Microwave Radiometer Spatial Resolution Enhancement

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Abstract—Scanning radiometer data processing can allow enhancing the limited intrinsic spatial resolution. This is important for data fusion. Mathematically, the problem to be solved is an inverse ill-posed problem. In this paper we compare the classical Backus–Gilbert inversion method with the truncated singular value decomposition (TSVD) one. A one-dimensional intercomparison is accomplished using an hypothetical sensor configuration. Results show the superiority of TSVD inversion method.

Index Terms—Fredholm integral equation, inverse problems, microwave radiometry, remote sensing, singular value decomposition (SVD).

I. INTRODUCTION

MICROWAVE radiometers are key sensors to globally monitor environmental parameters. Unfortunately, they suffer from low and nonuniform intrinsic spatial resolution, which changes from channel to channel and is determined by the antenna gain. For instance, if we consider the Special Sensor Microwave/Imager (SSM/I) radiometer we have that the intrinsic spatial resolution varies from approximately 100 to 10 km [1]–[3].

The low resolution is itself a limiting factor in some applications of radiometer measurements. Multisensor data fusion techniques are very attractive for geophysical applications and call for a coregistration of different dataset. Further, data fusion among various radiometer channels is of interest. It is, therefore, advisable to enhance the intrinsic spatial resolution. Alternatively, a resolution degradation can be used to match different spatial resolutions but this is very ineffective. In all these cases, a correct time/spatial matching of different measurements is a key element to effectively support geophysical parameter retrieval. Therefore, the capability to generate microwave radiometric measurements with enhanced spatial resolution respect to the intrinsic one is a key factor.

It is important to note that the class of problem which describes the relationship between the coarse but partially correlated radiometer measurements $T_A$ and the temperature to be retrieved on the high-resolution grid $T_{AP}$ belongs to the class of Fredholm integral equations of first kind whose kernel depends on the antenna gain [4], [5].

Throughout this paper we assume that atmospheric effect can be neglected so to confuse the apparent temperature with the scene brightness temperature [4]. Obviously, this does not affect the significance of the paper. From now, to avoid confusion, we refer to the surface brightness temperature $T_B$ instead of $T_{AP}$.

Mathematically, it is an ill-posed linear problem and must be inverted carefully [6]–[8]. Proper regularization techniques must be considered to take into control noise amplification when inversion is accomplished [6]–[8]. In other words, surface brightness temperature reconstruction at enhanced resolution calls for the solution of an ill-posed inverse problem [6]–[8].

In literature some methods to enhance the intrinsic radiometer spatial resolution have been proposed [1], [4], [9]–[16]. The Backus–Gilbert (BG) inversion method is the classical inversion method in microwave radiometry [9]–[12]. This method was first developed for the inversion of gross earth data [17] and then used to yield atmospheric temperature fields from microwave and infrared measurements [4], [18]. Then, this method has been suggested and adapted by Stogryn for retrieving brightness temperature fields $T_B$ from a set of radiometer measurements inverting a set of antenna temperature measurements $T_A$ [4].

In [1], the task of attempting to enhance the spatial resolution is made by an ad hoc method which is implemented as level 2 of the algorithm chain. In [12], a resolution enhancement method based on the scatterometer image-reconstruction (SIR) is investigated for the spatial resolution enhancement of SSM/I data. The SIR method is compared to the BG one, and it is shown that the two methods achieves similar resolution enhancement, though SIR requires significantly less computation [12]. In [13], a modified version of the BG inversion method, named DBG, is investigated by means of a theoretical one-dimensional (1-D) analysis. In order to optimize the implementation of the DBG algorithm the singular value decomposition (SVD) is exploited [13]. In [14], an inversion method based on a Wiener deconvolution filter is used to enhance the spatial resolution of SSM/I data. In [15], an enhancement method based on multiresolution wavelet transform is proposed. In [16], the authors proposed the truncated singular value decomposition (TSVD) as an inversion method to enhance the spatial resolution of radiometer data.

In this paper, we illustrate and compare the BG inversion method and the TSVD inversion method.

The BG inversion method reconstructs the surface brightness temperature by means of a weighted sum of the radiometer measurements. The coefficients depend on the antenna scanning configuration and on the antenna gain. The noise amplification is minimized by a proper choice of two tuning parameters which affect the coefficients [4], [10]–[12].

The TSVD inversion method we propose, as an alternative to the BG one, expresses the brightness temperature as a weighted
sum of radiometer measurements but now the coefficients are
directly related to the singular values associated to the problem.
In this case, noise amplification is taken into control by a proper
choice of the number of retained singular values [19], [20].

The two inversion methods are very attractive in terms of
computational efficiency since in both cases the expansion coef-
ficients depend only on the system configuration and not on
measurements. Therefore, at least in principle, the coefficients
must be determined only once, given the radiometer configura-
tion, and the inversion procedure reduces to a weighted sum of
the measurements. We note that the TSVD method, as detailed
in the following, calls for the evaluation of a single set of coef-
ficients while the BG one calls for a multiple set of coefficients.
However, only a simple and unexpensive hardware is required
to implement the inversion methods. All this makes the two in-
version methods suitable for real-time applications.

Physically, the two inversion methods are based on the char-
acteristic of a scanning microwave radiometer which makes par-
tially correlated measurements of the observed scene. In fact, a
scanning radiometer makes multiple observations of the same
scene under different view angles. Real aperture radiometer em-
ploys two typical scanning configurations, i.e., linear and con-
cical scanning [21]. In any case, resolution enhancement methods
exploit the availability of coarse but partial correlated measure-
ments to reconstruct the surface brightness temperature [21].

Here we present a numerical study conducted for the 1-D
case, i.e., the scene is modeled by a brightness temperature
field and the radiometer antenna scanning is linear. The study ana-
yzes the performance of the two inversion procedures in terms of
brightness temperature field reconstruction by means of objec-
tive norms. Realistic noise sources, i.e., instrumental noise,subscale spatial variability noise, cross-polarization coupling,
feed-horn spill-over, and edge bias, are considered [1], [2].

II. THEORY

In this section, we summarize the theory which is at the basis
of the two inversion methods.

Let us say $T_{Ai}$ the single (noise-free) radiometer measure-
ment acquired when the antenna gain observes the scene in
accordance to the pointing angle $\rho_{0i}$. $T_{Ai}$ can be written as follows:

$$
T_{Ai} = \int \tilde{G}(\rho, \rho_{0i}) T_B(\rho) d\rho
$$

(1)

where $\tilde{G}(\rho, \rho_{0i})$ is the radiometer antenna gain, and $T_B(\rho)$ is
the surface brightness field corresponding to the specific radiometer
channel [11]. Note that in (1) a further integration is due if we
want to emphasize the radiometer dwell time [4]. Equation (1)
corresponds to neglect the variation of the observed brightness
temperature field during the integration time of the instrument.
Since this is usually the case in the following we proceed accord-
ingly [10]. Although real radiometers usually scan the earth’s
surface by means of conical and linear scanning configurations,
in this study we consider a 1-D hypothetical sensor configura-
tion as in [13]. The scanning geometry is illustrated in Fig. 1
where a set of $N$ measurements are considered. The set of $N$

radiometric measurements $T_{Ai}$ is referred as $T_A$. The bright-
ness temperature field reconstructed at enhanced resolution is
referred as $T_B$. 

A. Backus–Gilbert Inversion Method

The Backus–Gilbert is an inversion method for solving
integral equations. When it is employed to determine surface
brightness temperatures from integrated, overlapping antenna
patterns, the BG method produces a weighted least squares es-
timate of the surface brightness temperature field with a spatial
resolution enhancement respect to the intrinsic resolution of the
sensor’s channel [12].

The algorithm estimates the brightness temperature field $T_B(\rho)$ at each scanning position $\rho_{0i}$. For this purpose, the BG
inversion method uses a linear combination of $2I + 1$ nearby
measurements [4], [12]

$$
T_B(\rho = \rho_{0i}) = \sum_{I=-I}^I a_I T_{Ai}
$$

(2)

where the coefficients are dictated by the measurement configu-
ration and the noise-correlation matrix [10]. Note that these co-
ficients depend on the pointing angle $\rho_{0i}$, due to the varying
antenna geometry over the swath. Let us now show how the $a_I$
coefficients are determined. Inserting (1) in (2) we have

$$
T_B(\rho_{0i}) = \sum_{I=-I}^I a_I T_{Ai} = \left[ \sum_{I=-I}^I a_I \tilde{G}(\rho, \rho_{0i}) \right] T_B(\rho) d\rho
$$

(3)

It is important to note that due to the unavoidable finite
number of available measurements, brightness temperature
reconstruction is by definition approximate. This leads to look
for the best solution in least square sense [4].

As a matter of fact, the problem to be resolved is the choice
of $a_I$ which minimize the following functional:

$$
Q_{0i} = \int \left[ \sum_{I=-I}^I a_I \tilde{G}(\rho, \rho_{0i}) - F(\rho, \rho_{0i}) \right]^2 J(\rho, \rho_{0i}) d\rho
$$

(4)

with the natural energy preservation constraint [4], [10], [18]

$$
\int \sum_{I=-I}^I a_I \tilde{G}(\rho, \rho_{0i}) d\rho = 1.
$$

(5)
Note that this inversion method allows reconstructing the temperature field at a number of points equal to \( N \). Therefore, to reconstruct the temperature field at any \( \rho \), i.e., \( T_B \), we need to perform a data interpolation. Here we use a cubic spline. It is in fact a simple and popular interpolation scheme, e.g., [10]. The interested reader can find a full study on alternative interpolation schemes in [22].

One important feature of the BG inversion method is to directly incorporate the noise model. Since real radiometer measurements are primarily affected by an additive Gaussian noise \( \mathcal{N}(0, \Delta T_{\text{rms}}^2) \), i.e., characterized by a zero mean and a \( \Delta T_{\text{rms}} \) root mean square (rms), in the hypothesis that noise is uncorrelated, we have that the variance noise \( \epsilon^2 \) over \( T_B(\rho_0) \) is [4]

\[
\epsilon^2 = (\Delta T_{\text{rms}})^2 \sum_{i=1}^{l} a_i^2.
\]

Therefore, the function to be minimized is now

\[
Q_i = Q_0 \cos \gamma + \epsilon^2 w \sin \gamma
\]

where \( w \) and \( \gamma \) are two tuning parameters. \( \gamma \) is the noise tuning parameter and ranges from 0 to \( \pi/2 \) controlling the tradeoff resolution enhancement and reconstruction noise level. The \( \gamma \) value can be subjectively chosen but it has been shown that is dependent on \( \Delta T_{\text{rms}} \) and the functions \( J(\cdot) \) and \( F(\cdot) \). Therefore \( \gamma \) is expected to change from channel to channel [10]–[12]. The dimensional parameter \( w \) is an arbitrary scalar factor which is independent of \( \gamma \) [12].

In order to assist the users, some studies have been developed to objectively define the best tuning parameters \( w \) and \( \gamma \), and the BG functions \( J(\cdot) \) and \( F(\cdot) \). In particular, some studies have been conducted to define an objective procedure, based on the maximum correlation between the 85-GHz channel and the channel of interest, to select \( \gamma \) for 19-, 22-, and 37-GHz SSM/I channels [12]. In other studies the values of \( w = 0.001 \) and \( \gamma \) included from 0\(^\circ\) and 30\(^\circ\) have been suggested [11]. Obviously, the \( F(\cdot) \) and \( J(\cdot) \) must be also specified to fully characterize the BG inversion method [11].

Backus and Gilbert have demonstrated that the physical significance of the theory do not depend on the exact choice of tuning parameters [4].

As a matter of fact, the set of \( a_l \) coefficients \( a \) is given by

\[
a = V^{-1} \left[ \bar{v} \cos \gamma + \frac{(1 - \bar{u}^T V^{-1} \bar{v} \cos \gamma)}{(\bar{u}^T V^{-1} \bar{u})} \bar{u} \right]
\]

where

\[
V = G \cos \gamma + (\Delta T_{\text{rms}})^2 w \sin \gamma,
\]

\( \Delta T_{\text{rms}}^2 \) is the noise matrix, and \( G \) is a matrix whose elements are

\[
G_{ij} = \int \tilde{G}_i(\rho) \tilde{G}_j(\rho) J(\rho, \rho_0) d\rho.
\]

\( u \) and \( v \) are vectors whose components are given by

\[
u_l = \int \tilde{G}_l(\rho) d\rho,
\]

\[
\psi_l = \int \tilde{G}_l(\rho) F(\rho, \rho_0) J(\rho, \rho_0) d\rho.
\]

### B. TSVD Inversion Method

Let us now describe the TSVD inversion method. With this respect, it is useful to cast the problem in matricial form, i.e.,

\[
y = A x
\]

where \( y \) corresponds to the \( M \)-long vector of observables data, and \( x \) is the \( N \)-long vector of the unknown field to be reconstructed. Note explicitly that in this case the reconstruction points are not constrained to be equal to measurements. Therefore, the real matrix \( A \in \mathbb{R}^{M \times N} \) with \( M \geq N \) must be inverted. Let us decompose \( A \) in accordance to the SVD; hence

\[
A = U \Sigma V^T = \sum_{i=1}^{N} u_i \sigma_i v_i^T.
\]

\( U \) and \( V \) are matrices with orthonormal columns so that

\[
U^T U = V^T V = I_N
\]

\[
\Sigma = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}
\]

with \( D = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_N) \) organized in decreasing order; \( u_i \) and \( v_i \) are, respectively, the left and right singular vectors of \( A \) [23].

Here we make explicit reference to the 1-D case since it is pertinent to the numerical experiments presented in Section III and it can be shown that the two-dimensional case can be decomposed into two 1-D problems [16].

Two important properties of the SVD of a matrix \( A \) must be emphasized. The condition number of \( A \) is given by the ratio between the largest and the smallest singular value and the singular values are always well conditioned with respect to perturbations [7].

The inversion of the problem as expressed in (13) can be simply managed noting that

\[
U \Sigma V^T x = y
\]

and therefore

\[
\Sigma V^T x = U^T y
\]

and finally

\[
x = V^+ U^T y
\]

where \( V^+ U^T = A^+ \) is the pseudoinverse of \( A \) [24]. Equation (19) is the unique minimal norm solution of the considered problem [22].
To facilitate users we explicitly note that
\[ U^T y = [(u_1.y) \ldots (u_M.y)]^T \] (20)
and
\[ \Sigma^+ U^T y = \left[ \begin{array}{c} (u_1.y) \\ \sigma_1 \\ \vdots \\ (u_M.y) \\ \sigma_M \end{array} \right]^T. \] (21)
In conclusion we have
\[ x = \sum_{i=1}^{N} \frac{(u_i.y)}{\sigma_i} v_i. \] (22)

It can be demonstrated that smaller \( \sigma_i \) values are less reliable and must be discarded [7], [8]. This leads to the truncated SVD which can be simply written as
\[ x_K = \sum_{i=1}^{K} \frac{(u_i.y)}{\sigma_i} v_i. \] (23)

Such a singular values truncation regularizes the inversion procedure [7], [8].

Before proceeding further two points deserve special care. First, the matricial form of the radiometric problem to be inverted is slightly different. In fact we have that (13) in our case is
\[ T_A = A^T T_B \] (24)
where the real matrix \( A' \in \mathbb{R}^{M \times N} \) with \( M < N \). As a matter of fact, the above-mentioned inversion method must be applied once that \( A'^T \) is considered and \( U \) and \( V \) are interchanged [7].

Second, the definition of the number of singular values to be retained is a key point in the TSVD inversion method.

The use of TSVD in radiometric resolution enhancement has found very limited attention. In any case, the choice of where truncate the SVD series has been solved by means of rule-of-thumb approaches [13], [16]. Although this may be effective, it is useful to support such a choice by means of a theoretical framework.

On this purpose it is useful to note that two cases may occur. In the first case the matrix to be inverted is characterized by a cluster of small singular values. This implies that the matrix contains almost redundant information which can be discarded to regularize the inverse problem. In this case the singular values truncation is straightforward. In the second case we have that the matrix to be inverted is characterized by a gradual decay of the singular values. In such a case we have that the discrete Picard condition is satisfied [7], [8]. The rationale at the basis of the discrete Picard condition is based on the analysis of \( (u_i, y)/\sigma_i \) in (22) and in particular on the decay of the coefficients \( |w_i/y_i| \) relative to the decay of the \( \sigma_i \). In this case the singular values truncation is given by the intersection of the curves.

Mathematical background theory shows that to such a second class of problems belongs the inverse problem associated to discrete Fredholm integral equation of the first kind [25]. In the following, we show that the real problem in question may belong to both classes of problems. This key fact has never been acknowledged. It is useful to note that although the TSVD inversion method originates to deal with the first class of problem it can be tailored to the second class of problem as well.

Physically, we can say that if we retain a greater number of singular values the reconstruction is capable to better describe the high-frequency part of the temperature field to be reconstructed but this is counterbalanced by an increased reconstruction noise.

III. NUMERICAL EXPERIMENTS

In this section, we present and discuss the numerical intercomparison of the BG inversion method and the TSVD one. A hypothetical 1-D sensor configuration is considered; see Fig. 1. A sensor at an altitude of 833 km scanning a field of 1400 km is considered. We consider 65 measures and, therefore a nominal sampling of 25 km is in question [1], [2], [12]. The antenna temperature measurements have been simulated on the basis of the algorithm described in [1] and [2]. In order to make the simulation noise as realistic as possible, as in [12], the high-resolution brightness temperature field used to generate the set of antenna measurements is affected by subscale spatial variability noise [12]. Further, a radiometric measurement error modeled as an additive Gaussian noise of 0.5 K rms is considered.

Three antenna gains have been considered in this study: a sinc gain, a Gaussian gain and a realistic gain. In Fig. 2 their normalized plots are represented with reference to the nadir looking case. All of them are characterized by a 3-dB main-lobe of about 70 km. The realistic antenna gain has been excerpted by [1]. A maximum antenna gain of 43 dB is considered.

As detailed in Section II to apply the BG inversion method the tuning parameters \( \gamma, w \) and the functions \( F(\cdot), J(\cdot) \) must be specified. In this study, their choice has been made on the basis of the fundamental works described in [10]–[12]. Hence the tuning parameters have been set as follow \( w = 0.001 \) and \( \gamma = \pi/6 \) [10]–[12]. We also note that in the numerical experiments shown in Section III we have experienced that the reconstruction quality is stable with \( \gamma \) (examples not shown). In any case,
in accordance to [10]–[12], we have selected $\gamma = \pi/6$ since it is considered as the optimum tradeoff between reconstruction and noise amplification. To maximize the resolution enhancement, $F(\cdot)$ has been set equal to 1 over the element of interest and zero elsewhere. For $J(\cdot)$ we have considered the constant function 1 except when we consider high boundary errors, i.e., edge bias. In these cases, in order to minimize boundary reconstruction errors, we have considered the function $(\theta - \theta_0)^2$ as suggested by [13]. Further, we have considered three nearby measurements since Stogryn in [4] shows that they are sufficient for the reconstruction of 1-D temperature fields and our study has shown that increasing $I$ reconstruction is not improved (examples not shown to save space). For reader completeness, we note that in [12] a different criterion for the selection of $I$ is suggested. In any case, the selection of $I$ must consider the radiometer characteristics, e.g., the half-power main-lobe width and the scanning step.

With reference to the TSVD method the selection of the truncation point, i.e., $K$, has been made on the basis of what detailed in Section II. We note that the parameter choice is fundamentally a function of the antenna gain. This is witnessed by the different plots trend of the normalized singular values reported in Fig. 3. Since the singular values are not affected by noise, once the antenna characteristics have been set, the value of the optimal $K$ is independent to the particular noise source. Of course, this is implicitly assumes that the considered measurement model is reliable.

The numerical experiments are organized in subsections in accordance to the type of error source. In order to provide an objective analysis of the reconstruction results some norms must be introduced. In some cases they have been referred to low-resolution brightness fields [10]. Here we make reference to the high-resolution ones although we must expect that the corresponding norms show poorer values. In particular, we refer to the total relative absolute (TRA) error defined as follows:

$$
\zeta = \sum_i \left| \frac{T_i^{\text{rec}} - T_i^{\text{ref}}}{T_i^{\text{ref}}} \right|
$$

where the high-resolution reference and reconstructed brightness fields are considered. Further, the objective norm based on the fields correlation [12] is employed. In particular, we refer to the correlation coefficient between the reference and the reconstructed field defined as follows:

$$
\rho(T^{\text{ref}}, T^{\text{rec}}) = \frac{\text{cov}(T^{\text{ref}}, T^{\text{rec}})}{s(T^{\text{ref}}) s(T^{\text{rec}})}
$$

where cov$(\cdot)$ is the covariance operator and $s$ is the standard deviation.

A. Antenna Gains

In this section, a set of brightness temperature reconstructions as function of the antenna gains are presented and discussed. In particular, the antenna gains shown in Fig. 2 are considered. Before showing the numerical experiments it is useful to present the behavior of the singular values in the three cases; see Fig. 3. We note that the antenna gain makes the singular values plot very different and this affects the truncation criteria. This makes interesting to investigate the behavior of the sinc and Gaussian cases, which can be meant as two extreme cases.

In this first set of experiments, only the radiometric measurement error and the subscale spatial variability noise are considered. As reference field a hot spot over a uniform ground is taken into account. Two sizes of the hot spot are considered: 28 and 18 km. Note that such a temperature field is very challenging to be reconstructed: abrupt changes corresponds to infinite bandwidth and cannot be exactly reconstructed by bandlimited remote sensing system [21]. Notwithstanding, we make reference to such cases, since they are often used [10]–[12].

Reconstruction results are shown in Figs. 4–9. Solid lines refer to the reference noise-free fields, while the hatched lines refer the reconstructed ones. The BG inversion method results are shown on the left side, and the TSVD ones are shown on the right side.

Note that, while the reconstruction is over a profile of 1400 km, to emphasize reconstruction performances, we show only the central 800-km part. The same format is used in subsequent figures.

As first subjective analysis of the results we can say that in all cases reconstructions are capable to detect the hot spot even if its actual width is much smaller than the intrinsic resolution. We note however that TSVD-based reconstructions are better than the BG ones. If we focus on the TSVD reconstructions we note that, as expected, the Gaussian antenna gain works much better than the sinc one due to its negligible side-lobes.

In the BG case the use of the Gaussian gain has a lower impact.

More interesting is to note that the TSVD reconstructions relevant to the realistic antenna gain are remarkable.

Obviously in these latter cases the reconstruction noise is larger if compared to the Gaussian case.

In order to accomplish an objective analysis the TRA error and the correlation have been measured; see Tables I and II. Further, the reconstructed hot spot width can be exploited to define an additional objective norm; see Table III. The reconstructed
Fig. 4. (Left) BG and (right) TSVD reconstructions with Gaussian gain —28 km hot spot.

Fig. 5. (Left) BG and (right) TSVD reconstructions with sinc gain —28 km hot spot.

hot spot width is defined as the null-null width once that the noise-free background temperature is removed.

Note also that in these tables not only the cases shown in Figs. 4—9 are illustrated but also the case in which the reference noise-free field is characterized by a 18-km hot spot and a 5-K rms radiometric error (figures not shown to save space).

Both the TRA error and the correlation show that the TSVD inversion method better behaves. On this purpose, it is particularly useful to consider Tables II and III. In the TSVD case reconstructions relevant to the realistic gain show a limited TRA error, an high correlation and a good capability to detect the hot spot even when the rms radiometric level is raised to 5 K.

Finally, we note that the BG reconstructions provide less ringings with respect to the TSVD ones. This is a direct consequence of the enhanced resolution achieved by the TSVD. In fact, it is well-known that “...all resolution-enhancement techniques...provide improved resolution at expense of an increased noise level in the images” [12]. This is also witnessed by the fact that the product spatial resolution by radiometric sensitivity is practically constant. In such a case it is appropriate to consider the spatial resolution in terms of the 3-dB definition and the radiometric sensitivity as the smallest change in the average brightness temperature that can be detected and, following generally accepted criteria, this smallest change is taken as the standard deviation of the random perturbation of the reconstructions [26]. For instance, if we consider the cases shown in Fig. 6, we have that in the BG case this product is equal to 55.36 while in the TSVD case is equal to 54.80.

Whenever the application calls for a limited ringing, this can be taken into account in the TSVD inversion procedure; see for example real data case.

B. Additional Sources of Error

In this subsection we consider a set of measurements which have been simulated taking into account not only the radiometric measurement error and the subscale spatial variability noise but other sources of error which can be encountered in real radiometric systems [1], [2]. From now on only the realistic antenna
Fig. 6. (Left) BG and (right) TSVD reconstructions with realistic gain — 28 km hot spot.

Fig. 7. (Left) BG and (right) TSVD reconstructions with Gaussian gain — 18 km hot spot.

Fig. 8. BG (left) and TSVD (right) reconstructions with sinc gain — 18 km hot spot.
Fig. 9. (Left) BG and (right) TSVD reconstructions with realistic gain – 18 km hot spot.

<table>
<thead>
<tr>
<th>Inv. Methods</th>
<th>BG</th>
<th>TSVD</th>
</tr>
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<tr>
<td>G</td>
<td>S</td>
<td>R</td>
</tr>
<tr>
<td>hotspot 28 km 0.5 K</td>
<td>2.152</td>
<td>3.084</td>
</tr>
<tr>
<td>hotspot 18 km 0.5 K</td>
<td>1.662</td>
<td>2.275</td>
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<tr>
<td>hotspot 18 km 5.0 K</td>
<td>2.476</td>
<td>2.279</td>
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Table I

Total Relative Absolute Errors Between the Reference Temperature Fields and the Reconstructed Ones

Fig. 10. (Hatched line) BG and (dotted line) TSVD reconstructions with additional cross-polarization error.

<table>
<thead>
<tr>
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<th>TSVD</th>
</tr>
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<tbody>
<tr>
<td>G</td>
<td>S</td>
<td>R</td>
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<tr>
<td>hotspot 28 km 0.5 K</td>
<td>0.684</td>
<td>0.550</td>
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<tr>
<td>hotspot 18 km 0.5 K</td>
<td>0.569</td>
<td>0.445</td>
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<tr>
<td>hotspot 18 km 5.0 K</td>
<td>0.368</td>
<td>0.443</td>
</tr>
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</table>

Table II

Correlation Coefficient Between the Reference Temperature Fields and the Reconstructed Ones

The reference noise-free field is now composed by three spots (from left to right) of 29, 28, and 18 km over a background; see Figs. 10–13. In particular, we consider three different shapes to evaluate the reconstruction performance in presence of various, less abrupt, vertical variations and different reciprocal spacing. In all these figures, the reference brightness temperature field (solid line) and the BG reconstructed field (hatched line) and the TSVD one (dotted line) are shown altogether. The first source of additional error is the cross-polarization one, which has been simulated as suggested in [1] and [2]. Reconstructions results are shown in Fig. 10. Subjective analysis shows that the TSVD shows better results although a noticeable reconstruction error is shown. The BG reconstruction is not capable to follow the reference noise-free field and shows a smoother behavior. TRA error and correlation have been evaluated in this case and reported in Table IV. We note that subjective analysis is best supported by the correlation. In fact, we have that if we compare the TRA errors relevant to numerical experiments of Section III-A (Table I) and Section III-B (Table IV) shows that such an error is able to describe the complexity of the reconstruction, i.e., smoother reconstructed fields, as generally in the BG case, are characterized by lower TRA errors.

The second source of additional error is the spillover which has been simulated as suggested in [2]. Reconstructions results are shown in Fig. 11. Subjective analysis shows, especially for the BG case, a reconstruction behavior similar to the case shown in Fig. 10. In any case, the TSVD reconstruction appears to be more reliable. Objective analysis reported in Table IV confirms the subjective analysis.

The third source of additional error is the edge bias which has been simulated as suggested in [2]. This systematic kind of error may arise of because the feedhorn partially sees the cold space reflector at the edge of the earth-viewing portion of the scan [2]. In this case, since some authors [13] have reported that
in order to minimize boundary reconstruction errors the most suitable choice for the \( J(\cdot) \) BG function is \((\theta - \theta_0)^2\) we have accomplished the BG reconstruction accordingly. Reconstructions results are shown in Fig. 12.

Subjective analysis shows a very different behavior with respect to two former cases. As matter of fact, we have now not only a spot broadening but even a misplacement. Objective analysis reported in Table IV clearly shows that the correlation is now, as expected, lower with respect to cases shown in Figs. 10 and 11.


Finally, we have considered all former additional sources of error altogether; see Fig. 13. Subjective analysis clearly shows the presence of the three sources of errors. Similarly to the case shown in Fig. 12 we have that the quality of the TSVD reconstruction appears to be degraded. Objective analysis reported in Table IV shows that all the error norms confirm the poorer reconstruction behavior. It is however clear that even in this case the TSVD reconstruction is better that the BG one: according to the correlation the TSVD reconstruction achieves a 10% improvement over the BG one.

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### Table IV

<table>
<thead>
<tr>
<th>Inv. Methods</th>
<th>TRA Error</th>
<th>Correlation</th>
<th>TRA Error</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 10</td>
<td>2.666</td>
<td>0.617</td>
<td>3.535</td>
<td>0.863</td>
</tr>
<tr>
<td>Figure 11</td>
<td>2.786</td>
<td>0.618</td>
<td>3.718</td>
<td>0.865</td>
</tr>
<tr>
<td>Figure 12</td>
<td>2.677</td>
<td>0.617</td>
<td>5.317</td>
<td>0.729</td>
</tr>
<tr>
<td>Figure 13</td>
<td>3.162</td>
<td>0.587</td>
<td>5.803</td>
<td>0.698</td>
</tr>
</tbody>
</table>

### C. Real Data

In this subsection we consider a reconstruction based on real data. The real data are relevant to SSM/I, 19 GHz, vertical polarization, F11 satellite, date 1992, day 210, top left corner image (latitude, longitude): (51.2, 14.9), bottom right corner image (30.9, 37.8). In accordance to the 1-D simulations previously described, we have extracted some cross-track lines. Since the physical results of these experiments are similar we detail only a single reconstruction case.

The corresponding high-resolution reconstructions are shown in Fig. 14.

Subjective analysis shows that the BG reconstruction is smoother than the TSVD one. This seems to be a peculiar behavior of all BG reconstructions.

In this case, we have also considered a TSVD reconstruction in which the number of retained singular values \( K \), has been on purpose wrongly selected. Five additional singular values have been considered for this reconstruction which is shown in Fig. 15 where the BG reconstruction is kept for reference. Subjective analysis clearly shows the importance of the correct selection of \( K \) in order to limit noise reconstruction.

Before moving to the objective analysis, it must be noted that when real data are in question one of the difficulties in evaluating the reconstruction results is that it is not available a reference high-resolution field. This fact has been clearly addressed in literature, i.e., [10] and [12]. Hence, some others methods of validation have been proposed [10], [12]. In particular, in [10] it is suggested to consider a self-consistency test in order to limit

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**Fig. 11.** (Hatched) BG and (dotted) TSVD reconstructions with additional spillover error.

**Fig. 12.** (Hatched line) BG and (dotted line) TSVD reconstructions with additional edge bias error.

**Fig. 13.** (Hatched line ) BG and (dotted line) TSVD reconstructions with all additional sources of errors.

**Fig. 14.** (Hatched line) BG and (dotted line) TSVD reconstructions.
Fig. 14. (Hatched line) BG and (dotted line) TSVD reconstructions.

Fig. 15. (Hatched line) BG and (dotted line) TSVD reconstructions.

Fig. 16. Low-resolution brightness fields: (+) reference field and (o) BG one.

Fig. 17. Low-resolution brightness fields: (+) reference field and (o) TSVD one.

Fig. 18. Low-resolution brightness fields: (+) reference field and (o) TSVD one.

In order to best detail the objective analysis the low-resolution brightness temperature fields are shown in Figs. 16–18. In particular, in Fig. 16 the available low-resolution, i.e., nonenhanced, temperature field (+) and the low-resolution BG one (o) are shown. In Figs. 17 and 18, the same format is used to illustrate the TSVD cases. In Figs. 17 and 18 it is shown the TSVD case in which $K$ is correctly (wrongly) selected. Corresponding objective analysis is reported in Table V. As expected, we have now that the use of low-resolution fields generates very low TRA errors and very high correlation values. We also note that now the differences between the BG inversion method and the TSVD one.

TABLE V

<table>
<thead>
<tr>
<th>Inv. Methods</th>
<th>TRA Error</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG Fig. 16</td>
<td>1.122</td>
<td>0.958</td>
</tr>
<tr>
<td>TSDK Fig. 17</td>
<td>1.115</td>
<td>0.966</td>
</tr>
<tr>
<td>TSDK Fig. 18</td>
<td>1.844</td>
<td>0.948</td>
</tr>
</tbody>
</table>

channel to channel variability. Here we follow this approach to objectively evaluate the reconstruction results.
are attenuated. It is clear, however, that the use of wrongly selected $K$ generates high reconstruction noise as experienced by the authors even by simulated experiments (not shown).

It is also useful to read the reconstruction results shown in Fig. 17 in a different perspective. In this case the TSVD reconstruction at low resolution no shows ringings and the objective quality norms are remarkable. As formerly mentioned, the absence of ringings can be useful in application where main focus is not on spatial resolution. In other words, this clearly shows that the TSVD procedure is flexible to best match different needs.

IV. CONCLUSION

A comparative study on the TSVD inversion method employed to enhance radiometer spatial resolution has been shown. Reference has been made to the classical BG inversion method. Results have shown that the TSVD reconstructions achieve higher spatial resolution improvement. A theoretically sound criterion to define the SVD truncation has been presented. As a result the procedure is stable and reliable. It has been also shown that the TSVD inversion procedure is also effective when applications call for reconstructions with limited ringings and spatial resolution enhancement.

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REFERENCES


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