Pricing of reservations for time-limited spectrum leases under overbooking

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Abstract—With spectrum trading the owners of usage rights on wireless spectrum may transfer them temporarily to other users. A secondary market for spectrum may be implemented through the use of reservation mechanisms, by which prospective consumers of spectrum portions may first book them and decide later whether to actually buy the usage rights. A possible strategy for the owners of usage rights is to practise overbooking, i.e., to allow for more reservations than what they could sell (and then pay a penalty if they cannot satisfy the requests). A relevant problem for the owners of usage rights is to set the price for reservations. An algorithm is proposed here to price reservations in the presence of overbooking. The algorithm is based on the iterative use of the Cox-Ross-Rubinstein approach to price financial call options. The algorithm is then applied to a number of cases and the dependence of the price on the overbooking probability and the penalty is analysed.

Index Terms—Spectrum trading, Wireless services, Real options, Overbooking.

I. INTRODUCTION

WIRELESS services represents an ever growing fraction of communications services. Such services rely on the availability of spectrum, which is typically assigned to service providers on a (semi)permanent basis through licensing agreements. A number of studies have shown that such long-term assignments may lead to inefficient use of the spectrum (a scarce resource by itself), and that the possibility of reselling the licenses or to temporarily lease spectrum portions would lead to a more effective usage of that resource (see, e.g., [1] and [2]). Such possibility would spur the creation of a secondary market for spectrum, where that resource could be exchanged.

A number of technical and business arrangements have been considered to deploy such secondary market, and some of them are a reality in several countries (see, e.g., [3] [4] [5] [6] [7] [8] [9]). Most studies envisage a spot market, where spectrum is sold on-the-fly, e.g., through an auction [10]. However, spot selling is the hardest implementation of a real time market, since the seller does not know the offers in advance and the ownership rights are to be switched from the seller to the buyer in a very short time. On the other hand, in [11] it has been suggested that the owner of usage rights may opt for a less demanding intermediate solution, by adopting a selling arrangement based on two stages through the use of reservations, where a payment is carried out first for the reservation itself and then possibly for the actual purchase. Though this two stage mechanism allows the selling side (the owner of usage rights) to carry on a minimal planning activity while cashing in on the option sale, it has been envisaged that the owner of usage rights may adopt an overbooking strategy, selling more options than the number of available spectrum blocks, but facing the possibility of paying penalties if it cannot satisfy the requests of some prospective buyers. In [11] that strategy has been analysed to determine its terms of convenience for the seller of usage rights, but the price to be paid for the reservation has been considered as a parameter. However, setting such price is a relevant issue, which directly influences the profit conditions for both parties. At present, no solutions have been proposed to price the rights associated to the reservation.

In this paper we approach the reservation price setting problem in the overbooking scenario and propose a pricing algorithm based on an iterative use of the Cox-Ross-Rubinstein (CRR) method, a well known tool employed for pricing options. The CRR method is not directly applicable, since in our case there is a mutual relationship between the option price and the penalty. Here we provide a description of the algorithm and apply it to examine the dependence of the price on the probability of overbooking and the penalty.

The paper is organized as follows. In Section II we review the literature on spectrum trading. In Sections III and IV we describe instead the trading arrangement we propose based on reservations, and show that it can be modelled through the use of the option device employed on financial markets. We describe our pricing algorithm in Section V and report the results of its application to a number of cases in Section VI.

II. SPECTRUM TRADING AND TIME-LIMITED LEASES

Wireless services require the use of portions of electromagnetic spectrum, which is a limited resource. Its allocation is therefore subject to strict rules both at an international level (set up by the International Telecommunications Union) and at a national one. For public services, blocks of frequencies are first assigned to service providers (through a licensing procedure), that in turn use them to provide services to end users. Currently, the prevailing assignment mechanisms to service providers fall into one of the following three categories: 1) Administrative authorization (which requires compliance with basic terms and obligations); 2) Beauty contest (which is based on the evaluation of the technical quality of proponents); 3) Auction. Whatever the assignment mechanism, spectrum portions are typically assigned on a semi-permanent basis (e.g., over 15-20 years) and over a nation-wide area.

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Such assignment conditions are widely considered to be quite inefficient (and lead to low spectrum utilization) for several reasons. A major cause of concern is that many licenses remain idle, e.g., because their holders are not able to carry out the needed investments: Gruber estimated that, as of the end of 2005, 25% of assigned licenses were unused in Europe [12]. In addition, the spectrum occupancy, even in densely populated areas, may be quite low, as reported in [13], and with large variations throughout the day, as shown in [14].

It is thus natural to consider the possibility of allowing the owners of spectrum rights (i.e., those who were assigned the licenses) to resell those rights, giving rise to a secondary market for spectrum, a possibility advocated in [1] and [2]. A number of regimes can be envisaged for the management of spectrum usage rights (see [15] for a thorough review), and secondary markets for spectrum have already been introduced in the national legislations of several countries [3] [4] [5] [6] [7] [8] [9] [10] [11]. Usage rights on spectrum may be transferred either on a (semi)permanent or a temporary basis (a taxonomy of trading instances for spectrum is reported in [17]). As to the former possibility, it has been shown in [18] that the possibility of reselling the licence represents a put option and adds a quantifiable value to the license itself. On the other hand, a secondary market acting on smaller timescales and with a strictly limited duration can be created. Due to the limitation on time, the transaction taking place is actually a lease rather than a sale, since the usage rights go back to their owner as soon as the lease duration expires. In addition to the limitation on time, the granularity of usage rights traded on secondary markets could be as small as desired. The definition of the resource whose usage rights are transferred has been first approached by De Vany, who introduced the concept of TAS (Time-Area-Spectrum) packets, whereby the owner of rights on a TAS has the exclusive right to produce electromagnetic waves for a specified period of time over a specified geographic area and a specified range of frequencies [19] [20]. Doyle and Forde have then refined that concept into that of Frequency-Space-Time (FST) block, where the notion of space, rather than area, is used to take into account the three-dimensional nature of electromagnetic radiation [21]. In the following we will refer to the FST block as the basic quantity exchanged on the secondary spectrum market. A number of techniques have been proposed to cater for the possibility of letting spectrum for a limited time, under the collective name of dynamic spectrum access technologies [22] [23] [24] [25].

Though the actors involved in spectrum trading are many (see [26] for some examples of the value chains occurring in this context), we refer hereafter to the simple (yet general) case where the owner of primary rights on spectrum releases them to a secondary user, for a limited time period and over a limited portion of spectrum (which may even be a single frequency). The temporary transfer of rights takes place after a trading procedure, for which basically two possibilities exist: either the spectrum is sold on the spot (e.g., through an auction) or as the result of a two-stage mechanism, involving a reservation phase. In both cases pricing the spectrum portion on sale is a very relevant issue. Dynamic auction mechanisms and pricing policies for spot selling have been investigated, e.g., respectively in [27] and [28]. On the other hand, reservation mechanisms, though well known and adopted to resolve the contention issue in multiple access systems (see, e.g., [29] or Chapter 11 in [30]), have not been addressed thoroughly yet in this context.

III. RESERVATIONS AND THE OVERBOOKING STRATEGY

As seen in the previous section there are various ways to implement a secondary market for spectrum. In this paper we focus on a two stage selling mechanism, where FST blocks are first reserved and then possibly bought. In this section we provide details about this mechanism and describe the overbooking strategy that sellers of spectrum rights may adopt.

The proposal for a reservation mechanism in spectrum trading has been put forward in [11]. As in any market we have basically two counterparts here: the buyer of temporary spectrum rights and the seller. In the reservation case the prospective buyer reserves an FST block for a specified time in the future. When the time comes to actually exploit the reservation and use the FST block, the prospective buyer may decide to actually buy the FST block (and pay the corresponding price) or to let the reservation expire. Since the reservation introduces a constraint for the seller, we may expect that the prospective buyer will have to pay for it. There are then two economic transactions directly associated to the reservation procedure: the prospective buyer first pays a price for the reservation and then, if it decides to purchase the FST block, pays a predetermined price for the block itself.

In this procedure the seller receives the reservation payment but faces the possibility of receiving nothing else if the prospective buyer lets the reservation expire. In this case the seller ends up with a small amount of money (as we expect the reservation payment to be) and an unsold FST block. Since this state of things may be unsatisfactory for the seller, an overbooking strategy has been investigated in [11] as a hedging device against the risk of remaining with unsold FST blocks. In the overbooking strategy the owner of the usage rights accepts more reservations that it can handle, i.e., more reservations than the number of FST blocks it owns and can actually sell. If, when the time T in the FST comes, the number of claimed blocks is not larger than the number of sellable blocks, the seller has no difficulty in satisfying all the buyers and give them the FST blocks they required. But, if the number of reservations that turn into actual purchase requests is larger than the number of sellable blocks, then the requests in excess cannot be satisfied, and the seller has to compensate the unsatisfied prospective buyers. An important issue here is the amount of money that has to be paid back to the prospective buyer to compensate it for the failed purchase (the penalty). On reasonable grounds, we can assume that this amount lies somewhere between the reservation price and the value of FST block (which in turn we can assume to be linked to the profits it can generate when used by the prospective buyer), i.e., between the actual loss and the potential profit. In [11] the compensation has been set equal to a multiple of the reservation price. Here we follow the same assumption.
IV. A REAL OPTIONS MODEL

In Section III the reservation procedure has been described. In [11], where the overbooking strategy was introduced, it has been shown that a reservation amounts to a real option on the spectrum usage rights, hence the tools of financial options can be employed to set the price to be paid for the reservation. In this section we review the notions of financial and real options, and define the correspondence between the real options model and our spectrum trading arrangement including reservation and overbooking.

Options are a well known device in financial markets. An option is a contract between two parties, a buyer and a seller, that gives the buyer the right (but not the obligation) to buy or to sell a particular asset (the underlying asset) at (or within) a later time at an agreed price (the strike price or exercise price) [31]. Options that must be exercised at a precise time are named European options, while contracts that provide a time framework to exercise the option rather than a strict date are named American options. In return for granting the option, the seller collects a payment (the premium) from the buyer.

Many complex types of option can be devised: the two basic ones are often named plain vanilla call and put options. A call option gives the buyer the right to buy the underlying asset, while a put option gives the buyer of the option the right to sell the underlying asset. When the time comes to exercise the option, the buyer may choose to exercise this right (and the seller of the option is then obliged to sell or buy the asset at the strike price) or not to exercise the right and let it expire. The underlying asset may be a piece of property, or shares of stock, or some other security.

When the underlying asset is an investment project rather than securities, the term real option is used. In that case the present value of the underlying asset is the Net Present Value (NPV) of the project in the absence of the flexibility embodied by the option. This assumption is known as the Marketed Asset Disclaimer (MAD) [32]. The correspondence between financial and real options is complete if we consider the investment needed for the project as the equivalent of the exercise price, and the time at which the investment decision will have to be taken as the expiry time of the option. In our spectrum trading case the exercise price is then the price at which the spectrum consumer will be able to buy the FST block from the owner of usage rights, and the expiry time is the time at which the spectrum consumer will decide whether to actually buy the block.

As shown in [11], the possibility of reserving a portion of spectrum for a later purchase represents a European call option. However, the possibility of overbooking makes this option defaultable. It is then what is known in the financial literature as a naked or vulnerable option.

In the financial sector the concept of vulnerable option (an option that is subject to the risk of the option seller’s default) has first been described by Johnson and Stulz [33] and later analyzed by Klein [34]. In our case the equivalent of the default condition is the inability of the option seller (the service provider) to provide the required block of frequencies (because overbooking has occurred). Though the literature on

the subject has expanded in the latest years, in this context we limit ourselves to these two papers. In fact, as we show below, neither of these papers is directly applicable to our case, but more recent contributions move even farther from applicability.

In particular, we examine the payoff under default, and the conditions under which the option cannot be exercised (default condition). As to the latter issue, in both models of [33] and [34] the default occurs when the value of the company’s assets fall below a threshold. That threshold is the value of the option at expiry in the model by Johnson and Stulz, and the overall amount of money due by the option seller in Klein’s model (this amount includes both the value of the option at expiry due to the option holders, and the debt towards other creditors). The two models also differ as to the actual payoff received by the option holder, which is either the value of the company’s assets (Johnson and Stulz) or a fraction of the value of the option at expiry (Klein). Instead, in our case, the default condition is due to an exogeneous condition, i.e., the occurrence of overbooking, which is not related to the degradation of the financial conditions of the option’s seller but rather to an external condition, the excess of demand. And the payoff in the case of overbooking is a multiple of the option’s price (i.e., the value of the option when it is sold, rather than at expiry). Both the methods proposed are therefore not directly applicable to our case.

V. THE BINOMIAL METHOD APPROACH TO PRICING

In Section IV we have seen that the reservation can be modelled as a real option. We can therefore employ the tools of real options valuations to set its price. A well known tool for this purpose is the Cox-Ross-Rubinstein (CRR) method [35], also known as the multiplicative binomial model. However, this method, in its original formulation, doesn’t allow for option vulnerability, and hence cannot be used to set the option price in the presence of overbooking. In this paper we propose an iterative algorithm, based on the CRR approach, to solve the pricing problem under overbooking. In this section we first describe the basic CRR method and then provide the details of the iterative algorithm we propose.

In the original CRR approach a binomial tree is built to model the evolution of the value of the underlying asset till the expiry time. The option price is set as the result of a procedure that traverses that tree twice, first forward and then backward. A full description of CRR is provided in [31]; here we skip a number of details (and proofs), focussing on the most relevant expressions. In CRR the evolution of the value of the underlying asset (typically modelled as a lognormal random walk) is approximated through its discrete-time version, a binomial model. We indicate the time at which the option has to be underwritten by 0; the option’s expiry date is then $T$. The time to expiry is divided into $N$ time intervals of duration $\Delta T = T/N$. We expect the approximation to be better as the subdivision gets finer, i.e., as the number of intervals grows. We index the intervals by $i = 0, 1, \ldots, N$, so that the value of the investment into the FST block at the end of the $j$-th interval is $S^{(j)}$. As already stated, the initial value $S^{(0)}$ of the project is the Net Present Value of the
project in the absence of any flexibility, i.e., in the absence of the buying option. While we suppose to know $S^{(0)}$ (though it is actually estimated), the values $S^{(i)}$, $i > 0$, are random quantities, described by a binomial process. Over any single time interval we suppose that the project value can move from its previous value to one of two alternative values
\[ S^{(j+1)} = \begin{cases} uS^{(j)} \\ dS^{(j)} \end{cases} \]
where the two up and down factors $u$ and $d$ are respectively $u = 1 + \sigma \sqrt{T}$ and $d = 1/u$, and $\sigma$ is the standard deviation of the value of the investment in the FST block (a.k.a. the volatility).

Since over the time $T$ we have $N$ such time intervals, we end up with $2^N$ possible values for the investment into the FST block at the option expiry date. In order to account for all the possible values we indicate by $S^{(j,i)}$, $i = 0, 1, \ldots, j$ and $j = 0, 1, \ldots, N$, the $i + 1$-th smallest value of the project at the $j$-th time interval (then $S^{(j,0)}$ is the lowest possible value at the $j$-th stage). Since we have $u = 1/d$ (and therefore $ud = 1$), the tree is recombining and the possible alternative values of the investment at the expiry date are actually $N + 1$. A sample tree is shown in Fig. 1.

![Event tree for a binomial process over three time intervals ($N = 3$)](image)

Since we have $S^{(j,i)} = S^{(0)}u^id^{j-i}$, at the expiry date ($j = N$) we are able to compare the values of the investment at that date with the exercise price $E$ of the option. When the investment’s value lies above the exercise price, the prospective buyer finds it worth buying the FST block, i.e., exercising the call option. If the investment’s value lies below the exercise price, the option is not worth exercising and its value is 0. We are then able to associate a decision (concerning the exercise of the option) to each of the investment’s possible outcomes, and to compute the possible values of the option at the expiry date, which are $W^{(N,i)} = \max(S^{(N,i)} - E, 0)$. The forward traversal of the event tree maps therefore the present value of the investment $S^{(0)}$ into $N + 1$ possible option values at expiry $W^{(N,i)}$, $i = 0, 1, \ldots, N$.

The present value of the option (which sets its price) is obtained by traversing the binomial tree backward and repeatedly using the expression
\[ W^{(j-1,i)} = p^*W^{(j,i+1)} + (1 - p^*)W^{(j,i)} \]
where
\[ p^* = \frac{1}{2} + \frac{r\delta T}{2\sigma} \]
is the so-called risk-neutral probability. In fact, the resulting option value can be seen as the expected value of the option under the risk-neutral probability, discounted at the risk-free rate $r$. The correct price $V_0$ for the option is then its present value, i.e., $V_0 = W^{(0,0)}$.

So far, we have described the CRR approach, which provides the value of the vulnerability-free option (i.e., with no overbooking). We modify the basic CRR method to take into account the possibility that the option cannot be exercised at expiry because of overbooking. For this purpose we introduce the probability of overbooking $\mathbb{P}_{\text{ovb}}$. This probability is determined by the correlated decisions of the prospective buyers. A normal copula model has been proposed in [11] for it, but in this paper we do not enter into the details of that model and simply consider it to be an exogeneous quantity. For each non-zero value of the investment at expiry, the option value at expiry is then either the penalty $X = \alpha V$ (with probability $\mathbb{P}_{\text{ovb}}$) or the no-overbooking result $\max(S^{(N,i)} - E, 0)$ (with probability $1 - \mathbb{P}_{\text{ovb}}$). The value of the option at expiry may then be set as the expected value
\[ W^{(N,i)} = [\mathbb{P}_{\text{ovb}} \alpha V + (1 - \mathbb{P}_{\text{ovb}})(S^{(N,i)} - E)] I_{S^{(N,i)} - E > 0}, \tag{4} \]
where we employ the indicator function $I_x$, which is equal to 1 if the subscript condition $x$ is satisfied and 0 otherwise. But in expr. (4), the value of the option at expiry depends on the option price $V$, i.e., the present value of the option $W^{(0,0)}$. The self-dependence character of expr. (4) can be overcome by adopting an iterative procedure, i.e., cycling through the following steps: 1) Traversing back the binomial tree, through the repeated use of expr. (2), starting with the option value at expiry to get the option price and then the penalty; 2) Putting the option price back into expr. (4) to get the option value at expiry. We obtain therefore a sequence $\{V_0, V_1, \ldots\}$ of approximations to the actual option price, starting with the option price $V_0$ in the absence of vulnerability. The full algorithm is summarized as Algorithm 1.

VI. RESULTS

In Section V we have described an algorithm, based on the binomial approach introduced by Cox, Ross, and Rubinstein [35], to determine the value of the option on an FST block when the penalty to be paid by the option buyer is a multiple
Algorithm 1 (Estimation of option price)

1: Compute value \( V_0 \) for option price under no vulnerability through CRR
2: Set threshold \( Tolerance \) for accuracy in the estimation of option price
3: Set \( Update \leftarrow V_0 \)
4: Set the index for the number of iterations \( i = 0 \)
5: \textbf{while} \( Update > Tolerance \) \textbf{do}
6: \hspace{1em} Update the number of iterations \( i \leftarrow i + 1 \)
7: \hspace{1em} Compute penalty \( X \leftarrow \alpha V_{i-1} \)
8: \hspace{1em} Compute the value of the option at expiry through expr. (4) with \( V = V_{i-1} \)
9: \hspace{1em} Update the present value of the option \( V_i \) through the backward tree in CRR
10: \hspace{1em} Compute the variation in the estimate of the present value of the option \( Update \leftarrow \left| V_i - V_{i-1} \right| \)
11: \textbf{end while}

of the value of the option itself. In this section we report the results of the application of that algorithm in some relevant scenario. In particular we focus on the dependence of the option price on the probability of overbooking and the penalty.

In our evaluation we consider the following values for the parameters of interest:

- Risk-free rate \( r = 0.05 \) (yearly)
- Volatility \( \sigma = 0.2 \) (yearly)
- Current value of the FST block \( S = 100 \) (Value of the underlying)
- Price of the FST block at the option expiry \( E = 100 \) (Exercise price)
- Time to expiry: 1 week, 1 month

The values adopted for \( S \) and \( E \) are purely conventional, since the option value will typically depend on their ratio. Instead the risk-free rate is typically determined as the interest rate paid by non-defaultable securities, such as short-term government bonds; hence its value depends on the current macro-economic situation. Instead, determining the volatility is quite harder. We refer to [18] for a deeper discussion on the subject. The value we adopt here is rather conservative, being near to the estimate \( \sigma = 0.18 \) suggested in [36].

We first examine the dependence of the option price on the probability that the set of FST blocks is fully sold (overbooking probability). We recall that in the case of overbooking the service provider (the option buyer) cannot provide the secondary user with an FST block and has to pay the penalty. We expect therefore that the option price decays as the overbooking probability grows. Here we report, in Fig. 2 and Fig. 3, the option price curve when the penalty is set at five times as large as the option price (\( \alpha = 5 \)), with a time to expiry equal to 1 month and 1 week respectively. The reference value, i.e., the option price when there is no chance of overbooking, is 2.51 (expiry time of 1 month) and 1.154 (expiry time of 1 week). We can see that the decrease in the value of the option is quite sharp (and sharper as the expiry time gets shorter). In fact, the option price in the case of overbooking gets down to one tenth of the no-overbooking value when the probability of overbooking is as low as roughly 0.075 (1 month expiry time) and 0.06 (1 week expiry time). As stated earlier, the general trend of the price curve is as expected, but the decay rate is quite large and may make the option nearly valueless even if the overbooking probability is moderate.

![Fig. 2. Impact of the overbooking probability on the option price (1 month)](image)

![Fig. 3. Impact of the overbooking probability on the option price (1 week)](image)

We now turn to the dependence of the option price on the penalty value, namely, the ratio between the penalty and the option price itself. Again we consider the two cases where the time to expiry is 1 month (Fig. 4) and 1 week (Fig. 5). As expected, a large penalty amounts to a large reward in the case of overbooking, hence it represents a safeguard for the option buyer and adds value to the option. The growth rate (of the option value as a function of the penalty/price ratio) appears to depend largely on the expiry time. In fact, the growth is quite limited on the shorter period (1 week). But in both cases there is a limit value for the penalty/price ratio, beyond which the evaluation algorithm stops converging. What happens in that case is that the penalty value becomes dominant with respect to the nominal payoff, so to make the overbooking more convenient for the option buyer than the actual purchase of the FST block. This limit ratio is slightly more than 40 for the 1 month case and slightly less than 90
for the 1 week case (the corresponding portion of the curve is not shown in Fig. 4 and Fig. 5).

![Fig. 4. Impact of the penalty on the option price (1 month)](image)

![Fig. 5. Impact of the penalty on the option price (1 week)](image)

VII. CONCLUSION

An iterative algorithm has been proposed to set the price of reservation for spectrum when overbooking is considered. Such algorithm allows to determine the reservation price when the reservation conditions are stated (i.e., the price to be paid for the spectrum block, the expiry date for the option, and the penalty in the case of overbooking), and the market conditions are estimated (i.e., the volatility and the risk-free rate). The possibility to set a correct price for reservation is fundamental for the deployment of spectrum trading through a two-stage arrangement. The algorithm has been employed to analyse the dependence of the reservation price on the overbooking probability and the penalty. The reservation price results to be a fast decaying function of the overbooking probability, with a decay rate that gets larger as the expiry time gets shorter. Instead, increasing the penalty/price ratio (the penalty is set as a multiple of the reservation price) leads to increased reservation prices, though the effect may be minimal over very short timescales.

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