Pradesh, India.*
*** Department of Mathematics, S.V. University, Tirupati, A.P. India.*
**** Principal, Annamacharya Institute of Technology and sciences (Autonomous), Rajampet, Andhra Pradesh, India.*

ABSTRACT

This paper deals with the numerical study of an unsteady MHD free convection and mass transfer flow of dissipative fluid past an infinite vertical porous isothermal plate with heat generation and homogeneous chemical reaction. As the problem is governed by coupled nonlinear system of partial differential equations, exact solutions are not possible, hence explicit finite-difference method is employed. The effects of Magnetic parameter M and chemical reaction parameter K are examined on Velocity, Temperature and Concentration. The results are discussed through graphs. It is noticed that Velocity decreases with the increase in Magnetic parameter M and chemical reaction parameter K and Concentration decreases with the increase in K.

Keywords: MHD, Free convection, Chemical Reaction, Mass Transfer.

INTRODUCTION

In recent years, the problems of free convective and heat transfer flows through porous medium under the influence of a magnetic field have attracted the attention of a number of researchers because of their applications in many branches of science and technology, such as in transportation cooling of re-entry vehicles and rocket boosters, crosshatching on ablative surfaces and film vaporization in combustion chambers. On the other hand, flow through a porous medium have numerous engineering and geophysical applications, for example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. In view of these applications, the unsteady MHD free convection flows of dissipative fluids past an infinite plate have received much attention .This problem was first solved by Siegal (1958) without taking into account viscous dissipative heat and MHD by integral method. The experimental conformation of these results were presented by Goldstein and Eckert (1960). Other papers in this field are by Gebhart (1961), Schetz and Eichhorn (1962), Menold and Yang(1962),Sparrow and Gregg(1960),Chung and Anderson(1961),Goldstein and Briggs(1964), etc. In all these papers, the effect of viscous dissipative heat and MHD was assumed to be neglected.

However, Gebhart (1962) has shown that when the temperature difference is small or in high prandtl number fluids or when the gravitational field is of high intensity, viscous dissipative heat should be taken into account in steady free convection flow past a semiinfinite vertical plate .Following this assumption, Soundalgekar et.al (1979), (1997) studied the effects of free convection currents on the flow past an impulsively started infinite isothermal vertical plate. Raptis (1982) has studied free convection and mass transfer effects on the flow past an infinite moving vertical porous plate with constant suction and heat sources when free stream velocity is an oscillatory function of time. Agarwal et al. (1980) have discussed the combined buoyancy effects of thermal and mass diffusion on MHD natural convection flows. Vajravelu (1978) has studied the problem of free convection heat transfer between two long vertical plates moving in opposite directions.

Magneto convection plays an important role in various industrial applications, such as magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals. Diffusion rates can be altered tremendously by chemical reactions. Muttucumara swamy et.al (2006) studied thermal radiation effects on moving vertical plate with chemical reaction. Chambre and Young (1958) have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al (1979) .Chamkha et al.(2006) considered the effect of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium. Umavathi et al. (2008) investigated an unsteady magnetohydroynamic two fluid flow and heat transfer in a horizontal channel. Combined Effect of Heat Generation or Absorption and First-Order Chemical Reaction on micro polar Fluid Flows over a Uniformly Stretched Permeable Surface was studied by Magyari et al. (2010). Patil and Chamkha (2010) considered unsteady combined heat and mass transfer from a moving vertical plate in a parallel free stream. Raju et al. (2012) studied radiation and mass transfer effects on a free convection flow through a porous medium bounded by a vertical surface. Raju and Varma (2011) considered unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature. Reddy et al. (2009) studied thermo diffusion and chemical effects with simultaneous thermal and mass diffusion in mhd mixed convection flow with ohmic heating. Chemical reaction and radiation effects on unsteady MHD free convection flow near a moving vertical plate was considered by Reddy et al. (2012). In this paper the authors investigated an unsteady MHD free convection and mass transfer flow of dissipative fluid past an infinite vertical porous isothermal plate with heat generation and homogeneous chemical reaction.

1. Formulation of the problem

Unsteady free convection flow of viscous incompressible conducting fluid past an infinite vertical porous isothermal plate is considered. The flow is assumed to be in the x' - direction, which is taken along the vertical plate in the upward direction and the y' -axis is taken normal to the plate. It is assumed that at time t'>0, the surface of the plate is maintained at a uniform constant temperature w T'_w and a uniform constant concentration C'_w, of a foreign fluid, which are higher than the corresponding values T'_∞ and C'_∞ respectively, sufficiently far away from the plate. Initially the temperature of the fluid and plate are the same. A uniform transverse magnetic field H_0 is applied in the y' direction. As the magnetic Reynolds number is considered to be very small, the induced magnetic field and applied electric fields are neglected.

By the above assumptions the equations governing the flow are given below

$$\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \sigma\frac{\mu_e{}^2 H_0^2}{\rho}u' \quad (1)$$

$$\rho C_P\left(\frac{\partial T^1}{\partial t^1} + V^1\frac{\partial T^1}{\partial y^1}\right) = \kappa\frac{\partial^2 T^1}{\partial y^{1^2}} + \mu\left(\frac{\partial u^1}{\partial y^1}\right)^2 + q^*\left(T^1 - T^1_\infty\right) \quad (2)$$

$$\frac{\partial C'}{\partial t'} + V'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - K_l\left(C' - C'_\infty\right) \quad (3)$$

With the following initial and boundary conditions

$$\begin{aligned} &t' \le 0,\ u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \text{ for all } y' \\ &t' > 0,\ u' = 0,\ T' = T'_W,\ C' = C'_W \text{ for all } y' = 0 \\ &u' = 0,\ T' \to T'_\infty,\ C' \to C'_\infty, \text{ for all } y' \to \infty \end{aligned} \quad (4)$$

Using the on dimensionless variables used by Siegal (1958) and Schetz (1962), the governing equations reduce to the following form

$$\frac{\partial u}{\partial t} - \gamma\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta + NC - Mu \quad (5)$$

$$\mathrm{P}_r\frac{\partial\theta}{\partial t} - \gamma\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} + P_r E\left(\frac{\partial u}{\partial y}\right)^2 + P_r H.\theta \quad (6)$$

$$S_c\frac{\partial C}{\partial t} - \gamma\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - K\ S_c C \quad (7)$$

The initial and boundary conditions are

$$\begin{aligned} &t \le 0,\ u = 0,\ \theta = 0,\ C = 0 \text{ for all } y \\ &t > 0,\ u = 0,\ \theta = 1,\ C = 1 \text{ at } y = 0 \\ &u = 0,\ \theta \to 1,\ C = 0 \text{ as } y \to \infty \end{aligned} \quad (8)$$

2. Solution of the problem

Equations (5), (6) and (7) are coupled non linear Partial differential equations and are to be solved by using initial and boundary conditions of equation (8). However exact or approximate solutions are not possible for this set of equations. And hence we solve these equations by explicit finite difference Method. The equivalent finite difference scheme of equations for (5), (6) and (7) are as follows.

$$\left[\frac{u_{i,j+1}-u_{i,j}}{\Delta T}\right]-\gamma\left[\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right]=\left[\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta y)^2}\right]+\theta_{i,j}+NC_{i,j}-Mu_{i,j} \quad (9)$$

$$P_r\left[\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}\right]-\gamma\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y}\right]=\left[\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta y)^2}\right]+P_rE\left[\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right]^2+P_rH\,\theta_{i,j} \quad (10)$$

$$S_c\left[\frac{C_{i,j+1}-C_{i,j}}{\Delta t}\right]-\gamma\left[\frac{C_{i+1,j}-C_{i,j}}{\Delta y}\right]=\left[\frac{C_{i+1,j}-2C_{i,j}+C_{i-1,j}}{(\Delta y)^2}\right]-K\,Sc\,C_{i,j} \quad (11)$$

Here, index i refer to y and j to time t. The mesh system is divided by taking $\Delta y = 0.1$. From the initial condition in (9), we have the following equivalent

$u(0,0) = 0$, $\theta(0,0) = 1$, $u(i,0)=0$,

$q(i,0) = 0$ for all i except $i = 0$ (12)

The boundary conditions from (9) are expressed in finite difference form as follows

$u(0,j) = 0$, $\theta(0,j) = 1$, $C(0,j) = 1$ for all j

$u(1,j) = 0$, $\theta(1,j) = 0$, $C(1,j) = 0$ for all j (13)

Here infinity is taken as $y = 1$.

First the velocity at the end of time step namely $u(i,j+1)$, $(i=1,10)$ is computed from equation (9) and temperature $\theta(i,j+1)(i=1, 10)$ from (10) and concentration $C(i,j+1)$ $(i=1,10)$ from (11). The Procedure is repeated until t = 1 (i.e. j = 800). During computation Δt was chosen as 0.00125. These computations are carried out for $P_r = 0.71$, $E = 0.1$, $S_c = 0.3$, N=0.2, $\gamma = 0.4$, $M = 5$, H = 5, K=0.2,2 and t = 0.2, 0.4. To judge the accuracy of the convergence of the finite difference scheme, the same program was run with smaller values of Δt i.e, $\Delta t = 0.0009$, 0.001 and no significant change was observed. Hence we conclude the finite difference scheme is stable and convergent.

3. Results and discussion

Numerical computations have been carried out for different values of magnetic field parameter M, chemical reaction parameter K on velocity, temperature and concentration and the results are shown through graphs from Figures 1 to 6.

In Figure 1 variation in velocity is shown for M=1, 3, 5. It is noticed that velocity decreases with the increase of magnetic parameter M.

In Figure 2 effect of time t on velocity is shown for fixed values of P_r=0.71, γ=0.2, N=0.2, H=5, M=5,E=0.5 and K=0.2. It is noticed that velocity increases with the increase of t. In Figure 3, variation in velocity is shown with the variation of K and t, it is noticed that velocity increases with the increase of t.

In Figure 4, variation of velocity profiles is shown with the variation of chemical reaction parameter K. It is noticed

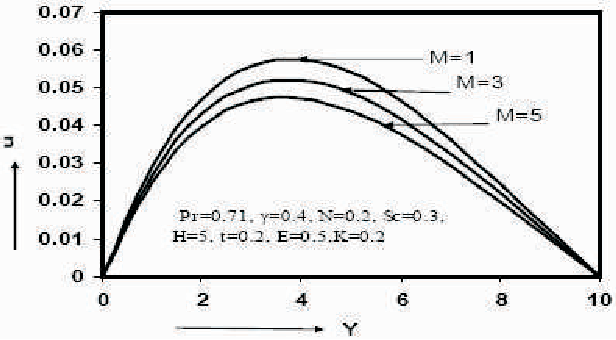

Figure 1. Effect of M on Velocity profiles

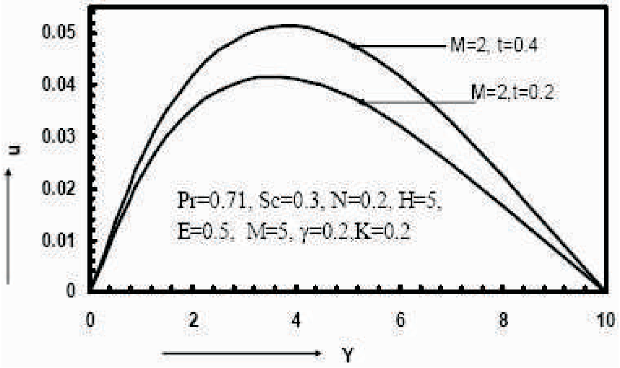

Figure 2. Effect of M and t on Velocity profiles

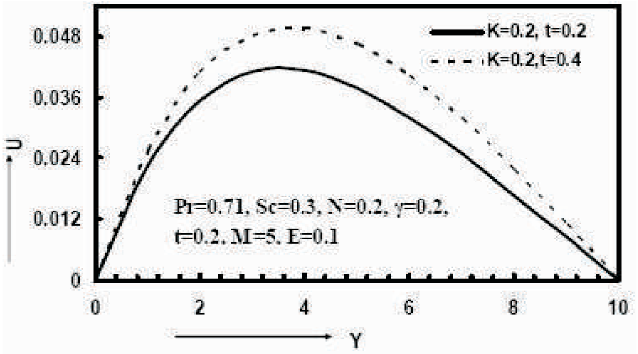

Figure 3. Effect of K and t on Velocity profiles

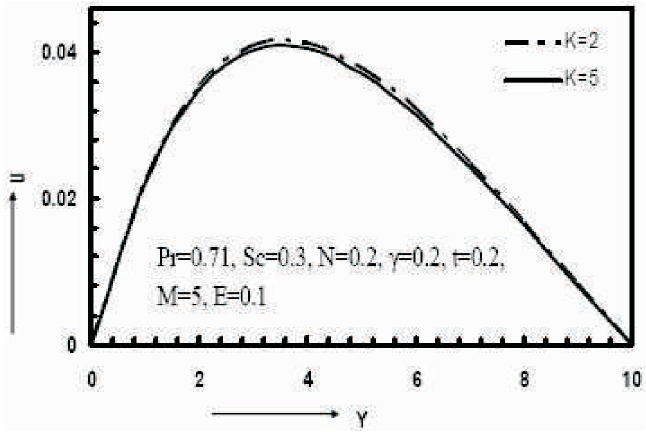

Figure 4. Effect of K on Velocity profiles

that velocity decreases with the increase of K. significant variation is seen near the centre and very low variation is observed near the plates.

In Figure 5, temperature profiles are shown with the variation of M and t. It is observed that temperature increases with the increase of t. In Figure 6, concentration profiles are shown with the variation of chemical reaction parameter K. It is noticed that concentration decreases with the increase of chemical reaction parameter K.

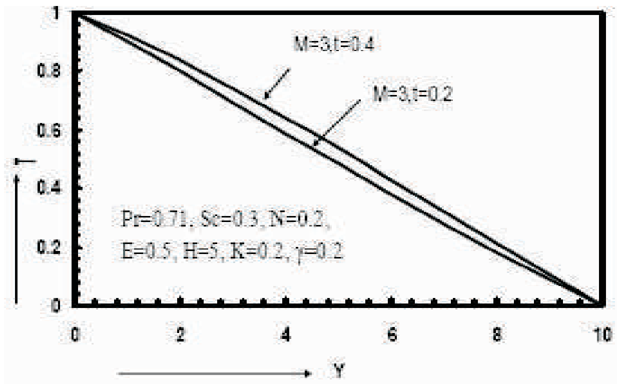

Figure 5. Effect of M and t on Temperature profiles

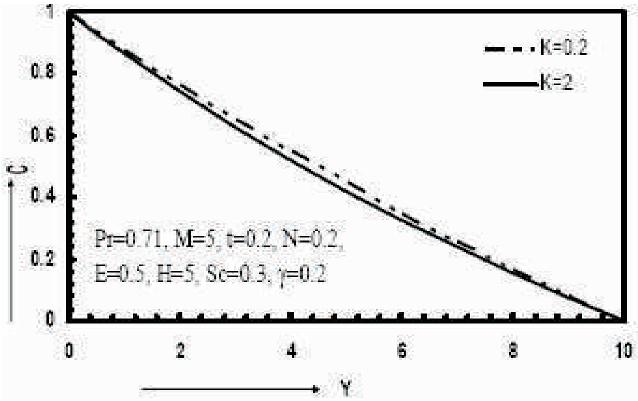

Figure 6. Effect of K on Concentration profiles

Conclusion

In this paper unsteady MHD free convection and mass transfer flow of dissipative fluid with heat generation past an infinite vertical porous plate with homogenous chemical reaction has been studied numerically by applying explicit finite difference method. From the present study, the following conclusions are summarized.

- Velocity decreases with the increase in Magnetic parameter M and chemical reaction parameter K.
- Concentration decreases with the increase of K.

Nomenclature

u' : Velocity of the fluid in x' -direction,
x' : Coordinate axis along the plat
y' : Coordinate axis normal to the plate,
T' : Temperature of the fluid near the plate,
T'_w : Temperature of the plate,
t' : Time,
C_p : Specific heat at constant pressure,
D : Chemical Molecular diffusivity,
D_1 : Coefficient of thermal diffusivity,
H_0 : Magnetic field of intensity,
C : Dimensionless concentration,
N : ratio of mass transformation,
P_r : Prandtl number,
S_c : Schmidt number,
E : Eckert number,
t : Dimensionless time,

u_0 : Velocity of the plate,

U : Dimensionless velocity,

T_R : Reference time,

L : Reference length,

M : Magnetic field parameter,

Kl : Chemical reaction parameter

H : Heat source parameter,

q* : Heat generation constant,

C′ : Concentration in the fluid near the plate,

C'_w : Concentration of the plate,

y : Dimensionless coordinate axis normal to the plate,

T'_∞ : Temperature of the fluid far away from the plate,

K : Dimensionless chemical reaction parameter

C'_∞ : Concentration in the fluid far away from the plate,

Greek symbols

β : Coefficient of volume expansion,

β^* : Coefficient of species expansion,

υ : Kinematic viscosity,

σ : electrical conductivity of the fluid,

γ : Suction parameter,

θ : Dimensionless temperature,

p : Density of the fluid,

κ : Thermal conductivity of the fluid,

μ_e : Magnetic permeability,

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ABOUT THE AUTHORS

M.C. Raju is currently working as Head of the Department of Humanities and Science of Annamacharya Institute of Technology and Sciences, Rajampet. He is a doctorate in Mathematics from Sri Venkateswara University Tirupati, Andhra Pradesh, India. Earlier he worked as the principal of Annamacharya PG college of Computer Studies, Rajampet. His area of research is Fluid Dynamics, Magneto Hydrodynamics, Heat and Mass transfer. He presented several papers in national and international conferences. He published 34 papers in National and International Journals. He is guiding few students for M.Phil and PhD in Mathematics. He is the reviewer for various National and International Journals. He is the life member in Indian Mathematical society and other bodies.

S.V.K. Varma, a Senior Professor in the Department of Mathematics, Sri Venkateswara University Tirupati Andhra Pradesh, India. He has vast experience in teaching and administration and also in research. His area of research is Fluid Dynamics, Magneto hydrodynamics, Heat and Mass transfer. He presented several papers in various conferences at national and international levels. He guided several students for PhD and M.Phil He is the life member of various bodies. He is the member of Board of Studies of various universities and Autonomous Institutions.

R. Ramakoteswararao did his B.Tech and M.Tech from Andhra University, Visakhapatnam. He received doctorate from Sri Venkateswara University Tirupati. He served in S.V.University at various administrative and teaching positions for 30 years. Presently he is working as Principal of Annamacharya Institute of Technology and Sciences, Rajampet. His research area is Hydrodynamics studies in3-Phase Specialization and heat and mass transfer. He guided one student for Ph.D and three students are duning Ph.Ds under his guidance. He has supervised 30 M.Tech projects in the area of Chemical Engineering. He published several papers in various national and international journals. He also attended various conferences, seminars and workshops.