The emergence and development of young children's personal mathematical inscriptions

Worthington, Maulfry Dobromila

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The Emergence and Development of Young Children's Personal Mathematical Inscriptions: The evolution of graphical signs explored through children's spontaneous pretend play

Maulfry Worthington

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The Emergence and Development of Young Children's Personal Mathematical Inscriptions

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Maulfry Dobromila Worthington

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promotor:	prof.dr. H.J.M. van Oers
co-promotor:	dr. M. Dobber
promotiecommissie:	prof.dr. M.E.J. Raijmakers
	prof.dr. M.H.A.M. van den Heuvel-Panhuizen
	prof.dr. L. Verschaffel
	dr. A. Boland
	dr. R. Keijzer

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Introduction

> We need to concentrate not on the product of development *but on the very process* by which higher forms are established […] Consequently, the study of rudimentary functions must be the point of departure for evolving a historical perspective […] *To study something historically means to study it in the process of change;* that is the dialectical method's basic demand. (Vygotsky, 1978, p. 64-65, emphasis in the original)

Until now, little research has been conducted showing how young children gain their earliest understandings of mathematical signs, how these early signs support their transformation into the abstract symbolic language of mathematics, or how effective contexts for their learning can be arranged.

The aims of this dissertation are to investigate the evolution of young children's graphical signs and texts, chosen and used freely by them to communicate ideas. The research began by determining if the children explored aspects of mathematics in their pretend play, and whether young children's existing interest in using graphical marks and signs contribute to their growing use of personal inscriptions to communicate their (mathematical) ideas. To achieve this, the study documents children's interest in exploring and communicating through their literacies, and the types of signs young children use to represent their thinking, including those to communicate their mathematical thinking. The main focus of the study is children's use of their *Mathematical Graphics*[1]. The dissertation also identifies some of the processes that are involved in the course of children's developing use and understandings of mathematical signs, children's mathematical abstraction and the role of intertextuality and mathematisation.

Rather than viewing young children's mathematics from a single, subject-based discipline, this study takes the child's perspective through a holistic, transdisciplinary approach, drawing on research into pretend play, children's social literacies, multimodality, language acquisition, early

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[1] Children's mathematical signs and representations are variously termed *external representations* (contrasting with *internal* signs*), inscriptions*, *notations*, *cultural*, *psychological* or *symbolic tools, emergent models, schematisations, visual signs,* and (from Carruthers & Worthington, 2005), *Children's Mathematical Graphics.*

childhood mathematics and semiotics[2]. The title and focus of this dissertation reflect Vygotsky's assertion of the importance of using a genetic approach to understand the *higher forms* of behaviour (1978).

Made spontaneously in social-communicative contexts, the examples in this study of children's early graphical texts include a range of heterogeneous marks and signs of their own choice, revealing their emergent conceptions and their need to communicate about their thoughts and feelings. The children's choices of *whether* and *how to* communicate through inscriptions (see Latour, 1990) are freely made, their inscriptions originating from their desire to communicate within their personally meaningful narratives and activities. In this research, the marks, signs and symbols the children chose to use are not mechanically copied, and neither did adults impose or directly suggest how the children should represent their meanings[3]. However, the influence of their cultural knowledge from home and their nursery school is unmistakable in each example.

Theoretical Framework

Alphanumerical Signs

It was Luria who first engaged in research into the "pre-history" of young children's writing (4-7 years) before they began school, contributing significantly to our understandings of their early beginnings (Rocco, 1998, p. 62). Luria (1983) found that during this early phase, methods the child adopts, "pass through a number of trials and inventions […] the child now build[s] up new, complex, cultural forms" (p. 193); revealing that "a whole series of little inventions and discoveries [the child] made […] enabled him gradually to use this new cultural tool" (p. 150). Vygotsky (1978) acknowledged the development of gestures and symbolism (object substitution) in pretend play as precursors of written language (pp. 97-99), see for example, Worthington (2010). Vygotsky (1983) states that the

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[2] Transdisciplinarianism is "an approach to curriculum integration which dissolves the boundaries between the conventional disciplines and organizes teaching and learning around the construction of meaning in the context of real-world problems or themes" (*United Nations Educational, Scientific and Cultural Organization* [UNESCO], IBE Glossary of curriculum technology, 2013).

[3] Teachers contributed symbolic tools and (later) mathematical procedures through frequent modelling, embedding them in contexts that are personally meaningful to the children: these models contribute to the children's *graphical repertoires* or *lexicons* (see chapters 4 & 5, and Worthington, 2020a). Pedagogy is not a focus of this study, but rather the children and their thinking, pretend play, and their inscriptions. However, pedagogy and the role of the teacher in supporting children's mathematical inscriptions is addressed where relevant.

relationship between symbolic representation in play and “a particular form of speech at an earlier stage […] leads directly to written language” (p. 285). It appears that this also holds true for young children’s early acquisition of graphical signs and symbols of mathematics, made in contexts that can be considered mathematical, even when the children themselves do not label them as such. Just as babies notice and begin to adapt and refine the babbling sounds they make, in order to approximate the speech to which they are attuned, so too do young children gradually adapt and refine their earliest marks and signs in approximation of the written mathematical signs and symbols they see their peers and adults use. According to Vygotsky, approximation is a significant personal drive, and also important is the young child’s experience of being able to create mutual understanding, with the jointly shared signs and symbols.

Analogously, the signs and symbols of mathematics can be viewed as sharing some similarities with writing in their early inception and development, and it is on the theoretical foundations of Vygotsky that the current study into young children’s mathematical semiosis is based. According to this approach, both writing and mathematical symbol use share their root in the human beings’ need to communicate. The similarities between the two symbol systems of writing and mathematics have been identified by Collins and Laski (2019) who observe that early literacy and mathematics share “deep structures” concerning “symbolic mapping and relational reasoning” (p. 1). *Symbolic mapping* refers to young children’s growing ability to readily access the names of standard symbols in both domains, (i.e., letters, numerals and later, operators such as “+”). Symbolic mapping is “regarded as foundational in both domains”, whilst *relational reasoning* refers to children’s ability to make comparisons and to determine connections across the symbolic systems of literacy and mathematics (p. 202-203). Recommending dramatic play and block play, Collins and Laski maintain that both children’s literacy and mathematics can benefit from these play contexts. Other studies including those by Tolchinsky (2003) and Teubal et al. (2007), investigated the relationship between written and mathematical signs, showing that young children could use their personal graphical marks and signs to make meanings and to represent and communicate their thinking. From their earliest beginnings with marks made in contexts that can be understood as mathematical, children’s developing understanding and facility with signs gradually encompasses the orthodoxies of the signs of their culture, the *literate* aspects of mathematics. The present study therefore can be regarded as a study of early mathematical literacy.

Emergent Writing Research

In 1975 Marie Clay published the outcomes of research in which she had analysed the personal writing of five-year old children, in some respects an expanded study of that which Luria conducted in 1983. Clay's research led to increasing interest in young children's informal and intuitive beginnings with writing (see for example, Bissex, 1980; Ferriero & Teberosky, 1979; Lancaster, 2003, Teale & Sulzby, 1992), developing to include studies into multimodality (e.g., Kress, 1997), and emphasising the social aspects of all literacies. This widely researched *emergent* or *developmental* tradition grew to embrace (emergent) mathematics (e.g., Atkinson, 1992; Carruthers, 1997; Carruthers & Worthington, 2005; 2006; 2011; Cook, 1996; Gifford, 1997; Hughes, 1986; Hopkins, 2007; Munn, 1994; Whitebread, 1995), young children's alphanumerical signs originating in young children's drawings (Lancaster, 2003; 2014). The word *emergent* expresses that "the child's understanding is gradually developing, and is being produced through interactions within a culture", their understandings and knowledge expanding through activity (Gillen, 2003, p. 19).

Researching Young Children's Beginnings with Signs in Mathematical Contexts

Drawing at first on Piaget's work (1952), Martin Hughes (1983) conducted clinical trials with 4-year-old children into "the development and evaluation of a method for introducing simple arithmetical symbolism to pre-school children" (p. 163). Hughes (1983) concluded that prior to starting school, children have considerable competence with numbers, but that "what skills they do lack *appear to be primarily linguistic in nature* – such as understanding how symbols can be used" (p. 172, emphasis added). Extending his 1983 study resulted in the publication of Hughes's seminal text (1986) in which he reported a large-scale study with ninety-six children aged 3-7 years. In a game with tins and bricks, Hughes invited children to "put something on the paper to show" the quantity of bricks in each tin (p. 54). The findings evoked a range of responses revealing the children's understanding of their signs. In conclusion Hughes advised that, to make connections between their earliest marks and the standard signs of mathematics, "children need to develop links – *or ways of translating* – between this new language and their own concrete knowledge" (p. 51), arguing that "these translations are of fundamental importance in understanding mathematics" (p. 51-52, emphasis in the original). Moffett and Eaton (2018) observe that Hughes's research constituted a "significant theoretical shift in perspective […] [marking] a movement towards an alternative conception of the use of representation within the mathematics classroom" (p. 550), particularly in the teaching of signs and representations and children's opportunities to use them.

Researching the early development of literacy and numeracy, Munn (1994) similarly argued that the functional use of signs used by children in mathematical contexts is "essentially *a literate strategy*" (p. 13, emphasis added). Cook (1998) also began her research into children's mathematical sign-use from an emergent literacy perspective, enhancing provision in research settings by adding resources related to a birthday party. Cook (2001) explored the children's signs and symbols within a Vygotskian framework and from an *authentic learning* perspective (van Oers & Wardekker, 1999, p. 246, emphasis in the original). Munn (2001) found that after her intervention the children's spontaneous use, recognition and construction of number increased, arguing that the nature of pretend play offers opportunities for children to explore aspects of "familiar socio-cultural activities" (p. 61).

As teachers Carruthers and I had become concerned with the difficulties children often experienced with "written" mathematics, and the ways in which it was widely taught in the early 1990s. Wishing to improve children's understandings of the abstract symbolic language of mathematics, coupled with our prior pedagogical experiences of emergent writing and knowledge of Hughes's seminal work[4], led to our expanding research into *Children's Mathematical Graphics* (CMG)[5] and other modes of graphicacy[6].

Mathematical Literacy

The Organisation for Economic Co-operation and Development (OECD, 2017) defines mathematical literacy as: "formulating, employing and interpreting mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (p. 1) and begins with problems that are located in reality. Citing English (2002, p. 8), Yore, Pimm and Tuan (2007) point out that to be mathematically literate:

> involves fundamental literacy and the independent abilities to use mathematical thinking, construct understanding, and solve

[4] During the 1980s and 1990s Carruthers and Worthington taught children (in different settings and schools) from 3-7 years of age.
[5] Graphics include children's drawing, maps, writing, and those signs and symbols that are made in contexts that can be understood as mathematical.
[6] Carruthers and Worthington's research (e.g., 2005, 2006) includes examples from children of 2-8 years of age, from their earliest marks to number, calculations, problem solving, measuring and data handling.

> problems. Mathematics as human and social activity requires mathematical literacy to be functional and to prepare people to live, understand, and act critically in a modern, mathematised society. (p. 574)

Mathematising and *mathematical literacy* are interrelated, mathematical literacy characterising "how mathematicians *do mathematics*" (OECD, 2003, p. 27, emphasis in the original).

Noting that the OECD's focus is on students of 15-16 years, an important issue of research into the beginnings and early development of mathematical thinking is acknowledged, of how to connect these characteristics of mature mathematical thinking with young children's ways of thinking and talking about the world. Although the OECD (2003) did not aim at describing a developmental theory, it highlights important issues that can be seen as fundamental in the course of children's mathematical progression:

> The ability to read, write, listen and speak a language is the most important tool through which human social activity is mediated […] Analogously, considering mathematics as a language implies that students must learn the design features involved in mathematical discourse (the terms, facts, signs and symbols, procedures and skills). (p. 26)

According to B. D. El'konin, (the author of an article about D. B. El'konin; quoted in El'konin, 2001), "The sign is a kind of gift. A gift serves as a reminder of the giver. That is why a sign is social, and that is why it organizes behaviour" (p. 11). Thus, when a teacher models some signs or symbols in context, in a sense such *gifts* may be added to the children's personal (internal) sign lexicons; individual children may subsequently take one or more of these new signs and use them (or not) when they see fit to do so, making them their own. "The adult [is] middleman, […] This *mediating*, i.e., *concerted, open*, action is the system of coordinates within which child development has meaning. This action is never given directly: it is by nature one that must always be found and constructed" (p. 17, emphasis in the original). Ilany and Margolin (2010) emphasise that both writing and mathematics have a grammar, and that learners need to develop "mathematical-linguistic literacy" (p. 141), which will help transform their earlier signs to formal, symbolic mathematical signs. The grammaticisation of mathematical inscriptions is one aspect explored within my research.

The use of the word "mathematisation" in this thesis refers to the socially collaborative activity in which sign-use, from children's initial level of intuitive and informal use of signs, becomes increasingly sophisticated as they move towards the culturally defined system of abstract mathematical symbols. Mathematisation is realised through a combination of the children's progressive awareness of the usefulness of the culturally accepted graphical languages, through negotiating meanings, and supported by adults who model graphics for realistic purposes throughout each day. Such authentic purposes are expected to have personal meaning and relevance for children and are often related directly to something they have been doing within another context. Van Oers (2013a) observes also that a necessary condition for mathematisation is that teachers "really believe in children's potential to mathematise with their own notions", the breadth of mathematics "and to communicate about them with others" (p. 200).

Communication and Graphical Inscriptions

Communication through gesture and oral language is widely considered an instinctive aspect of human behaviour, and, whilst we were not genetically programmed to understand and use abstract symbolic languages such as mathematics, drawing or writing, these became possible through the re-deployment of existing neural structures in the brain (Jablonka et al., 2012; Wolf, 2010). However, citing Rommetveit, Wertsch (2003) contends that rather than a matter of "simple transmission of information from speaker to listener", communication is considered "a process of building a fragile, temporary, and partially shared intersubjectivity in a deeply pluralistic world" (p. 2). Observing that all communications require a context, van Oers (2000) emphasises it is the context that contains "the implicit rules that make the meanings and symbols comprehensible (Hundeide, 1985)" (p. 171).

The process of creating and representing meaning (i.e., *semiosis*) refers to the relationship between a sign and its meaning. In transposing an open literacy approach to children's mathematics, it is clear that not only do children's own emergent signs and symbols strengthen their aptitude with mathematical semiosis and support mathematisation, but that their use appears to establish connections with young children's developing sense of communication. The use of inscriptions to convey meanings dates to between 10,000 and 40,000 years ago (von Petzinger, 2009). Long before written language appeared, abstract symbols for recording quantities began: the habit to consistently record quantities began with notched bones, those in Africa dated at 35,000 and 30,000 years ago (Boyer & Merzbach, 2010, p. 3). Marks were later scored on clay tokens in the Middle East for accounting or tax purposes (Robinson, 2007), and tallies relate closely to these origins, with numerals developing later.

In contemporary times graphical communications appear to dominate our worlds, and it seems likely that at home young children are more often involved in literary practices in which *written* and *visual* (or pictorial) texts have a role (which can include mathematical signs), rather than in those in which only mathematical texts feature. Yet children's cultural mathematical knowledge and desire to communicate (mathematical) ideas is evident from a very early age (e.g., Aubrey, 1993; Baroody et al., 2006; Carruthers, 1997; Doig et al., 2003; Gelman & Galistel, 1986; Ginsburg et al., 2008; Hughes, 1986; Perry & Dockett, 2002; Resnick, 1987; Sophian, 1998; Wynn, 1998; Young-Loveridge et al., 1997), provided educators are open to children's meaning making through their graphical signs and texts.

In pretend play and other open contexts young children communicate their thinking through graphics, gradually (where their learning culture and teachers support and value this) choosing to communicate their thinking through drawing, writing and their *Mathematical Graphics*. They do this through personal marks and a diverse range of signs as they move towards what Goldin-Meadow (2002) describes as "the linguistic frames of that language" (p. 128).

Van Oers (2000) asserts that "the connection between communication and knowing can only become imaginable by understanding symbolic meaning as a means of the system of human activity" requiring "*negotiation of meanings"* (p. 134-135, emphasis in the original). The children's examples in this thesis (chapters 2-5) highlight their participation as they make and negotiate meanings through their inscriptions. Participation in the communicative act originates from the fact that humans are inherently social, in line with Lave and Wenger's (1991) appreciation of the significance of cooperation.

Tomasello (2008) writes that humans' skills of linguistic communication originate from many sources; evolutionary processes depending on *shared intentionality* and *cooperative communication*. Shared intentionality (integral to communication) creates changes in language over time, particularly in socially collaborative contexts: this allows us to achieve shared reference. Tomasello asserts that the key role in the communicative act "is that of the communicator" (p. 14) which, when combined with the communicator's intentionality, assists the recipient in that communication. Specific aspects of communication relate to grammaticisation, a *usage-based* theory of language acquisition (e.g., Langacker, 2008). Tomasello (2005) emphasises that language structure emerges "from language use […] patterns of use emerge and become consolidated into grammatical constructions" (pp. 5-6). Children's *intention-reading* results in the "patterns" identified between various inscriptions. We assume that these factors hold true not only for spoken language communication, but also for graphical communications. In turn, *pattern-finding* supports the grammaticisation of children's mathematical inscriptions and their increasing mathematisation.

Van Hiele (1985) considers the appropriation of symbols as principally a process of communication, although in van Oers's view (2000), this is unlikely to be straightforward "since symbols *may mean different things to different participants*" (p. 135, emphasis added). This coincides with Cassirer's (1923) theory that the abstract is *projected into* objects by seeing them *from a specific point of view,* and by the same token neglecting all aspects that fall outside this focus (see also chapter 4). Rommetveit (1990) too comments on "perspectival relativity", claiming it as "an inherent characteristic of human cognition and part and parcel of its situatedness […] the very identity of any given state of affairs is *contingent upon the position from which it is viewed*" (p. 87, emphasis added). Writing on a "psychology of the second person" as a communicative genre, Rommetveit (2003, p. 214) emphasises that mental activities are socially distributed. Rather than "owners" of "our common language" we are "shareholders" in it, *co-authorship of meaning* comprising shared knowledge and language. Meanings are not ready made: inhabiting "a shared cultural habitat" (Rommetveit, 1985, p. 215), the words spoken are situated and co-authored, their *meaning-potential* dependent on the perspectives of those engaged in the dialogue. The "speaker monitors what he is saying in accordance with what he assumes to be the listener's outlook and background information, whereas the *second person* [the listener] […] makes sense of what he is hearing by adopting what he believes to be the speaker's perspective" (p. 189-190).

Bakhtin (1986) writes "each speech genre in an area of speech communication, has its own typical conception of the addressee, and this defines it as a genre" (in Morris, 1984, p. 87). Van Oers (2012a) observes that that in addition to dialogue, communication involves "creative digestion and innovation of meanings for participation in cultural practices" (p. 139).

Natural Languages

The languages humans speak or write are known as *natural languages* and consist of vast inventories of words that may be combined in flexible and infinite ways within the grammatical frames of those languages. In contrast, the formal standard mathematical language "is more precise and less flexible" (Ilany & Margolin, 2010, p. 139). Schleppegrell (2007) emphasises, "learning the language of a new discipline is a part of learning the new discipline […] the language and learning cannot be separated" (p. 140). Lemke (2003) proposes that a semiotic (meaning creating) perspective helps us understand the relationship between natural languages, and how mathematics and other graphical representations combine to form a consolidated meaning-making system: "mathematics is used and can only be learned and taught as an integral component of a larger sense-making resource system"; this includes "natural language and visual representation" (p. 1).

Mathematical Symbols

The role of mathematical symbols is a critical one for participation in mathematical practice, for encoding and communicating meanings and mediating understandings. Mathematical signs are integral to the discipline of mathematics, playing an important role in school learning. They are *symbolic tools,* signifying something other than the marks of which they are constituted. To engage in meaning making with graphical signs is to understand the value of sign systems such as those in writing, drawing or mathematics and use them to meaningfully represent and communicate ideas.

In contexts that can be considered mathematical, children must determine how best to communicate their ideas from the diversity of graphical signs available to them, grappling with at least three sign systems, e.g., written, mathematical and drawn (Brizuela & Gravel, 2013; Carruthers & Worthington, 2006). Children's signs arise naturally from their interests and cultural knowledge (Worthington, 2018), mediating learning and impacting on their thinking, and in time they will come to learn the formal symbolic language of mathematics, a system that is precise and intended to be unambiguous. Luria (1976) observed that:

> As the basic forms of activity change, as [mathematical] literacy is mastered, and a new stage of social and historical practice is reached, major shifts occur in human mental activity. These […] involve the creation of new motives for action and radically affect the structure of cognitive processes. (p. 161)

Aubrey (1993) found that before young children start school, they possess considerable understandings that help them to gain access to mathematics. She concludes, "in early stages of learning mathematics […] children should be prompted to extend the range of strategies at their disposal so that their natural inventiveness is not undermined by a struggle to find the one single, convergent and acceptable response" (p. 39). For teachers[7]this could mean looking at one child's or several children's inscriptions in a given situation and valuing their different approaches, discussing their signs and meanings and the various ways or procedures they have used to represent their particular ideas. According to Salmina (1988) children should have opportunities to "experiment with symbols and meanings, and invent and explore their own forms of symbolization […] before they are led to the generally accepted mathematical symbols" (van Oers, 2000, p. 155, citing Salmina).

Being able to freely decide *if*, and *how* they might represent their ideas and strategies means that young children use their chosen symbols and signs with personal sense and aptitude (Carruthers & Worthington, 2006; Munn 1998), helping establish deep understandings of the purpose, role and use of formal signs for mathematics later in school. Like Carruthers and Worthington, Terwel et al. (2009) emphasise the importance of learners *taking ownership* of mathematics, such ownership potentially leading to what Hoyles (2010) describes as learners' "engagement, confidence and empowerment" (p. 2). Indeed, in the light of the socio-cultural basis of this process, such ownership may be termed *co-ownership*.

In early childhood, young children's mathematical inscriptions are those in which they use informal marks, signs and symbols, drawings, writing- or numeral-like signs, tallies, formal (standard) letters and numerals or grids to communicate a mathematical idea or to signify something in a mathematical context (although young children themselves do not necessarily see these as mathematical)[8]. A number of researchers have investigated the emergence of symbol systems in early childhood. For example, Tolchinsky (2003) made an extensive study of young children's understandings "of writing and numbers before being taught", investigating children's own ideas about notational systems and the impact of culture on their signs. Children's earliest writing and mathematical inscriptions relate to Lancaster's (2014) research findings of the marks and signs very young children (under 3 years) employ when *drawing* freely. Correspondingly, Levin and Bus (2003) also acknowledge that both drawing and written signs emerge from a "common core" of foundational graphical marks" (p. 892, see also Worthington, 2009).

[7] In the setting in which data was gathered for this study, teachers and early years' professionals work together, but for the sake of brevity, the term "teachers" is used throughout.
[8] When their learning culture supports this meaning making.

Mathematical signs form a concise cultural system integral to communication in mathematics. Yet the meanings of the standard visual signs of mathematics are not readily transparent to young children and, if transmitted in isolation, have been found to create problems for them. Taught traditionally, children's early understandings of the symbolic abstract graphical language of mathematics often cause them difficulties and a growing confusion about mathematics. These problems arise especially when the signs are presented without a context and lack personal meaning for them (e.g., Carruthers & Worthington, 2006; Ginsburg, 1982; Hughes, 1986; Poland, 2007). Hiebert (1984) referred to the need for children to represent and manipulate symbols in school mathematics according to prescribed rules:

> formalization is essential, but it also presents a serious learning and instructional problem. Many children do not connect the mathematical concepts and skills they possess with the symbols and rules they are taught in school. I shall argue that it is the *absence* of these connections that induces the shift from intuitive and meaningful problem-solving approaches to mechanical and meaningless ones. (p. 498/501, as quoted in Hughes, 1986, p. 175, emphasis in the original).

Mathematics Education

Developing good understandings of mathematics in early education is important for children's facility with mathematics later in school, and their achievement in this subject in the preschool phase is a useful predictor of later success in school (Clements & Sarama, 2011; Nguyen et al., 2017). In their recent review of evidence for the Early Years Foundation Stage (EYFS), Pascal et al. (2019) highlight the findings of the Early Intervention Foundation (EIF) (2018): "*Mathematical achievement in turn is consistently found to be the strongest predictor of children's overall school achievement and their success in entering the workforce",* emphasising the importance of *young children's comprehension of symbolic numbers.* (p. 22, emphasis in the original)."

Van Oers (2013a) posits "mathematics nowadays indeed has become a core element in people's cultural functioning" (p. 183). Mathematically literate citizens are needed "to deal with a very complex and rapidly changing society" (OECD, 2003, p. 27). Mathematics is integral to almost all educational disciplines and vital for individuals intellectually, supporting problem solving and (in their future lives) increasing vocational opportunities. It influences every aspect of society, including economic, social, business, scientific, agricultural, medical, industrial and technological, and contributes to improved living standards (Fatima, 2014). Fry (2018) asserts that the power of the abstract symbolic language of mathematics lies in the way that language and symbols have allowed us to manipulate the world. However, in England and elsewhere, pedagogical strategies generally fail to consider the significance of meaningful introductions to this critical and largely overlooked aspect of representing mathematical ideas in early childhood, current practices predominantly focusing on direct teaching of skills and assessment of discrete and disconnected aspects of mathematics, or adult-planned play.

Contrasting with traditional approaches, the discourse in this thesis explores an open and meaningful approach to support young children's beginnings with mathematical signs and representations, from their earliest inceptions to their beginnings with formal, standard mathematical symbols. It endeavours to trace the nature and essence of young children's marks made in what adults would acknowledge as mathematical contexts. But the children's marks, signs and symbols themselves tell only a part of the story: the democratic culture of the setting featured in this dissertation; the teachers' deep understandings of pretend play, early mathematical development, graphicacy and emergent learning; and the children's cultural knowledge constitute other significant aspects and are reflected in my research. The social and cultural features of young children's meaning making, features of abstraction and usage-based language acquisition all contribute to our understanding of this complex development.

In order to understand the process of acculturation in the mathematical domain and children's gradual adoption of conventional mathematical symbols, an important role for intertextuality to understand the use of signs in diverse contexts is assumed, seen when children adopt symbols from others to integrate them into their own texts. Together with features of cultural learning, these processes suggest ways in which young children's understandings and more advanced cultural knowledge of mathematical signs transform and mathematise over time. This concerns the need to '*bridge the gap'* identified by Hughes (1986, p. 170), the 'gap' suggesting a deep gulf between children's concrete, practical understandings and their use of formal symbolic numerals and ways of representing their mathematical thinking. Moffett and Eaton (2018) conclude that assisting children in forming "crucial links" between the two "is arguably the single most important task in early mathematics education" (p. 549). Regrettably Hughes's "*important message* […] was lost when the national numeracy strategy [in England] was introduced in 1999" (Sutherland & Ching Yee, 2011, para. 3, emphasis added).

Approaches to Teaching and Learning Mathematical Inscriptions

To avoid problems identified with traditional methods of teaching mathematical signs and symbols in early childhood, a number of researchers have investigated ways of connecting children's informal mathematical understandings to its formal abstract cultural system. Three of these will be discussed in this section.

1. Realistic Mathematics Education (RME)

This is a Dutch instructional design founded by Freudenthal and his colleagues in the early 1970s. RME was based on Freudenthal's idea "that mathematics – in order to be of human value – must be connected to reality, stay close to children and should be relevant to society" (Freudenthal, 1977, p. 9). The belief is that through the use of realistic contexts, students "learn mathematics by developing and applying mathematical concepts and tools in daily-life problem situations that make sense to them" (van den Heuvel-Panhuizen, 2003, p. 9). Subsequently Treffers (1987) introduced "horizontal" and "vertical" mathematisation. *Horizontal* mathematising refers to children's use of mathematical tools in everyday life (i.e., from real life to the world of symbols), whilst *vertical* mathematising refers to moving *within* the world of symbols. Van den Heuvel-Panhuizen (2003) explains that in RME models are understood to bridge the gap between informal understandings and the formal systems of mathematical signs (p. 13). She concludes "it is not the models in themselves that make the growth in mathematical understanding possible, but *the students' modelling activities"* (p. 29, emphasis added). Rather than being given "ready-made" models by their teacher, children explore personally meaningful mathematical problems (such as a birthday party or travelling on a bus) and develop their own models, while teachers gradually introduce more efficient models for children to use, guiding them through the use of RME textbooks.

2. Schematising

Also in the Netherlands, van Oers has developed *schematising*, an approach embedded in play contexts that are personally meaningful to the children, and which depend on teacher interventions. Van Oers (1994) describes schematisations as "the making of diagrams" (p. 21). Poland et al. (2009) explain schematising as "every cognitive activity accomplishing the construction and improvement of symbolic representations of an element of the physical and sociocultural reality" (p. 307). It is "an emergent approach that integrates both the child's personal constructions and the educator's pedagogical responsibilities" (Dijk et al., 2004, p. 71). Research by Poland (2007) investigated the *effects* of schematising in early childhood, showing how children's ability to meaningfully represent layouts (positions, relationships, quantities) can be developed through introducing schematisations, which "promote a deeper understanding" and "generate improved learning" (p. 149). Poland concludes that schematising underpins children's use of mathematical representations, improving "their mathematical understanding in later development" (p. 151).

3. Children's Mathematical Graphics

In England a third approach has been developed by Carruthers and Worthington (e.g., 2005; 2006) who, like Munn, and Cook, began by exploring the beginnings of the abstract symbolic language of mathematics from emergent literacy (writing) and semiotic (meaning making) perspectives. Their joint research revealed that young children could represent mathematical meanings in a variety of social contexts, drawing on their own and others' signs and representations (including those modelled by adults)[9]. Acknowledging children's early marks, drawings and signs made in contexts that can be understood as mathematical, they analysed many hundreds of examples of children's graphical inscriptions, charting children's progression (e.g., Carruthers & Worthington, 2005, 2006) as they move from their earliest marks to which they attach meanings, to increasingly use standard mathematical symbols. This research revealed children's confidence and competence in using a variety of signs to represent quantities and their personal strategies for calculations, problem solving and other aspects of mathematics (such as measurement), their inscriptions mathematising over time. The approach is inclusive, children representing their thinking in their own ways.

Summarising Carruthers and Worthington's approach, Carruthers (2015) emphasises that:

- Children are given genuine choices in their play and in the ways in which they represent their thinking;
- Their home cultural backgrounds are understood by their teachers as rich resources for their learning;
- Teachers value children's play and recognise its mathematical potential;
- Children have *ownership* of their mathematical problems, which are sufficiently open to be tackled in a variety of ways;

[9] Whilst teaching, and prior to learning of published research on modelling (e.g., Gravemeijer, 1999), Carruthers and Worthington independently developed their use of modelling mathematical signs and symbols in meaningful contexts.

- Children's thinking is valued and supported;
- Together children and adults co-construct mathematical understandings.

Like RME and schematising, this view acknowledges the importance of collaborative dialogue among teachers and peers about the nature and meanings of their signs and representations, and their value for satisfying their need for communication[10]. In Shor and Freire's words (1987):

> Dialogue is not a mere technique to achieve some cognitive results; dialogue is a means to transform social relations in the classroom […] a way to recreate knowledge as well as the way we learn […] [it] rejects narrative lecturing where teacher talk silences and alienates students. In a problem-posing participatory format, the teacher and students transform learning into a collaborative process to illuminate and act on reality. This process is situated in the thought, language, aspirations, and conditions of the students [and] is also shaped by the subject matter and training of the teacher. (p. 11)

Teachers base their responses on their observations of children's behaviours, talk and inscriptions, and their understanding of mathematics in early childhood; their learning and teaching "are intertwined" (Carruthers, in process). Moreover, adults working with young children need to be "on the inside of children's learning" in order to privilege their "cultural practices, meanings and purposes (Wood 2010, p. 11)" (Carruthers & Worthington, 2011, p. 156).

Examining the role that emergent models play in developing formal signs and representations, Gravemeijer (1999) argues "what is aimed for is a process of gradual growth in which formal mathematics comes to the fore as a natural extension of the student's experiential reality" (p. 156). These three approaches are reform movements that challenge traditional and mechanistic approaches[11]. Whilst they share certain similarities in their underlying philosophies and values, their pedagogical approaches differ.

Pedagogical Concerns

Nowadays pedagogical strategies employed in early years' classrooms are often dependent on direct teaching of narrow mathematical skills, and increasingly advance the start of formal mathematics to the youngest

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[10] Emma, one of the teachers in this study, now works with children of 2-3 years of age. Emma's interest and deepening understandings of the importance of graphicacy prompted her to develop the same culture with these younger children. She found that even at this age, many of their graphics suggest emerging mathematical thought.

[11] Both *schematising* and *Children's Mathematical Graphics* are underpinned by Vygotsky's research.

children (van Oers, 2013a). However, this is not a new situation, since it was identified by Hiebert as early as 1984. Since this time a conspicuous lack of interest from curricula writers and many others, in children's use and understandings of the beginnings of the abstract symbolic language of mathematics has been notable. As Vellom and Pape (2000) observe, typical written mathematical tasks involve colouring-in and one way of copying or completing. They propose that such "production" of representations lack meaning and offer no opportunities from which relational statements can be drawn (p. 125). Such tasks have been found to limit understanding and impede subsequent learning in mathematics (Åberg & Taguchi, 2005; Ahlberg, 2001; Pramling Samuelsson & Mauritzon, 1997). These difficulties can result in a growing dislike and alienation from the subject, compounded as children move through school (van Oers, 2012a). Munn (1998) argues that children's difficulties may eventually lead to them resorting "to coping strategies that alienate themselves from an understanding of number" (p. 70), that has resulted in a widely accepted "can't do" attitude to mathematics in England (Department for Children, Schools and Families [DCSF], 2008a, p. 71). Holmes and Dowker (2013) observe that there is a strong connection between young children's early difficulties and those they experience later. In a similar vein, Bynner and Parsons (1997) found that most adults with serious numeracy difficulties had already struggled with mathematics by the age of seven. The outcome of this disaffection is the high percentage of adults in England for whom an aversion to mathematics often originates in education (Boaler, 2009).

Hughes (1986) pointed to the severe disparity between the prescribed symbolic system that children are expected to learn and their own impromptu conceptions, affirming his belief in, "the immense capacity of young children to grasp difficult ideas", provided they accord with their interests and make sense to them. "Instead of nullifying and ignoring young children's strengths […] [we should] bring them into play and build on them" (p. 184). Hughes advocated a need for a "reorientation and rethinking" and to respect their invented symbolism (177). Vellom and Pape (2000) write "in more realistic learning contexts, students may make sense of complex phenomena through their efforts to construct and through the use of graphical representations of these complex systems" (p. 125).

Pretend Play and Cultural Knowledge

My study is replete with examples of pretend play (and some open, small group contexts), and mathematics, interweaving young children's graphics and their cultural knowledge to reveal the holistic nature of their learning. Following Vygotsky (1978) and drawing on the work of Carruthers and Worthington (2006), in this study play is understood as social pretend play, and in the nursery school in which data were gathered, the children freely and spontaneously initiate their play indoors and out. The nursery's philosophy and culture empower children to voluntarily play with friends of their choice, wherever they wish, and to freely use any resources they choose in their play narratives without asking an adult's permission, playing without following any adult plans or requests to use graphical signs. The headteacher's and teachers' understandings of graphicacy (including children's inscriptions made in contexts that can be understood as mathematical) are well understood and supported. The children frequently and spontaneously choose to communicate through inscriptions with each other and with their teachers, who value these inscriptions and often engage in shared dialogue, discussing the children's meanings of their signs and symbols in the context of their play narrative[12]. The teachers' have developed a deep understanding of pretend play, their involvement collaborative and supportive.

Young children learn best when they are able to make sense of what they do, connecting their existing understandings to new knowledge. According to Vygotsky (1978), it is *pretend* (or symbolic) play that is of particular value for their learning development, play in which children may substitute one item to signify another (such as using a bowl to wear as a hat), and subsequently use marks and signs as *symbolic tools* to aid and communicate their thinking. Recent research has shown how young children will freely explore problems in their pretend play and other open contexts, drawing on their existing cultural knowledge (e.g., Carruthers & Worthington, 2006; van Oers, 2010; Worthington, 2018). From the perspective of pretend play, learning can be understood as *cultural-conceptual*. Rather than adults engineering restrictive play situations with mathematical outcomes in mind (such as a predetermined setting of a shop), open contexts are likely to furnish greater opportunities for spontaneous pretend narratives of their own, in which children can explore *everyday* concepts through their already appropriated cultural knowledge.

Adults' Constructions of the Young Child

Ferriero and Teberosky (1979) emphasise that not only do young children need time for free and unstructured play, but "to understand children we must hear *their* words, follow *their* explanations […] and listen

[12] The staff in the nursery provide a rich range of resources indoors and out, including a range of papers, notebooks, child-height whiteboards, pens, pencils crayons, chalks, paint, etc.,

to *their* logic" (p. xii, emphasis in the original). Alldred and Burman (2005) caution that "we need to ask through what cultural understandings of children are the words of any child "heard" and how our account of them will be heard" (p. 19). For Dahlberg et al. (2007) how we construct the young child and early childhood, matters: they argue for a view of children as "a *rich child,* active, competent and eager to engage with the world", maintaining that the constructions of children that teachers and educators have, carry "enormous consequences for how we relate to children pedagogically" (p. 7/137, emphasis in the original). Formulating a view of the child as *rich* means that rather than a transmission of knowledge and skills to individuals, learning can be understood as a multi-faceted social, cultural and collaborative process inherently connected to children's will to participate self-dependently in cultural practices of their community.

Ladson-Billings (1995) argues "all children can be successful in mathematics when their understanding of it is linked to meaningful cultural referents, and when the instruction assumes that all students are capable of mastering the subject matter" (p. 141). Tensions occur between voluntary choices for child and adults' demands; between adult and child-initiated play and activities, and between structured and unstructured pursuits. Competing struggles between play and *subjects* such as mathematics in early childhood are, Brooker (2010b) argues, also determined by adults' differing views of children:

> as [either] immature, inexperienced and ignorant people whose learning depends on the tutoring of more mature, experienced and capable adults, or as competent individuals who are capable of making meaning from their experiences of the world, in collaboration with others and with the support of cultural tools […] such views are themselves fundamentally cultural (p. 44).

Bringing a positive perspective of children's learning opens considerable possibilities for them. Carruthers and Worthington (2011) argue that adults working with young children need to be "insiders" (p. 156), Carruthers (2015) maintaining that "if we understand that young children have mathematical competencies derived from their own self-learning based on their rich cultural experiences at home and in the community […] teaching should encompass children's mathematical questions, ideas and lines of thought" (p. 315). This view reflects Freire's (1970) *conscientização* (awareness) and literacy empowerment.

Play and Mathematics in the English Curriculum

In the curriculum for the early years in England (from birth-5 years), the Department for Education [DfE] (2017), describes "characteristics of effective teaching and learning", including "*playing and exploring*, *active learning* and *creating and thinking critically"* (p. 10, emphasis in the original), explaining the focus of mathematics as "providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measure" (p. 8).

However, as a result of the government's increasingly narrow emphasis on children's *skills*, and on assessment and achievement of those skills, there appears to be scant connection between the characteristics of effective teaching and learning. In the mathematics section of the curriculum, young children's use and understanding of mathematical signs and inscriptions is not mentioned. Of considerable concern is that mathematical curricula and teaching seldom focus on *how* young children learn, *how* they might best explore the meaning of graphical signs and representations (as a means for communication), or the relationship between play and subject learning. The curriculum proposes "areas of learning and development must be implemented through *planned, purposeful play*" (DfE, 2017, p. 9, emphasis added). This suggests that learning can only develop through *teachers' plans*: moreover, the idea that play can be "*planned"* repudiates the wealth of research that points to play being largely owned and shaped by children.

Opportunities for children to initiate and shape their pretend play allow them to both draw on their cultural knowledge and avoids the restrictions that can come if goals and purposes have been fixed in advance. When children have ownership of their play and their cultural knowledge, new goals come about during their play, negating the need to give room to establish goals and purposes in advance. However, although play is ostensibly given a role in the English early childhood curriculum, opportunities for children to play freely have been considerably marginalised in recent years (e.g., Cameron & Moss, 2020; Hedges, 2010; Neum, 2016; Pascal et al., 2019; Roberts-Holmes, 2015; Rogers, 2010a; 2010b; Waters, 2020; Wood, 2014; 2019). The outcome of this is that not only are opportunities for free play often restricted, but that direct teaching often takes precedence and teachers may not have time to observe or participate in children's play. As a result, children's vocal and graphical communications made in their pretend play are likely to be unheard or unseen, and are rarely acknowledged.

Carruthers (in process) confirmed in her doctoral research that nursery teachers, and especially the reception class teachers, identified Ofsted[13] inspections as the most significant issue impacting their pedagogy. These teachers *particularly emphasised* the negative influence the government's "Ofsted" inspections had on play in their classrooms[14]. Raising concerns regarding the amount of control wielded by these inspections, Carruthers stresses that:

> Ofsted inspectors' influence is imbalanced and certainly not contested, continuing to be the greatest issue influencing early education in England. Whereas governments have previously given some positive curriculum guidance for early childhood education, lack of understanding by Ofsted inspectors has culminated in the narrowing of teachers' professional understandings, to which the teachers in her research referred (Worthington, 2020a,)[15].

In 2008 the government in England published the results of a two-year investigation into the teaching of mathematics in early years' settings and primary schools, the chapter on early years' mathematics emphasising the significance of pretend play and the approach developed by Carruthers and Worthington (DCSF, 2008a). However, a subsequent change of government resulted in a shift of emphasis from mathematics to synthetic phonics, and the findings of this report failed to result in any changes for the teaching and learning of mathematics in the early years.

More recently a review of research evidence relating to early childhood education in England (Pascal et al., 2019), includes a comprehensive section on mathematics, endorsing Carruthers and Worthington's curriculum concept of *Children's Mathematical Graphics* (2004; 2005; 2011) and supporting the holistic nature of young children's development[16]. The authors of this review urge teachers to "allow and actively support opportunities for children to freely explore how they represent their

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[13] Ofsted is the government's Office for Standards in Education, Children's Services and Skills. They inspect services providing education and skills for learners of all ages, also inspecting and regulating services that care for children and young people.

[14] In England children generally start school at 4 years of age, and in the Reception class are of 4-5 years of age. Confusingly, the curriculum for this age group (from birth-5 years) bridges pre-school and nursery schools and reception classes, and it is because reception children are in school, that the "top-down" pressure of the primary curriculum negatively influences pedagogy and children's experiences in England.

[15] This lack of understanding appears to be particularly acute in relation to play and children's graphics, but could, as Carruthers (in process) suggests, be resolved with further training for Ofsted inspectors.

[16] This is a collaborative document, with contributions and endorsements from twelve professional early childhood organisations in England.

mathematical knowledge and understanding by drawing on their personal and cultural knowledge in pretend play" (p. 36). Citing a number of Carruthers and Worthington's publications, and Worthington and van Oers (2016), Pascal, Bertram and Rouse (2019) assert that these studies "demonstrate the benefits of teaching that allows children to represent their mathematical understanding [in ways of their choice, permitting] personal and cultural knowledge to become tools" (p. 35).

Meaningful Early Learning Cultures

Wood (2010) maintains that open and democratic learning cultures allow learning, "to go beyond surface culture towards understanding deep culture" (p. 21). When adults write something in a mathematical context (such as a shopping list) in conjunction with oral explanations, children have a sense of reality and of cultural significance: the notations make *real sense* to them, helping them in the meaningful appropriation of the formal abstract system of mathematical signs and representations.

Supporting young children's freely made signs and representations in this phase can contribute to the improvement of mathematics education, especially at a time when – in England and globally – there is such anxiety about children's achievement in mathematics. In this nursery school the culture and ethos of the learning environment is influenced by the headteacher's philosophy, beliefs, values and knowledge of children's development and subject knowledge, and shared with the teachers and early childhood practitioners: together they have a profound impact on young children's learning.

The early years' curriculum in England (DfE, 2017) refers to *children's interests* (p. 9); as Wood (2009) observes, their interests are "often driven by their fascination with the world of adults, and their motivation to act more knowledgably and more competently" (p. 37, see also Worthington, 2018). Rogoff (1990) similarly states that, throughout the history of humans, young children have played and learned alongside and through participation in cultural activities, through "guided" or "intent participation" that builds on "young children's eagerness to be involved" (p. 180). However, Anning et al. (2009) caution: "the folklore and practice of designing a curriculum for young children in the UK has been "to follow the interests of the children", though in reality it has reflected the adults' constructs of childhood" (p. 13).

This thesis also highlights some of the processes involved in this progression and the means by which children's access to their own early signs mesh with those of the formal mathematical system of signs, to *bridge the gap* and support their mathematical understandings in school. These processes include imitation, emulation and peer learning (chapter 4), and intertextuality (explored in chapter 5). Van Oers (2010) proposes that meaningful learning is:

> created in children in a process of guided co-construction. Consequently, we have reason to assume that this will finally yield meaningful mathematical symbols that may turn out to be more functional for the development of mathematical thinking than conventional symbols imposed onto the child's mind (p. 33).

Drawing on Leont'ev (1975), van Oers (2010) explains *meaningful learning* as having a dual definition, firstly its cultural meaning relating to the children's cultural knowledge from home and the culturally accepted signs (or cultural tools) of formal mathematics, and secondly to *personal sense* "often expressed in interests, special attitudes or dispositions […] In order to be meaningful and to be stimulating for development, learning necessarily should be meaningful in this double sense" (p. 26).

Research Questions

This study explores the hypothesis that, provided very young children are able to draw on their cultural knowledge of mathematics in a meaningful environment that enables them to play freely, their experiences can support them in making use of emerging graphical signs and everyday concepts. Open social situations allow them to communicate ideas in meaningful (mathematical) contexts and to freely make connections with this existing cultural knowledge, their understandings supported by their teacher, other early years' professionals and peers. The main questions posed by this research are:

- What evidence of mathematics can be found in the children's free pretend play, and how does their cultural knowledge influence their thinking? (Chapter 2)
- What early graphical inscriptions do young children of 3-4 years of age spontaneously employ in the context of various literacies in their nursery school, and to what extent does their personal cultural knowledge strengthen their understandings? (Chapter 3).
- How do the children's inscriptions support their emergent abstractions? (Chapter 4).
- What evidence is there of intertextuality, particularly with respect to the use of graphical signs the children made in contexts that can be understood as mathematical, and how does this impact on their mathematisation? (Chapter 5).

Hypothesis

Mathematics requires children to communicate, using personal language and/or the standard language of mathematics, both oral and represented. In educational contexts where children's ideas and spontaneous pretend play are supported by their learning culture, young children will readily and confidently communicate their ideas through their emerging graphical signs and representations. Children's earliest marks and informal signs underpin future mathematical thinking and understandings of increasingly formal symbolic mathematical inscriptions, and, based on previous research (Carruthers & Worthington, 2004; 2006) and the examples in this thesis, it is reasonable to presume that they will provide strong and effective foundations that are of benefit in their later learning of mathematics in school. Contexts such as spontaneous pretend play and open group sessions can both support rich and meaningful *cultural-conceptual* learning, enabling children to interpret the mathematics from their perspective. Children in this study are expected to explore a range of ideas, solving problems using a variety of signs and strategies to communicate: such activity contributes indirectly to the establishment of what Vygotsky (1986) termed *everyday* or *spontaneous* concepts, allowing children to move towards *academic* or *scientific* concepts.

Hughes (1986) recognised the challenge in early mathematics education, arguing that if we are to reduce the difficulties young children experience, we should acknowledge their aptitude to comprehend complex ideas, provided that "they are presented in ways which interest them and make sense to them […] so that, instead of nullifying and ignoring children's strengths, we are able to bring them into play and build on them" (p. 184). Underpinning the approach used in the nursery school in which the data for this study were gathered, is research by Carruthers and Worthington (e.g., 2005; 2006), and, as headteacher, Carruthers's vison and philosophy concerning pedagogy.

The central hypothesis of this thesis is that the children in this study are likely to benefit from engaging in meaningful social contexts in which they can freely explore aspects of their cultural knowledge of mathematics. In such situations, children will often spontaneously communicate their contextual meanings and understandings of number, numeracy, relations, space and measurement through graphicacy, their marks and signs originating from their interest through social and collaborative learning, supported by their teacher's involvement and teacher-modelling. A basic assumption in this hypothesis is that the development of mathematical thinking in young children originates from their meaning making communicated through their early graphical marks to which they attach meanings, which gradually become mathematised through the use of signs, symbols and other mathematical texts borrowed from adults or more knowledgeable peers when solving problems.

The Genetic Law of Cultural Development

How can we theoretically understand this process? To examine the origins and history of learning, Vygotsky (1978) employed *genetic* analysis, arguing that to understand the *higher forms* of behaviours such as children's mathematical inscriptions "is to understand all its idiosyncrasies and differences. In short, we need to understand its origin […] *To study something historically means to study it in the process of change*" (p. 64-65, emphasis in the original). Vygotsky (1991) designated the historical development of children's cultural knowledge as "a sociogenesis *of the higher forms of behaviour*" (p. 40, emphasis in the original). Vygotsky's cultural-historical theory merges in a *psychosemiotic* perspective, defined by Smith (1999) "as the study of how humans learn, understand and use the signs of culture" (p. 19), its practicality and relevance found in cultural practices including those of mathematics.

Young children's cultural knowledge develops through their participation in everyday activities in and around their homes. For example, Rogoff (1991) described the supportive engagement that Mayan mothers had with their children, providing numerous opportunities to pay attention to, and engage in everyday cultural activities, through which they "become skilled practitioners in the specific cognitive activities in their communities" (p. 351). This situated knowledge permits "legitimate peripheral participation" in which learning is fashioned through children's gradual engagement as a full participant in a sociocultural practice (Lave & Wenger, 1991). While young children participate peripherally in cultural practices, some of this situated knowledge will be mathematical and some focused on graphical communication. Such situated and meaningful opportunities furnish children with their developing *funds of knowledge* or cultural knowledge (Moll et al., 1992) on which they subsequently draw in their pretend play (Riojas-Cortez, 2000; Worthington, 2018), and in their early attempts to communicate about mathematical problems and solutions. The culture of the early childhood educational setting also contains important elements that influence children's mathematical understandings. The outcomes of this research confirm that in meaningful learning contexts, young children are able to build on their previously appropriated cultural knowledge, using their own signs and representations to communicate their mathematical ideas in the context of pretend play and other open activities.

Research Design, Sampling and Situation

This research project addresses young children's freely chosen engagement with aspects of mathematics and their spontaneous graphical communications during their final year in nursery school when they were 3-4 years of age. The aim of the study is to demonstrate that in educational settings in which children's own meanings are valued and actively supported, children will freely choose to talk about and explore problems and their solutions, using mathematical notions. This only ensues when the culture of a setting or class and its teachers value the children's inscriptions and their meanings, and when the teachers model different signs and representations for their literacies (i.e., drawings, maps and writing, and those in meaningful (mathematical) contexts[17]. Secondly it aims to show that young children will spontaneously communicate though graphical signs and representations in contexts in which aspects of mathematics (such as counting) arise, thereby demonstrating features of their sign making and the processes by which advancement through mathematisation to the formal symbolic conventions is achieved. The research focuses on young children making and communicating meanings in the context of their spontaneous pretend play and other open activities in which aspects of mathematics present themselves.

This study involves longitudinal, ethnographic case studies of children, data gathered during the course of one year. In accord with the qualitative data and the nature of the children's signs and representations, the design of the study involves naturalistic interpretive methodologies, with the addition of some quantitative data. Ethnographic case studies were chosen to provide insights into the children's developing cultural knowledge and understandings at home and in their nursery school, and the features that appear to influence them. It was understood that a longitudinal study would best provide a range of data and provide insights into the processes of change. The chosen methodology was considered appropriate for answering the research questions. Employing largely qualitative research methods in order to investigate "a social human problem", allows the researcher to "conduct a study in a natural setting and builds a whole and complex representation by a rich description and explanation as well as a careful examination of informants' words and views" (Diaz Andrade, 2009, p. 43). Qualitative research methodologies also provide insights that are more realistic than statistical analysis alone could do, providing flexible means of collection, analysis and interpretation of data (Boodhoo &

[17] Young children also sometimes emulate signs from their peers' graphical texts (see chapter 4).

Purmessur, 2009, p. 6). O'Connor and Netting (2005) argue that whilst interpretative analysis is not generalisable interpretive work, it "can provide complex, context-based deep understanding" (p. 17). However, this study does not rely exclusively on qualitative research, but also uses some quantitative methods, Burke Johnson and Onwuegbuzie (2004) and Schoonenboom (2018) arguing that in such a mixed methods approach both methods complement each other.

The children attended a nursery school in a large, multicultural city in the southwest of England in which the head teacher had introduced and led the educational concept of *Children's Mathematical Graphics* (originated by Worthington & Carruthers, 2003), leading professional development for all the staff. Under her expert guidance and over a period of years, the staff has developed a good understanding of young children's mathematical development, including their use of mathematical inscriptions. Together, the headteacher and staff of the nursery school had created a culture that supports children's rich play and all aspects of graphicacy, and mathematics has a high profile there.

The headteacher introduced this approach to the nursery school and has provided ongoing professional development for the staff members, at the same time promoting and valuing teacher- and practitioner-research. The teachers directly involved in this research developed their understandings and practice to a high degree and can be regarded as "expert" teachers. Mathematics in this nursery has a very high profile and the children's meaning making and pretend play are valued and well supported. The teachers in this nursery school are very skilled at noticing and listening to the children, tuning in to their play and activities as much as they are able in the time available. Whilst I had previously visited the nursery school, until I began this research project I had not met with the children or their parents. I was aware of the risk of bias, and although I had met the two teachers involved in the study before, I had not discussed with them any aspect of this research until the onset of this study. I knew the headteacher through our shared research and writing into *Children's Mathematical Graphic*s over a number of years, but she was not in any way involved in this research, either in the selection of the seven children, through discussions with the teachers or parents, the gathering of data, or data analysis[18].

At the time of data collection for the current study, the children were aged between 3-4 years.

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[18] The exception to this were the occasions when inter-rater reliability checks on a proportion of examples were made, as reported in chapter 2, data analysis section; chapter 3, data analysis section and chapter 4, data collection and analysis section.

Since this thesis focuses on children's graphical communications, the children's prior interest in communicating through drawing or writing was considered likely to be advantageous to their subsequent use of graphical signs and representations in mathematical contexts. The teachers were asked to select three children as *focal children,* those who had exhibited a particular interest in freely made drawing and/or writing in the preceding year. Four other children were randomly chosen for comparison, and it was expected that the findings might reveal differences in the performances of focal and non-focal children. In the nursery school the children are divided into groups of children of similar ages: the teachers of the two groups were Emma and Hugo, both qualified teachers and (in English early years' education) known as a "key person" (Elfer et al., 2011).

Initial Aims

Carruthers and Worthington had not previously been able to follow the progression of children over time to trace their mathematisation. The initial aim of this current research was to gather data from a group of children for one year during their final year in the nursery, then to follow them during their reception year when they were 4-5 years of age, and into year 1 (aged 6 years of age), for a total of 2½ years[19]. However, with a growing emphasis on "skills-based" teaching and the imposition of increasing pressures of assessment in England since the start of this study, children's experiences in reception classes had become progressively circumscribed. Due to increasing "schoolification" (see for example Bingham & Whitebread, 2018), play became increasingly marginalised in reception classes, and children seldom had opportunities to use their own signs for writing and mathematics. Furthermore, few of today's teachers have been educated in or understand *emergent* or *developmental* approaches to writing or mathematics, so that children's own ways of making meanings were largely unacknowledged. These changes frustrated any possibility of following the progress of the children's mathematisation in a context that afforded meaningful learning over an extended period, and my original plans to gather data for this doctoral study for more than one year had to be abandoned.

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[19] Reception' classes are required to employ the same early years' curriculum as nursery schools and pre-schools. However, because they are sited in primary schools, reception teachers experience considerable pressure to use more formal methods.

The Research Project

This research project began in September of 2009. The headteacher of the nursery school (Carruthers)[20] had generously offered me the opportunity to gather data for this dissertation. The research followed existing practices within the nursery including the teachers' daily use of written observations of children's play and learning in the *learning diaries* for each child. For these reasons no intervention for the children or adults was necessary.

Ethics

At the onset and throughout the research process, ethical guidance from both the UK (British Educational Research Association [BERA], (2011) and the Netherlands (Vaste Commissie voor Wetenschap en Ethiek [Scientific and Ethical Review Board, 2016] Vrije Universiteit), regarding educational research was carefully considered. This guidance concerns informed consent of the headteacher, parents, teachers and children, confidentiality, observation of the children, information and use of data. By informing the headteacher, teachers, children and their parents on the aims of the study and their rights to withdraw from participation, these aspects were fully met.

Data Collection

Data for this study were gathered in the nursery school over the course of one year. The greatest proportion of the observations and children's inscriptions were written and gathered by the teachers during the children's pretend play, with some additional observations from open contexts such as small groups at other times. In these groups the teachers introduce a focus (and sometimes bring new experiences through resources), which the children can freely explore in their own ways, communicating through dialogue and sometimes through graphicacy if they so choose.

In total, the teachers made 191 written observations, documenting them in the children's *learning diaries*, an established part of their daily practice. Observations were made as the child played (or was engaged in an activity) and include transcripts of any words spoken by the child exactly as they were said, their actions and interactions with other children and adults. The observations are informal and aim to capture the children's behaviours and speech in as much detail as possible, as they happen (see

[20] Together Carruthers and I have collaborated in researching and writing about *Children's Mathematical Graphics* for more than 20 years.

figures 1.1 and 1.2). The teachers' observations were supplemented by the 47 observations I made on my visits to the nursery. There were no prescribed number of observations each day. Figures 1.1 and 1.2 are examples of Emma's observations

Figure 1.1

Observation of Ayaan at the forest[21]

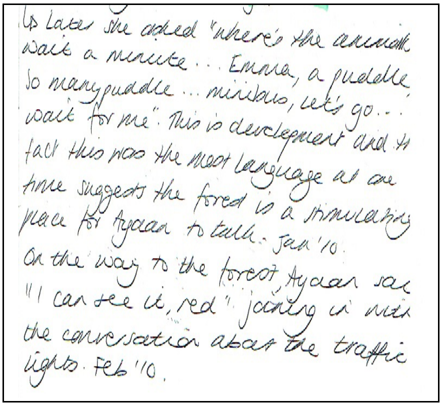
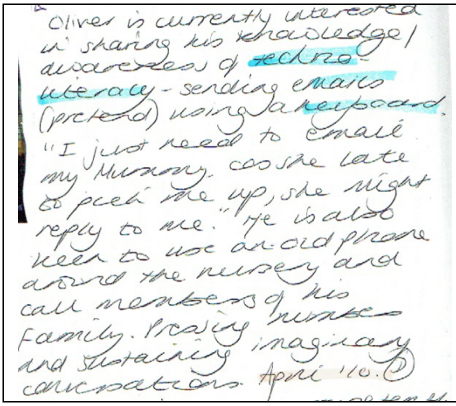

Later she added 'where's the animal wait a minute... Emma, a puddle so many puddle... minibus, let's go... wait for me". This is development and in fact this was the most language at one time suggests the forest is a stimulating place for Ayaan to talk. Jan '10.
On the way to the forest, Ayaan sai "I can see it, red" joining in wi the conversation about the traffic lights. Feb '10.

Figure 1.2

Observation of Oliver sending an email[22]


Oliver is currently interested in sharing his knowledge / awareness of techno-literacy - sending emails (pretend) using a keyboard. "I just need to email my Mummy, cos she late to pick me up, she might reply to me." He is also keen to use an old phone around the nursery and call members of his family. Pressing numbers and sustaining imaginary conversations. April '10.

I transcribed the 238 handwritten observations of teachers and those I had made. The observations included details of the case study children's play and their graphical communications through drawing, maps, writing and those made in mathematical contexts, in addition to those in which they referred (orally) to aspects of mathematics. A method of "event sampling" was followed, also known as *everyday experience methods* (Reis & Gable, 2000, p. 190) in which the time unit is the duration of the play episode (or group activity). This enabled investigation of the children's experiences as they occurred. Reis and Gable (2000) emphasise the value of

[21] Figure 1.1 *transcription of Emma's documentation:* "Later she added, "Where's the animal, wait a minute […] Emma, a puddle, so many puddle […] minibus, let's go […] wait for me." This is a development and in fact this was the most language at one time suggest[ing] the forest is a stimulating place for Ayaan to talk. On the way to the forest Ayaan said "I can see it, red", joining in with the conversation about the traffic lights. Feb. 10."

[22] Figure 1.2 *transcription of Emma's documentation:* "Oliver is currently interested in sharing his knowledge and awareness of techno-literacy, sending emails (pretend) using a keyboard. "I just need to email my mummy 'cos she late to pick me up, she might reply to me." He is also keen to use an old phone around the nursery and call members of his family, pressing numbers and sustaining conversations. April 10."

event sampling in that it offers a "detailed, accurate and multifaceted portrait of social behaviour embedded in its natural contexts" (p. 190). Together with the photographs of the children's graphics (taken at the same time as the written observations), the transcribed observations formed the main body of the data, subsequently analysed for chapters 2-5.

I also made *home visits* to each of the children's homes. Such visits are used in many early childhood settings in England and are a practice accepted by staff and parents in this nursery school, enabling parents and their child to meet their child's key person on their own territory and to become acquainted[23]. Following my visits to each child's family, I made informal notes on the children's play and interests at home. Additional data consist of field notes I made in the nursery and notes during each of my visits in discussion with the teachers, on aspects of the children's play and learning.

I also gave each child a scrapbook for their use at home (either to use directly, or for a parent to paste in drawings and writing): the children's use of these varied, and therefore they were less reliable as a data source.

Analysis

To analyse and understand the children's communications with their mathematical inscriptions, data are collected from a number of perspectives. In general, the chosen methodology can be characterised as observational and holistic, enabling the capture of conditions for development relating to "societal, institutional and individual perspectives" (Hedegaard & Fleer, 2008, p. 7).

Interpretive (qualitative) analyses of the data are supported by *computer assisted qualitative data analysis software* (CAQDAS), to systematically code as the data were processed, to provide answers to the research questions[24]. Additionally, some quantitative analysis is included. The specific analysis methods for each study are described separately in chapters 2-5.

Since the nature of the studies do not allow empirical generalisation with powerful external validity, the outcomes of the different studies will be drawn together in a *theoretical generalisation* in chapter 6, that binds together the empirical findings in a construct that describes the interrelated processes of early mathematics learning within a "best practice" environment. In employing qualitative analysis, the advice of Akkerman et al. (2006) was followed, who state that, "where one can not rely on

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[23] For personal family reasons it was not possible to visit one of the children at home.

[24] See for example https://www.surrey.ac.uk/computer-assisted-qualitative-data-analysis

standardized strategies and procedures", three criteria need to apply to decisions relating to qualitative research so that quality is assured (p. 258). These criteria are:

- visibility, which refers to the need to be transparent in the decisions made regarding how the research is conducted and communicated;
- comprehensibility: which means that linkages between data gathering and analysis should be substantiated;
- acceptability: which means that the decisions should be "logically and scientifically acceptable".

These criteria are taken into account in the current study.

Outline of the Thesis

In chapter 2 children's free and impromptu pretend play is investigated as a meaningful context for young children to explore aspects of mathematics. Identifying the total number of pretend play episodes in which the children engaged during the year, those that included evidence of the children drawing on their cultural knowledge of mathematics were established. Using open coding, coding for specific aspects of mathematics in the teachers' observations, such as number, quantities and counting; money, time and measurement were conducted. The percentage of the total number of pretend play episodes that included evidence of mathematical exploration was identified, as were episodes in which the children also chose to communicate through mathematical inscriptions. Quantifying references to aspects of mathematics in the children's play also provided information of those aspects of mathematics to which the children alluded most frequently, revealing the extent of their understandings. Finally, play episodes in which the children also used mathematical inscriptions were coded.

Chapter 3 explores the children's graphical communications through their freely-made literacies (i.e., drawing, maps and writing), considering the relationship between family members' literacy practices at home and the literacy events the children instigated within their pretend play in their nursery school. This enabled us to assess features of their meaning making in conjunction with the children's visual texts and the teachers' written documentation. The number of observed pretend play episodes for each child is given, and the percentage of episodes in which each child engaged in a literacy event within them. In a process of open coding all episodes in which there was evidence of the children's use of graphical marks, signs and symbols to communicate ideas through drawing, maps and writing (given as a percentage of the total of each child's pretend play episodes) were noted. Finally in chapter 3, analysis turns to multimodal features of the children's texts that contributed to their play and understandings. Using interpretive analysis and looking for recurring patterns of signs, codes were generated from these, and percentages of the various graphical marks and signs identified. We assume that this communicative use of graphical signs will transpire to be useful for mathematical inscriptions too. Chapter 3 also answers the emerging need to become more specific about the contents of children's communications in order to focus particularly on children's mathematical inscriptions described in the following chapters.

In chapter 4 the emergence of mathematical abstraction in young children's inscriptions and its contribution to our understandings of the evolution of the abstract symbolic language of mathematics is investigated. Following Cassirer (1977), a conception of abstraction is explained, investigating both observations of the children's pretend play episodes and a limited number of adult-led small group sessions, giving numerical figures for each child's mathematical texts. Analysing the various signs used by the children to communicate their mathematical thinking, led to percentages for signs in the three Peircian categories, *iconic, symbolic* and *indexical*, preceded by an additional category, *early mathematical marks/early explorations with marks – attaching mathematical meanings* (Carruthers & Worthington, 2005; 2006). In order to gain further insights into the children's signs and texts, analysis of the children's inscriptions (and inscription patterns) from a grammaticisation (or *user–based*) language acquisition perspective was made, focusing on *intention-reading* and *pattern-finding* (details explained later, in chapter 4). Finally, the total number of graphical signs in all of the children's sign lexicons were quantified, followed by the quantity of iconic signs and abstract symbolic numerals across the three terms, to provide insight into the development of the children's knowledge and use of signs use during a year.

In chapter 5, investigation of young children's mathematical inscriptions is extended to address the role of *intertextuality* and its role in *mathematisation,* children's expanding use of signs contributing to their rich graphical sign lexicons or repertoires. The term intertextuality is used to describe the way in which an author of a text uses existing signs from elsewhere (their own or others). At first young children appear to do this subconsciously, re-using signs they have used previously in other contexts and also those that they have seen adults and their peers use in meaningful contexts: through these means they gradually come to understand and use conventional signs in appropriate textual contexts. Identifying specific signs within the data to establish examples of intertextuality relating to quantity, evidence of mathematisation was considered, tracing mathematisation from the children's earliest beginnings with marks to the formal system of abstract mathematical symbols and strategies. Over time, children develop understandings of sign-use for mathematical problem solving, their informal signs progressively mathematising into the formal abstract symbols of mathematics. In conclusion, the final chapter draws together knowledge presented in the foregoing chapters, closing with an overview of the study and a discussion of the findings, limitations and suggestions for future research. This chapter also considers some of the impediments to productive mathematics in early childhood education (receptive to emergent learners), and the sort of transformations that are needed in England (and in many other countries), if young children are to develop a good understanding of mathematical semiosis. This closing chapter will propose a theoretical generalisation of the findings (rather than an empirical generalisation), as a contribution to theoretical development regarding the process of early mathematics learning. In this we considered how various *processes* support mathematisation, revealing an apparent relationship between Carruthers and Worthington's "bi-numeracy" model (2006) and Tomasello's "ratchet effect" (2005). Since individual chapters have been submitted to various international peer-reviewed journals as independent articles, there is some unavoidable repetition between chapters.

Empirical Explorations

- 2 -

Pretend Play and the Cultural Foundations of Mathematics[25]

Abstract

The aim of this study is to uncover the emergence of cultural mathematical understandings and communications in young children's spontaneous pretend play. It is based on Vygotskian cultural-historical perspectives and social-semiotic theory, informed by research into "funds of knowledge" (e.g., Moll et al., 1992) and considers how children's informal knowledge of family practices enriches their play and cultural mathematical understandings. Longitudinal, ethnographic data were gathered in an inner-city nursery in the south-west of England. Data include written observations and mark making of seven children aged 3-4 years of age engaged in social pretend play. The findings reveal that many episodes included aspects of mathematics and that these increased through the year: they show how the children's home cultural knowledge underpinned their pretend play and informed their mathematics. The children's mark making to communicate mathematics within their pretend play was also evident. The findings show also that where children are immersed in mathematical- and graphical-rich environments, bridging home and early childhood cultures becomes a natural feature of their pretend play. They will add to our understanding of cultural mathematical knowledge in young children.

Background to the Study

From birth children are immersed in organized cultural environments with a strong propensity to communicate, using a rich and multi-faceted cultural background. Munn and Kleinberg (2003) emphasise that children need to learn the cultural rules concerning "how to use a system, and what its role is in our culture" arguing, "these cultural rules are possibly the most important things that children learn"; without understanding them children "risk becoming stranded in a sea of meaningless activity" (p. 51-3).

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[25] Previously peer-reviewed and published as: Worthington, M. & van Oers, B. (2016). Pretend play and the cultural foundations of mathematics. *European Early Childhood Education Research Journal, 24*(1), 51-66.

Where in ontogenesis do the cultural foundations of mathematics originate? Research has highlighted young children's home knowledge (e.g., Aubrey 1997; Carruthers 1997), yet Munn and Kleinberg (2003) maintain that in educational settings there remains "such a mystique about maths as a cultural activity" (p. 109).

Van Oers (2001) emphasises children "are from the beginning of their life a member of a community that extensively employs embodiments of mathematical knowledge" (pp. 59-60). Children participate naturally in cultural practices that include mathematical talk and representations, and parents introduce infants and young children to counting and numbers, through toys, songs, games and numbers for birthdays. We should expect to see evidence of this cultural knowledge in play. However, reflecting on her informal observations, Gifford (2005) comments: "children's role-play was concerned with the larger themes of life, like love and power, rather than mundane things like the price of potatoes", concluding, "a laissez-faire approach to children learning maths […] does not work" (p. 2) since children fail to take advantage of opportunities provided. Other researchers have found a similar lack of mathematics in play. Munn and Schaffer's (1992) study of ten Scottish nurseries found children's use of number without adult involvement was very rare, and observed no maths in role-play. Ewers-Rogers and Cowan (1996) found children playing "fast food" scenarios "used no numbers" arguing, "for ordinary English pre-schoolers, money may have little significance" (p. 23). Brannon and van de Walle (2001) concluded from their research, that "It may be that for young children, number is not automatically a salient dimension of the environment" (p. 75). A systematic study of the experiences of children in ten early childhood settings in England confirmed these findings (Moyles & Worthington, 2011). However, it appears unlikely that young children could omit one aspect of their experiences such as numbers from their play: where mathematical thinking is not readily observed in pretend play the implication points to dominant discourses and practices of play. An exception to these findings comes from a study by Cook (2006), which, like research by Carruthers and Worthington (2006), began from an emergent literacy perspective. At the onset of her study Cook intervened by adding play resources that included number symbols (such as birthday cards with numerals) to the role-play area in a nursery. The most significant outcomes were the consequent increase in the children's use of number symbols in their play. In turn this stimulated the use of child-initiated mathematical utterances and "encouraged the children to *play about* what they knew" by drawing on existing knowledge (2006, p. 65, emphasis in the original).

Brooker (2011) emphasises that in many settings, genuine *child-initiated* pretend play, “voluntary, goal-less, spontaneous – but which for children is entirely serious” (p. 162) has been “displaced by a more systematic induction into social patterns of meaning” (Parker-Rees, 1999, p. 61; cited in Rogers & Evans, 2008, p. 36). Rogers and Evans argue that this creates tensions between children’s:

> natural and powerful propensity to play in ways that transform and find new meanings […] the pedagogical imperative to reproduce real life – the café, the shop, the doctor’s surgery – so that requirements in literacy and numeracy can be met” (p. 37). This highlights practice common in most of the world where adults choose, plan and resource themed role-play areas, revealing *adults’ perceptions* of children’s interests, rather than children’s authentic and immediate interests that have personal cultural meaning. However, since we support the ideology of child-initiated play, is it realistic to assume it could include aspects of mathematics?” (Worthington & van Oers, 2016, p. 52).

The present study builds on research by Carruthers and Worthington, into *Children’s Mathematical Graphics*, (their personal marks, symbols and representations), to focus on the emergence of mathematics in pretend play. Carruthers and Worthington developed the educational concept of CMG from “emergent writing” in which children use their own informal marks and graphical signs to represent and communicate meanings. Their research reveals a continuum from young children’s earliest marks to which they attach mathematical meanings, to written calculations (2005; 2006). Two specific questions are addressed:

1. *What evidence of mathematics can be found in young children’s free pretend play, and what is the breadth of their mathematical interest?*
2. *To what extent do children draw on their personal cultural knowledge in their pretend play, and how does this influence their mathematical thinking?*

Theoretical Framework

Cultural Knowledge and Mathematics

Mathematics is a human product inseparable from its cultural context (Brandt & Tiedemann, 2009). As humans we learn through participating in cultural practices, (Rogoff 2008), Leont’ev (1981) asserting that through activity humans “assimilate the experiences of humankind” (p. 55).

Basically, Vygotsky (1987) identified two ways of appropriating culture: through directly experiencing cultural situations and practices (leading to *spontaneous*, empirical, everyday concepts), and through instruction (leading to schooled, *scientific* concepts). Spontaneous concepts lay a foundation for later elaborations into more scholarly concepts. “The development of scientific concepts begins in the domain of conscious awareness and volition […] the development of spontaneous concepts begins in the domain of the concrete and empirical. It moves towards the higher characteristics, towards conscious awareness and volition” (p. 220).

Pretend play furnishes opportunities for the development of everyday concepts and, in Vygotsky’s view, provides a “bridge” between spontaneous and scientific concepts (p. 238). Munn and Kleinberg (2003) emphasise that whereas children can readily “be taught the mechanics of arithmetic, if they lack any wider sense of purpose of these activities then their spontaneous learning will be hindered” (p. 52). However, in many situations home and school cultures are viewed as mutually exclusive (Abreu et al., 1997). Yelland and Kilderry (2010) argue “it is difficult for children to link mathematical skills and concepts taught only in isolation, since ways concepts are taught in school are frequently very different to their use in everyday life” (p. 93).

As active participants of family cultural practices children draw on their personal knowledge in their play, enabling their previously acquired body of cultural knowledge, their “funds of knowledge” to come to the fore (Moll et al., 1992, p. 133-4). Taking this focus Riojas-Cortéz (2000) analysed role-play, reflecting it ‘provided a naturalistic picture of the linguistic and cultural repertoires’ children possess (p. 305). For Hedges and Cullen (2005a) this highlights how children’s current cultural knowledge contributes “to their growing content knowledge” as they move “from novice to expert […] emphasises the importance of constructing new knowledge based on existing evidence” in collaboration with others (p. 4).

Mathematical Communication in Early Childhood

Communication is significant in mathematics, but where formal vocabulary and written notations are introduced without meaningful cultural contexts their use will fail to make sense. From a Vygotskian perspective, mathematical thinking in ontogeny begins within participation in communicative cultural practices that have personal meaning for the children (van Oers, 2012b).

Pleas for greater emphasis on the cultural significance of mathematics have been voiced (Bishop, 1991; Saxe, 1991), and for a more discursive approach that recognises children’s own understandings, (for example,

Krummheuer, 2013). Two streams of research have explored early mathematical notations from these perspectives. In the Netherlands van Oers (e.g., 2012b) and Poland (2007) have demonstrated how children's "schematizing" in play can support abstract thinking that is so important for mathematics. In England Carruthers and Worthington have researched early childhood mathematics from the child's perspective, (2006), beginning with the premise that, as with emergent writing, young children can use their own Mathematical Graphics
to explore and communicate their mathematical thinking. Children's need to elaborate ideas by symbolic means is rooted to a great extent in their desire to communicate some aspect of reality and can be realised through their pretend play. As they imitate and explore mathematical ideas and culturally specific tools and symbols in which they have been involved at home, children clarify their ideas and elaborate their goals in culturally meaningful ways.

Pretend Play and Mathematics

Vygotsky (1978) acknowledged play as the "leading activity" for young children, proposing "as in the focus of magnifying glass, play contains all developmental tendencies in a condensed form and is itself a major source of development" (p. 102). Social pretence and imagination offer potentially rich contexts that "situate" learning and allow children to explore their existing cultural knowledge of mathematics. Vygotsky related play to creating an "imaginary situation":

> A reproduction of the real situation takes place […] only comprehensible in the light of a real situation that has just occurred. Play is really more nearly recollection of something that has actually happened than imagination. It is more memory in action than a novel imaginary situation. (p. 103)

Whilst the most effective play appears spontaneous, it does have its own internal "rules of behaviour" that "stem from the imaginary situation" (1978, p. 95), evident in the examples in this study. In the following section we describe an empirical study exploring the occurrence of mathematics in children's pretend play related to their personal background knowledge ("funds of knowledge"). For the purposes of this study pretend play episodes are defined as social play involving two or more children, engaged either in pretend play or making maps involving elements of pretence and imagination.

Characterisation of the Study

In order to answer our research questions, we conducted an ethnographic study, focusing on "detailed accounts of the concrete experience of life within a particular culture and of the beliefs and social rules that are used as resources within it" (Hammersley & Atkinson, 1995, p. 9). Ethnographic research allows researchers "to get alongside children in their environment", to "enter the world of the participants", and is a method "befitting the exploration of the meanings and constructions held by research participants of their social world" (Emond, 2005, p. 124-5). In our study the semiotic focus is on children's graphics and reflects this aspect of social semiotics: the study did not investigate other multimodal dimensions such as body language.

According to Geertz (1973), data gathered are qualitative and suggestive of a "thick description", expressed as, "a multiplicity of complex conceptual structures, many of them superimposed upon or knotted into one another, which are at once strange, irregular, and inexplicit". A researcher must somehow contrive such description in order to be able "to grasp and then to render" (p. 10).

Research Setting and Participants

The research setting is a nursery school within a Children's Centre in a large city in the southwest of England, in an area designated one of the 30 percent most deprived in England. It welcomes families from various ethnic cultures, providing many services to support families, babies and young children in the locality. Sixty children attend nursery sessions each morning and afternoon, and thirteen different languages were spoken there at the time of data collection.

The children are free to initiate their own ideas in play, time for play constituting the greater part of each session. It was expected that in a setting in which mathematics and graphicacy have high profiles, children would be likely to explore these aspects in their play. However, at the onset of the study it was not clear if *all* children would choose to communicate through graphicacy, and the teachers were asked to identify several children who did so. They based their judgments on their knowledge of the children and their previous written observations, identifying Isaac and Shereen, both 4 years of age, and Elizabeth, 3 years, 6 months.

In addition to these focal children, four additional children (Oliver, David, Ayaan and Tiyanni) were randomly chosen in order to determine if they might also choose to communicate through graphics, providing a total of seven case study children, three boys and four girls. All the children were in their final year at nursery, and at the beginning of the academic year their ages ranged from 3 years, 2 months, to 4 years of age.

Data Sources

The main body of data is taken directly from teachers' written observations[26] or "learning stories" (Carr, 2001), which "document the learning culture […] this is what we value here" (p. 103). The data also include graphics from the nursery and children's homes, and transcripts of discussions with the teachers: field notes made in the nursery and during my home visits to each child's family provided additional information. Six of the seven children were visited at home: it was not possible to visit the remaining child due to personal family circumstances. Visits were informal, the main aim to see the child within the social and cultural context of their home and family, and to observe any spontaneous play or graphics in which they engaged at the time. Short discussions with parents during the home visits focused on the child's play and graphicacy. Data focused on a period of one academic year.

Procedure

The teachers' established practice is to write ongoing observations of children's play behaviours, actions and talk. As participant observer I made written observations of play during regular visits to the nursery. As a methodology, Dunn (2005) maintains that naturalistic observations used in ethnographic research "enable us to study children in situations that have real emotional significance to them […] [they] provide invaluable evidence on children's real-life experiences and their reaction to those experiences" (p. 87).

In order to eliminate bias towards specific research outcomes, two factors are significant. Firstly, the teachers' standard practice in this nursery is to write observations of all aspects of the children's play and learning throughout each day, writing for children, staff and parents rather than for the purposes of this research. Secondly, to ensure that not only observations of pretend play which included evidence of mathematics were selected, written observations of *all* pretend play episodes are included as data for analysis.

Data Analysis

[26] Whilst the word "teachers" is used throughout, they work closely with all the qualified early childhood professionals in the nursery, who also contribute to the children's learning diaries.

Hammersley & Atkinson (1995) write that analysis involves "interpretation of the meanings, functions, and consequences" (p. 3) of the children's play and graphicacy. ATLAS-ti software was used to support systematic data analysis of the written observations, and for the purposes of this chapters, data were coded to identify evidence of *pretend play episodes*. Within these episodes the following aspects were identified and coded:

a. The children's cultural knowledge, showing the mathematics they explored – *coding evident aspects of the children's home funds of knowledge;*
b. The mathematics explored – *coding the specific mathematics that could be identified, such as children's use of numbers, references to time or measurement;*
c. The children's use and understanding of mathematics – *coding the role of the mathematics in the context of the children's play and how they appeared to use and understand mathematics to communicate to their peers and to further their play narratives.*
d. Within ATLAS-ti, transcripts of each observation were examined for evidence of the child's known cultural knowledge, based on home visits I made, discussions with the child, their parents and teacher. Relevant sections of text were highlighted and the code "a" assigned: this was repeated for "b", for those observations that contained evidence of mathematical talk or representations. Evidence of mathematics pointing to the child's mathematical understanding, and which appeared to serve a relevant role within the play narrative was coded "c".

In order to strengthen reliability and validity, an additional researcher conducted independent coding of the data, applying it to 10 percent of randomly chosen observations. Agreement was reached for over 94 percent of the codes assigned, a significant level of consensus.

Ethics

Data collection was guided by BERA's ethical principles (2011) and includes voluntary informed consent, openness, the right to withdraw and privacy. The parents were consulted at the onset of the research and their permission sought to observe their child and collect data. Using everyday language, the research was explained to the children and their agreement sought. One family withdrew their child from the study early in the period of data collection[27], and none of the data pertaining to this child have been

[27] The study began with eight children.

used. Questions of power concerning myself to gather data is acknowledged: staff at the nursery continuously make written observations in the context of their work, activities with which parents and children are familiar. Parents all gave signed consent for their child's first name to be used in publications relating to the research, whilst confidentiality and anonymity are protected since no surnames are used.

Presentation of Findings

Cultural Influences and Children's Interest in their Pretend Play

Factors Influencing Variation between Children

Analysis of the data shows the children engaged in a total of 146 pretend play episodes over three terms[28], with wide variation from 51 episodes for Isaac, to eight for both Tiyanni and Elizabeth. Variations in social pretend play depends partly on children's confidence in interacting with others to initiate and maintain it. Isaac obviously built this confidence in the year he had already spent in nursery. Tiyanni's lack of confidence may have related to some home difficulties. Elizabeth had attended nursery since she was one-year old and was very confident: in the previous year she had often engaged in pretend play but developed other interests during the year of data collection. Ayaan's mother described her as "very shy", in marked contrast to her more outgoing siblings. Ayaan's first language is Somali and during her first two terms at nursery she spoke very little English, engaging in no role-play during this period. By the third term Ayaan's confidence in speaking English and understanding the nursery's culture had grown, and she often chose to initiate role-play with her peers. Cultural differences related to adult attitudes and understandings of play may have been a factor for Ayaan (Somalian) and Tiyanni (West Indian).The children's play also appeared to be influenced by the extent of their direct involvement in cultural experiences at home or work and was especially marked for Isaac who had accompanied his father to work since he was small and had also been directly involved in the conversion of their home, work that included extensive use of mathematical tools and talk.

Children's Interest and Involvement in their Play Narratives

All children showed deep interest and a high level of personal involvement, controlling what Hughes (2001) refers to as "the intent and content of their

[28] Of the seven case study children, data were collected in only two terms for Shereen, since she did not begin to attend the nursery school until the January.

play" (cited in Brown, 2012, p. 68). Moreover, the children often developed themes and ideas over many days, and in some instances over several terms, allowing increasing complexity of ideas. All pretend play episodes were observed in the nursery, individuals spontaneously initiating their play narratives during the extended free play sessions that are an integral aspect of the nursery's practice. The children's unfolding play narratives revealed that in every episode they drew on their funds of knowledge and suggested also a subtle interweaving of new cultural knowledge from the nursery.

Home Cultural Influences

Shereen's family is from the Philippines, and shopping, preparing and eating meals together have special cultural importance. Elizabeth enjoyed camping with her family and Oliver was interested in trains. Shopping with her mum Ayaan knows a lot about paying for goods, and sees her dad count the money he takes each day from working as a taxi driver. At home she loves helping to prepare ingredients and helps care for her siblings. Isaac's father worked as a builder: Isaac sees his dad setting out wood for carpentry involving calculating, helps his father measure timber, mark out squares and angle cuts and understands the use of a spirit level. Isaac's father now runs a local brewery involving deliveries, invoices, payments and counting cash, and Isaac is involved in all these activities. Isaac is also very knowledgeable about a wide range of technologies, vehicles, maps and camping, and his father also shares his interest in motorbikes and trains.

The following two observations exemplify aspects of children's cultural funds of knowledge, revealing the embedded nature of their mathematics. Ayaan was just beginning to explore mathematics in her play, whereas Isaac's longer episode contrasts in its complexity. The following two transcripts are taken directly from the teacher Emma's written observations[29].

> For two weeks Ayaan had been playing in the gazebo, offering pretend ice cream through the window to children. Today when a child replied, *"yes",* Ayaan answered, *"no left",* adding, *"I make more".* Collecting stones and pretending to make ice cream, Ayaan asked Tariq if he wanted any. She passed him an imaginary one, then pressed buttons on the till saying, *"it's 50 minutes."* Shortly afterwards Ayaan drew dashes in a notebook without comment. Next time Ayaan played ice cream shops she asked*, "50 minutes please."* When a child offered *"£1.00",* Ayaan replied*, "that's*

[29] The only change made to these transcripts, is the substitution of "Emma" for "I" and "she".

> *£50.00 please."*

Appreciating his interest in security and locks, Isaac's teacher Emma bought a small safe into the nursery for the children to investigate.

> A few days later Jaydon and Isaac moved a small cupboard to create a safe, placing a keyboard and clipboard on top. They transported wooden blocks on the trolley and when another child removed one, Jaydon wrote wavy lines on his clipboard. Taking his paper, he placed it in the safe, tapping several keys on the keyboard and repeating this each time a child removed a block. Isaac announced, *"this is the safe. There's a key, only one - you press it here and it opens. It has a number and no one else knows it, "one, one, eight, seven, zero, six." It's rather difficult to remember."*
>
> Jayden put some real coins and play cheques in their safe and Isaac stuck a calculator on the cupboard door adding*, "you need to press the buttons to get in the safe […] it's "four, nine, seven, nine."* Jayden pressed some numbers making, *"beep, beep"* noises as he opened it, then closing the doors asked, *"what's the closing number?"* saying, *"one, nine, five, two,"* as he pressed buttons on the calculator. Later Jaydon said, *"you need to give me "one, nine, five, two",* and when Emma explained that she didn't have enough cash but could write a cheque, Isaac replied, *"I need hundreds of pounds!"* Emma managed to find a selection of coins in her purse and Jayden responded, *"okay! We need to fill the box: you need to give me £1,500.60."* After several days playing with their *"*safe*",* Isaac decided to write down the number of blocks being taken from the block areas, *"one, two, three, gone! Gotta write it down and put it in the safe."*

Reflecting on her observation, Emma wrote: "*both boys were using their experiences and understanding of numbers in a real situation. Isaac showed huge awareness of numbers and combinations* [and] *Jayden, in awe of large numbers in relation to money, knew that £1,560.00 was a large amount of cash. Isaac understands that number sequences can be unique for codes on safes and that longer sequences would be harder to remember and less likely that unwanted people will be able to get into the safe."*

Sustained play episodes appeared to be most effective in allowing ideas to be explored and developed, and sometimes re-visited over time. Isaac was most likely to do this, the children with whom he played clearly benefitting from his initiatives, contributing their own ideas and peer-models of graphics to their joint narratives.

Mathematical Use and Understanding

Incidence of Mathematics

Over 44 percent of all pretend play episodes included evidence of mathematical exploration: this exploration was most often evidenced through the children's dialogue, the children using this to communicate mathematical ideas to further their play. Transcripts of Isaac's play showed 19 occasions on which he made reference to mathematics, to only three for Elizabeth. However, Elizabeth often engaged in mathematics independently of pretend play, showing mature understanding that suggested more conscious enquiry. For all children, incidence of mathematics in pretend play increased throughout the year.

The Range of Mathematics

The transcripts of Ayaan and Isaac's play included above provides full details of their play behaviours and talk as documented by their teacher. Table 1 shows the range and quantity of the children's use of mathematics in all their pretend play episodes.

Table 1

The children's use of mathematics within their play (numbers represent the quantity of play episodes that included references within each category)

Number, quantities and counting	money	time	length and distance	direction	speed	weight	temperature	shape, space & capacity	Data handling	**Totals**
31	24	15	4	2	4	3	3	10	4	100

Table 1 shows that the children included references to *number, qualities and counting* most frequently. The high number of references to *time* and *money* also underscores their significance in the children's home experiences. Isaac for example made thirty-two references to mathematics within his play (32 percent of the total made by all children). Whilst Ayaan only began to engage in pretend play during the third term of the year in which data were gathered, she also made a surprising twenty-two mathematical references. The tables below summarise children's use of mathematical notions in relation to different pretend contexts.

Table 2

Showing the contexts of the children's play and their references to number, quantities and counting

Contexts	**References to mathematical notions**
Playing "car park entry" with Isaac in the sandpit, Oliver made a sign for the gate.	Oliver explained: *"These are ticks. When there are three ticks you can go, when there are two you can't go that way. I've made two ticks - that means you are not allowed. People allowed in that way."*
Ayaan playing "mother and baby".	Ayaan used numbered raffle tickets as buttons on a remote control for her *"television".*
Playing "deliveries", Isaac filled a wheelbarrow with bottles, paper and a watering can.	Isaac explained: *"I'm delivering wine, I can't remember which house number but I remember which street and what the house looks like."*
Several children wanted to get inside the car in which David and Remi were playing, but there was no room.	David drew several large crosses on a piece of paper, saying*, "no more children getting in our car."*
Shereen, playing shops	Shereen counted 20 items on her shopping list with one-to-one correspondence.

Money

Transcript of observation exploring money:
David and Isaac were talking down the phone to each other. Isaac decided to use a diary as a "booking book" for a campsite, and explaining that two people were staying, made two marks in the diary. Isaac used the phone to take more bookings, telling David, *"One hundred million people are staying!"* David replied, *"I want to stay for two nights."* But Isaac said, *"no. I'll put you down for two million nights, but don't worry – it's only £1.00 a night."* He then wrote it down in his *"booking book",* this time making many marks and David also took a diary and made his own symbols (circles and vertical lines).

Table 3

Showing the contexts of the children's play and their references to money

Isaac and David drew a collaborative, imaginary road map.	David explained*: "You have to pay money. It's £2.00 to park here."*
Isaac, playing builders with several boys in the small house outside.	Isaac wrote a cheque for £500.00 *"for all the jobs done.* […] *£1000.00 to hire generators so we're getting electricity to the building."*
Oliver and Remi have a large cardboard box with interesting shaped holes on the top, into which they post plastic "buttons". Remi announces it's a "spaceship".	Later they changed the function of their *"spaceship"* to a *"cash point",* Remi says*: "we're making money".*
Shereen, playing shops.	Shereen wrote a receipt for a customer.

Table 4

Showing the contexts of the children's play and their references to time

Playing builders with several boys, in the small house outside, Isaac fitted a clock on a hook on the wall.	Isaac said: *"This is so we can time how much building we're doing. We're building a builders' yard - we've got 50 minutes left."*
Isaac and David drew a collaborative, imaginary road map with a beach.	Isaac explained: *"here's where you park your lorries for two hours while you sit on the beach."*
Tiyanni spooned uncooked rice from a baking tray into a cup and put it in the toy oven.	*"This is going to cook for an hour. Hours and hours and hours."*
David held a small clock in his hand and pretended to sleep.	David tells Remi: *"Five minutes to go […] two minutes to go […] morning time! We can watch TV now."*

Shape, Space and Measures

Exemplars of the remaining areas of mathematics the children explored in their pretend play are given below.

Length and Distance:

- Playing builders, Isaac and Jaydon estimated the length of the pretend house they were building, and Isaac used a tape measure to see how far away to put the next block for the house he was building.
- On another occasion (also playing builders), Isaac measured a large box with a tape measure, saying*, "sixty metres."*
- While drawing a building plan with Isaac, Jaydon remarked, *"my house is getting bigger and bigger."*

Speed and Direction:

- David referred to the need for a road sign designating the speed limit on the imaginary map he drew with Isaac, showing lorries *"rushing to the beach".*
- Isaac explained the arrows he'd drawn on their map, *"these are arrows to say 'go this way.'"*
- Isaac used a compass, referring to the compass points *"north and south*" and related them to locations in the city.

Size/Area/Capacity:

- Isaac and Jaydon built a doorway, Isaac commenting, *"Lefty* (a character from "Sesame Street" on television) was *"too big outside and couldn't get through the door. You'd have to measure it to make sure."*
- An additional child was desperate to sit inside the pretend "camper van" that several children had built, but Isaac explained, *"There's only room for two - not three!"*
- Making a joint plan of a house, Isaac announced, *"this is my big eating room."* Jaydon added, *"my house is getting bigger and bigger."*

Weight:

- David, Oliver, and several other boys are outside mixing soil, sand and bark chippings in the wheelbarrow. Oliver said, "*we're making chocolate cake!"* As they stirred their mixture, David commented, *"it's too heavy - we can't lift it."*

Temperature:

- Isaac explained a feature on the map he'd drawn with David, *"there's the sandy beach - it's as hot as chicken!"*
- Tiyanni removed her pretend cake from oven, "*It's not warm like yours – it's cool now."*

Data Handling

- David, Jayden and Isaac are by the door into the nursery with clipboards, paper, pens and a calendar, checking people in and out. Isaac uses vertical marks for entries, then a letter "X" for a member of staff who he says, will soon be leaving and David writes a mark in her hand saying, *"that means you work here."*

Children's Mathematical Graphics

In over 46 percent of all play episodes that included mathematics, the children also spontaneously used their *Mathematical Graphics* to communicate: these included arrows to signify direction; specific symbols to signify "you work here"; marks and abstract symbols to represent quantities, to show food eaten and to signify cinema open and closed; crosses to signify "no more children" and a personal abstract symbol for "£". In young children's informal mathematical representations, drawings of arrows and hands have been shown to signify the operations of adding and subtracting, (Hughes, 1986; Carruthers & Worthington, 2005, 2006; Poland et al., 2009).

Graphicacy

Shereen's graphics were often mature with clear representational drawings and some use of standard letters of the alphabet, although like all the children she often used scribble-marks as "shorthand" for writing within her play. Isaac had a highly developed interest in signage and writing in the community, and in the nursery and at home frequently drew on this knowledge in play.

The amount of graphicacy the children used is insufficient to allow clear conclusions to be drawn: for example, whilst Elizabeth only once used graphics within her few episodes of pretend play, she showed some mature use in a range of other contexts. All of the seven children used personal abstract symbols to convey meaning, and the remaining four children showed their developing understanding of various symbolic systems.

Adult Roles

Parents and other family members influenced children's narratives and concepts. In some homes pretend play was encouraged and valued**,** which was evident during home visits. For example, Elizabeth created an elaborate tent with her brother. Isaac set up an "ice cream van" in his dad's van, something he often played with his dad. Isaac also made a "register" for his family, an aspect of the nursery's culture that had travelled back to home. Graphicacy appeared to be valued in all the children's homes, evident in their "home scrapbooks" and during home visits.

Teachers' Roles

There appear to be contrasting adult responses to children's play: either, without adult intervention, or by adults directly mediating in play. Whilst teachers' roles are not a feature of this study, it was clear from the data that adults in this nursery value and support children's free pretend play and interests. Rather than intervening in play[30] they mediate learning in other ways: through creating mathematically- and graphically-rich environments; modelling graphics for authentic purposes throughout each day, and through collaborative dialogue they support and extend children's thinking.

Discussion and Conclusion

The aim of this study was to uncover the emergence of cultural mathematical understandings and communications in young children's spontaneous pretend play. As an answer to our first research question, we can say that evidence of mathematics was found within all the children's play and that it extended beyond number and quantity to span the breadth of the mathematics curriculum. No remarkable differences were found between the focal and the randomly selected children.

Drawing on their home cultural mathematical knowledge the children imitated and extended ideas that fused reality with imagination. The children's use of their spontaneous concepts from the mathematical domain contributed to their play in ways that made sense both in the contexts and development of their narratives. These findings are in direct contrast to almost all findings from research into pretend play and mathematics referenced earlier, and may be accounted for as differences between the cultures and philosophies of various early childhood settings.

In the nursery in which the data for this study were gathered, the headteacher and staff have developed an open and unstructured culture in which children are encouraged and supported as learners, and their emerging understandings valued. Adults have clear philosophies of young children as learners and of play and mathematics, and have developed deep knowledge of learning and significant pedagogical skills to support children's thinking and learning.

Answering our second research question, analysis of the findings revealed that the children drew extensively on their personal cultural knowledge in their pretend play, exploring and elaborating their mathematical knowledge within the context of their unstructured pretence and imagination. Their cultural knowledge influenced their mathematical thinking by providing coherent contextual and mathematical meanings within their chosen play narratives. Written observations of the three children selected by their teachers for the study showed that they

[30] At the time of writing this chapter it was understood that the teachers did not take part in the children's play. However, subsequently it became clear that they did so, participating as players in children's play narratives.

continued to develop their personal interest in graphicacy throughout the year, in both play and a range of child-initiated contexts.

Paradoxically this research reveals that whilst the adults in this nursery did not plan for mathematics within pretend play, the children's self-initiated play triggered their cultural mathematical understandings. Why are these findings in such sharp contrast to those cited earlier? Play scholars have identified the widespread lack of understanding of play, which results in pretend play that lacks clear connections to the children's personal experiences of life (e.g., Rogers, 2010b; Brooker, 2011). Consequently, Fleer (2010a) argues, concepts are "*conceptually disembedded* from the practices and the imaginary situation being played out by the children" (p. 75, italics in the original).

Rogoff (2008) highlights learning through participation in cultural activities, involving "three inseparable processes" (p. 58). Apprenticeship through participation in cultural events is evident in examples such as Ayaan's involvement in shopping with her mother and Shereen's participation in preparing and sharing family meals. Isaac's guided participation in his father's work is unambiguous, as are teacher-models of graphics for authentic purposes. Participatory appropriation allows "change through involvement"; visible throughout the children's increasing understandings in pretend play and in other contexts during the year.

As Göncü and Gaskins (2007) emphasise, "a child's biology, culture and experiences all influence behaviours and play" (p. 10): for adults in early childhood settings, it is important to understand these influences and maximize opportunities for effective learning. The challenge facing teachers is to determine contexts that will most effectively enable children to employ and explore their cultural knowledge, and to develop "spontaneous" concepts that will gradually connect with scientific mathematical concepts (Vygotsky, 1978). Understanding mathematics in cultural practice enables children to "bridge" home and early childhood cultures (Carruthers & Worthington, 2006). It begins between people on an *interpsychological* level, and subsequently within the child on an *intrapsychological* level: "all the higher functions originate as actual relations between human individuals" (Vygotsky, 1978, p. 57).

On the basis of our findings, we can only endorse Brooker's (2011) conclusion that "we need only to offer children spaces in which they can undertake activities which are important and meaningful to them […] Increasing mediation from the adults and children around them and from the cultural resources" enables children "to increase their participation repertoires, hone their skills and move from being peripheral members of the group to full membership" (p. 162). Effective pretend play should be ecologically valid and offer optimum spaces and contexts for the social interactions that will induct children as apprentices and participants into the cultural knowledge of mathematics.

Children's Social Literacies: Meaning Making and the Emergence of Graphical Signs and Texts in Pretence[31]

Abstract

This study builds on recent research into young children's pretend play. Social literacy practices and events in which the children engaged were investigated to uncover features of their meaning making. Drawing on Vygotsky's view of the social nature of symbol use and writing, it stresses the significance of cultural and social features of meanings and literacies within the children's play narratives. Data were collected for case studies of seven children aged 3-4 years in an inner-city maintained nursery school in southwest England, as part of a larger longitudinal ethnographic study. Data comprise written documentation of the children's play and their visual representations, and analysis follows an interpretive, social semiotic multimodal paradigm. The findings make a compelling case for greater appreciation of pretence as a potentially valuable context for the enculturation of literacies, highlighting the diversity and richness of children's spontaneous meaning making and self-chosen literacy events. Informed by cultural and literacy practices of home and nursery, the children's communications show how meanings and signs are carried across time, space and contexts. Rich and sustained play supported the children's self-initiated literacies in which they explored a heterogeneous range of textual genres, uncovering their developing semiotic understandings and expanding repertoire.

Introduction

An important aspect of children's cultural development is the process of sense-making in which young children attribute personal meaning to available cultural heritage (van Oers, 2012a). The abstract symbolic systems of writing and mathematics, for example, are important for both individuals' and society's success and are consequently a focus for learning in many early childhood curricula. Both systems are acknowledged as literacies (United Nations Educational, Scientific and Cultural [UNESCO],

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2006), this change affirming a shift from a monomodal, written literacy to multiple literacies.

Within their families and communities, children engage in a continuous stream of cultural events where literacy is embedded in children's everyday activities at home rather than being just a school subject (Barton, 1994). Early childhood teachers[32] must consider which contexts best provide young children with meaningful experiences to explore and build on their powerful cultural knowledge and home literacies. Vygotsky and Leont'ev regarded pretend play (symbolic play) as the "leading activity" in early childhood, and according to Leont'ev (1981) the location of the most significant psychological changes, paving the way for "the child's transition to a new, higher level of development" (p. 369). Recent research has revealed the extent of children's *mathematical* interests and the role of cultural knowledge in social pretend play (Worthington & van Oers, 2016). The current study goes beyond the mathematical domain and interrogates the same data. Its aim is to investigate features of young children's meaning making and the breadth and nature of the literacies they use to communicate at home and in their imaginary play.

In this study we address the following research questions*:*

1. *What is the relationship between the children's cultural knowledge of literacy practices at home and their literacy events in the nursery?*
2. *What is the extent of children's literacies in pretend play?*
3. *Which features of the children's texts contribute to their play and understanding?*

Theoretical Framework

Rationale for Research

Hall (e.g., 1991), Meek (1991) and Wells (2003) identified the relationship between children's cultural knowledge of home literacy practices and their educational settings. This study draws on cultural-historical and multimodal, social semiotic theories: these different research traditions are underpinned by Vygotskian theory, informing the design and analytical approach employed. These research traditions support various aspects of the study including the home cultural knowledge the children bring to their nursery. The ways in which the children make meanings

[32] Both teachers and early years' practitioners work together in the nursery school, but for the purposes of this article, the word "teacher" is used throughout.

through symbolic tools, and their use of their literacies to communicate meanings in the social context of their play confirm the significance of children's spontaneous pretend play.

The headteacher of this nursery school emphasises the intellectual over academic and children's meaning making over "school-readiness". Continuing professional development [CPD] is promoted through research, and almost all staff members are involved in research projects. All the teachers are studying for Master's degrees and the headteacher is engaged in doctoral research. Of the two teachers involved in this study, Hugo was an experienced teacher and Emma was in her third year of teaching.

The nursery's culture places considerable value on children's self-initiated play and literacies, and there is no formal adult-directed writing or teaching of synthetic phonics[33]. Teachers provide a rich literate environment in which all aspects of play and graphicacy are encouraged and enriched through rich dialogue and numerous displays, and a wealth of graphical resources is available indoors and out (e.g., pens, chalks, papers, whiteboards).

Literacy as a Social Practice

This study conceives of literacy as an everyday social-communicative practice. Moreover, Gutiérrez and Rogoff (2003) explain that cultural ways of knowing show how children's experiences and interests "may prepare them for knowing how to engage in particular forms of language and literacy activities" (p. 23). Street et al. (2008) stress that literacy *practices* focus "on the social models of literacy that participants bring to bear upon those events and that give meaning to them […] to *link* them to something broader of a cultural and social kind" (p. 19, emphasis in the original). Literacy *events* refer to incidents in which children use literacies as an integral part of their interactions (Heath, 1982). Literacy events also include communicative acts that further the play narrative. Many children appear to communicate their ideas through graphicacy almost as frequently as they use speech, and children are the main players in these play episodes although occasionally adults are involved. Their play provides instances of embedded uses of literacies, imitated or adapted from home literacy practices, for example when shopping with their daughters, the mothers of

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[33] From 1975 the government in England introduced mandatory teaching of "synthetic phonics" to primary schools. This highly structured programme has since crept into many nursery schools and pre-schools in England. (https://www.gov.uk/government/publications/phonics-teaching-materials-core-criteria--and-self-assessment/validation-of-synthetic-phonics-programmes-supporting-documentation).

two of the children in the study regularly made shopping lists (a literacy *practice*): when the same children played pretend shops, they often made their own shopping list (a literacy *event*). In our view, all purposeful use of signs (including both the production and interpretation), are taken as "literacy".

Children learn from other people in the process of communication within culturally developed forms of activity in accordance with cultural-historical values (Valsiner, 1987; Wertsch, 1985). Vygotsky (1978) placed emphasis on the importance of culture and the way in which social contexts and cultural tools mediate learning, resulting in learning that is active, collaborative and emergent. Moreover, in the context of children's participation in cultural and social contexts, Rogoff (2008) employs the metaphor of *apprenticeship*. This stance reflects the work of Lave and Wenger (1991) who see apprenticeship learning as *situated* within a cultural community. In the current study children participate in their home cultural knowledge and, where favourable opportunities exist in their educational settings, they draw on their cultural understandings (Worthington & van Oers, 2016). Cultural-historical theory therefore underscores the importance of meaningful and shared social engagement in cultural practices. In early childhood educational settings, a potentially rich context for children's engagement in literacy events is pretend play.

Pretend Play

Many children engage frequently in pretence through talk, action and symbolic artefacts (as in Vygotsky's example of a child using a stick to signify a horse): this ability to substitute one meaning for another underpins symbolic representation through graphical marks, signs and symbols[34]. Vygotsky (1978) regarded pretence as highly significant for young children's development: cultural learning and the development of psychological tools within social pretend play also "furnish opportunities for the development of everyday concepts […] a "bridge" between spontaneous and scientific concepts" (p. 238). Hence, pretend play is a powerful context for the emergence of graphical signs that underpin development of symbolic activities such as literacy, mathematics and music.

[34] For the purposes of this research, *sign* is used to refer to graphical signs such as crosses, ticks, drawings, writing-like wavy lines, letter- and numeral-like signs. *Symbol* is used to refer to standard alphabetic letters and numerals. I use the terms *graphical signs* (or *graphical inscriptions*) as generic terms to refer to both signs and symbols (footnote in slightly adapted form).

Harris (2000) observes that children take a great deal of their conceptual knowledge to their pretend play (p. 8). Communication through talk and graphical representation helps further the development of the play narrative, not as adult planned tasks to fulfil curriculum goals, but as improved participation in meaningful communicative cultural practices.

The starting point should be to view play from the children's perspective (Rogers, 2010b), although research has shown that whilst many early childhood teachers make provision for pretence it is nowadays still largely adult-planned. The extent to which rich pretend play is realised in modern early years' practice is disputed (Brooker, 2010a; Rogers, 2010a; 2010b). Opportunities for genuine, *child-initiated* pretend play appears to be limited, a concern that has been highlighted elsewhere (e.g., Brooker, 2011; Parker-Rees, 1999, p. 61; cited in Rogers & Evans, 2008). Adult-planned play restricts children's genuine connections with their personal "funds of knowledge" (Moll et al., 1992). Worthington and van Oers (2016) highlight the practice common in most of the world:

> where adults choose, plan and resource themed role-play areas, revealing *adults' perceptions* of children's interests, rather than children's authentic and immediate interests that have personal cultural meaning. In contrast when children initiate and freely shape their play, their authentic and immediate interests that have personal cultural meaning. (p. 52, emphasis in the original)

Hall (1991) advocated the provision of rich literacy resources for pretence; however, whilst Hall and Robinson's 1995 publication includes some examples of children's spontaneous writing (albeit in adult-planned and resourced play areas), one teacher's creation of a garage - planned with the intention of stimulating contextualised writing - suggests that that she had specific curricula goals in mind. Hall and Robinson refer to this as *writing in association with play* (p. 70).

In line with these reflections, we identified as pretend play in our studies, all play activities of children that focused on imaginary or "possible worlds" in which the children freely imitated and explored cultural practices they had experienced. Integral to this play are features of children's meaning making. It is argued that in early childhood settings joint pretend play provides a potentially rich context for meaningful social participation, and, since the study focuses on children's communications, only social play episodes involving two or more children playing together were categorised.

This research focuses on children's communicative means for specific communicative purposes in pretence, embracing the diverse ways in which children make meanings: they invent new means of communication in situations that make sense to them and provide the freedom to do so.

Other researchers have identified this inter-relationship between pretend play and literacy, (e.g., Barrs, 1988; Hall, 1991; Isenberg & Jacob, 1983; Klenk, 2001; Kress, 1997; Meek, 1991; Pellegrini, 1980 and Vygotsky, 1978). Multiple ways of meaning making are increasingly referred to as *multimodal*. It was evident from the data that the children drew on multimodal ways of representing and communicating their meanings, and the increasing body of research into multimodality informs analysis of the set of visual data in this study.

Multimodality

The children's literacies are analysed from a multimodal, social semiotic perspective. Research by Kress and others (e.g., Jewitt & Kress, 2003; Pahl, 1999) encourages us to "re-think children's paths into writing" (Kress, 1997, p. xviii) and challenges conventional conceptions of literacy. Kress emphasises that whereas literacy as a social practice focuses on collaborative acts, multimodality attempts to understand the tools people use as they engage in joint social actions (2006). He proposes that we "cannot understand how children find their way into print unless we understand the principles of their meaning making" (p. xvii). A traditional, cognitive psychological approach to literacy learning normalises progress, matching it to an agreed developmental pathway for all children (Larson & Marsh, 2014, p. 4). In contrast, for Kress (1997) multimodality presents a more realistic view of young children's emerging literacies, treating *"individual speakers or writers not as language users but as language makers"* (p. xvi, emphasis in the original).

Young children's language making is essentially a multimodal process. *Modes* (forms of communication), *materiality* and *affordances* are significant aspects of multimodality. According to Bezemer and Kress (2008) a mode is "a socially and culturally shaped resource for meaning making" (p. 171). *Modes* (e.g., found and made artefacts, speech, drawings) are used to express particular meanings, doing "different kinds of semiotic work with different *affordances* – potentials and constraints for making meaning" (Bezemer & Kress, 2008, p. 171). *Materiality* refers to the "stuff" chosen to make particular meanings (for example, paper, plastic, sticks, pens).

Characterisation of the Study

In this study *literacies* refers to the marks and signs children choose to use: they are also all referred to as *graphics*[35] (Carruthers & Worthington,

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[35] Emig (1977) first used the term *graphics* in the context of symbol systems such as writing and mathematics.

2006), the term *graphical communication* emphasising the children's communicative intention in creating a text. The term *visual data* is also used to refer to the children's graphics in this study.

The observations of the case study children's play were analysed to uncover some processes in their ways of participation in literacies, especially to articulate how they develop meaningful communicative texts and how their understandings grow. Data collected through case studies as part of a larger, ethnographic study are interrogated from several perspectives, taking an interpretive stance combining social semiotic and discursive analyses.

In order to ensure that this stance is sufficiently rigorous and to support systematic data analysis of the written observations, *computer assisted qualitative data analysis* software "ATLAS-ti" was employed. Data were coded to identify evidence of *pretend play episodes* combined with *graphicacy*. Additional coding enabled the children's marks, signs and symbols to be located in the transcripts, and those occasions when a child referred to the meaning of their graphics to be identified. Transcripts were also coded for the children's different writing genres.

Although concerns have sometimes been raised regarding the generalisability of case studies (Anderson, 1990; Bogdan & Biklen, 1982), Simons (1996, as cited in Marsh, 1999), argues that they can provide "glimpses into wider socio-cultural patterns" (p. 130). Early and Cummins (2011) argue that case studies provide creditable data documenting authentic instances of practice that allow hypotheses to be formed and tested, thereby contributing to theory. Case studies "speak directly" to both theory and practice, and contribute to the knowledge base relating to literacy and education […] every bit as credibly as any quantitative research" (p. 19). The multiple case studies in this study also work together, highlighting similarities and differences in the children's play and literacies.

In the early stages of this study ethnographic research was identified as being particularly appropriate, firstly since it is an invaluable method to investigate children's cultural knowledge and practices in naturalistic settings, and secondly since it has been widely and successfully used in studies of literacies and multimodality. According to Griffin and Bengry-Howell (2008) ethnographic research is concerned with understanding people's cultural and symbolic behaviours within specific contexts (2008). For achieving this, we follow Geertz's (1973) idea that "ethnography is thick description" (p. 217) in order to allow the voices, emotions, behaviours and meanings "of interacting individuals are heard" (Denzin, 2001, p. 100). The diverse data and the different voices of the children, their parents and teachers contributed to this thick description.

Embedded in the children's literacy events are *artefactual literacies* (Pahl & Rowsell, 2014), also referred to as *literacy objects* by Neuman and Roskos (1992). From a Vygotskian perspective these *artefactual literacies* are also material cultural tools. According to Pahl and Rowsell (in Larson & March, 2014): Artefacts embody people, stories, thoughts, communities, identities, and experiences… They are valued or made by meaning makers and in a particular context […] They enable a different kind of learning, one that is located, drawing on personal and collective stories and heritage, and re-position learners as experts in the field of their own objects. (p. 99)

In this study the children appeared to understand *texts* as "things", "as objects with a history and as a material presence" (Pahl & Rowsell, 2005, p. 27), their texts also serving as objects within their play.

Methodology

Research Setting and Participants

The nursery school participating in this study is situated in a large city in southwest England: 60 children attend each morning and afternoon session and individual key people each lead a group of children. The nursery's culture and ethos support open ways of learning and teaching. Rather than anchoring practice in narrow curriculum goals, teachers encourage children to initiate ideas and support their complex thinking through rich dialogue. Pretend play in this nursery is neither themed in advance nor planned by adults and is a popular choice for many of the children, and graphical communication has a high profile.

The teachers value and support children's impromptu pretence and imagination, often staying close to the location of their play and sometimes drawn into children's dialogues. They frequently bring in resources to support the children's interests, enabling children to extend and deepen their ideas and narratives. Whereas some researchers focus on the provision of literacy resources relevant to adult-planned pretence (e.g., Bradford, 2015; Bröstrom, 1997; Cook, 2006), children in this nursery locate resources when they need from those readily available. The teachers collaborating in this study were asked to identify several children whom they knew - from their knowledge of the children and previously documented observations – to often choose to use graphics. They identified Isaac and Shereen, (both 4 years of age) and Elizabeth, (3 years, 6 months). These children will be referred to as focal children in the following. To determine if previous interest and experience in graphicacy were significant, the teachers randomly selected four additional children (Oliver, David, Ayaan and Tiyanni). The children were all in their final year at nursery, and at the onset of the academic year their ages ranged from 3 years, 2 months to 4 years of age.

Shereen's family comes from the Philippines and she speaks English fluently. Ayaan's family is from Somalia: Ayaan speaks Somali fluently and when she arrived in the nursery, she was unsure about communicating in English. Ayaan's teacher observed that the pretend play in which Ayaan increasingly participated encouraged friendships to develop, boosting her confidence in spoken English. The remaining children's first language is English. Tiyanni's family is from the West Indies. Several of the families live within walking distance of the nursery school, but Elizabeth, Isaac, Shereen and Tiyanni live at a distance, their parents explicitly choosing this particular nursery school. Elizabeth had attended the longest, first attending as a baby and providing long term familiarly with the culture and routines of the nursery.

Data drawn from my visits to the children's homes and informal discussions with their teachers, showed that their home cultural experiences included many everyday experiences rich in literacy practices.

Ethics

With the described measures and provisions, we endeavoured to conscientiously abide by the ethical regulations of the British Educational Research Association [BERA] and the Vrije Universiteit's *Ethical Review Regulations,* (Vaste Commissie voor Wetenschap en Ethiek [Scientific and Ethical Review Board, 2016]), and participants were consulted and informed at every stage. In order to sanction the ethical quality of the research, permissions were sought to gather data from all those who would be involved. The headteacher and teachers gave their consent for data to be gathered in the nursery. The parents were consulted at the onset of the research and their informed consent sought in writing to observe their child and collect data. The research was explained to the children using everyday language and their agreement sought. Since some of the children's graphics would be published, this was explained to both the children and their parents, and their consent given: the headteacher and teachers also gave their permission for their future publication. The parents were informed that they could withdraw their child from the research at any point, and one family did so early in the period of data collection: none of the data pertaining to this child have been used.

With the consent of the parent present and the child, several photographs were taken of each child playing or drawing at home, and short written notes made of information provided by the parent regarding their child's play and graphicacy at home, also with the parent's consent.

Data Sources

In order to answer our research questions, several types of qualitative data were collected: visual data comprising children's writing, drawings, maps and plans children use to play out pretend scenarios, and *Children's Mathematical Graphics* (CMG), (all referred to as literacies or *graphics* here)[36]; field notes I made; transcripts of informal discussions with parents and photographs of children engaged in play and literacy events.

The teachers' written documentation in the children's "learning diaries" complement the graphics and include children's talk and behaviours. I also documented the children's play and graphicacy during regular visits to the nursery school, through children's home scrapbooks (that I provided), and during visits to the children's homes. To ensure maximum validity the visual data is analysed in conjunction with their accompanying written documentations. The children's use of home scrapbooks varied, both by their frequency of use and the range of their graphics. The data from the scrapbooks were therefore less reliable, although they did reflect some of the range of children's choices and interests relating to their graphicacy at home.

Procedure

Informal home visits are an accepted practice in this nursery and familiar to all parents, so that expectations were already established. I approached the parents of each child during the summer term (by which time the parents had come to know me), to ask if I might make a visit at a time that suited them. Visits were of 30-45 minutes duration and entailed informal discussion with the child and parent: they focused on the child's interests and what their child played and activities in which they engaged at home, individually and with siblings and parents.

The teachers make daily, written documentation of children's learning, observing and photographing children's play and collecting children's graphics as part of their normal practice. Green and Hogan (2005) propose that "observing children engaging, in as natural way as can be arranged, in the types of activities that would be a typical part of their everyday lives, is surely a way for those children to be participants in the study rather than passive objects of research" (p. 115). The teachers' rich written observations provided the main written data of the children's play. They write open and unstructured accounts whilst observing children and include

[36] Whilst the children's *Mathematical Graphics* are understood as a literacy, they are not the main focus of this chapter.

as much detail as possible, focusing not only on an individual child but also on the context of the play and all the children involved, and document the behaviours, talk, actions and artefacts they used or made. In addition to the teachers' documentation, I made some additional observations: these were checked and discussed with the teachers to ensure that they were of a similar nature to those they routinely made and that they fairly represented the observed play. In order to eliminate bias, we avoided selecting only some written observations for analysis, and included documentation of *all* pretence during the year as data for analysis.

Data Analysis

In the current study, computer assisted qualitative data analysis software was used to support systematic data analysis of the written observations. The data codes were open and derived from the field. For the purposes of this chapter transcripts of each observation were coded to identify:

1) Children's engagement with literacies at home;
2) The extent of children's literacy events within their pretend play in the nursery. The rationale for quantifying these events, was to support identification of the frequency of the children's freely chosen literacies across all play episodes.

The visual data enabled aspects of the children's texts to be analysed in order to determine:

3) Multimodal ways in which the children communicated their meanings;
4) Specific semiotic features of the children's literacies.

For both the transcripts of written observations and graphics, features coded were then quantified and are presented in Table 1 and in the findings of question 2b. To strengthen reliability and validity an additional researcher independently coded 10 percent of randomly chosen examples. Agreement was reached for 90 percent of the codes assigned, a significant level of consensus.

Elaboration of the Research Questions

RESEARCH QUESTION 1*: What is the relationship between the children's cultural knowledge of literacy practices at home and their literacy events in the nursery?*

Interpretive analysis of the children's home scrapbooks, transcripts of informal discussions with parents and field notes made during home visits provided additional evidence, helping identify:

a. The children's cultural knowledge of literacies at home, *showing family members' literacy practices, and the self-initiated literacies in which the children engaged*;
b. The children's literacy events within their sustained pretend play, showing their *relationship with home literacy practices and events.*

RESEARCH QUESTION 2: *What is the extent of children's literacies in pretend play?*

Analysis of the written documentation was coded with the support of "ATLAS-ti" software and identified;

a. The number of pretend play episodes for each child - *coding each child's play episodes;*
b. Pretend play episodes in which children engaged in literacy events – *coding all play episodes that included evidence of children's use of graphical marks, symbols and texts.*

RESEARCH QUESTION 3: *Which features of the children's texts contribute to their play and understanding?*

Features of the *children's play* were investigated in conjunction with the children's graphics and the teachers' written documentation, highlighting,

a. Multimodal ways in which the children communicated their meanings, *modes, materiality* and *affordances*;
b. Specific semiotic features of the children's literacies: graphical marks, symbols, signs and their *affordances.*

Results

In the following we discuss the collected data in relation to the different research questions.

RESEARCH QUESTION 1: *What is the relationship between the children's cultural knowledge of literacy practices and their literacy events at home; and their literacy events in the nursery school?*

a. Children's engagement with literacies, showing family members' literacy practices, and the children's literacy events at home

The extent to which a parent (sometimes with their child) engaged in literacy practices for authentic purposes appears to be especially valuable, clearly demonstrating the role and power of literacies in ways that make personal sense. For example, one of Ayaan's older brothers was learning Arabic, and at Ayaan's request her aunt had started to teach her Arabic using a large blackboard. Other experiences originated from a parent's work: for example, from a very early age Isaac had been involved in his father's work, first as a builder and then managing a micro-brewery. Oliver's father often worked on his computer at home and Tiyanni saw her mother studying. The richness and diversity of these 'funds of knowledge' (Moll et al., 1992) provided meaningful models of contextual literacy practices and communications, models on which the children drew, imitated and expanded in their pretend play.

The children also initiated their own literacy events at home, as when Elizabeth made an invitation for her birthday party. Isaac's played pretend "ice-cream vans" with his dad (in his father's real van): his play included several literacy events using written symbols, including a large "M" for "McDonald's" and a double-headed arrow "to show where to go". A large map Isaac had drawn was stuck on the wall of the stairs in his house: Isaac's interest grew from several old maps his father displayed on the walls at home, and together they often consulted contemporary maps when going out in the city or on a longer journey. Elizabeth's birthday invitation, Isaac's "ice cream van" and his map were themselves "texts" full of meanings (Pahl, 1999; Pahl and Rowsell, 2014). These examples show how some of the children's play narratives at home were embedded in meaningful contexts that required literate communications.

Most of the children freely engaged in some graphical activity (visible in their scrapbooks) at home, for example Shereen and Elizabeth wrote and drew individual letters, numerals and a wide range of other symbols such as arrows, crosses and ticks, smiley faces, spirals, circles, hearts and stars, sometimes including them as graphical elements in their drawings. Tiyanni also enjoyed drawing, also experimenting with graphic symbols and letters. Oliver's mother encouraged him to copy writing his name and he enjoyed drawing people, rockets and characters from television. David's extensive engagement with literacies in his play suggests home influences[37].

[37] For personal family reasons it was not possible to visit David at home for this research, or to discuss his graphics with his family.

b. Children's literacy events within pretend play in the nursery

Analysis of the data revealed the significant relationship between the literacy practices the children experienced at home and their literacy events within their pretend play at nursery. The following transcriptions exemplify the children's cultural and literate knowledge evident in every narrative and text.

Pretend Play Narrative 1: Car Park Entry

Isaac and his dad had recently swiped a plastic card through an electronic card-reader in a city car park. Isaac connected this concept with the business cards his dad used for work: Isaac's own interest in technologies, electricity and security measures, and his considerable knowledge of environmental signage were also evident in this and many other play narratives. In the nursery the following illustrative events occurred:

> Rapidly drawing marks on a sticker, Isaac announced, *"you need to have a business card to get in here. I'm fixing the gate so it has electric. You have to have a business card to swipe. I don't need one - I use my hands."* Isaac gave a piece of paper to Oliver, *"Here's your business card",* then writing more marks on a label announced*, "this says "swipe here with your special code card."*
>
> As Oliver swiped his card, Isaac noticed another child enter without one. He stuck a smaller sticker with scribble-marks on the fence, *"this is the bell if you don't have a sticker, someone can let you in. It says, "press here". Someone will come and open the gate."* He added a third sticker in the centre of the gate, *"This is for lorries and deliveries - it opens automatically - it's a camera."*
>
> Oliver quietly listened and observed before deciding to participate, drawing dots followed by several ticks, explaining: *"these are ticks. When there are three ticks you can go, when there are two you can't go that way. I've made two ticks - that means you are not allowed. People allowed in that way."*
>
> Oliver wrote his name, *"O, L, I",* then wrote *"E" for Ellie (his sister)* and *"D"* for Daddy, before attaching them to the fence, appreciating the power of signs when another boy followed his verbal instruction and sign by walking where directed.

Isaac's texts drew Oliver's interest and the complexity of their signage evolved through their joint play. The boys used graphics intentionally for authentic and contextualized purposes, Isaac using scribble-marks as shorthand for the information he wished to communicate. Oliver used a combination of dots, scribble-marks, several letter-like signs and ticks to convey the meaning of his car park sign. The term “scribble-marks” refers to marks which adults would find difficult to interpret without the child’s explanation, and is used by Carruthers and Worthington in the context of CMG (e.g., 2006). Others also use the term “scribble” when referring to children’s early marks for writing (e.g., Baghban, 2007; Department for Education and Skills [DfES], 2008; Kaderavek et al., 2009; Klenk, 2001). The term *scribble-marks* is also used in the context of early drawings, (e.g., Pinto et al., 2011). In this chapter this term is differentiated from the wavy or zigzag lines that some children used to signify writing.

Confusingly the term “mark-making” (though seldom defined), is increasingly used as a generic term to encompass all of young children’s earliest scribble-marks, signs, symbols and drawings (e.g., DfES, 2008). Carruthers and Worthington (2005) chose instead “graphics” to embrace the full range of marks, signs and visual texts.

The *modes* used and their *affordances* met the rapidly changing pace of the children’s play, for example Isaac’s placed his marks centrally on his stickers, perhaps for impact and certainly for brevity. Accompanying their sign making with dialogue enabled the children to make their meanings transparent to their co-players. Machón (2013) emphasises, “Oral language… more developed than drawing in children of these ages, comes to the aid of graphic language to create a mixed language that results from the combination of both symbolic systems” (p. 258). Each textual artefact had its own role to play in this episode; for example, a card to gain entry and an imagined bell with written instructions “press here.”

Pretend Play Narrative 2: Playing Cafés

Buying, preparing, ordering and eating food together have significant roles in Shereen’s family. Drawing on her personal and social knowledge of cafés, Shereen approached her friends for orders, creating wavy lines, and a drawing of a mushroom in response:

> After a while she returned to ask her teacher Emma, *“what you want: rice, chocolate, cake, chicken?”* Emma said she didn’t want chicken and Shereen wrote a mark for “chicken” and drew a cross by it, clarifying, *“It says “x” - no chicken*.” Later Emma said she would have chicken, but pointing to the “x” she had written, Shereen said *“Look! No chicken! You want mushroom*?” Then pointing to her drawing of a mushroom explained*, “Look. A tick, that mean we got some.”*

Figure 3.1

"Look. A tick, that mean we got some."

Shereen approached her customers with a clipboard, ready to take orders as a waitress would: her written orders became an aide-memoire, enabling her to revisit them as she discussed menu options. Figure 3.1 highlights Shereen's expanding repertoire of graphical symbols. Kress (1997) emphasises "signs arise out of our *interest* at a given moment […] This interest is always complex and has physiological, psychological, emotional, cultural and social origins" (p. 11, emphasis in the original). It appears also that the children were also aware of a *need* to communicate to maintain the integrity of their play narrative, reflecting Vygotsky's (1978) concern that writing (and by analogy, all literacies):

> must be "relevant for life", [they] should be meaningful for children, that an intrinsic need should be aroused in them, and that writing should be incorporated into a task that is necessary and relevant for life. In the same way as children learn to speak, they should be able to learn to read and write […] Reading and writing should be necessary for her in her play. (p. 118)

In both play narratives the children's cultural knowledge of home and community permeated their pretence in a functional way, melding their lived experiences and cultural knowledge with imagined possibilities and underscoring that meaning is the foundation of semiotic behaviours. The children chose to be involved in emulations of everyday practices that required communication. They recontextualised literate genres and texts (e.g., business cards, car park signs, menus), exemplifying "the lifting of particular genres, or texts from one context, to another" (Pahl & Rowsell, 2006, p. 6), in Dyson's (2003) words, "re-mixing" and "energizing the children's appropriations of cultural texts" (p. 25) for their play. The seven children's literacies revealed many other genres including persuasive letters, cheques, registers, bookings for a campsite, "open" and "closed" signs, maps, plans, shopping lists and receipts.

RESEARCH QUESTION 2: *What is the extent of children's literacy events in play?*

a. Pretend play episodes identified for each child

Table 1 shows individual children's engagement in pretend play and the percentage of episodes in which individuals engaged in literacy events.

Table 1

The relationship between the number of pretend play episodes and children's engagement in literacy events (the focal children are marked by *).

	Total number of pretend play episodes observed (n=146)	Percentage of play episodes in which children engaged in literacy events (n=64)
Shereen*	12	91.6%
David	29	65.5%
Ayaan	16	43.7%
Isaac*	51	35.2%
Oliver	22	27.2%
Tiyanni	8	25.0%
Elizabeth*	8	12.5%

Note: This table represents all literacy events explored in the children's pretend play, including children's mathematical signs and texts.

A total of 146 episodes of pretend play were recorded over three terms, with variation in the number of episodes for each child, from 51 episodes for Isaac to eight for both Elizabeth and Tiyanni. Such variability between the quantities of pretend play episodes for each child was unexpected and could not have been predicted prior to the beginning of the study. However, since four of the children were randomly selected, these differences appear unlikely to have distorted the findings.

Pretend Play Episodes in which Children Engaged in Literacies
The data showed that pretend play often provided contexts in which the children freely chose to communicate through literacy events. Almost 44 percent of all pretend play episodes included children's literacy events, increasing during the year (five during the first term, 27 in the second term and 32 in the third term). This is an interesting finding; particularly since neither their pretend play themes nor their literacy events were adult-planned or resourced.

Of the focal children, Shereen engaged in literacy in all but one of her pretend play episodes, and Isaac in 35 percent of his. Whilst Isaac showed considerable cultural and social use of literacies, much of his pretence occurred in the large outside sand pit and involved talk, actions and other artefacts rather than graphical communications. Elizabeth seldom engaged in pretence during the year and was observed to communicate through graphicacy in only one play episode. However, according to her teacher and documented observations, Elizabeth frequently used literacies in other child-initiated contexts, her understanding and use of signs showing considerable maturity. The data reveal that no remarkable differences appear between the focal children and the randomly chosen children. David and Ayaan in particular (non-focal children) made extensive use of literacies in their pretend play.

RESEARCH QUESTION 3: *Which features of the children's texts contribute to their play and understanding?*

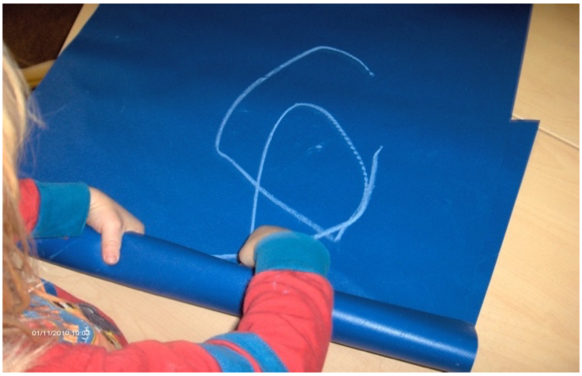
a. Multimodal ways in which the children communicated their meanings, *modes, materiality* and *affordances* were investigated: (in conjunction with the teachers' documentation) highlighting;
b. Specific semiotic features of the children's literacies: graphical marks, symbols, signs and their *affordances.*

A number of multimodal features were evident in the children's texts. For young children, writing includes the *choice* of marks and signs used (e.g., scribble-marks, letters), their size and arrangement within a frame. The children in this study placed most of their written texts centrally on their writing surface. Exceptions included curved or zigzag lines (to represent writing) and occasions when children wrote their names: Shereen was the only child who wrote letters in a linear, left-to-right arrangement (writing a shopping list) in a column arrangement, rather than placing them centrally on her notepad.

The children created their maps using spatial arrangements of lines, and in the process, recounted highly complex narratives. Their maps were made on *large* sheets of paper: in the case of maps this afforded the boys space to physically construct the extensive networks of roads. In their "registers" some children used both random arrangements of symbols and marks, whilst others inclined towards a linear (left-right), column arrangement. When used alone, single symbols (e.g., on paper, in the sand, in snow) tended to be larger than those they used within texts, the size of the symbols, the children's spoken words accompanying them emphasising their importance. Figure 3.2 shows the marks Isaac used for his "building plan" appeared very similar to those he used in the letter he wrote to Oliver; however, for his map Isaac selected a large sheet of blue plastic that he could readily roll, as he had seen his father a builder, do, whereas he wrote his letter on a sheet of A4 paper.

Figure 3.2

Isaac's rolled "building plan"



In terms of *materiality* the children chose a range of surfaces, including sand, paper (colour and size), plastic, old diaries, envelopes and stickers, sometimes using clipboards or child-height whiteboards. Large diaries may have been chosen to lend gravitas and emphasise an adult's activity, whilst stickers could readily be stuck to many surfaces. When pens were not to hand (e.g., in the sand) the children used sticks or a spade. The *affordances* of both the chosen modes and materials arose from the children's knowledge of "what works best" combined with what was readily available.

b. Multimodal, semiotic features of the children's literacies: marks, symbols and signs and their affordances

Analysis of the children's texts revealed a range of specific, multimodal marks, symbols and signs. Whilst there was variation between individual children's use of literacies to communicate within their pretence, there were no significant differences between the focal and non-focal children's use.

Scribble-marks. A little over 20 percent of the children's texts included predominately these marks. Whereas some children used graphic symbols, letters or drawings in *other* child-initiated contexts during this period, in their pretence they appeared to sometimes use scribble-marks as "placeholders" to denote specific meanings, suggesting that such rapidly made marks allow the course of play to proceed uninterrupted.

Interpretations of Children's Graphical Symbols

Machón (2013) emphasises that between the ages of 3 and 4 years, children begin to use distinct graphic symbols, concurring with the findings of this research. Vygotsky (1978) identified children's ability to substitute the meaning of an artefact with an alternative in pretence: this ability unpins symbolic languages where graphic symbols carry abstract meanings unrelated to their shape or form. For Machón "wavy lines, loops, zigzags circles, squares, crosses, grids radials […] give rise to equivalents [that] are at a midpoint between graphic symbols and writing signs" (p. 322); children develop "a graphic vocabulary" (p. 430) that span representational systems. According to Machón this period of experimentation and expansion of graphic symbols "*is undoubtedly the most important in the entire graphic development*" (p. 95, emphasis added).

Crosses are one of a number of abstract symbols children choose to use to signify both similar and different meanings (Carruthers & Worthington, 2006; Worthington, 2009; Magnusson & Pramling, 2011). For example, David drew a cross in the sand, explaining "the bumblebee died here", later writing crosses on paper, saying, "No more children getting in our car". In addition to crosses (the most frequently used graphical symbol in this study), the children used ticks, arrows, circles, vertical lines and zigzags within their play. These symbols show the "multifunctional nature of signs" (Valsiner, 2001, p. 92), described by Werner and Kaplan (1963) as *multireferentiality* or *plurisignificance* (p. 216). This kind of graphical symbol appeared in just over 34 percent of all cases in this study.

Signs for Symbolic Languages

Children use symbols to signify a likeness to something (a *denotative* function), as in historically early pictographic language systems. For example, David "read" his diagonal, zigzagging marks as writing, "I'm going shopping to get sausages, beans and peas." Shereen used a single wavy line to signify "shop open" followed by two similar lines (one above the other), commenting "shop closed". Machón determined that such "writing-scribbles" begin around the age of 3 years when children notice and imitate adults' linear arrangements of writing that appear similar to their own scribbles.

In the current study some children also used letter-like signs and standard letters they knew (a little over 32 percent of the total). For example, while playing "builders" Isaac wrote zigzag lines on a play cheque book, followed by a letter-like "a" announcing, "a cheque for £500.00, for all the jobs I've done at my house." When the totals for *graphic symbols* and *signs for symbolic languages* are combined, they show that the children included one or more of these in just over 67 percent of their texts.

Drawings

Drawings were the least chosen graphics the children used in their play. For example, Ayaan told Zalluyah, "My baby need TV" and taking a piece of paper wrote a capital "A" and drew a grid of intersecting lines, then fixed her "television" to the wall. As she did so, Ayaan again explained "my baby need TV" and taking a strip of raffle tickets, carefully placed the doll in a chair facing her "television" and pressed the numbered raffle tickets (as a remote control) to turn it on. On the same occasion Ayaan also drew ice creams, pretending to lift them from the paper. Isaac drew a "builder's plan" with identifiable features and the boys drew several maps. Drawing in pretence appeared to be reserved for *objects*, whereas writing in all its modes was used to explain or recount information. Drawing is *showing*, whereas writing is *telling* and more often used to communicate within pretend play.

Combining Modes

As previous research (Carruthers & Worthington, 2006) has shown, children's use of *code switching* in graphical texts is significant, enabling them to select the most appropriate symbols for their immediate purpose. Whilst several children used one or two *different* marks or signs within one text, Figure 3.1 is notable in its inclusion of drawings (a fish and a mushroom) in association with other marks, symbols and signs. Shereen appeared to know that the meanings of crosses and ticks are unambiguous, and the different modal signs she chose to use each offered very particular *affordances*. Their texts revealed many different genres including persuasive letters, registers, bookings for a campsite, "open" and "closed" signs, maps and receipts.

Limitations

The range of children's graphical signs, symbols, genres and textual artefacts accord with those found in previous studies by Carruthers and Worthington of children of the same age (e.g., 2006; 2011; Worthington, 2009), however, in this study, the small number of case study children may limit generalisations. In defence of case studies Early and Cummins (2011) assert that they "are the mainstream of scientific inquiry." The implications for policy are equally direct […] The logic […] underlying case studies] can be simply stated: actuality implies possibility […] [Moreover] case studies have immense power to effect change both in the instructional choices made by teachers […] policy makers [… and] students" (p. 19).

Several factors could contribute to the variation in the findings. The seven children had attended the nursery school for differing amounts of time, from 3 years for one child, to another who had just begun to attend at the onset of data collection: these differences may have impacted on individual's level of confidence and familiarity with the nursery's culture. Selection effects (for both focal and randomly selected children) include personal characteristics of the children and of the teachers involved in the study. Variation in the seven families' home cultures and backgrounds are likely to have also contributed to differences in individual children's experiences of home literacy practices and opportunities: these differences were not possible to predict or to select for prior to the onset of the study. Due to the nature of this study, children who did not readily choose to engage in graphical communication were not included: this important aspect will be worth investigating in the future.

Discussion and Implications for Practice

The aim of this study was to reveal features of young children's meaning making and literacies within their pretend play. The findings contribute to theories and practices concerning pretend play and emerging literacies, extending understanding of the creativity and capabilities of young children. In contrast, Roberts-Holmes (2015) emphasises that current pedagogy in England,

> has increasingly narrowed to ensuring that children succeed within specific testing regimes which interpret literacy and numeracy in very particular ways […] the impact of such school-based testing

regimes has the potential to subvert the early years from being a unique child-centred and play-based educational stage in its own right to that of subserviently preparing children for school. (p. 303)

To answer the first question, the parents' use of literacies for authentic and contextual purposes clearly contributed to children's understandings of the role and purposes of literacies, although there appeared to be variation in the children's level of participation and apprenticeship in these situated literacy practices. All the children appeared to freely engage in self-initiated literacy events at home, although the amount they did so varied.

The three focal children often chose to explore meanings thorough play and graphical texts at home, Shereen and Isaac also engaging in rich literacy experiences within their pretence at nursery, whilst Elizabeth engaged in literacies in many other self-initiated contexts. The transcripts of two pretend play episodes included here exemplified the significance of the children's sustained narratives, their home cultural knowledge both framing their play and revealing their textual understandings, and, as the children explored and expanded their home cultural knowledge, clear links between both became evident.

Second, our findings show that many of the children's pretend play episodes included literacy events, with sustained play proving especially rich and complex: particularly noticeable was that the children so readily chose to communicate through their literacies. The children's communicative meaning making clearly arose from their experience and involvement in cultural and social literacy practices at home and their *interest* and perceived *need* to communicate to further their play. Shereen used graphic symbols in all but one of her play episodes, and, although there was variation in the extent to which individuals used graphics to communicate, no significant differences between the focal and randomly selected children were found.

Third, the children's use of different modalities and materials showed the extent of their growing understanding of the affordances of various artefacts and graphical signs. Particularly significant for this study is the children's growth of *graphic symbol and sign-use* within their play narratives, underpinning their emerging symbolic languages such as writing and mathematics. Machón (2013) emphasises that "graphic symbols are not merely a cultural product which children borrow" from their sociocultural environment, "but a genuine construction of theirs […] a highly personal mode of representation of the child" (p. 251). The period between 3-4 years of age appears to be a liminal one for children's literacies, children continuing to use their early marks, yet also experimenting with graphical symbols and signs as they move towards standard, abstract symbolic written languages.

Finally, the findings also reinforce the impact that high ratios of well-qualified staff have on the quality of children's play and literacies in early childhood education. Drawing on cultural-historical, ethnographic and social-semiotic, multimodal methodologies, reveals early meaning making and communication through literacies as a continuum that evolves through everyday practices within the interlinking cultures of family, community and nursery school. Understanding is exhibited not through interventions of special programmes or techniques, but through children's meanings and literacy events within naturalistic contexts.

Children bring their culture and experiences to nursery, but the opportunity for using them may be lacking, and whilst pretend play is not the only social context in early childhood education, it offers significant potential for children to explore and link their home culture to nursery school or school.

Carruthers and Worthington (2011) emphasise that young children invent and adapt from different symbol systems, using "visual signs and texts in incredibly powerful ways" (p. 42). The children's literacies and shared social interactions with peers and adults combine to mediate symbolic understanding. According to Vygotsky (1978), "In this context, we can use the term *higher* psychological function […] as referring to the combination of tool and sign use in psychological activity" (p. 55, emphasis in the original). Freely situating literacies within their pretence permits children to develop dynamic and reciprocal relationships, allowing existing cultural knowledge to flourish, and fulfilling Vygotsky's concern that writing should be "cultivated" rather than "imposed" (p. 118).

What are the Implications for Early Childhood Education?

The research findings challenge current views of a single "skills-based" literacy with its narrow emphasis on synthetic phonics in England and restricted views of pretence. Recommendations for literacies suggest a need to develop a multimodal stance; to appreciate young children's existing knowledge in all its complexity and to support the emergence of children's literacies in meaningful social contexts.

The findings show how spontaneous social pretend play can create a rich social-ecocultural niche in early childhood (Worthington, 2015), promoting the emergence of a variety of literacy events through social engagement in pretend play contexts. Acknowledged by a considerable body of research and highlighted in the current study, pretend play is an important aspect of the early years' curriculum that demands to be understood in more than words. The implications of the findings point to a need to elevate the status of play during this important phase of childhood.

This study underscores the importance of adults' understanding the significance of children's meaning making in all its guises, enabling them to link their new knowledge with their existing home cultural knowledge, and highlighting social pretend play as a context for making meanings and communicating through literacies. For adults working with young children, increased understanding of the value of free and spontaneous pretend play is likely to contribute to richer and sustained narratives. To achieve greater understanding of these important aspects it is recommended that policy documents and teachers' and practitioners' professional development be reassessed, so that both high quality, rich and sustained pretend play and literacies are fully understood by all engaged in early childhood education, so that their true potentials are valued.

The complexity, richness and diversity of the multimodal texts and literacies highlighted in this article also have implications for young children's emergent mathematical understandings and the beginnings of communication through their *Mathematical Graphics* in early childhood education settings (e.g., Carruthers & Worthington, 2011; 2006; 2005). The visual data collected for the larger, doctoral study of which this forms a part, includes examples of *Children's Mathematical Graphics* and will be interrogated in the future: one aspect of this will examine the interrelationship between the processes and contexts involved in the various literacies analysed here, and those young children use to communicate their mathematical thinking.

- 4 -

The Development of Mathematical Abstraction in the Nursery[38]

Abstract

The aim of this article is to document the types of signs that young children make to represent their mathematical thinking, and to determine the extent to which features of usage-based language acquisition are evident in children's early graphical communications made in mathematical contexts. Studies of young children's symbolic principles in ontogeny and research into the acquisition and development of language, provide insights into the rich foundational knowledge on which they build their early mathematical inscriptions. The study conceives of children's mathematical abstractions as emergent cognitive representations, originating in their need to communicate within personally meaningful contexts. The collected ethnographic data comprise mathematical inscriptions from seven children aged 3-4 years in their nursery school and written observations from their teachers and those I made. Analysis follows an interpretive, social-semiotic paradigm; the inscriptions analysed to show how they convey their emerging mathematical understandings, and how this supports their emergent abstractions. The findings illuminate children's strategies as they communicate their thinking, indicating the importance of symbolic number knowledge in acquiring the abstract graphical language of mathematics.

Introduction

An important aim of education is teaching children to communicate about their abstractions in a culturally agreed form. However, the formal representations of mathematical abstraction are widely recognised as challenging for children to achieve, Ginsburg (1977) identifying difficulties in mathematics as partly relating to their problems with written symbolism. Hiebert (1984) concurred, declaring that school mathematics:

> is much different from the intuitive and informal mathematics the children acquire […] many of the children's observed difficulties can be described as a failure to link the understandings they already

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have with the symbols and rules they are expected to learn. (p. 498/501)

In school, children are confronted with formal mathematical language without explanation of the point of view from which these can be seen as valid or consistent. Traditionally taught mathematics "becomes mainly a process of manipulation of numbers" (Nunes, 1993, p. 197-198), often lacking meaningful connections between children's personal cultural knowledge and understandings. Van Oers (2012a) warns that early imposition of abstract signs can cause alienation from school subjects (p. 137), resulting in children adopting superficial features, unable to transform them into a personally meaningful system (Ernest, 2005, p. 25). In contrast our study argues that an approach in which children can socially connect their existing cultural knowledge and integrate signs introduced by their teacher, contributes useful foundations for mathematics in school. Indeed, Morgan (2006) contends that from a social semiotic perspective, learners' thinking and meaning-making "is not simply set within a social context but actually arises through social involvement in exchanging meanings […] the relationship between the individual and the social compatible, Hodge and Kress (1988) argue, with the theories of Volosinov and Vygotsky" (p. 221).

Considerable changes in government policies in England (e.g., *Department for Education* [DfE], 2017) imply that children's inscriptions are unlikely to be recognised, or their meaning-making supported (Carruthers, 2015; Moffett & Eaton, 2018). Based on the belief that children's learning is always socially constructed, this omission makes it essential that this current study be conducted.

An extensive body of research exists into young children's emergent beginnings with writing (e.g., Clay, 1975; Kress, 1997), with some studies into emergent signs for mathematics (e.g., Brizuela, 2004; Carruthers & Worthington, 2005, 2006; Hughes, 1986; Tolchinsky, 2003). Munn (1994) upholds a view of children's functional use of signs for mathematics as "essentially a literate strategy" (p. 13). However, neither the *beginnings* of children's own notations made in mathematical contexts, nor their progress into abstract sign-use have previously been interrogated. Investigating children's signs from a Peircian perspective (Buchler, 1955), coupled with a usage-based view of language acquisition (Langacker, 2008; Tomasello, 2005), may help establish their emergent mathematical abstractions (which for young children extend beyond understanding of numbers).

Abstraction

Abstraction is widely conceived as a human ability to focus on relationships through formal language and symbolic representations.

Mathematical signs are analogous with *inscriptions*, *notations*, *symbolic tools, emergent models, representations,* and (from Carruthers & Worthington, 2005) *Children's Mathematical Graphics*.

An important aspect of mathematics is its cultural symbolic system in which mathematical symbols execute a dual function, supporting personal thinking and providing communicative tools. The German philosopher Ernst Cassirer (1923) suggested a seminal solution to explain *abstraction,* proposing that abstraction is *projected into* differing objects by seeing them as related *from a specific point of view*, consistently focusing on a specific relationship that can be seen as connecting all objects attended to, and, *by the same token neglecting all aspects that fall outside this focus*. Words, drawings and marks help children to focus on relationships they see as relevant for representation. As we will show, children's inscriptions transpire to be productive in assisting them to express their (abstract) view on their world.

Peirce's Perspective

The present study draws on Peirce's semiotic theory to analyse and interpret children's early signs, investigating ways in which they indicate meanings recognised as *mathematical* by others in their community.

In Peirce's terms *iconic signs* have some resemblance to the object signified. The term *symbolic* refers to conventional signs (e.g., letters, numerals). *Indexical* refers to something directly connected to that which is signified. Children's ostensive signals draw attention to and help clarify their intentions, rendering the "act of reference […] a social one" (Werner & Kaplan, 1963, p. 43). Csibra and Gergely (2011) established that infants' sensitivity to others' ostensive gestures such as direct eye contact and speech, prepares them "to identify and interpret others' actions as communicative acts specifically addressed to them" (p. 1150). Children's often-isolated early signs develop over time into rule-based structures (Langacker, 2008), to be interpreted as a process of *grammaticisation*, following a *usage-based* theory of language acquisition. According to Tomasello (2005), language structure emerges "from language use […] patterns of use emerge and become consolidated into grammatical constructions" (p. 5). The mathematical realisation of this theory provides insights into children's growing ability to engage in intention-reading, locating (grammatical) patterns in their graphical communications.

Language Acquisition

Using signs for expressing an abstract view, people often combine different signs according to some basic rules. Joint attention facilitates the first feature *intention-reading,* in which individuals focus on others' behaviour and speech, helping determine how they might contribute to their shared activity.

The second feature is functional, meaning-related *pattern-finding*: "to learn the conventional use of a particular word [or sign] the child not only must discern across instances that it is the same […] but must also see the way adults use a particular form communicatively across different usage events" (Tomasello, 2005, p. 30-31). Tomasello likens pattern-finding to children's ability "to create analogies (structure mappings) across two or more complex wholes, based on similar functional roles" (p. 4), understood here as *how this particular sign "works" to convey my meaning*.

Lancaster (2014) found 2-3-year-old children's beginnings in using a systematic structure, enabled them to devise "independent and original solutions […] in the face of complex symbolic problems" (p. 45), identifying "marks such as lines, dots, or zigzags being used to show […] quantity, as with number" (p. 37). Worthington (2009) identified patterns of sign-use 3-4-year-olds adopted in drawings, suggesting *a continuation of sign-use* from children in Lancaster's study, and indicating that this usage-based theory may also be successfully investigated in graphical contexts such as mathematics. Significant for the current study is the finding that between the ages of 3-4 years the children had begun to spontaneously use abstract signs, a feature also identified by Machón (2013).

In studying the emergence of mathematical abstractions, we assumed children's participation in everyday cultural practices supported their mathematical communications. Hence, we first investigated the cultural foundations of mathematics (Worthington & van Oers, 2016). Sarama and Clements (2008) highlight "five major mathematical topics […] number and arithmetic, geometry, measurement, patterning and algebraic thinking, and data and graphing" (p. 67), all of which may include aspects of number. MacDonald (2013) investigated children's ideas about measuring length. Worthington and van Oers (2016) found that aspects of mathematics the children most frequently explored were number, time and money, followed by wider aspects of mathematics, all made on their own initiative[39].

Exploiting their existing cultural knowledge of mathematics and sign-use, the children expanded their understandings in pretend play. For example, at home Isaac and his dad often went camping: subsequently in one play episode Isaac focused on the number of nights booked, the fee for camping and the number of people staying. Interrogating the same data, a second

[39] The children's frequent references to *time* and *money* are considered to underscore their significance in the children's home experiences (Worthington & van Oers, 2016, p. 59).

study explored the children's drawings, maps and writing in which they made considerable use of their cultural knowledge of authentic literacy practices at home (Worthington & van Oers, 2017) in meaningful literacy events within pretend play, locating a similar range of *modes*, *materiality* and *affordances* to those identified in the current study. The present study is situated within current research on semiotics and multimodality related to mathematics (e.g., Cobb et al., 2000). Presmeg (2006) highlights the potential of *semiotic chaining* "to bridge [the] apparent gap" between learners' existing understandings and taught concepts (p. 164), identified here in some of the children's own notations which themselves become peer models, contributing to their *cumulative collective repertoires*[40].

Young Children's Graphical, Mathematical Semiotic Activity

Young children often exceed adults' expectations of their ability to represent abstract symbolic thinking (Brizuela, 2004). Research into children's sign-use to communicate mathematical ideas include Hughes's seminal work (1986), establishing that young children make personal sense of mathematical notations relating to number, provided they are free to represent them in personally meaningful ways. This, he proposed, could help children translate between "this new language" and their concrete knowledge (p. 51).

Drawing on Vygotsky's work, van Oers (e.g., 2012a) has investigated children's schematisations as foundations for later mathematical representations. This dialectical, social process is based on teachers' interventions in their play, in which children construct useful means (signs, inscriptions) for communication, the relationship between concrete and abstract elaborated through dialogue (van Oers, 2001). Also from a Vygotskian perspective, during the 1990s Carruthers and Worthington (e.g., 2005; 2006) began investigating mathematical signs and texts of children from 2-8 years in natural, everyday contexts of homes and classrooms. They found that children's signs are often integral aspects of contexts in which they are free to use their own representations, such as pretend play.

Vygotsky (1978) understood pretend play as "a major source of development" (p. 102-103) for young children, laying the foundations of abstract symbolism. Children learn to use socially valued conventions related to mathematical signs from family members and teachers who

[40] Comment made by the teacher Emma.

frequently model signs (within multi-sign utterances)[41]. This study extends Carruthers and Worthington's earlier research, building on that of Worthington and van Oers (2016; 2017) and addresses the following research questions:

1. *What range of signs do pre-school children use to communicate their mathematical thinking?*
2. *How does children's intention-reading relate to increases in acquisition of the abstract symbolic language of mathematics?*
3. *How does children's pattern-finding support their increasing grammaticisation?*

Regarding the outcomes of our investigations, we expect that the answers are predicated on the conditions that children are allowed to freely engage in graphicacy in all contexts.

Methodology

Research Setting and Participants

The research setting is a nursery school located in Bristol, a large multicultural city in south-west England: 60 children attend each half-day session, individual key persons leading each group of approximately 14 children. Led by the headteacher, this nursery school had pioneered the approach to mathematical representation developed by Carruthers and Worthington: mathematics and graphicacy have high profiles, children frequently using their own signs to communicate ideas[42]. The headteacher of this nursery school co-researched and developed the educational concept on which this study draws. Working with the headteacher, the teachers developed their understandings of mathematics and sign-use over several years. Worthington had also previously led professional development on this approach at the nursery, both teachers in the study attending.

The nursery advocates a democratic culture, valuing and supporting children's ideas, choices and decisions and how they express them. As the only nursery school identified sharing these values, this nursery was

[41] Over time teachers attach explicit mathematical meanings to the children's signs, engaging with them in dialogue and referring to where (from contextual clues or the child's vocal explanation) their marks and signs suggest aspects of mathematics.

[42] The headteacher is engaged in doctoral research on the pedagogy of this approach (see Carruthers, *in process*).

selected for this study[43]. Children self-initiate their mathematical ideas through play indoors and out, and in adult-led small groups[44]. As more knowledgeable others, adults notice and recognise children's language and graphics as mathematical; the teachers involved in this study acknowledged as "expert" teachers in this.

Ethnographic data were gathered of seven children. To determine if previous interest in graphicacy influenced their interest in communicating through inscriptions in *mathematical* contexts, teachers Emma and Hugo were asked to identify several children who often chose to draw or write. Nominating Isaac, Shereen and Elizabeth as *focal children*, their teachers randomly selected four other children for comparison (Oliver, David, Ayaan and Tiyanni): their ages ranged between 3 years, 2 months to 4 years and they would start school the following year. Shereen's family is from the Philippines and Tiyanni's from the West Indies: both speak English fluently. Ayaan's family is Somalian: she speaks fluent Somali, her confidence in spoken English is growing. The remaining children's first language is English.

Research Design

This is a longitudinal, ethnographic study: Zaharlick (1992) describes ethnography's value as aiding understanding of "beliefs, attitudes and behaviours of sociocultural groups" (p. 122), to support improvements in education. Field notes on the children's home cultural knowledge were gathered from parents and teachers. Geertz (1973) asks how we might "frame an analysis of meaning - the conceptual structures individuals use to construe experience - which will be at once circumstantial enough to carry conviction and abstract enough to forward theory" (p. 313). According to Geertz we must isolate elements of the culture studied, establishing its inner relationships to identify "the ideological principles upon which it is based" (p. 17). This idea influenced this study and its analyses, revealing a broad appreciation of the symbolic principles involved, and also previously hidden elements of grammaticisation as children moved towards conventional signs of the established mathematical culture.

Multiple naturalistic and unstructured observations of seven children are supplemented by information from parents and teachers. Due to their ecological validity and grounding in authentic contexts, observations are

[43] During two years (prior to collecting data in this nursery) I made numerous visits to a total of 12 classrooms (in seven schools), with the intention of data gathering. However almost no useful examples of children's spontaneous mathematical notations were identified in other classes.

[44] Examples from the data were gathered from both contexts.

particularly suitable for our ethnographic research[45]. The teachers' established practice is to record children's behaviours and talk as they occur. This is a theory-driven observational study of one case, of one specific nursery school, in which we focus on seven children.

Procedure: Data Sources

Data were collected during the course of one year, primarily from the teachers' written observations in the children's *learning diaries*. I also made observations during a total of 65 visits to the nursery[46]. According to our view of abstraction as a process of consistent perspective taking, naturalistic observations can be considered relevant as they can reveal the children's point of view on a situation. Our theory-driven observations make our *interpretations* of these observations conceptually valid. Most observations are of pretend play, with several from adult-led small groups (e.g., figure 4.1). The headteacher led regular professional development, staff members developing their skills in documenting observations. Conforming to government requirements, the headteacher regularly moderated their written observations.

In addition to the written observations, the data comprise photographs of the children's graphics and field notes of informal discussions with parents and teachers. To ensure maximum validity the children's graphics are analysed in conjunction with the written observations. Single visits to each child's home enabled me to become acquainted with the parent/s. Together these sources provided background information on the children's experiences and cultural knowledge.

Data Collection and Analysis

Data are examined from several perspectives to affirm how children create meaningful communicative signs, especially for those parts of their everyday lives generally acknowledged as mathematical. Accordingly, our study takes an interpretive stance, combining social semiotic and language development perspectives. The first codes (relating to research question one) arose from Peirce's categories; *iconic, symbolic,* and *indexical* (Buchler, 1955), necessitating a new code for *early mathematical marks* and sub-codes to differentiate between the various modalities identified. To confirm

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[45] In England throughout the early years' foundation stage (0-5 years), daily use of written observations is implemented as formative assessment, and has been established for some years at this nursery school. Early years' settings are required by the government to collect and submit details of each child's achievement in respect of specified areas of learning.
[46] Occasionally I observed with one of the teachers, discussing afterwards what they had understood from their observations and identifying a similarity in them.

inter-rater reliability, a second coder coded a randomly chosen 20 percent of the children's inscriptions, achieving a 100 percent consensus. For questions two and three interpretive analyses allowed identification of episodes in which *intention-reading* and *pattern-finding* were evident. Coding signs throughout all their literacies was achieved through using computer-assisted qualitative data analysis software, enabling consistent identification. A second coding of the children's signs made in mathematical contexts highlighted those using iconic signs and those representing abstract numerals and indices.

Data were open and derived from the field; elemental coding employed to identify the children's mathematical inscriptions, which were then coded as a distinct set of data for the purposes of this study.

Ethics

Guided by BERA's[47] ethical principles (2011) and those of the Vaste Commissie voor Wetenschap en Ethiek [Scientific and Ethical Review Board, 2016] Vrije Universiteit Amsterdam, participants were informed and consulted at every stage, and permissions sought from all who would be involved. The headteacher and teachers gave their consent for gathering data in the nursery. At the onset of the research parents' informed consent to observe their child and collect data was sought in writing. The research was explained to the children using everyday language and their agreement requested. Parents were informed that they could withdraw their child from the research at any time, and one family did so early in the period of data collection: no data pertaining to this child have been used.

Results

To determine which of the children's graphical communications were mathematical, contexts in which they referred to aspects of mathematics were identified. Especially notable is that their mathematical signs and texts combined simplicity and utility rather than elaborate drawings. They explored a wide range of mathematical genres to communicate comprehensive aspects of mathematics. Of the three focal children, Shereen favoured shopping lists, receipts and orders in cafés, whilst Isaac made signs for a car park, campsite bookings and maps and plans. Elizabeth's interests were broad and the proportion of all her graphics less often mathematical[48]. David (a non-focal child) also communicated his

[47] The British Educational Research Association.

[48] Elizabeth's abiding interest in signs for writing was evident in her earlier learning diaries when she first attended nursery before she was 2 years old.

mathematical thinking through a high number of mathematical texts (Table 1). Thirty-three percent of all the graphics were considered mathematical.

Table 1

Quantities of each child's mathematical texts in the year

Isaac	Shereen	Ayaan	David	Oliver	Elizabeth	Tiyanni	Totals
19	13	6	17	12	10	2	79

RESEARCH QUESTION 1: *What range of signs do pre-school children use to communicate their mathematical thinking?*

Early Mathematical Marks

Of all the inscriptions 19 percent were scribble-marks, acknowledged by Carruthers and Worthington (2006) as *shorthand* for communicating meanings, and suggesting they allow play to continue without interruption (Worthington & van Oers, 2017). Werner and Kaplan (1963) refer to such symbols as *"protosymbols"* that "directly *"present"* a meaning rather than *"represent it*" and "extremely important in the genetic process of symbolization" (p. 42-43, emphasis in the original). Price et al. (2015) consider the "meaning" of such marks "initially only available to the child […] as their symbolic understanding progresses [they] become more recognizable to others" (p. 132), for example:

> Standing on the bathroom scales David looked at the numeral on the dial, saying, *"I'm "15", so I need to write it down"* and made some letter and numeral-like signs on the whiteboard. Making rapid scribble-marks on paper (figure 4.1), Isaac used his emergent knowledge of various measuring units explaining, "*David weighs 700 kilos, he's 60 metres heavy".*

Figure 4.1

"David weighs 700 kilos"

Taking a specific point of view, the children's intention focused on communicating about number and weight, *neglecting all aspects that fall outside this focus:* from our view of quantity and measurement, scribbles can be seen as an early form of abstraction. However, these are not *abstract symbols*, consistently referring to specific meanings from a certain point of view, and Isaac's use of the language of measurement does not imply that he had established conceptual understanding of the amounts to which he referred. In early childhood, children's meaning-making shows this is a very versatile process, their willingness to communicate through signs more significant than the ability to write neat numerals (Brizuela, 2004). Analysis of the data from a Peircian viewpoint highlighted the percentage of signs in three categories that the children used when approaching their environment from a specific (mathematical) point of view.

Iconic Signs

Just over 49 percent of the children's signs were iconic, Fay, Ellison et al. (2014) emphasising their relationship relating "via either perceptual resemblance or natural association to the referent"; as such they are semantically motivated signs that "act as a crutch to help people establish shared meanings" (p. 244). The children's iconic signs included tallies, wavy or zigzag lines (signifying writing), letter and numeral-like signs, crosses, and dots (often used to refer to uncounted quantities), ticks and arrows, letters and numerals[49]. Occasionally a child's drawings resembled concrete items to which they referred, constituting an *equality relationship* between object and sign (in terms of Cassirer's 1923 perspective). Emma had modelled tallies in meaningful contexts and subsequently several children explored their use: for example, Isaac represented the quantity of pancakes each child in his group wanted, making a series of circular marks (abstractions of concrete objects). On another occasion David, Isaac and Jayden decided to check people in and out as they entered or left the nursery. Using a range of marks and signs, Isaac drew a cross, explaining to an adult who entered "*That means you work here".* In this instance Isaac used a cross as a tally.

Another day David enumerated members of his family, naming each line he drew. Shereen drew tallies for items on her shopping list, counting each with confidence to twenty. Tallies are usually regarded as mathematical, and associated with one of Gelman and Gallistel's (1986) five counting principles; in one-to-one counting, objects are itemised and only one number assigned to one item. The children's use of tallies indicates that their signs are based on their point of view (counting itemised objects), *neglecting all aspects that fall outside this focus*. For these children tallies appear to have the same value in terms of mathematical development. Tallies, however, are more abstract than drawings (iconic signs); their meaning depends on a person's point of view of enumeration. The use of tallies indicates movement towards abstraction from Cassirer's perspective (1923) suggesting objects are now seen from a specific point of view, i.e., as countable items, rather than depicted as an amorphous collection.

Numeral-like Signs

The abstract signs of writing and mathematics are discrete yet share some features, and children may occasionally confuse signs of their systems (Bialystok, 1992; Carruthers & Worthington, 2006; Tolchinsky, 2003): understandings gradually emerge "as they learn to differentiate between their shapes" (Neumann & Neumann, 2014, p. 1144).

Playing builders, Isaac wrote a letter or numeral-like sign on a play-cheque (figure 4.2) explaining *"A cheque for £500.00, for all the jobs I've done at my house"*; Isaac's focus on the monetary aspect of the situation makes it abstract. As a young child beginning to communicate through

[49] This sentence has been very slightly amended.

graphicacy, Isaac had only recently begun to use alphanumerical signs, and we cannot assume that he consistently used the same sign to represent "£500.00", or that he had a conceptual understanding of what this amount represents. Isaac's sign is conceived as a sign in an early stage of development.

Figure 4.2

Isaac's cheque

As with writing, children must confront multimodal aspects of mathematical notations and the potentials or affordances of particular signs (Worthington & van Oers, 2017). Signs' orientation serves as graphic organisers, impacting on how they are understood, such as the position of numerals in two-digit numbers.

Symbolic Signs

Almost 13 percent of signs were symbolic, their meanings (especially when viewed from their contexts) more readily transparent to others. Fay et al. (2014) explain that complex representations "become graphically simpler, and the iconic components depict[ed] […] become more symbolic" with repeated use, "iconicity moving from the level of the sign to the level of the system" (p. 251-252).

Of the three focal children Elizabeth and Shereen wrote numerals to nine, Shereen also writing "14". Large numbers and quantities often intrigue young children, and proud that she could also write "100", Elizabeth used the written number as an abstraction representing (for her) a "big number", although we cannot assume that she understood the exact amount represented. David and Oliver had also begun to write standard numerals. This is of particular interest since multiple researchers have identified relationships between children's early use of numerical symbols and subsequent longitudinal achievement (Merkley & Ansari, 2016). In her study in preschools, Munn (1995) identified a relationship between children's understandings and achievement in recognising numerals and letters: the progress made during their first year of primary school "strongly related to the understanding of symbols they had brought with them at school entry", suggesting, "that the important developments taking place concerned the children's understanding of symbols as communicative systems" (p. 217).

Indexical

Almost 18 percent of all the children's signs were indexical, most children drawing attention to features and meanings of their signs.

Figure 4.3

Shereen's map

Shereen's arrow (figure 4.3) signifies direction in her "treasure map": Shereen described the route, pointing to indicate the direction. Isaac drew arrows on his collaborative maps, explaining *"These are arrows to say, "go* *this way"*, and at home drew a double-headed arrow indicating turning left or right at the "T" junction at the end of his road. Isaac's cultural knowledge of maps originated from his father's, many maps were displayed and frequently discussed at home. During my home visit Isaac proudly showed an informal map he'd drawn of the city's roads, *pointing to* his home's location and the route travelled to his nursery school each day. Isaac's interest in directions and locations extended to using a compass. Visiting the forest, Isaac told an adult, *"I think we're going west […] That way is south"*. Concerned that they might be lost Isaac asked*, "Are we north or south? This is a mystery path! I don't want to go south – it will go to Africa and my bedtime's at six o'clock".* In his subtraction (following figure 4.5) David *pointed to* the marks he was about to rub out, his pointing an example of indexicality. In our view pointing is the simplest and most clear form of abstraction for young children as it creates joint attention and articulates/highlights one (and only one) point of view.

Our data suggests that the range of signs the children employed across Pierce's semiotic modes appear to contribute to their understandings as they moved towards an abstract symbolic mode. More research is needed.

RESEARCH QUESTION 2: *How does children's intention-reading relate to increases in acquisition of the abstract symbolic language of mathematics?*

Joint activities such as collaborative pretend play enable children to discern another's intentions (Tomasello, 2005, p. 5-6), helping them discern how they might contribute to their shared play through *imitation* or *emulation. Imitation* (of behaviour) suggests faithful copying, whereas *emulation* (of actions) points to the child's adaptive use of signs. In our data we looked for events that show how children observed others in order to find how they might use the signs in their own activities and communication.

Oliver was playing with Isaac (a focal child), their pretence triggered by Isaac's recent experience of using an electronic card-reader in a city car park, and linked to his considerable knowledge of technologies.

> Isaac used scribble-marks to signify his parking signs, explaining *"This says 'swipe here with your special code card'.* Adding further marks, he explained, *"This is the bell if you don't have a sticker and someone can let you in. It says, "Press here". This is for lorries and deliveries. It opens automatically, it's a camera."*

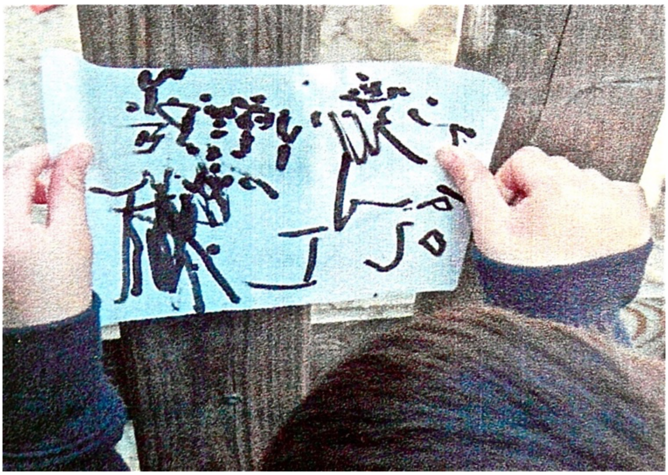
Oliver's teacher noted, "*Oliver watches and waits before deciding to participate*", seemingly to determine Isaac's intentions before he contributed to their play. Although Oliver had not shared Isaac's experience of visiting the same car park, his ability to *read* Isaac's intentions enabled him to decide how he might contribute to their shared play.

> Oliver begins by drawing dots and letter-like signs, followed by several ticks (figure 4.4), explaining, *"These are ticks. When there are three ticks you can go, when there are two you can't go that way* [pointing]. *I've made two ticks – that means you are not allowed",* then pointing in the opposite direction, *"people allowed in that way"*. Continuing their play, when another child attempted to access their car park, Oliver referred to his sign.

Oliver had begun to focus on specific meanings, *neglecting all aspects that fall outside this focus and* emulating Isaac's *idea* of making a sign but not imitating his marks. *Emulation* suggests pedagogy through peer learning (Csibra & Gergely, 2011)[50]. Oliver's use of ticks shows that he is consistently working from a point of view of communicating quantity in this context.

Figure 4.4

Oliver's car-park entry sign



[50] Csibra and Gergely (2011) propose that the cognitive mechanisms enabling "the transmission of cultural knowledge *by communication* between individuals […] represent[ing] an evolutionary adaptation along the hominin lineage", maintaining that children are the most obvious beneficiaries of such a system (p. 1149).

Early Calculation

In another example of joint attention Shereen (a focal child) represented her thinking about items sold in her pretend café (figure 4.5), her teacher and David watching.

Figure 4.5

Subtracting cakes

> Pointing to the people she'd drawn, Shereen explained "*This is me and my Daddy at the café*". Drawing a flower and a heart above them, and five cakes on the left, she asked a friend "*You like some cake?*" and following her friend's affirmation, Shereen rubbed out one cake to show it had been sold[51]. Repeating the same question, when her teacher also replied "*yes*" Shereen rubbed out another cake remarking "*Three left*".

Whilst exploring a calculation was unexpected for such a young child, Shereen's combination of meaningful elements highlights children's innovative early strategies for *written* calculations. Shereen's action of rubbing-out (i.e., subtracting) cakes, suggests its function as an operand. Tomasello (2005) observes that for young children "linguistic competence is most accurately characterized [...] as an inventory of relatively isolated,

[51] It was unclear to what the number "14" at the top referred.

item-based" constructions: development "proceeds gradually and in piecemeal fashion, with some constructions becoming abstract more rapidly than others" (p. 140/142).

Using the same strategy, David drew himself and Shereen in a café and made several small marks, detailing items to which each referred. Inviting Shereen to "visit" his café she "ordered" one cake and a cold coffee. David pointed to several small marks, then rubbed them out to signify their removal saying, *"here you go. I have to rub them away 'cos they're gone from the café*". David had clearly benefitted from *reading* Shereen's intentions; within their joint attentional frame his "ability to culturally (imitatively) learn the intentional actions of others" (Tomasello, 2005, p. 3). Both Oliver and Shereen's examples show some movement towards abstract signs (construed by a previously taken point of view).

The findings of question 2 highlight the importance of pretend play in supporting a form of social learning. Such contexts provide a reciprocal relationship between children playing and interacting, showing how intention-reading relates to increases in their acquisition of the abstract symbolic language of mathematics.

RESEARCH QUESTION 3: *How does children's pattern-finding support their increasing grammaticisation?*

To identify examples of pattern-finding we searched the children's graphics (drawing, writing, maps and mathematical), finding that several included dots, crosses and arrows for specific purposes, identifying these repeated sign-function units as examples of pattern-finding. Grammaticisation was identified as the structure of the patterns.

Tomasello (2005) regards pattern-finding as categorisation, children using their skills "on the functional (or meaning) side of things" (p. 30). To learn conventional, culturally accepted uses of particular signs, children need to distinguish signs in many examples in various situations, but "also see patterns in the way adults use a particular form communicatively across different usage events" (p. 30-31). Children appear to understand that only a specific graphical sign will *fit* their immediate communicative purpose, intuitively selecting the most suitable from their personal lexicons. This can be seen in the data, children generally use crosses to signify "none" or "no", dots to signify *lots* and arrows to denote direction. As the findings of question 1 show, many children using crosses appeared to understand *absence* or *nothing,* prior to appreciating that they can represent these concepts by zero (Merritt et al., 2012).

Identifying patterns of contexts in which crosses are used, individuals sometimes modified their signs for emphasis: for example, whilst playing café Shereen asked Madison if she wanted some food. When Madison shook her head, Shereen wrote a *series* of crosses, retorting "*You not very hungry*?" On another occasion David wrote *two large* crosses on paper, asserting "*No more children getting in our car!*" both examples signifying emphatic negatives, highlighting multimodal aspects of their signs[52]. These examples show children's use of crosses as *logographic signs* in which the meaning of a word or phrase is encoded in the visual symbol. Shereen's calculation (figure 4.5) implies a pattern of layout and a means of subtracting that David was able to discern and emulate. Tomasello (2005) explains that children "cut and paste" functionally appropriate pieces of language" (as David did) that they have learned or created (p. 321). Children seem to recognise that adults expect them to use graphical signs to convey meaning, although adults may not always understand the meanings of their inscriptions without their verbal explanations.

Lexicons

Our data suggest that children of this age already have a repertoire of signs that they subsequently combine in mathematical communication. There was variability in the children's use of signs: Elizabeth and Shereen (two focal children) used most graphical signs across all their literacies, and the greatest number of *abstract symbolic* signs: both wrote letters of the alphabet (capital and lower-case), and most abstract numerals. Isaac's (a focal child) particular strength was his knowledge of environmental signage on which he often drew in the nursery. The remaining non-focal children varied in their use of signs across all literacies, Oliver and David doing so most often, and (of the non-focal children) having the largest lexicons of signs, although the quantity of their abstract symbolic numerals was small.

Whilst the sample size is small, the findings suggest a shift over time to increasingly using abstract signs from the established mathematical system, highlighting how "basic conceptual categories that derive from everyday experiences, develop into predictable structures that are used automatically" (Lancaster, 2014, p. 35). Though we have to avoid sweeping conclusions, these findings affirm our expectation that as children freely engage in graphicacy in all contexts, they are likely to progress towards standard mathematical patterns of signs.

[52] Such signs are suggestive of emphasis in oral speech.

Discussion

This study is part of a larger research project into the genesis of mathematical semiosis in early childhood. The aim of the current study is to investigate the emergence of abstract thinking about aspects of mathematics in young children's graphical communications, by thoroughly interrogating data from a nursery school in which an emergent approach to learning mathematics is well understood and supported.

Early childhood is an important period in children's lives. In our earlier study (Worthington & van Oers, 2016) we found that free pretend play of young children often triggered spontaneous interest in and discussions about aspects of their mathematical cultural knowledge.

There has long been recognition that when traditionally taught, young children find the abstract symbolic language of mathematics challenging. Our data-analysis showed that the children's use of signs in mathematical contexts increased during the year: all used iconic signs and by the end of the year some had begun to adopt standard abstract symbols. Frequent sign-use (including letters and numerals) across all the children's literacies, appeared to be an important factor in developing *mathematical* abstractions for communication about quantity, transformations, measurement and space, etc. On the basis of Munn's article (1995) we can speculate on substantial links between early understanding and later achievement.

This study ascertained that feature of usage-based language acquisition were evident in the children's early mathematical inscriptions, and how intention-reading related to their acquisition of the abstract symbolic language of mathematics, helping them understand, imitate, and emulate visual signs of more mature users. Analysis showed that some children identified patterns of signs that best fitted their communicative intentions, employing individual signs across diverse texts. For example, children employed crosses in various contexts to signify absence or nothing, used arrows in various directional contexts and similar layout and strategies for their subtractions: other examples include dots, ticks, letters and numerals.

Given occurrences of such transferrals we cautiously conjecture that children's expanding lexicons benefit their ability to select appropriate signs from one context to "fit" in another, and that this expansion contributes to the grammaticisation of mathematical inscriptions. Tomasello (2005) proposes that it is best to see children's signs as "*growing gradually in abstraction over time* as more and more relevant exemplars are encountered and assimilated" (p. 316, emphasis added). These findings draw attention to the compelling value of the usage-based theory for understanding children's early mathematical abstractions, and have implications for the mathematisation of children's early signs (Worthington et al., 2019) in future practices.

The emergence of mathematical abstraction is viewed here from an early point in children's progression towards the fully abstract symbolic language of mathematics. Children's home cultural knowledge and effective socio-cultural contexts contribute to the free exchange of ideas through speech and text, regarding objects that are generally acknowledged as mathematical (although at this age children themselves will not acknowledge their signs as mathematical). Together with our earlier studies (e.g., Carruthers & Worthington, 2006; Worthington & van Oers, 2017), these findings suggest a close relationship in learning the two alphanumeric symbol systems, provided children have freedom to use personal graphical communications to signify mathematical meanings. Moreover, the finding that several children made use of standard numerical symbols, (predictive of subsequent mathematical achievement in school, Munn, 1995), points to the value of this approach and the teachers' expertise.

We consider that the values and democratic culture of this nursery school precipitated the children's interest in the use of signs to communicate their thinking in mathematical contexts. The headteacher created a rich community of learners, instigating a culture of staff research and dialogue that can be seen in the following characteristics:

- Adults' support of children's self-initiated ideas and how they express them provided positive messages, confirming for them that their communicative use is meaningful, relevant and valued, and contributing to their agency;
- Teachers' frequent modelling of signs within authentic contexts provided new signs on which the children might draw, increasing their sign repertoires;
- Contexts in which children experimented with signs and "read" others' intentions helped them discern patterns of sign-use across all their literacies;
- Children's frequent graphical inscriptions stimulated their increased use of letters and numerical symbols for communicative purposes.

The teachers' deep understandings of pretend play contributed to its quality, providing meaningful opportunities for children to explore their cultural mathematical knowledge: their understanding of the children's home cultural knowledge supported this.

These findings commend learning cultures in early years' classrooms in which graphicacy, mathematics and pretend play are highly valued and understood.

Limitations

Data were gathered in a nursery school embracing open approaches to graphical inscriptions and the teaching of mathematical signs: since we know of no other nursery schools in England consistently working in this way, and due also to the small numbers of children, empirical generalisations are not possible. The problem of "transferability" can be solved in three ways (1) through "professionalisation" of teachers of how to interact with young children and their use of marks etc., (e.g., Pompert, 2012); (2) through methodological criteria, giving in-depth specifications on visibility, comprehensibility and acceptability (Akkerman et al., 2006), and (3) through 'theoretical generalisation'[53]. Working closely with teachers to develop their professionalism Carruthers and Worthington (e.g., 2011; Carruthers, 2012) identified many instances of transferability that provided comparative results. Willig (2013) posits that conducting studies "in sufficient numbers can give rise to statements about general trends and the typicality of occurrences" (p. 109). Hence, up-scaling the study with a larger number of pupils is necessary to determine if our outcomes can be reliably reproduced. In spite of these limitations, the findings from this study will add to knowledge of the emergence of abstraction, the early evolution of children's mathematical communications through graphical signs, and some of the processes involved in their developing understandings.

Conclusion

This study focused on documenting the types of signs young children make to represent and communicate about their mathematical thinking, and to determine the extent to which features of usage-based language acquisition are evident in children's early mathematical inscriptions. The empirical findings, analysed from our point of view of "abstraction", can indeed highlight the appearance of abstract thinking in young children, a previously unknown finding. The children's movement from iconic towards abstract symbols suggests they are making connections between their existing and new knowledge. "Reading" their peers' intentions and intuitively locating patterns of sign-use appears to be integral to this development. The findings suggest that these personally meaningful and authentic beginnings support the emergence of the symbolic language of mathematics early in childhood. They indicate that problems identified by Ginsburg (1977), Hughes (1986) and many others may be avoided, allowing children's mathematical potentials to be more fully realised. Especially

[53] We worked on all three ways to ensure that this study could further the fields of educational practice and research.

interesting is that the children who most often used graphical signs to communicate in all literacies most often used numerical symbols, a feature identified as predictive of subsequent success in mathematics in school.

Curriculum reform based on recent research in this significant area of mathematics is sorely needed to further pedagogical understanding, and to deepen teachers' appreciation of children's beginnings with mathematical signs, and the development of their powerful mathematical thinking.

- 5 -

Intertextuality and the Advance of Mathematatisation in Young Children's Inscriptions[54]

Abstract

A fundamental question in early childhood mathematics concerns the relationship between young children's own informal signs and the formal abstract symbolic language of mathematics. This study draws on recent research investigating the genesis of mathematical semiosis from a Vygotskian cultural-historical (social-semiotic) perspective. It is a part of a larger investigation into the emergence and development of young children's own mathematical inscriptions. In this study we look at the premature stages of dealing with quantity and their relationships. Our aim is to reveal the interweaving of young children's sign-use and to consider the role of intertextuality in mathematisation. Longitudinal, ethnographic data were gathered from case studies of seven children aged 3-4 years in an inner-city nursery school in England, documenting observations of their spontaneous pretend play. The data are interrogated through interpretive analysis and show that some graphical signs moved between individuals' texts, also borrowed from others including the teacher, and woven together. Children's progressive understandings of mathematical sign-use appear to be attained through intertextual exchanges in social contexts such as pretend play.

Introduction

Mathematics is now a widely accepted component of early childhood curricula, understood as a means of initiating children into the established mathematical culture. An important aspect of mathematics is its culturally developed, abstract symbolic language. In 1986, Hughes identified the difficulties young children have when confronted by formal mathematical symbols in school, maintaining that "children need to develop links – *or ways of translating* – between this new language and their own concrete knowledge" (p. 51, emphasis in the original).

Our focus is primarily on young children's marks and graphical inscriptions, used to build links between signs and symbols (of any sort) and

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their understandings of quantities and number. In the wake of this sociogenetic process, children develop their understandings starting from their earliest marks, over time transforming these into the conventional cultural signs such as standard numerals, through borrowing signs from others. To many adults, young children's beginning marks bear little relationship to items counted or quantified, or any resemblance to the formal symbolic signs of mathematics (such as 2, 3, + or =). However, Kress (2003), stresses that "children's early meaning making is governed by the very fact that they do not [at first] use ready made signifiers" (p. 155).

Whilst young children require experiences with diverse mathematical ideas (e.g., measurement, classification, shape and space), Jung et al. (2013) maintain that:

> Number relationships are arguably among the most important mathematics concepts in number and quantity, and they must be developed throughout the early years of life (Baroody, 2000; Jung, 2011; Ma, 1999). Number relationships, which go far beyond counting skills, refer to the ability to represent a quantity in multiple, flexible ways. (p. 166)

This helps provide children with a firm foundation for developing greater understanding of number and quantity, allowing them to develop greater adaptability and responsive means of representing numbers. The more that young children use graphical signs to communicate their thinking, the more they will come to understand the conventions of standard symbols and texts. This flexibility helps children appreciate the interconnectedness of numbers, and that they can be used in meaningful ways to communicate their thinking in mathematical contexts.

Brizuela and Gravel (2013) emphasise that tally-type marks depend on one-to-one correspondence, and that "this correspondence may be obtained by icons, tallies dots, crosses, letters or even numerals" (p. 119), and the same has been established in this current study. When tallying small quantities one-to-one-correspondence is often taken as an indicator of early mathematical thinking (Gelman & Gallistel, 1986), and although it would not be possible with huge numbers, young children appear to be open to this means of recording a count. Using the one-to-one relationship is also often understood as an indication of mathematising.

Vygotsky (1987) emphasised dual routes to the appropriation of cultural knowledge, first, through children's home cultural experiences, second in pretend play. He argued that this play supports spontaneous concepts, establishing foundations for the subsequent formation of scientific concepts (p. 220/238). Significantly, pretend play also provides a context for communication with others, such communications preceding communication with oneself, or thinking (Veraksa et al., 2016). The communication that takes place in social pretend play also allows elements of others' texts to meld in new texts. Vygotsky (1978) showed how the child's cultural development to higher functions appears twice, "first, between people (interpsychological) and then inside the child (intrapsychological) [...] All the higher functions originate as actual relationships between individuals" (p. 57).

The examples in this study show children communicating their ideas to others, thus providing insights into their interpsychological development. The children's play narratives frequently focused on everyday home practices and related cultural knowledge, extended and adapted as they explored their existing "funds of knowledge" (Moll et al., 1992; Worthington, 2018), meaningfully connecting to their existing understandings and sign-use in their nursery. In such contexts, children will occasionally use number, quantity, order, relations or other mathematical-like objects in their own ways.

Without recourse to relational understandings, learning is often superficial, with negligible connection between children's existing and informal mathematical inscriptions and the formal signs of school mathematics (Carruthers & Worthington, 2006; Hughes, 1986; Worthington et al., 2020a). Therefore, Hughes (1986) suggested that teachers[55] "*build on children's own strategies*" and "*respect their invented symbolism*" (p. 176/177, emphasis in the original). Most studies on early childhood concept formation suggest that it involves a process of transformation based on previously acquired knowledge. In our study, we attempt to investigate how this process takes place in young children, addressing issues of intertextuality and mathematisation in pretend play.

The data in this study show that the children represented their thinking in contexts (within their play) that could be understood as mathematical. From their own and others' utterances (e.g., word combinations, drawings, signs), young children appear to subconsciously consider those features that have the potential to effectively communicate their current thinking, sometimes linking a sign used in one context intertextually, with the same sign used in a different context.

[55] Both teachers and early years' practitioners work in the nursery school, but for brevity the word "teacher" is used throughout.

Theoretical Framework

The Case for Intertextuality

Intertextuality refers to "the complex relationship between a text and other texts taken as basic to the creation or interpretation of the text"[56]. Signs mediate human actions and thus may result in learning (Vygotsky, 1978). Bartolini Bussi and Mariotti (2008), emphasise that in social contexts in which there is a task to be achieved, joint signs are developed, their use related both to accomplishing their task and its content to be mediated. From their early attempts at communicating, children blend different (self-made, adopted or adapted) signs and symbols together, rendering their expressions a text-like character, and presumably helping them develop greater understanding of number and quantity. This flexibility helps children appreciate the interconnectedness of numbers, and that they can be used in meaningful ways to communicate their thinking in mathematical contexts. Rather than randomly assembling signs from elsewhere, inserting others' signs into one's own utterances is a rule-guided occurrence, governed in part by grammaticisation, a usage-based process in language acquisition (Langacker, 2008).

Kristeva (1980) argued that authors do not create their texts from their own mind, but compile them from existing texts and signs: texts become "a permutation of texts […] in the space of a given text, several utterances, taken from other texts, intersect" (p. 36). Kristeva's work on intertextuality grew from Bakhtin's (1981) *dialogicality* or *multivoicedness* where "the word in language is half someone else's" (p. 293). With Bakhtin, Kristeva sees each text as an *intertext* in a succession of texts already written or yet to be written (1980), the flow of iterated inscriptions creating lengthy torrents of changes, revealing "successive traces" (Latour, 2014, p. 347).

Intertextual exchanges intuitively add to the social means by which young children enlarge their repertoire of graphical signs, augmenting the signs modelled by their teachers, and those they imitate and emulate. A text's author is not an individual but a social unit: children learn single signs and multi-word texts mainly by emulating everyday cultural activities. From a Bakhtinian (1981) point of view, children can be observed to appropriate and modify signs, which become their own when they populate them with their own intention, their own accent, adapting them "to their own semantic and expressive intention" (p. 293). Intertextuality contributes to symbolic diversity within the social group, providing multiple perspectives

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56 {https://www.merriam-webster.com/dictionary/intertextuality}

of signs and enriching children's expanding mental sign-lexicons with the help of adult signs and texts.

Mathematisation

Mathematics is an act of mathematising, at the core of which are problem solving activities in which children construct relations among symbolically represented notions of quantity, order, operations. Mathematisation highlights children's increasing use and sophistication of mathematical signs and multi-sign texts from the established mathematical culture. Mathematising was first identified by Freudenthal (1973), who asserted that "there is no mathematics without mathematizing" (p. 134). Freudenthal's work on mathematising was subsequently developed by Treffers (1987) and others as an integral feature of *Realistic Mathematics Education* (RME) in the Netherlands. Freudenthal (1973) emphasised that "mathematics is applied by creating it new each time […] This activity can never be exercised by learning mathematics as a ready-made product [arguing that] the opposite of ready-made is 'mathematics *in statu nascendi*'"[57] (p. 118). The pupil himself should re-invent mathematics" (p. 118). Freudenthal (1971) believed that mathematics is a "human activity", mathematising an

> activity of solving problems, of looking for problems, but is also an activity of organizing a subject matter. This can be a matter from reality which has to be organized according to mathematical patterns if problems from reality have to be solved. It can also be a mathematical matter, new or old results, of your own or others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach. (p. 413-414)

Van Oers (2014) argues that "*productive mathematisation* is to be conceived as an essentially playful activity that has its roots in young children's playful participation in cultural practices" (p. 112, emphasis in the original), and it is in their play that children gain access to cultural activity (including mathematising), by emulating this activity. Carpay and van Oers (1999) emphasise that "thinking and making sense […] has to be conceived of as sociosemiotic process in which oral and written texts [...] constantly interact in order to bring about improved texts on the part of the interlocutors" (p. 303). Young children's texts may comprise a single utterance or sign, or a combination of signs as they make choices about

[57] In the state of being born; nascent; emerging but not yet formed {https://www.merriam-webster.com/dictionary/in%20statu%20nascendi}

how best to communicate their meanings. In early childhood, children's informal sign-use is personal and often intuitive.

Pretend Play

Social pretend play provides contexts for children to interact with peers who may be either novice or expert (Razfar & Gutiérrez, 2003, p. 40), fostering intertextual exchanges. Our previous studies (Worthington & van Oers, 2016; 2017; Worthington et al., 2019) focused on children's spontaneous pretend play and several adult-led small groups, for communicating about mathematics through graphical inscriptions. Pretend play offers potentially rich contexts for mathematising, and Pahl (1999) commends close observation of graphicacy and play for the insights they provide "into the flow of children's thoughts" (p. 106). Children's early signs play a significant role in their developing mathematisation as they gradually integrate formal signs into higher, more powerful operations (Treffers, 1987). Vygotsky (1978) viewed development from babbling to speech as a metamorphosis, "a qualitative transformation from one form to another" (p. 73), explaining his view of the development of signs whereby, "sign-using activity in children is neither simply invented nor passed down by adults […] [it becomes] one only after a series of qualitative transformations". Children's relative freedom in pretend play is a fruitful context for probing meanings and signs.

Research Question

Until now, research has not fully determined how children's own early signs and representations relate to standard forms (Gifford, 2005; Purpura et al., 2013). To clarify young children's emergent mathematical thinking, we studied young children's activities from the perspective of intertextuality and mathematisation. In this chapter we focus on the following research question:

> *What evidence is there of intertextuality, particularly with respect to the use of mathematical signs in the inscriptions of children made in nursery school, and how does this impact on their mathematisation?*

Previous Empirical Research

Children's Mathematical Graphics

Through observing difficulties experienced in mathematics by the young children they taught, Carruthers and Worthington (e.g., 2005; 2006) developed the educational concept of *Children's Mathematical Graphics* (CMG), a cultural-conceptual approach that privileges children's cultural knowledge, ways of thinking and representing. Given the opportunity, young children will communicate meanings through a combination of emergent and culturally constructed sign systems for drawing, maps, early writing, and notations. A significant feature of this approach is to provide authentic or realistic social contexts that have personal meaning to the children, an element also advocated by Freudenthal (1973), Gravemeijer and Terwel (2000) and van Oers (2014)[58].

Carruthers and Worthington originally based this concept on the emergent or developmental approach to writing they had already established in their classes. In respect of mathematics, they found that children's use of increasingly formal signs is explained in part by teachers frequently modelling mathematical signs and strategies in meaningful multi-sign contexts. In this approach, teachers always use contextually meaningful examples when modelling inscriptions, modelling frequently but never with an expectation that the children will immediately copy this modelling with their own inscriptions.

Recent studies have extended Carruthers and Worthington's research; firstly, through investigating 3-4-year-old children's spontaneous references to aspects of mathematics in their impromptu pretend play (Worthington & van Oers, 2016). The findings showed that children drew on their cultural knowledge from home, freely exploring different mathematical topics such as number and measurement. A second study investigated the same children's meaning making through their literacies (drawing, maps and writing), and the relationship between their cultural knowledge of home literacy practices and their literacy events in the nursery (Worthington & van Oers, 2017)[59]. Using the same data set as Worthington & van Oers (2016; 2017), a third study investigated the development of mathematical abstraction in the same nursery (Worthington et al., 2019). Drawing on Peirce's semiotic theory, its findings highlight children's significant use of iconic signs as they moved towards symbolic or formal numerical signs, showing how isolated early signs develop into grammatical structures over time through grammaticisation (Langacker, 2008; Tomasello, 2005). An arresting finding was that the greatest proportion of the children's signs was iconic, indicating progression towards the formal signs of mathematics.

[58] Carruthers and Worthington developed their approach before reading the work of these researchers.
[59] From our second study we found that the same communicative use of inscriptions is useful for children's signs made in mathematical contexts.

The current study builds on these three studies to further investigate the same data set by focusing on mathematisation and intertextuality.

Methodology

Characterisation of the Study

This is a longitudinal, ethnographic study: its aim is to gain a genetic perspective of young children's communicative interactions relating to mathematical ideas. Fetterman (2009) writes that ethnography examines social life as it unfolds "through the complex world of social interaction […] telling a credible, rigorous and authentic story" (p. xi/2). Willis and Trondman (2000) list a number of important features of ethnographic research, including understanding and representing experience, presenting and explaining the culture in which experiences are located, but also acknowledging that "experience is entrained in a flow of history" (p. 6). Qualitative, ethnographic data were gathered over a period of one academic year through case studies of seven children of 3-4 years. The following section examines these aspects in greater detail. The value of these approaches for answering the research question of this study is addressed in the following section.

Ethnographic Case Studies

Gathering longitudinal ethnographic data through case studies is especially relevant for answering the research question in this study. Moschkovich (2019) maintains that ethnographic methodologies require a "naturalistic paradigm" which "are still systematic and have to be consistent with the theoretical framing for the study" (p. 61), analysis starting "with observations in a setting where cognitive phenomena occur regularly without intervention" (p. 63). Such theoretical or analytic generalisation (see Yin, 2010) builds a hypothetical model that best describes the relations of the observed phenomena in a case study. Einarsdóttir (2007) emphasises that listening to what children say whilst they represent their ideas should be prioritised, so that the children's meanings and explanations of their graphics are clear.

Pound (2006) highlights that observing in authentic and natural contexts increases opportunities for researchers to "really see children operating at their most effective". Moreover, children's mathematics is richer and children are more communicative "when they have real things that are important to them to discuss" (p. 122). The first author's discussion with the parents and a visit she made to each child's home, helped provide understanding of the source of the children's cultural knowledge of mathematics and their communicative interactions. Data gathered in the children's nursery school provided understanding of the social and cultural context there.

In the context of the current study, these case studies are expected to shine a light on aspects of value to furthering our understanding of young children's graphical communications in contexts that are considered "mathematical" by adults.

Research Setting and Participants

The nursery school participating in this study is in an inner-city location with a large multi-ethnic population in the southwest of England, and thirteen different languages were spoken there at the time of data collection. Sixty children attend each morning and afternoon session, individual key persons[60] leading a group of approximately fourteen children. The nursery advocates an open and democratic culture, valuing and supporting children's self-initiated ideas, choices and decisions through rich dialogue. Mathematics, graphical communication, pretend play and collaborative dialogue all have high profiles in the nursery. This nursery school is replete with printed numbers, indoors and out, with number lines (up to 100, 1000, negative number lines and some written in the home languages of children in the nursery), add to numbers displayed in realistic contexts and children's examples displayed on the walls. Sign-types identified here are comparable to those of Carruthers and Worthington (2005, 2006); Hughes (1986); Munn (1994); Papandreou (2009) and Rogers (2008).

During a period of twelve years'[61] the efficacy of mathematics teaching in this nursery school has been repeatedly demonstrated through the children's higher than expected achievement in mathematics, and corroborated in a mathematics subject inspection by the government's Ofsted inspectors, who judged almost every aspect of mathematics teaching and learning in the nursery as "outstanding" (the Office for Standards in Education (Ofsted), 2010; see also Knowles, 2017, p. 52-55).

The teachers collaborating in this study were asked to identify several children whom they knew often chose to communicate through their own graphical texts, identifying Isaac and Shereen (both 4 years of age) and Elizabeth (3 years, 7 months), these three children are referred to as *focal children* in the study. To determine if the children's previous interest and

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[60] These "key persons" are qualified teachers and other early years' professionals.
[61] This period covers the time in which Elizabeth Carruthers was headteacher of the nursery school.

experience of communicative interactions through graphical sign-use were significant, the teachers were asked to randomly selected four additional children (Oliver, David, Ayaan and Tiyanni). The children were all in their final year at nursery, their ages ranging from 3 years, 2 months, to 4 years of age at the onset of the academic year.

Data Sources, Collection and Analysis

The teachers' daily practice is to document all aspects of children's play and learning in each child's learning diary. The children's oral communications and specific language they used within their play and relating to their inscriptions were recorded. Photographs of children's notations provided the visual data, and to ensure maximum validity, were analysed together with the written documentation.

The first author coded, modified and refined coding as the data were processed. Computer assisted qualitative data analysis software (CAQDAS) was used to locate the data for this question. Data were open and derived from the field; elemental coding employed to identify the children's mathematical inscriptions, which were then coded as a distinct set of data for the purposes of this study. Data were examined from several perspectives, looking at the premature stages of dealing with quantity and their relationships. The data are interpreted from naturalistic interpretive methodologies, with Dahlberg et al. (2007) sharing a view of the child as "a rich child, active, competent and eager to engage with the world" (p. 7). The previously mentioned research question can now be elaborated as follows:

RESEARCH QUESTION (reformulated): *What evidence of intertextuality is there in the children's graphical inscriptions of tallies and numbers made in contexts that can be understood as mathematical?*

Ethics

Engaging in research requires that researchers attend to ethical considerations, and research with young children requires special sensitivity. The ethical principles of the British Educational Research Association (BERA) (2011) and those of Vaste Commissie voor Wetenschap en Ethiek [Scientific and Ethical Review Board, 2016] Vrije Universiteit Amsterdam, provide clear guidance concerning informing and consulting with participants (children, teachers and parents).

As part of the teachers' established practices, observing the children and collecting data were already familiar to the parents, who readily gave written consent for data collection for this research. Using everyday language, I explained the study to the children and sought their permission to observe and involve them. It was envisaged that some of the children's graphics would later be published and this was also explained to the children and parents, and their consent given for this study. The children's oral communications and specific language they used within their play and those relating to their inscriptions were documented.

The parents were advised that they could withdraw their child from the research at any point: one family did so early in the period of data collection and none of the data pertaining to this child have been used. With the parents' and child's consent, I took several photographs of each child's graphics (if any) during a home visit, and made short written notes of information provided by the parent regarding their child's play and graphics at home.

Results

Examination of table 5.1 enables individual children's graphical inscriptions to be traced over a period of time, and indicates a diffusion of signs between children and the teacher. What the children's signs in this study are *not* able to show, is the multiplicity of signs the children employed in other contexts such as drawing, maps and writing. The implication of the children's numerous and varied use of graphical inscriptions is that *all* of the children's graphical signs flowed among and between the children, influencing their choice of signs and impacting on those they chose to use in *mathematical* contexts. showing how early marks and signs migrated from one text to another.

Table 5.1 shows the teachers' documented observations of the children's play, and their use of tallies the children used to communicate their thinking about counting one-to-one.

Table 5.1

Examples of children's intertextual use of signs signifying tallies (n=8 occasions)

Month	**Description from documented observation of the children's play**	**Intertextual connections**
Septem-ber	David (3 years, 9 months) made small, tally-like marks on paper, counting, "*one, two, three, four, five, six, seven, eight, nine, ten, fourteen, fifteen*."	Following the teacher Emma's modelling of tallies, David was the first to use tally-like marks.
March	Ayaan (3 years, 11 months) now uses some recognisable letters (from the Latin alphabet) when writing her name. In this example (figure 5.1), pointing to each of the letters "A" she'd written (and using them as tallies), she named the six members of her immediate family (writing two more "As" after this photo was taken).	Ayaan used a sign she knew well (the first letter of her name) as tallies. She appeared to have taken on the *idea* of tallies from others, but not yet the tally marks modelled by her teacher into her personal sign lexicon.

March	Seeing Elizabeth's "register" this morning, Tiyanni (3 years, 11 months) made her own. Tiyanni's teacher Hugo asked if she had everyone written down? Counting her letter-like signs she replied, "*There's two more* [to come]."	Tiyanni employed letter-like signs. Her teacher had modelled letters and numerals in various contexts, and in this instance, it was these that she adapted, maintaining the idea of tallying.
April	Shereen (4 years, 7 months) drew tallies as shorthand for numbers of items on a shopping list, counting each line with confidence up to 20 (figure 5.2). She also wrote the numeral "6", though did not explain its meaning.	This was the first time that Shereen had used tallies, blending those she'd seen others use, to express the meanings of her list.
May	Isaac, David and Jaydon (3-4 years) are by the door into the nursery with clipboards, paper, pens and a calendar, checking people in and out. They used scribble-marks, crosses and a tick, Isaac using vertical marks as tallies for people who come in and out. On the same occasion Isaac (4 years, 8 months) wrote an "x", explaining *"that means you work here."*	The boys adapted the concept of tallying, (modelled by their teacher, and intermingled with signs from their peers). In addition to crosses and ticks, Isaac used tallies, perhaps borrowed from David or Shereen.

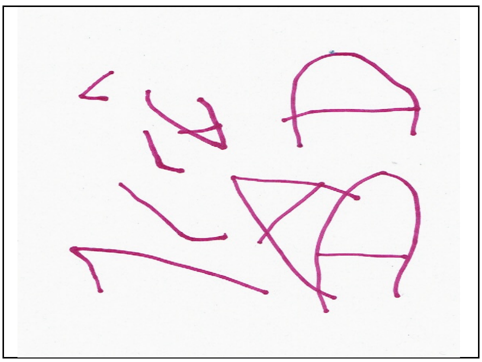
May	Isaac (4 years, 8 months) decided to use a diary as a "*booking book*" for a campsite, explaining that two people were staying, and making two marks in the diary. Isaac then used the phone to take more bookings, telling Oliver "*One hundred million people are staying!*" Oliver said "*I want to stay for two nights*." But Isaac replied, "*No. I'll put you down for two million nights, but don't worry - it's only £1.00 a night.*" Isaac then wrote it down in his "booking' book", this time making many marks. During the same pretend play episode, Oliver (4 years, 8 months) also took a diary and made his own signs (circles and vertical lines) to signify campsite bookings.	Isaac's first tallies signified two people who had booked to camp, the two marks (like tallies) matched one-to-one. Isaac's fascination with large numbers (*one hundred million people*) led to him to freely make numerous small marks. Oliver decided to use his own signs as tallies, neither Isaac's many marks nor Oliver's signs corresponding to a specific quantity, both boys' tallies suggesting intertextual exchanges.
May	Playing shops, Oliver (4 years, 8 months) asked Emma, "*What do you want me to buy?*" Emma asked for "*four apples*" and he made four small marks.	Oliver has understood the tallies Emma modelled, and used one tally for each of his four apples.

July	In the forest, the children arrived at a curious shelter, with a wooden pallet in the doorway. David thought an elephant lived there. Some other children joined them, and Shereen (4 years, 10 months) watched as the children went inside, then drew elephants and tallies on her hands, as she counted children going into the shelter. She counted the marks on her hand up to 10, and showed everyone the marks.	Shereen again used tallies to count 10 children, integrating others' use of tallies she had seen and repeating her own use in April.

Other children used alternative signs to tally, including circles, crosses, ticks and letters. For example, Ayaan's tallies (figure 5.1) show how she fused her understanding of tallies with her knowledge of the first letter of her name, show how this single letter subsequently moved across diverse contexts and left successive traces. Ayaan's first language is Somali, and at this time Ayaan's aunt was teaching her the Arabic alphabet at home. It is not known why she used ticks in this example, however young children do not always explain their graphics, added to which Ayaan was just beginning to speak English at this time.

Figure 5.1

Ayaan tallies the members of her family



Together with the capital letter "A" of her name, ticks and crosses were the first formal signs Ayaan used. Ayaan also used the "A" of her name at other times, and in other contexts[62] in a succession of texts. For example, in July Ayaan when playing families, Ayaan represented a "television" by combining two capital letters "A" connected by lines to form a grid, explaining "*my baby need TV*", then announcing "*CBeebies still on*"[63]. She then stuck her "television" on the wall, and taking a book of raffle tickets, pressed the numbers "4, 1, 9, 4" to change channels. In July, Ayaan again represented a "television", this time writing several letters "A", interweaving them with a series of other letter-like signs, ticks and crosses. Ayaan's frequent use of the letter "A" suggests Clay's (1975) *recurring principle*, "where children use any letters and words they know, over and over again" (Carruthers & Worthington, 2006, p. 63). It is interesting also to note the growth of the children's interest of using tallies in May, tallies disseminating rapidly amongst them in cascades.

Number and Quantity: Tallies

The children's heterogenous signs for tallies shows that on almost every occasion the children understood tallies as one mark for one item or person counted (i.e., one-to-one correspondence). It is worth noting that in their use of tallies, the quantities represented almost always corresponded with the number of items or people counted.

The vertical marks as tallies used by some children (not yet bundles of 4 lines crossed by a diagonal line), may have originated from their teacher's modelling, then circulated among several children. The findings show that the choice of vertical marks as tallies exceeded all other signs the children used to tallies. For instance, figure 5.2 shows Shereen enumerating items on her shopping list.

[62] Ayaan sometimes made other writing-like marks and signs, referring to them as her name.
[63] CBeebies" is a popular children's channel on television in England.

Figure 5.2

Shereen's shopping list

Table 5.2 provides examples of children's intertextual use of signs signifying numerals, from a range of contexts.

Table 5.2

Examples of children's intertextual use of signs signifying numerals (n=17 occasions)

Month	Description from documented observation of the children's play	Intertextual connections
October	Elizabeth (3 years, 8 months) rolled a piece of paper into a cylinder and wrote part of her name on it with a standard number "4", explaining *"I'm going to be four soon 'cause it's my birthday soon."*	Elizabeth knew and used a range of numbers, uniting the numeral "4" on the birthday party invitations she had recently made at home.

November	Noticing some small numerals painted on the garden path, David (3 years, 10 months) decided to write a standard numeral "3".	David's teacher frequently modelled numerals in meaningful contexts, added to which he could see a series of numerals painted on the path, (including a "3"). Furthermore, David knew how to write the "3" of his age.
December	Oliver (4 years, 3 months) spent time drawing, *"It's a puddle, a bee, a number one"* (referring to the numeral "1" he'd written) and then next to it he wrote a zero, saying *"It's a number 10 now!"*	Oliver may have recalled the number "10" his teacher had modelled, and noticed this number from one of the number lines displayed in the nursery.
January	Shereen (4 years, 4 months) wrote numbered "buttons" (1, 2, 3, 4), reading *"one, two, three, four".* Then wrote several letters above, reading *"drawing".*	Shereen's numerical symbols circulate between her different texts at home and in the nursery.
January	Shereen (4 years, 4 months) drew a circular clock, with numerals from one to twelve in order around the clock face.	Shereen's intertextual use of numerals shows a succession of traces within her own graphical texts.

February	During "Talk Time"[64] featuring kitchen weights and bathroom scales, David had stood on the bathroom scales. Isaac (4 years, 5 months) used his understanding of a variety of measuring units to talk about David's weight. As he made circular scribble-marks on paper, Isaac explained, "*David weighs 700 kilos, he's 60 metres heavy."* Next David (4 years, 1 month) stood on the scales, and looking at the dial he announced, "*I'm 15, so I need to write it down*." He made some letter-like signs (as numerals) on the whiteboard.	Isaac's scribble marks were indicatory, whereas David's intention was to represent the numerals for "15", showing intertextual links with some of the letters he had seen his teacher and peers write.
February	When playing builders, Isaac (4 years, 6 months) wrote a reversed letter "a" on a pretend cheque, to signify *"£500.00*".	Isaac wrote a letter-like sign he knew, generated by his teacher's modelling and his peers' writing.
March	Tiyanni (3 years, 11 months) and her friends went into the garden to the gazebo, "*it's our house"* Tiyanni said. Her teacher asked if it had a number outside? Tiyanni	Tiyanni has observed her teacher modelling, and other children write numbers. Drawing intertextually on these, this was her thoughtful

[64] The nursery school had instigated "Talk Time", "where the focus is on conversation" (Carruthers, 2012, p. 31), and where drawing and writing resources are always available: often the focus of this special time relates to an aspect of mathematics.

	replied "*the number needs to go inside*" and wrote her numbers on the wall in chalk (figure 5.3), explaining "*that's the number eight*" (pointing to the large, almost enclosed circle on the right of the photo) "*and the other number's nine.*"	approach to writing a two-digit number.
March	When taking pretend food orders in the group, Shereen (4 years, 6 months) represented the various items her friends ordered as wavy writing-like lines and swirling, vertical scribble-marks, and wrote the standard numeral "8".	Shereen drew intertextually on her personal rich sign lexicon of numerals in writing the standard numeral "8".
March	Shereen (4 years, 6 months) was playing cafés, and taking orders for meals. She used several different sign-types during this play episode, including a drawing of a mushroom, but it was only the cross and the tick that related to quantities.	A number of children have used ticks and crosses, sometimes modelled by their teacher. In this instance, Shereen used a cross to indicate "no" or "none", her cross traversing intertextually from other crosses in non-mathematical contexts.
April	Isaac (4 years, 7 months) filled a register by making many marks on the page, and showing Emma he explained, *"Emma, there are ten thousand million people in purple group."*	Several children used small marks to signify a quantity (either uncounted or counted) and Isaac appeared to have borrowed this idea.

May	Shereen (4 years, 8 months) started a play theme of going to the café, using drawings (including a specific number of cakes). This led to her discussing with David, the number of cakes that had been sold, as she rubbed one out each time (chapter 4, figure 4.5). Soon after this, David (4 years, 4 months) responded to this by drawing himself and Shereen in the café. He made small marks, referring to them as, *"two cakes, coffees, hot coffees, cold coffees, crisps."* David asked Shereen to visit his café and she gave him an order for one cake and a cold coffee. David said, "*Here you go, I have to rub them away now, cos they've gone from the café*."	Appropriating Shereen's idea, David used small marks intertextually (as Isaac had done) to signify named items of food in his café, rubbing them out to "take away" (subtract) items.
May	Oliver (4 years, 8 months) wrote zigzag, writing-like lines, letter- or numeral-like signs and a numeral "3" on post-it notes (figure 5.4), and stuck them on a paper bag.	Trying out various graphical signs, Oliver included a numeral "3" that he had seen his teacher and some of his peers write.
May	On the first author's visit to Shereen's home, her mum invited me up to Shereen's bedroom, and	Shereen's interest in alphanumerical signs persists at home, moving intertextually from one

	Shereen (4 years, 8 months) pointed to where she'd written numerals one to nine on a cupboard door, with drawings of three stars beneath.	context to another.
May	Elizabeth (4 years, 3 months) admired her brother's "Super Mario" game and decided to make her own (figure 5.5). Her game had a screen with a drawing of the character of Super Mario. Below it she'd drawn buttons with numerals, "3, 2, 1, 2, 0, 2, 2, 0, 0", some reversed.	Elizabeth drew on numerals she'd seen her teacher model, and in this instance, wove together the numerals she'd read on her brother's game.
June	Inside a pointed shape she'd drawn, Elizabeth (4 years, 4 months) wrote a line of letters and numerals "i, i, 2, 1 4, 9, 2, 6, 7", remarking "*He's amonster with one eye.*" Hugo asked her to spell the symbols out and she read, *"a, a, c, l, l, l."*	Elizabeth has seen her teacher model both written and numerical texts, and seemed to be thinking about the differences between the two systems.
July	Elizabeth (4 years, 5 months) sat next to Cameron in the gazebo, both with large diaries. She drew a line of standard number symbols across the top of the left-hand page, including "5, 6, 7, 8, 9". On the right she wrote "2, 0, 3, 5, 1, 2, 1, 5, 2, 2, 0, 9", some of the numerals	This example was the first time Elizabeth had explored the concept of written "teen" numbers, integrating her cultural knowledge from home to expand her existing knowledge of numbers.

reversed (figure 5.6). She read pairs of numerals as "teen" numbers, beginning to represent two-digit numbers in a pattern.

Children's Intertextual Use of Signs Signifying Numerals

Influences from Writing

The intertextual traces in children's inscriptions showed evidence of a range of graphical responses, many intersecting with the children's emerging understanding of writing. As in the findings of Carruthers and Worthington (2005; 2006) and the current study, Ferriero and Teberosky (1979) also identified that some children used *letter-like signs, letters to signify numerals* or *numeral-like signs* in their progress towards standard numerals, the connection between letters and numbers developed though several important conceptual stages. The influences of written words and texts seemed strong, and whilst several of the children's inscriptions in table 5.2 included wavy or zigzag writing-like lines to signify the appearance of writing, these seemed to be brief placeholders to reduce interruptions in their play, rather than signifying quantitative meanings or numerals. Tolchinsky (2003) found that whilst young children have extensive knowledge of differences between the systems of alphabetical and numerical signs, this "does not preclude their crossing the frontiers of the respective territories […] to fulfil communicative purposes [...] When formal distinctions [are] relaxed, boundaries between systems [are] opened" (p. 159/181).

Standard Symbolic Numerals, *including reversed numerals*

Figure 5.3 shows Tiyanni had written the number "89" for her house number, writing her large numeral "8" on the right, and then added the "9" on the left, Clay (1975) referring to a *directional principle,* "the starting point [on a drawing surface] appears to be critical" (p. 25). However, her understanding of this two-digit number was clearly transformed by convergence with other texts from her teacher and peers.

Figure 5.3
Tiyanni's door numbers

Figure 5.4 shows Oliver chose post-it notes and a paper bag. He used a variety of means to signify writing, his numeral "3" reflecting the fact that he was only just 4 years of age. His numerical symbol had also blended signs from others' inscriptions, including a numeral his teacher had modelled and David's writing of "3" in the garden.

Figure 5.4

Oliver's post-it notes

Elizabeth's "Super Mario" game (figure 5.5) combines free play and graphical signs (some of her standard numerals reversed): however, it seems to go further, allowing not only the "buttons" on her game to be pressed, but the lid to be opened and shut. Elizabeth showed her game after she made it, and in its creation, it seemed to be the making of it that was more important rather than any pretence to play it.

Figure 5.5

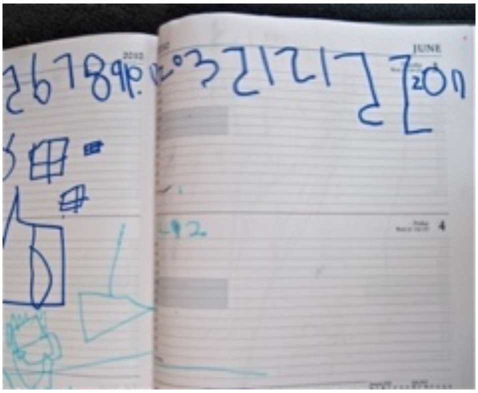
Elizabeth's "Super Mario" game

Figure 5.6 (below) shows the first time Elizabeth had explored the concept of written "teen" numbers, suggesting that she understands that the teen numbers require two digits, as she read them (from the left-hand page and across the right) as "*13, 12, and 15*", which may have intersected with written numbers at home. Lai and Fung (2018) point to the difficulties young children have with fluently counting the teen numbers. By July Elizabeth had confidently and freely used numerals to 10 in many diverse contexts, and proudly wrote the number "100" twice. Tolchinsky (2003), (and many other authors) maintain that "children's realization and the meaning of tens and units is only very slowly generalized to hundreds and thousands" (p. 126), yet from 3 and 4 years of age, some children are fascinated by infinity, as large numbers as Isaac's references to "*ten thousand million people in purple group"* and *"one hundred million people are staying!"* show.

Oliver was another child who wrote a teen number, beginning with writing a "1", then adding a zero announced, "It's number 10 now!", and Tiyanni's "89" for her house number the other. These examples show that for the children who have begun to explore numbers beyond nine, their symbols interweaving with each other.

Figure 5.6

Elizabeth's "teen" numbers



Of the case-study children, Shereen and Elizabeth (two of the focal children), used most signs across all their literacies and wrote many standard numbers, Shereen representing her mathematical thinking most often. Shereen and Elizabeth also engaged most frequently in graphical inscriptions in their home scrapbooks[65], frequently including standard letters and sometimes numerals, which they had used previously in a variety of situations. This suggested that many of their signs may have iterated from the children's families (e.g., parents, siblings).

Processes of Mathematisation

The traditional view of children's "written" mathematics is that they move from their informal marks and signs to the formal abstract signs of

[65] For the purposes of this research, the children had been given scrapbooks to use at home (for drawing, writing and other graphics), but their frequency of use and the range of their graphics varied. The data from the scrapbooks were therefore less reliable for analysis, although they did reflect some of the range of children's choices and interests relating to their graphicacy at home.

mathematics as a one-way journey. However, Sophian (2007) writes that when children accept symbolic methods without understanding, solving problems with graphical signs become susceptible to errors, and may often be implausible. Hughes (1986) identified a "dangerous gap" between children's concrete experiences of mathematics and use of informal signs, and formal written symbols and strategies, a rift also established by Treffers (1991), who proposed that teachers' "strong models" can provide children opportunities to "bridge the gap between informal, context-bound work and the formal, standardized manner of operation through the constructive contribution of the children themselves" (p. 33), but as we have seen, children's own models also play a powerful role in young children's developing understanding of graphical signs.

Laland (2017) underscores the fact that cultural evolution is not linear, that is to say, learning does not "progress from simple to more complex over time" (p. 284). Pascal et al. (2019) cite Göbel et al., (2018) who emphasise the importance of a "holistic approach to teaching mathematics", that recognises "the *non-linear* nature of children's learning", cautioning against traditional methods of mathematics education, "where this holistic nature of learning can become overlooked" (p. 34, emphasis added). Vygotsky (1978) believed that:

> child development is a complex dialectical process characterized by periodicity, unevenness in the development of different functions, metamorphosis or qualitative transformation of one form into another, intertwining of external and internal factors, and adaptive processes which overcome impediments that the child encounters. […] To the naive mind, revolution and evolution seem incompatible and historic development continues only so long as it follows a straight line. Where upheavals occur, where the historical fabric is ruptured, the naive mind sees only catastrophe, gaps, and discontinuity. History seems to stop dead, until it once again takes the direct, linear path of development. (p. 73)

Tomasello (1999) emphasises that mathematics is likely to have developed over historical time by "cumulative cultural evolution" (p. 37). He argues that in ontogeny, cultural learning creates, "especially powerful forms of social-collaborative creativeness and inventiveness, that is, processes of sociogenesis in which multiple individuals create something together that no one individual could have created on its own" (p. 6).

On the basis of our studies, we interpret this movement as a process of intertextuality through which formal signs (borrowed from others) are woven into the children's personal texts. Vygotsky's (1978) observation that "between the initial level (elementary behaviour) and the higher levels (mediated forms of behaviour) *many transitional psychological systems occur*" (p. 46, emphasis in the original) appears highly relevant to this study.

Vygotsky observed that transitional signs are "born from the interweaving" of children's informal signs and those "of sociocultural origin" (p. 46). It must be noted here that for Vygotsky the development towards higher forms of thinking is not a case of replacement of the informal marks by formal mathematical signs, but are essentially *transformations* of spontaneously developed informal thinking.

Hence, we conjecture, that young children increasingly chose to integrate into their textual communications the formal signs of the language of mathematics borrowed from more knowledgeable others. Inscriptions acquire modifications to more advanced knowledge over time as understandings become increasingly complex and integrate with children's informal (everyday) concepts and, therefore, require more powerful and sophisticated communicative means to exchange and elaborate this advanced knowledge.

Discussion

This study sought to investigate the mathematisation of young children's freely made and spontaneous inscriptions used to communicate mathematical ideas, showing how their signs gradually transform into formal signs using the abstract symbolic language of mathematics. The examples of children's mathematical sign-use, is, like their signs in drawings, maps and writing, embedded in meaningful social practices, findings that challenge any separation of sign-use in literacies, into writing, drawing, or of a single skills-based mathematics in early childhood (Worthington & van Oers, 2017).

The research question asked, *what evidence is there of intertextuality, particularly with respect to the use of mathematical signs in the inscriptions of children made in nursery school?* The findings identified examples of intertextuality, signs circulating between texts and crossing to another's inscriptions. The intertextual sharing of signs showed that the children made use of quantities, number, order and relationships, realising mathematising *in statu nascendi*. I see this as an indication of growing mathematisation and development in mathematical thinking. Iterated signs enabled the children to appreciate the different contexts in which others used signs, broadening their understandings of sign-use and adding to children's sign lexicons. In the context of this study, children do not imitate disciplined mathematical activity as such, but everyday *cultural* activity, whilst occasionally using number, quantity, order, relations (or other mathematical-like objects) in their own ways. Through their pretend play children get access to any cultural activity (including mathematising) by emulating this activity.

In relation to the overarching ambition to promote mathematising, the results suggest a movement towards Clements and Sarama's (2009) description of mathematisation as "a critical learning process, involving redescribing, reorganizing, abstracting, generalizing, reflecting upon and giving language to that which is first understood on an intuitive and informal level" (p. 244). For young children the beginnings of mathematisation are rooted in the cultural (adult) sign-use that they integrate in their own texts, and through collaborative activity with others: mathematisation encompasses creating and using symbols and is achieved through negotiating meanings. Only later do children learn to recognise their own thinking, speaking and graphical signs and texts as mathematical.

Van Oers (2014) explains that within children's "genuine" experiences of play, "mathematising is provoked and encouraged in children as a way of dealing (collaboratively) with the quantitative and spatial dimensions of reality which surface during their participation in engaging and meaningful cultural practices" (p. 115). The children's inscriptions embedded in their play episodes (exemplified in tables 5.1 and 5.2), demonstrate "how mathematising emerges in play on the basis of learning how to communicate about number" (p. 118). Children employ graphical signs within contexts in which they refer to quantities, number or measurement.

The children's pretend play episodes clearly provided valuable social contexts, allowing the children to explore their cultural knowledge and thinking though dialogue and through their inscriptions, prior texts of others facilitating intertextual transformations.

Limitations

We believe that our study adds significantly to understandings of the intertextual process in children's mathematisation. However, the conclusions need to be taken cautiously, due to the limitations that necessarily inhered to our small-scale qualitative study. The number of children involved in the study, and the extraordinary, well-developed expertise of the teachers in this nursery school limit the external validity of the outcomes. Future studies have to be undertaken to enhance the generalisability and replicability of our model of young children's mathematical development. Such studies should focus on teacher professionalisation to implement the described classroom culture that allows children high degrees of freedom to invent new communicative means and explore the meanings of their texts. Further studies with a larger group of children should then be conducted to see if the conjectured dynamics can be repeated and will enhance children's development of mathematical thinking.

The use of symbolic tools to represent thinking is a significant aspect of mathematics and should be accorded a more prominent role in early childhood mathematics education. There is potential for this study to be replicated in other settings, provided that a similarly open approach to children's inscriptions is employed, and that staff value and understand children's meaning making and engage in appropriate pedagogy to support their early graphical communications.

Learning does not, in our view, depend on direct teaching of graphical signs and elementary operations with quantities, but on learning how to take part in the activity of mathematising and problem solving regarding varying quantities, suggesting that intertextuality plays a significant role in this. Carpay and van Oers (1999) propose, "the ultimate goal of any learning activity should be to establish *a new personalized* mode of speaking about the world (i.e., a new "narrative")", (p. 307, emphasis in the original), something that this study has sought to do.

Over a period of more than 20 years, research by Carruthers and Worthington, has demonstrated the value of this approach for young children, but until now a clear relationship between young children's own beginning marks and the formal abstract symbols and strategies of mathematics had not been established. Investigations in the current study have highlighted the significance of children's intertextual links in their increasing movement towards formal symbols, intertextuality appearing to play a significant role in supporting young children's semiotic understandings and advancing mathematisation.

It is not that children lack the willingness and interest to make personal sense of reality, to make connections with their cultural knowledge, or to communicate their thinking about aspects of mathematics in meaningful ways, but that opportunities to do so in England are often severely circumscribed, limiting young children's explorations and communications. This is even more acute for children of 4-6 years of age in English schools. Young children are powerful thinkers and the "gap" between children's concrete knowledge of mathematics and formal mathematical inscriptions may yet be bridged, provided their learning cultures support their meaning making and graphical communications.

- 6 -

Conclusion

> It is not understanding that generates the act [of writing], but far more the act that gives birth to understanding. (Luria, 1983, p.193)

Introduction, Research Questions and Hypothesis

Mathematics is widely considered to be an important aspect of learning in childhood, and the standard, culturally agreed symbols of mathematics are a significant aspect of this subject discipline. Indeed, van Oers (2000) maintains "the efforts of pupils to get a better grip on symbols in a meaningful way should be considered one of the core objectives of education, especially in the domain of mathematics" (p. 136). "Graphicacy" and the need to use signs and symbols in early years' mathematics are seldom the focus of curricula in the early years or in schools in England (and in many other countries). Accordingly, the focus of this thesis is a compelling one, given the significance of semiosis in mathematics.

The principal objective of this thesis is to identify where in ontogenesis the cultural foundations of mathematical inscriptions originate. Due to the social and cultural nature of young children's mathematics, the intention has been to investigate these origins within the children's social pretend play and other "open" contexts, and to identify some of the factors that contribute to their success. It also aims to establish whether children's existing interest in freely communicating through graphical signs for drawing or writing at 3 years of age, could contribute to the inscriptions they use to communicate their *mathematical* thinking during the course of the subsequent year. Central to this is children's meaning making in contexts that could be understood as mathematical. To achieve this, I investigated the social pretend play of seven children of 3-4 years of age[66] in their nursery school during the course of one year, interrogating the mathematics to which they freely referred, and the marks, signs and symbols they used spontaneously for various communicative purposes. Commencing the study, the main questions were:

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[66] The children were 3-4 years of age at the onset of the year in which data were gathered.

- *What evidence of mathematics can be found in the children's free pretend play, and how does their cultural knowledge influence their thinking?* (Chapter 2)
- *What early graphical inscriptions do young children of 3-4 years of age spontaneously employ in the context of various literacies in their nursery school, and to what extent does their personal cultural knowledge strengthen their understandings?* (Chapter 3).
- *How do the children's inscriptions support their emergent abstractions?* (Chapter 4).
- *What evidence is there of intertextuality, particularly with respect to the use of graphical signs the children made in contexts that can be understood as mathematical, and how does this impact on their mathematisation?* (Chapter 5).

This naturalistic study began by inviting two teachers in a nursery school in England, to select three children who had been observed to often freely choose to draw or write. My assertion was that those showing this interest would also continue to engage in graphics for various purposes during the year, including those made in contexts judged as mathematical, (without the children themselves framing this as such). These children are referred to as "focal children". The teachers were also asked to select four other "non-focal" children. The data largely comprised the teachers' documented observations of children's pretend play and other open contexts (supplemented by some I made), and photographs of the children's graphics. Additional data included field notes and visits to the children's homes that helped validate aspects of the data, and drawings and writing in the children's "home scrapbooks"[67]. The data were analysed using *computer assisted qualitative data analysis software* (CAQDAS), employing coding specific to each of the four main study chapters, with findings verified by a critical friend (a doctoral student) and colleague.

The research questions are answered in chapters 2-5 of this thesis. I anticipated that due to the rich social nature of pretend play in this particular nursery school, and the opportunities it affords children to explore their cultural "funds of knowledge" (Moll et al., 1992), that their spontaneous play narratives would provide potentially meaningful and authentic contexts for them to communicate their thinking through graphical inscriptions. In respect of mathematics, I anticipated that such contexts would often prompt the children to explore diverse aspects of

[67] Since only four of the seven children used their home scrapbooks, their combined content did not provide sufficient reliable data for systematic analysis.

their cultural knowledge, (without the children themselves framing this as mathematics), and rouse their need to communicate about this, and that subsequently their available mathematical inscriptions might enable them to build on and extend that knowledge.

This is a naturalistic study with data gathered largely from the everyday contexts of a nursery school. Following the teachers' established practices, no interventions were employed. Our approach took a sociocultural and social-semiotic perspective (Vygotsky, 1978). Our interest throughout this study has been to determine the evolution of young children's graphical signs and texts, chosen and used freely by them to communicate ideas, in the context of situations that may be understood as mathematical, through following Vygotsky's genetic approach (1978).

Graphical sign are symbolic tools that mediate understanding (El'konin & Vygotsky, 2001). By making their internal representations external, this allows visual feedback, enabling them to reflect on the signs and their meanings. External representations also allow their signs to become the focus of discussion with peers and teachers. The children's increasing quantity of heterogeneous signs and symbols extends their semiotic repertoires, promoting greater adaptivity in sign-use when faced with new and challenging communicative demands or problems to solve. Van Oers (2002) explains:

> mathematical activity is basically a special form of semiotic activity, i.e., an activity of reflecting on signs, meanings and the interrelationships between signs and meaning. Semiotic activity occurs in both play activity and learning activity, but in different forms with respect to their regulation, and with different levels of strictness and consciousness (p. 32).

Vygotsky wrote that full understanding of "higher psychological processes" (such as graphical signs), can only be thoroughly understood by establishing their origins and charting their history: transformations of sign-use "are historical in nature" (pp. 45-46). Tomasello (2019) contends that the "higher cognitive functions" to which Vygotsky referred, are not the consequence of individual learning, "rather, the result of humans' ability to create and internalize social practices, especially those concerned with the use of cultural artifacts and symbols" (p. 301). Tolchinsky (2003) emphasises that: notational systems are not only communicative tools that serve to convey a particular content, but they are also epistemic tools. In line with many others (Clark, 1997; Lee & Karmiloff-smith, 1997; Olsen, 1994; Vygotsky, 1986), I consider notations to be objects-to-think-with. […] On the one hand [graphical inscriptions] enable more efficient thinking for handling our thoughts. On the other hand, our cognitive functioning is "in a large part shaped and changed by the representational artifacts we ourselves create" (Wartofsky, 1979, cited in Wells, 1999). Furthermore, because work on our own thoughts is one of the sources of cognitive change, external representations as tools for cognitive change (p. xii/xxii).

Especially important for this study is an emergent perspective, acknowledging young children as *emergent learners*, their nascent understandings and use of graphical inscriptions in some ways similar to babies' intuitive desire to communicate with others, and their early acquisition of speech. The children's emergent thinking is informal, and as referred to by Ginsburg (2006) regarding everyday mathematics, it is "ubiquitous, often competent, and is more complex than is generally assumed" (p. 145). Fleer (2010b) argues that in "early childhood education one is presented by a dilemma – whether to guide and educate young children in relation to already established values or whether to give children room to become people in their own right" (p. 1). In England and globally, mathematics curricula and related teaching practices have become increasingly narrow, young children's personal mathematical inscriptions seldom acknowledged, understood or supported, and learning commonly teacher-planned and led (Carruthers, 2015; Moffett & Eaton, 2018; Williams, 2016).

Matthews (2006) raises concerns about educational initiatives that "merely add to the damage wrought upon children's emergent symbolization" (p. xiv). A growing threat to young children's meaningful learning in England is the increasing "schoolification" of teaching children up to 6 years of age (e.g., Bingham & Whitebread, 2018), which, according to the Organisation for Economic Co-operation and Development (OECD) (2006), has a tradition of early education focusing "strongly on cognitive development, early literacy and numeracy" (p. 136)[68] often teaching narrow skills through transmission approaches. This includes the growth of "Teaching for Mastery" (e.g., Boylan et al., 2018), an imported approach from Shanghai that is increasingly influencing nursery schools in England. "Teaching for mastery" (or "maths mastery")[69] requires whole class

[68] This has not always been the case. Since early years' pioneers such as Susan Isaacs promoted rich child-centred nursery education in England, her work accumulated interest and a positive reputation for nursery education over many years (Giardiello, 2013). The current Maintained Nursery Schools in England provide unique early years' provision and are government funded.

[69] The government in England has invested very considerable sums in introducing and supporting teachers in this approach. This has included funding to cover the cost of Chinese teachers coming to England to provide demonstration lessons in English schools, and of sending teachers from England to observe lessons in China (DfE, 2019).

teaching and repeated lessons, in which larger learning goals are broken down into smaller steps (*True Education Partnerships*) and children are seated in rows for mathematics (Boylan, 2019, p. 17). For Unger (2005) the government's support of this approach amounts to a "dictatorship of no alternatives" (p. 1)[70].

Several studies have investigated students' use of their own mathematical models, e.g., van Oers (e.g., 2010; 2013a) and Poland (2007) on schematisations for children of 5-6 years; Terwel et al., (2009) for children of 10-11 years; Pape & Tchoshanov (2001) in the context of high school students; and Carruthers and Worthington (2005; 2006) into *Children's Mathematical Graphics* (CMG), for children of 2-8 years. However, until now, no one has systematically researched *the very beginnings* and early development of young children's personal use of mathematical signs and symbols in depth. Since children's learning is socially constructed from the beginning, such an oversight makes this study imperative.

Children's early understandings of mathematics and sign-use arise in social contexts such as pretend play: their earliest marks are significant indicators of their genesis and development towards the abstract symbolic language of mathematics. My main interest in semiosis originates from my determination (with my colleague Elizabeth Carruthers), to understand children's beginnings and development with mathematical inscriptions and to identify ways that pedagogy might better support children's understandings and use of formal mathematical inscriptions. Matthews (2006) contends:

> Those of us who have devoted our lifetimes attempting to understand the origin and development of expressive, representational and symbolic thought in infancy and childhood, and how best to support it, quickly came to realize that the beginnings of linguistic and mathematical thought are embedded in rather commonplace actions and drawings made by the infant and young child. […] developmentally, these beginnings are of the most profound importance. (p. xiii / xiv)

At the onset of this research, I aspired to locate early years' settings in which the children freely engaged, in contexts that can be understood by adults as mathematical in their pretend play. However, I found little evidence of rich or sustained pretend play, and no indication of mathematics (in any pretend play that *did* occur) during more than one

[70] Unger's comments suggests that teachers feel that they have no alternative but to take on this approach, especially when their headteacher directs them to do so.

year of visits to a dozen schools. Bingham and Whitebread (2018) highlight the conflicting messages from the early years' curriculum in England that:

> acknowledges the "informality of the learning experience" in pre-school settings, yet calls for learning outcomes that can only arise from the formal teaching of basic literacy and numeracy skills […] The requirement for formal learning outcomes to be achieved is rigidly enforced via a draconian inspection system carried out by the Office for Standards in Education (Ofsted). (p. 373)

This formality impacts on pedagogy in the early years, and has negative repercussions for children's experiences and opportunities. Rather than viewing young children's mathematics from a single, subject-based discipline, this thesis takes *the child's perspective* through a holistic, transdisciplinary approach, drawing on research into early childhood mathematics; semiotics; pretend play; social learning; cultural knowledge; children's social literacies; multimodality; cultural evolution and language acquisition.

Increasingly my attention has been drawn to the identified and intertwined political issues, which continue to impede pedagogical strategies that might strengthen children's understandings of the abstract graphical language of mathematics. Rather than the widely employed transmission teaching of what is commonly referred to in England as "recording"[71] or "written mathematics", the strength of the approach outlined in this thesis, is that it appears to assist children in connecting their understandings of mathematics in concrete situations, with the formal symbols of the culturally accepted system of mathematics. Related to this has been my growing concern that mathematical practices in early years' education in England seem to be increasingly antithetical to evidence from research into young children's play (Wood, 2019) and into mathematical semiosis (Presmeg et al., 2016).

In the following section I highlight the findings of our studies. The theoretical gains of our research and implications for educational policy and pedagogical practice are discussed, and finally the limitations of this study and directions for future research are presented.

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[71] The use of the words *recording* (or *written maths)* in England are used extensively to refer to copying symbols and written strategies provided by the teacher, or writing something they have already done following a practical activity with objects. Children do not need to record something they have already worked out, or can work out mentally. Carruthers and Worthington found that recording has very limited value and involves lower levels of thinking, placing the emphasis on symbols, drawings and problem-solving strategies as *products* to be formally assessed. Most significantly, *recording* fails to enable the child to build deep understandings of semiotic representation for mathematics.

Summary of the Findings

In this section I provide an overview of the four studies we conducted in relation to our principal research questions, the answers helping reveal the evolution of young children's signs. The four research chapters all draw on the same data. This is followed by a discussion of the significance of the findings of my research project consisting of these four studies. A successive section "transformative change" addresses some of the challenges for teachers in England and is followed by a discussion of the findings; theoretical gains and implications for education, and consideration of transmission and holistic perspectives. Lastly the limitations of the study and recommendations for future research are made.

The Beginnings and Development of Young Children's Mathematical Inscriptions

Human communication (in essence a sort of information exchange) allows not only for speech and gesture, but for "contextualized integration" through the use of graphical signs or inscriptions (Harris, 1995, p. 4). Harris (2002) observes that graphical signs are notations for representing aspects of mathematics (e.g., numerals, quantities, operations), and are "said to represent not words but mathematical abstractions" (p. 159). These abstractions are understood as emergent cognitive representations (see chapter 4), through which, according to Cassirer, objects are seen from a specific point of view, and, by the same token neglecting all aspects that fall outside this focus. For Cassirer:

> None of these formations [symbolic forms] can be simply absorbed by another or derived from another, but rather each of them refers to a specific mode of mental apprehension, within and through which it constitutes its own dimension of the "real". (Eilenberger, 2020, p. 109)[72]

However, a growing body of research has shown that when young children are introduced to the formal symbols of mathematics in traditional teaching contexts, they experience difficulties in moving from concrete situations to abstract symbolic thinking (e.g., Carruthers & Worthington, 2005, 2006; Ginsburg, 1982; Hiebert, 1984; Hughes, 1986).

Carruthers and Worthington were previously unable to follow children's emergent and developing sign-use longitudinally, and in order to develop deeper understandings of the beginnings and progression of the

[72] The words in this quote are Cassirer's own. See Cassirer, E. (2001). *Philosophy of symbolic forms,* Volume I (Language), chapter 1.

mathematisation of inscriptions, I decided to follow Vygotsky's stance of taking a genetic perspective (1978, p. 46) in this thesis.

First Study (chapter 2): Pretend Play and the Cultural Foundations of Mathematics

To establish the extent to which children drew on their cultural knowledge and communicated their understandings in mathematical contexts, we interrogated multiple written observations of children's spontaneous pretend play episodes. Drawing on Vygotsky's (1978) cultural-historical theories of development and pretend play, our intention was to determine the influences of the children's home cultural knowledge on their play and developing understandings.

Analysis of the documented observations revealed that in almost half of their play episodes, the children explored their ideas in contexts that could be understood as mathematical, their interests encompassing all aspects of the mathematics curriculum. The impact of these cultural foundations on their pretend play was to enrich the play, expanding and advancing the child's original cultural experience and potentially moving them towards new semiotic understandings. Vygotsky (1978) writes that the difference between play and other types of activity is that "in play a child creates an imaginary situation" of relative freedom (p. 93)[73]. However, according to Vygotsky play also requires children's *engagement*, *rules* and *participation.* Following Vygotsky, van Oers, (2013b) identified children's high levels of involvement in play, that "allows the child the freedom to follow its personal sense and imagination" (p. 190), children following "some rules (either implicitly or explicitly), and who have some freedom with regard to the interpretation of rules, and to the choice of other constituents of an activity (like tools, goals, etc.)" (p. 191). Regarding the "rules" of play, van Oers identified *social, technical, conceptual and strategic rules* (2013b) that are governed by the *degrees of freedom* children are permitted in their choices (p. 191-192)[74].

In our study, the children all displayed deep enthusiasm and high levels of engagement in their play narratives, frequently developing ideas over many days and permitting development of increasingly complex notions. Analysis of the data confirmed our earlier hypothesis*:* clear evidence of the

[73] In the context of this thesis, the use of the term "free and spontaneous pretend play" refers to children's freedoms (agreed by their nursery school) to choose where they will play, with whom, what resources they use, the focus of their play episode, the duration of their play and whether they will engage in the same focus on other days, allowing them to further develop their understandings.

[74] See also Worthington et al. (2020).

children drawing extensively on their existing cultural knowledge was found, the culturally meaningful contexts of their play (coupled with the nursery school's philosophy and teachers' deep understanding of play) enabling them to explore and elaborate empirical concepts relating to mathematics in the social contexts of the nursery.

Previous studies suggest that young children rarely explore aspects of learning that adults view as mathematical within their pretend play narratives. For example, Gifford (2005) was unable to identify children freely engaging with mathematics in pretend play, and maintained, "opportunities may be there, but children do not necessarily take advantage of them" (p. 2). In respect of this finding, Gifford may well be correct in contending that "a *laissez-faire approach* to children learning maths in the "secret garden" of play does not work"[75] (p. 2, emphasis added), yet the issue may be more to do with children's typical experiences of pretend play, with a setting's culture and pedagogical practices, and teachers' understandings and support of pretend play and early childhood mathematics. A notable factor appears to be the fleeting or adult-curtailed nature of children's play narratives and their associated low quality: this outcome suggests adults' poorly developed understandings of pretend play (and the often-associated teacher-planned play), which may result in concepts that are "*conceptually disembedded"* (Fleer, 2010a, p. 75, italics in the original). A major New Zealand review of research into effective pedagogy in mathematics (Anthony & Walshaw, 2007), highlighted children's engagement in pretend play, suggesting that it appears to depend on the extent to which the children were interested:

> Spontaneous free play, while potentially rich in mathematics, is not sufficient to provide mathematical experiences for young children. Evidence from observational studies suggests that children's involvement in mathematical activities *appear to be moderated by their own interest and prior knowledge*. (p. 30, emphasis added)

However, Anthony and Walshaw's statement is opposed and exemplified by the high levels of children's interest and their prior knowledge embodied in the data for chapter 2, pointing once again to the strength of the nursery school's culture and pedagogical practices. Chapter 2 shows that it was precisely the children's *interest and prior [cultural] knowledge* that ensured their rich pretend play, in which they

[75] We concur with Gifford, that neither learning (or teaching) mathematics, nor pretend play should approached in "laissez-faire" ways.

communicated about aspects of mathematics[76]. Chapter 2 also shows that the children's many references to number, quantities, counting, money, time and all other areas of mathematics, underscored their significance in many of the children's home experiences, prompting them to communicate about these aspects in their play. Our findings from this first study revealed that in almost half of the play episodes, the children freely chose to explore ideas in contexts that adults could understand as mathematical, their interests encompassing all aspects of mathematics. These are significant findings, and contrary to those of previous studies.

The discrepancy between the findings of this study and previous studies of pretend play and mathematics appears to be due in part to the children's considerable freedom to self-initiate their play, and the provision of opportunities for meaningful and culturally relevant contexts for the children to explore and build on their cultural knowledge, which in turn provided authentic and meaningful foundations for further development of mathematical thinking in the nursery.

The findings of this first study confirm that the foci of the children's play narratives all originated in their home cultural knowledge. Drawing on their own readily comprehensible contextual and mathematical meanings within their chosen play narratives, it was clear that the children's cultural knowledge had considerably influenced their mathematical thinking. These emergent understandings broadened and strengthened the children's interactions, with numerous aspects of mathematics explored from across the English early childhood curriculum (Department of Children and Family Services [DCSF], 2008b; Department for Education [DfE], 2017). Moreover, they knew that their play and meaning making was valued and understood by their teachers.

The incidence of the children's play narratives that included aspects of mathematics increased during the year, their immersion in mathematical- and graphical-rich environments helping bridge home and early childhood cultures, such bridging (or "hybridity")[77] becoming a natural feature of their pretend play. It seems significant that none of the children's pretend play episodes analysed in this current thesis were planned by teachers, that there were no adult expectations that the children would explore aspects of mathematics, and that the children's play episodes were all impromptu and freely chosen. The findings of this study are corroborated in more recent research that has embraced Carruthers and Worthington's work into

[76] These strengths also ensured the nursery's high ratings in mathematics by Ofsted inspectors (see chapter 5).

[77] See also the section on "hybridity 2" in this chapter.

Children's Mathematical Graphics (e.g., Papandreou & Tisouli, 2020; Papandreou & Konstantinidou, 2020).

The Role of the Adult in Children's Mathematics

Throughout this study we have referred to both pretend play and other "open" contexts (such as teacher-led small groups) in which children are free to explore aspects of the mathematics that is the focus of the group, in ways of their choice. Early in our enquiries when we were still teaching, we (Carruthers and Worthington) developed a means of teacher-modelling in which mathematical signs and strategies are modelled in meaningful contexts in ways that children can readily understand. We identified a need to model inscriptions at times *other than* small groups in which the focus is an aspect of mathematics, and, with older children in school, at times *other than* mathematics lessons: we found that this ensures that children do not *copy directly* what the teacher has modelled (see chapter 4 regarding Tomasello's (2005) findings of *over-imitation*). Significantly, children intuitively add these modelled signs to their personal, mental graphical sign lexicons (Worthington, 2020a), thus enlarging the repertoire of signs on which they may draw in new situations. Teachers' non-intrusive participation in the children's pretend play, and their collaborative dialogue with children about aspects of their thinking and signs, are also significant factors in children's progression.

Investigating pedagogical practices that support children's mathematics in pretend play, Carruthers (2015) highlights "attunement" in the teacher-child relationship, which, in English nursery schools is created through the "key person" approach (Elfer et al., 2003). This is an important feature of the nursery school in which data for this thesis were gathered. In this role the child's teacher/key person develops a close relationship with the child and her family, enabling her to also tune into the child's home mathematics and the experiences that shape their cultural knowledge. *Attunement* is often associated with the literature on *attachement* (Berk, 2003) in which teachers have a deep knowledge of each child.

In my view, the significance of joint participation in meaningful everyday activities (including play) is that it is a productive context for meaningful learning, a view that is shared by Papandreou and Konstantinidou (2020); Rogoff (2008); Tomasello (1999; 2019) and Moll and Tomasello (2007). In these activities, the children's learning intentions are respected (Hedges & Cooper, 2018). Whilst it was not the aim of this study to focus on specific aspects of pedagogy, these are clearly aspects of a suite of "intentional practices", described by Wager (2013) as planning and preparing for mathematical learning; building on children's interests; cultural practices and understanding; and recognising and responding to mathematics that emerges in their play ("responding in the moment", p. 165-166). These are all aspects to which the teachers in this nursery school attend, and appear to be similar to what Pramling et al. (2019) regard as teaching that is "play responsive"; "*a mutually co-constituted activity* […] where children are equally important participants" (p. 175, emphasis in the original), and the "equal relationships" that are highlighted "between teacher and the child, teachers "conceptually and contextually connected with the children" (Hedegaard & Fleer (2013, p. 56)" (Carruthers, 2020, p. 37). If teachers are to have joint participation and support children's agency, they need to be proactive and self-determining, enabling them to act on their individual ideologies. At the same time, teachers need to continue to support children's cultural understandings so that they may participate as agents of their own learning (van Oers, 2015). However, as a result of mandatory curricula, van Oers warns that various threats to teachers' agency have resulted in "a growing discontent among teachers, who feel they have lost ownership of their professionalism and feel degraded to mere executors of imposed curricula (Wilkins, 2011)" (p. 19). As a consequence, it is likely that children will develop only restricted and "mechanic conceptions of learning […] and what it means to be a (life-long) learner" (p. 20).

Together, the key person approach and joint participation in play, in which teachers are play responsive, coupled with the culture of the nursery school and the teachers' deep understandings of social pretend play, mathematics and graphicacy, enabled the children in this study to explore and elaborate their considerable empirical concepts related to mathematics in the nursery. Significantly, this study illuminated ways in which spontaneous social pretend play can support children's understanding of the "subject" area of mathematics in ways that are appropriate for young, emergent learners.

Second Study (chapter 3): Children's Social Literacies: Meaning Making and the Emergence of Graphical Signs and Texts in Pretence

Continuing from our first study (chapter 2), in chapter 3 we addressed children's literacies as a social practice. Examining our data for evidence of the cultural knowledge the children had developed at home through their engagement with literacy practices, we expected to identify some spontaneous literacy events within their pretend play[78]. We also sought for evidence of multimodality (*modes, materiality* and *affordances*) in

[78] In this thesis we equate "literacies" with heterogeneous graphical inscriptions, such as drawing, maps, writing and those relating to mathematics.

children's graphical texts, and identified the range of marks and signs the children chose to use.

The findings showed that the extent to which parents engaged in literacy practices (sometimes with their child) for authentic purposes, impacted on the child's choice of pretend play narratives and their cultural knowledge of literacy events. These home literacy practices were mirrored in the nursery; for example, one of the children drew maps at home and in the nursery (linking with his father's interest in old maps displayed on the wall at home). Additionally, at home the children sometimes self-initiated literacy events (the focal children appearing to have done so most often): the more they freely did so at home, the more this enabled them to experiment with graphical signs and symbols and use them spontaneously in their nursery school. The children's play narratives reflected the literacy practices and events of home, as they freely and spontaneously communicated through speech and employed a range of graphical signs in contextually appropriate ways. Their cultural knowledge considerably enriched their understanding, revealing diverse genres including persuasive letters, bookings for a campsite, maps and receipts.

The findings of this study revealed that, in children's self-chosen pretend play narratives, almost 44 percent included literacy events (many of these in mathematical contexts), increasing during the year. This is a noteworthy finding, since all of these events were entirely child-initiated. Of the focal children, Shereen engaged in literacy in all but one of her play episodes: of the remaining two focal children, Isaac's pretend play interest more frequently centred on imaginative play with several friends in the large outdoor sand pit, focusing on his cultural knowledge of his father's building work, and not always including use of graphicacy. Rather than often engaging in pretend play, Elizabeth chose instead to involve herself in literacies in other self-initiated contexts. For example, she wrote a series of letter symbols (each drawn inside a box), and pointing to them explained, "that says all of my name" That's my brother's name and his second name and his third name. My mummy has also got a second name. My daddy also has a second name"[79].

This study also identified a number of multimodal features that were evident in the children's self-initiated texts, and highlighted specific semiotic features of the children's marks and signs. The children's texts showed variation in their choice of marks and signs, the size of individual signs and their spatial arrangement on a page or writing surface. They

[79] Elizabeth had named her brother's, mum's and dad's full names, but for reasons of confidentiality, they are omitted here.

appeared to recognise the affordances of various materials and signs, choosing those that they deemed appropriate in a specific context and to convey a particular meaning. An example of this is Isaac's "building plan" that he rolled up as he'd seen his father do (see chapter 3, figure 2). All the children developed their graphic vocabulary for their various literacies, from their beginning scribble-marks, to some including standard alphanumerical symbols.

Beginning with *early explorations with marks* (to which children attached meanings), the children used a diverse range of graphical inscriptions; a variety of graphical lines; shapes and signs such as arrows, ticks, circles and arcs; wrote crosses in various contexts to signify diverse meanings; and used both zigzag and wavy lines as "writing", also using letter-like signs. With the exception of map making, drawings were the least chosen graphics used to communicate ideas in their play. Elizabeth and Shereen (two of the focal children) used the greatest numbers of graphical signs, including standard symbolic letters and numerals, and applied them most frequently.

Throughout the year, all the children developed their proficiency in using graphical inscriptions to communicate. As Vygotsky emphasised, it is the social aspect of learning contexts that is important. The findings of this second study affirm that the graphical languages of various literacies (i.e., drawing, maps, writing and mathematical) originate from the same range of early marks, and that young children gradually attribute meanings to their early marks. Children's free and spontaneous use of their informal signs, and those that are modelled by teachers, contribute to their graphical repertoires, helping them as they move towards the standard symbols of mathematics (Worthington, 2020a).

Our findings and those of the studies listed here, confirm our premise that children's freely made graphical communications for all literacies are in line with the research into young children's early beginnings with drawing, emergent writing and multimodality[80]. Furthermore, the relationship between the various inscriptions and the frequency of their use, relate to their growing use of emergent *mathematical* signs and symbols.

[80] The *communicative aspect* of mathematical graphical signs and texts is a feature that is absent from any discussion of "recording" in traditional approaches to mathematical inscriptions or written strategies such as problem solving, and omitted from curriculum documents on early years' mathematics in England.

Third Study (chapter 4): The Development of Mathematical Abstraction in the Nursery

This study perceives children's emergent cognitive representations of graphical inscriptions as mathematical abstractions. Examining their development of mathematical abstraction through sign-use in contexts acknowledged as mathematical, the study investigated the extent to which Cassirer's (1923) theory of abstraction, that the abstract is *projected into* objects by seeing these objects *from a specific point of view,* was borne out by analysis of the data, children often communicating about the same focus, and representing that focus from their own point of view. From this perspective, children interpret contexts that can be viewed as mathematical from their personal point of view, representing them in distinctive ways.

Following the results of our second study (chapter 3) we aimed to identify the range of signs the children used to represent and communicate their thinking in mathematical contexts. Using a Peircian perspective, we first investigated the children's graphics as they moved towards the full abstract symbolic language of mathematics. Peirce's semiotic theory (Buchler, 1955) considers signs as either *iconic*, *symbolic* or *indexical* and to these we added an additional category, *early explorations with marks – attaching mathematical meanings*, in order to accommodate young children's earliest graphical marks to represent their mathematical thinking.

Features of grammaticisation (or usage-based language acquisition) highlighted the relationship between (oral) language acquisition and graphical, mathematical signs and symbols. Investigating abstraction from a Cassererian (1923) perspective and combining this with findings from research into language acquisition (e.g., Tomasello, 2005; Langacker, 2008) permitted insights into the processes that contribute to abstraction.

The findings revealed the range of signs the children freely chose to communicate their thinking in mathematical contexts, accentuating how their emerging understandings of graphical signs supports their transformation into abstract mathematical symbols. The children in this study all used a range of signs and texts, the quantity used by different individuals varying, and their inscriptions reflecting their interests, cultural knowledge and thinking (Worthington, 2018). All the children used iconic signs and towards the end of the year, some had begun to adopt standard abstract symbols, indicating their progression towards the fully abstract symbolic language of mathematics.

The findings affirmed that features of usage-based language acquisition (regarding signs and symbols relating to mathematical communications), were evident in the children's inscriptions, and that *intention-reading* was connected to their growing ability to acquire the abstract symbols of mathematics: imitation and emulation of their peers' signs (in addition to signs modelled by their teachers) were integral to this. This analysis enabled identification of examples of children "*reading*" others' intentions concerning their graphical inscriptions, and "*pattern finding"* of graphical signs across various inscriptions (Tomasello, 2005). Contexts in which children shared *joint attention* aided their pattern finding: for example, several children used crosses in various contexts to communicate diverse meanings, pattern-finding helping them appreciate when crosses were used to signify negation, absence, none or nothing, their use appearing to anticipate writing zero. As with the previous studies, this study also confirmed the significance of pretend play for social learning.

The findings of this study highlight strategies the children employed as they communicated their thinking, indicating the importance of symbolic number knowledge in acquiring the abstract graphical language of mathematics. All the children developed their inscriptional lexicons during the year, two of the three focal children using an extensive quantity of heterogeneous signs in their literacies (chapters 3 and 4), some also employing standard numerical symbols, (chapter 4); see also Worthington (2020a).

Tomasello (2005) maintains that children's signs grow *"gradually in abstraction over time* as more and more relevant exemplars are encountered and assimilated" (p. 316, emphasis added). Frequent use of a range of signs across all the children's literacies is an important factor in this development, and suggest that young children's own, freely made drawings should be valued and "listened to" as much as their writing and mathematical inscriptions. These findings direct attention to the importance of symbolic number knowledge in acquiring the abstract graphical language of mathematics, and highlight the appearance of abstract thinking in young children, a previously unknown finding.

Fourth Study (chapter 5): Intertextuality and the Advance of Mathematisation in Young Children's Inscriptions

The fourth study identified a number of graphical texts in which children and their peers used signs *intertextually* and in different contexts, so that their inscriptions became "a permutation of texts" (Kristeva, 1980, p. 36). Intertextuality can assist learners in understanding the relational role of individual signs (Skemp, 1976), contributing to deeper levels of understanding and increased conceptual competence. Drawing on Bakhtin's (1981) theory of "multivoicedness", and Kristeva's (1980) work on intertextuality, the findings provided evidence of intertextualisation through examples of signs moving between both an individual's and several children's texts, the same signs also moving between different graphical contexts (i.e., drawing, maps, written and mathematical). Children's use of signs in contexts that may be understood as mathematical, is embedded in meaningful social practices, findings that challenge a single skills-based mathematics through transmission teaching.

In this study we investigated the early stages of children's representations of quantity and their relationships, focusing on examples of their signs signifying tallies and numerals The children used tallies spontaneously, and there is evidence that signs including assorted shapes, wavy-line writing, ticks, crosses, letter- and numeral-like signs and letters also crossing intertextually for this purpose. These findings show that mathematisation can be understood as a combination of increasing grammaticisation and intertextuality.

Mathematisation in young children is, to a large extent, identified in children's increasing use and sophistication of mathematical signs and multi-sign texts from the established mathematical culture, including examples of early calculations. Intertextual transmission of formal Arabic numerals in children's own texts seems to play an important role in young children's developing understanding and progression and reflect the findings of chapter 5. Several of the children *spontaneously wrote Arabic number symbols* (SWANS).

The children used an extensive and growing repertoire of marks, signs and symbols to signify and communicate meanings in different contexts, some of these from the culturally established systems of writing and mathematics, and employed multimodal ways of representing, the children benefitting from freedoms to "participate fully and productively in the making of their meanings, in the ways that make sense to them" (Kress, 1997, p. 151). The children's interest in the use of inscriptions progressed during the year, children using a range of graphical signs in personally meaningful contexts. The nursery school's culture and philosophy, the headteacher's and teachers' associated positive beliefs about children, their professional knowledge and related pedagogical practices, provided the bedrock on which a range of activities in which the children engaged, and supported their mathematisation over time. Open opportunities for rich pretend play (and other open activities) in which children could draw on their cultural knowledge; the freedom to represent in their own ways, in drawings, maps, writing and mathematical inscriptions and social opportunities for intention-reading and intertextual exchanges of others' graphical signs and texts were important conditions for their mathematisation. A significant feature appears to be how the children productively drew on their personal "funds of knowledge" (Moll et al., 1992), their family's home literacy practices in their pretend play helping shape the focus of their play episodes and contributing understandings about mathematics and sign-use. The abundant diversity of children's graphical texts provided multiple opportunities for them to identify patterns of sign-use and their related significations, to imitate and emulate others' signs and to develop their understandings of mathematical abstraction. One of the most important outcomes of this thesis is the conjectured theoretical generalisation suggesting how the gap in understanding identified by Hughes (1986), which originates between "children's concrete understanding of number [… and] the written symbolism of arithmetic" (p. 53), and may be bridged through social learning. The social features of the children's play and open activities also provided extensive freedoms for intertextual exchange of signs in meaningful contexts.

My research has investigated young children's mathematics and their mathematical inscriptions within the particular context of pretend play, through the comprehensive study of specific cases. However, it is not possible yet, without further support of new teachers, to empirically generalise the findings of the seven case studies to a larger population. This would need further professionalisation of new teachers in their abilities to promote pretend play through open play activities that make sense to the children, and to find ways of talking with them about aspects of quantity, number, relationships within their play activities. The findings offer ideas to teachers and teacher educators, that have shown to be valid and useful for implementation in classroom practices that aim at promoting early mathematical thinking in children's pretend play. The findings showed that there was an appreciable difference in the number of pretend play episodes in which individual children engaged. The reasons for this may be due to the children's dissimilar levels of interest in doing so (and their preferences for other activities); their distinct personalities, agency and their diverse cultures may have also influenced their choice of play activities, none of which were the focus of this research. Against this, the children's contrasting home experiences certainly resulted in individuals often exploring diverse aspects of their cultural knowledge, and in turn, may have been a factor in their choices of engagement in pretence.

Since the focal point of this research project was the emergence and development of young children's mathematical inscriptions explored *through their spontaneous pretend play*, pretend play contexts were largely the centre of attention for the data. Those children who seldom engaged in pretend play did not miss out on mathematical experiences, since all the children in the nursery school have multiple experiences and opportunities of the full range of mathematics in other contexts, including ‘talk time’; open group times; snack time and story time[81]. Furthermore, these same multiple experiences provide numerous opportunities in which the children can explore diverse mathematical areas. This ensures that those children who explore only a limited range of mathematical domains within their pretend play, have many other occasions in which they may investigate the full extent of mathematics.

The children’s frequent and diverse communications through graphical inscriptions (along with the teachers’ modelling) permitted their intuitive and exponential development of personal graphical lexicons, thus providing an expanding source of signs and symbols on which to draw. In ways similar to the well-documented emergent writing, the children in this study moved from (and between) their early scribble-marks and standard symbols. The children’s earliest marks were of significance in allowing them to discern that even these may be used to communicate meanings, and assisted them in moving to iconic signs. Some of their signs (such as crosses) appeared interchangeable between drawing, maps, writing and those that could be understood as mathematical, and it is only with time and through the use of many graphical texts that children come to appreciate that these different graphical systems have their own signs, sometimes shared (as in letters for writing and for algebra) and sometimes context specific. The findings show that young children’s evolution of signs need time to develop: they require teachers’ support and understanding that traditional, transmission-oriented teaching practices that sees learning as reproduction, cannot fulfil. It appears to be the highly social nature of pretend play that made the learning exemplified in these studies possible. Van Oers (2013b) states that when learning implies “strict execution of tasks, following specific rules without the freedom to interpret the rules or to invent new ways of doing them”, this is not play, since it diminishes children’s “degrees of freedom” (p. 196).

Finally, this chapter considers the *processes* of mathematisation of children’s early inscriptions, proposing that Carruthers and Worthington’s “bi-numeracy” model (2006), supported by Tomasello’s concept of a “ratchet” (1999), submitting it as a theoretical generalisation for the process of mathematisation. This, we believe, might assist in answering Hughes’s (1986) question and exemplify how the *gap* between informal and formal mathematical knowledge can be bridged, or – as we see it – how

[81] At story time, picture story books often include aspects of number and wider aspects of mathematics.

informal knowledge can be gradually transformed into formal mathematical knowledge.

Summary of Main Findings

The findings of this thesis highlight a number of contrarian and novel findings.

Finding 1: The children communicated through dialogue and inscriptions within their pretend play, in contexts that could be understood as mathematical

Gifford (2005) asserted that opportunities for children to engage in aspects of mathematics in their pretend play are unlikely to be explored, due to a lack of interest. In conjunction with this, Anthony and Walshaw (2007) wrote that whilst spontaneous free play "is potentially rich in mathematics, [it] is not sufficient to provide mathematical experiences for young children" (p. 30). However, the findings of this thesis counter this, exemplifying high levels of children's interest and involvement (often arising from their funds of knowledge) in aspects of mathematical thinking and inscriptions. Moreover, it is telling that none of the children's pretend play episodes were planned by teachers, and there were no adult expectations that the children would explore aspects of mathematics: all the children's pretend play was impromptu and freely chosen by them.

Finding 2: The children employed a variety of early graphical marks to which they ascribed meanings

Levin and Bus (2007) describe the marks that children make up to the age of three as "a common core of *indistinguishable* nonrepresentational graphic products" (p. 892, emphasis added) that fail to convey the referent's meaning. However, the suggestion that young children's scribbles are "indistinguishable" is contradicted by the examples of children aged 3 years in this thesis[82], the children's scribble-marks including what Matthews (1999) describes as first and second "generational marks" (p. 23-27), and encompass many of the "basics scribbles" identified by Machón 2013, p. 191) and by Lancaster (2014). Furthermore, to support others' understanding, the children in this study always verbally explained the intended meanings of their scribble-marks.

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[82] Emma, one of the teachers in this study, now works with children of 2-3 years of age. Emma's interest and deepening understandings of the importance of graphicacy prompted her to develop the same culture with these younger children, many of their graphics suggesting early emerging mathematical thought.

Finding 3: Several children adopted crosses in diverse contexts to communicate distinct meanings, variously signifying negation, absence, none or nothing, their use appearing to anticipate writing zero

Cockburn and Parslow-Williams (2008) refer to some of the ways in which young children use the (spoken) language of zero, such as "nothing", "none left", "empty", finding these are dominant aspects of zero. The children's use of written crosses to signify the same concepts relate to these findings. Zero is also integral to negative numbers, and as a placeholder in developing understanding of numbers beyond 9. An instance of this is when David wrote the numeral "1", then added a zero, announcing, "It's number 10 now!" (chapter 5); (see also Anthony & Walshaw, 2004; Wilcox, 2008).

Finding 4: Those children who had developed a *rich lexicon of graphic signs* were at an advantage in representing their mathematical thinking, and were more likely to use standard symbols

In this research, those who used an especially abundant "vocabulary" or repertoire of iconic signs, seem to advance most in respect of their fluency and adaptivity in representing their mathematical thinking, and were more likely to use standard symbols[83]. This finding is contrary to that of Kato et al. (2002), who found that in their study of young children representing groups of objects, whilst the majority of children knew how to write numerals, they did not use them. From the data in this thesis, it was clear that those children with the richest graphical lexicons benefitted in respect of communicating through signs and symbols. Rather than the limited number of signs and symbols which, in a transmission model of teaching, are selected by the teacher and given to the children (often for a specific purpose), the findings here showed that in open, social contexts children are able to develop rich and diverse repertoires of graphical signs.

Finding 5: The children employed a *flexible range* of diverse signs

This flexibility, Jung (2011) argues, helps children appreciate the interconnectedness of numbers, and that they can be used in meaningful ways. It helps children make sense of more advanced mathematics concepts, (Baroody, 2004), but it also "prepares them to solve mathematical problems with understanding (Fosnot & Dolk, 2001)" (p. 166).

Finding 6: Children who had developed the most *extensive lexicons of graphical signs*, were also those who *spontaneously wrote Arabic number symbols* (SWANS)

[83] Shereen and Elizabeth knew and wrote a significant number of standard numerals.

Spontaneous focusing on numerosity (SFON), has been identified as indicative of future success in school: for example, comparing preschoolers' informal knowledge of numerals and number symbols (Purpura & Napoli, 2015) found that:

> Before children can apply their informal knowledge in the formal domain, they must map that knowledge onto the Arabic numeral system (Purpura et al., 2013; Sinclair, Siegrist, & Sinclair, 1983). This domain is both theoretically (Baroody & Wilkins, 1999) and empirically (Purpura et al., 2013) distinct from informal and formal numeracy skills and, critically, appears to have a considerable dependence on literacy-related skills (Purpura & Ganley, 2014). (p. 199)

Rathé et al. (2019) investigated "young children's tendency to *spontaneously* focus their attention on Arabic number symbols" (SFONS) in pictures they were asked to describe, to determine whether such a focus has a role in children's early mathematical development (p. 111, emphasis added). Their findings confirmed "significant associations between SFONS, early numerical abilities, and teacher ratings of mathematical competence" (p. 120), and that of children of 2-6 years, attend to Arabic numerals "without being explicitly guided to do so" (p. 120). Other researchers have identified relationships between children's early knowledge (recognition) of standard Arabic numerals and subsequent longitudinal achievement (e.g., Griffin et al., 1995; Habermann et al., 2020; Merkley & Ansari, 2016; Rubinsten et al., 2002). In connection with this, it became evident that those who used the greatest and most divergent range of graphical signs to communicate (Shereen and Elizabeth), also most frequently spontaneously *wrote* standard (Arabic) numerals. These findings suggest a direct relationship between the extent of children's graphical repertoires and *spontaneously writing Arabic number symbols*, which I refer to as "SWANS", a finding that extends beyond previously cited research. The difference is that these previous studies did not look at children freely and spontaneously *writing* Arabic numerals, suggesting that this is a significant and new finding of this study.

Finding 7: The iconic signs the children used suggest these serve as elements of their protolanguage, their use predating and transforming into standard symbols

Evidence from the data in this thesis and from Carruthers and Worthington's research (2005; 2006), suggests that between the ages of 2-4 years, the iconic signs of *Children's Mathematical Graphics* are features of their protolanguage[84] of (mathematical) inscriptions, that subsequently open the way for syntax in their mathematical texts. For example, Shereen's subtraction of cakes in her pretend café (figure 5, chapter 4), implied that she is beginning to use syntax - signifying subjects, verbs and objects - her visual text an intermediate and transitional one signalling future moves to increasingly standard forms of written calculation, and other aspects of standard written mathematics. Laland (2017) submits that syntax "introduces rules that eradicate ambiguities", clarity an essential feature of later "school" mathematics, that allows "an almost infinite flexibility in usage" (p. 195).

Finding 8: The children intuitively engaged in abstraction, representing their mathematical thinking in personal ways: this led to a range of heterogeneous inscriptions for the seven children

When analysed from Cassirer's (1923) perspective and our related own view of "abstraction", the empirical findings highlight the appearance of abstract thinking in young children, a previously unknown finding.

Finding 9: Intertextuality contributes to symbolic diversity within *the social group*.

Social interaction provides multiple perspectives of signs and, with the help of adults' and peers' signs and texts, enriches children's expanding sign-lexicons.

Finding 10: The children's pretend play episodes and their associated inscriptions were clearly underpinned by the *social practices* of their play, and rooted in their funds of knowledge

The findings of this thesis show that children's use of graphical inscriptions to make and communicate meanings are firmly embedded in social practices such as pretend play: children draw on their existing cultural knowledge, ensuring that they are based in realistic and meaningful contexts. This cultural knowledge helps children build on and connect their existing understandings with those of their new experiences and knowledge of the nursery school. Indeed, referring to a vignette on counting (from the same nursery school), Coles (2021) comments "I read in these practices a trust that, if children are allowed to follow their own lines of enquiry, they will become ever more sophisticated in their use of number" (p. 38), something justified by examples from Carruthers and Worthington's research (2004; 2005; 2006), relating to children's mathematics of children from 5-8 years (including calculations). For parents and teachers, it also shows how children's home knowledge and experiences are valued,

[84] Protolanguage, "[…] strings of words [signs] with no structural relationships or syntax" (Laland, 2017, p. 358).

demonstrating how culture is enmeshed in play. Children's funds of knowledge also provide valuable information for teachers, informing them "what children know and are capable of doing" (Riojas-Cortez, 2001, p. 39) and supplying indications that may guide teachers in supporting children's interests: an instance of this was when Emma brought in some small security safes to support Isaac's interest, subsequently leading to some rich pretend play connected to safes (chapter 2).

Finding 11: Pretend play contexts are invaluable in promoting collaborative, *social learning*, encouraging imitation, emulation and intertextuality of peers' graphical signs use in personally meaningful ways

A powerful aspect of social contexts is that they afford fresh circumstances to learn from others: Laland (2017), asserts that one advantage is copying others' ideas or ways of doing things (such as imitating or emulating), but whilst "social learning itself is not inherently beneficial […] *efficient* social learning" is (p. 127, emphasis in the original). Much of this copying occurs through intertextual exchanges. The outcome of increasing copying, Laland maintains, is "greater behavioural diversity [that can] increase the knowledge base of a population" (p. 75), enhancing valuable behaviours that can support young children's mathematics.

Finding 12: The *social learning* in which the children were occupied, ensured *social cooperation and collaboration*

Most significant for the children's mathematics, was the social learning in which they engaged, when the children in a play episode engaged in a shared focal point. Laland's research connects to "*the cultural learning hypothesis"* (van Schaik et al., 2011), which assumes that "social learning is more efficient than individual, or asocial, exploration and learning, and that individuals in practice tend to rely on social learning to acquire skills" (p. 1009): at the same time social learning permits innovative behaviours. Moll and Tomasello (2007) theorise that the intellectual skills necessary to devise "complex technologies, cultural institutions and systems of symbols for example – were driven by, or even constituted by, social cooperation", involving cooperative communication, joint attention and shared perspectives. They conclude that "regular participation in cooperative, cultural interactions during ontogeny leads children to construct uniquely powerful forms of cognitive representation" (p. 639; see also Tomasello, 2019).

Finding 13: The outcome of our study into intertextuality and mathematisation, reaffirm the importance for children's *social learning*, for cultural understanding.

Tomasello (2016) asserts that cultural learning alone allows individuals to learn collaboratively, co-constructing their understandings through one another.

Finding 14: Our findings have helped determine the range of processes and conditions that support children in building on their own strategies, adults respecting "children's invented symbolism" (Hughes, 1986, p. 177)

These measures help close the "gap" between young children's concrete experiences with mathematics, and the standard symbols of the culturally accepted system of mathematical symbols identified by Hughes. The children's foundational understandings of mathematical semiosis establish a solid base from which to move forward with "written" mathematics later in school (see for example, Carruthers & Worthington, 2004; 2005; 2006).

Finding 15: The findings of this thesis are interdependent upon the culture of the early years' setting

These findings are direct outcomes of the democratic culture and open ethos of the nursery setting, coupled with the teachers' deep understandings of pretend play; of early mathematical development; graphicacy and emergent learning that together supported children's learning. From this holistic view of learning, mathematics is seen within the context of all the child's meaning-making and learning, children having considerable agency as *active* learners. We envisage no other necessary conditions for the children to freely engage in aspects of mathematics, beyond teachers noticing, listening to and taking a genuine interest in the children's meaning making, dialogue, and in the breadth of their inscriptions. This enables teachers to participate in the children's pretend play and other activities with understanding, to collaborate and contribute purposely to their play and mathematical thinking, and to plan for future support.

Discussion of Empirical Findings

Reflecting on symbolic diversity and hybridity

In this thesis the term "hybridity" is applicable in two ways. Firstly (*hybridity 1,* below) to denote a child's text in which she/he used multiple and diverse signs. The second interpretation (*hybridity 2*) refers to the negotiated spaces of home and nursery school in which a child's text is a combination of textual practices arising within the various social contexts the child inhabits.

Hybridity 1: hybridity or multi-sign texts

Children's use of an increasingly diverse range of signs and symbols (or "symbolic diversity"), relates to the child's use of texts that include various signs and symbols. These "hybrid" or multi-sign texts, are those in which children included two or more categories of signs (e.g., combining early marks with iconic signs). This is reflected in the data of this research, Shereen and Elizabeth (both focal children), not only using the greatest number of signs and symbols, but sometimes combined them in one hybrid text, to communicate their thinking in increasingly mature ways. For example, in both of Shereen's hybrid texts she employed a diverse range of signs to communicate *different elements of meaning*, in an order in her pretend café, using writing-like marks, drawings (a fish and a mushroom), iconic signs (ticks and a cross) to signify her meanings (Figure 1, chapter 3). On another occasion and again when playing cafés, Shereen represented subtraction. She used a combination of drawings (of herself and her dad; a heart, five cakes and a flower); and the number "14" (its meaning unknown), subsequently "subtracting" cakes that customers had sold, by rubbing them out (figure 5, chapter 4). When combined, the different elements of her visual text suggest a grammatical sequence that contributes to her ability to communicate her understandings of subtraction as "taking away". This finding was previously identified by Worthington and Carruthers (2003); (and Carruthers and Worthington, 2006), in some of the examples from the classrooms in which they taught[85] and recently demonstrated in examples from research by Papandreou (2014; 2009)[86].

Gutiérrez et al. (2011) argue that children's hybrid texts are "more than multimodal practices; they are also mediated by unmarked and hybrid language practices that provide a safe harbor for children to express themselves, to create meaningful content, and […] the means to open up possibilities to engage more fully" (p. 243). Greeno and Hall (1997) maintain that it is vital that students "*learn to use multiple forms of representation* in communicating with one another" (p. 363, emphasis added), and use signs flexibly and adaptively. Hiebert and Carpenter (1992) contend that students' ability to develop conceptual understanding is dependent on the

[85] We identified examples of "*code switching*", as children changed between different graphical forms, such as their informal signs and standard symbols, or between drawings and standard numerals.

[86] Papandreou's 2014 and 2009 research in a Greek kindergarten was with children of 4-6 years of age.

connections they are able to construct between their semiotic resources (or graphical inscriptions).

Having a rich lexicon of graphical signs (individually, between peers and in a group or class) indicates that when faced with a new communicative situation or a problem to solve, both individuals and their peer group have a wider repertoire of graphical signs, and a wider range of textual responses on which to draw.

In an interesting study, Cohn (2012) examined the relationship between the structure and development of language and drawing, maintaining that both involve "many interacting components, including […] perceptual feedback, interaction with the drawings of a culture, social interactions and motivations". Van Oers (2013b) observes that "it is the *sense* that children make of a particular cultural situation that motivates their actions" (p. 190, emphasis in the original). According to Cohn (2012), the graphical lexicon "must include individual graphemes that compose the basic graphic parts of a representation (i.e., dots, lines, curves, circles, squares, etc.,)" (p. 168); echoing once again Lancaster's (2003; 2014), Matthews's (1999) and Machón's (2013) findings, that writing and mathematical signs arise from the elemental marks young children use in their earliest drawings.

Hoff and Naigles (2002) argue that a notable advantage of lexical richness is that "the more words [signs] a child hears [sees], the more words [signs] the child can potentially learn" […] with regard to children's language acquisition […] the richer the information, the faster the process occurs" (p. 430-431). Rather than giving children single symbols to copy, for uses specified by the teacher, teachers need to value children's symbolic diversity, and themselves model a wide variety of ways of representing thinking (informal and standard) relating to mathematics.

Our argument here is that traditionally taught signs largely result in the antithesis of diversity, in symbolic *uniformity* in which graphical signs and means of representing are restricted to a narrow range of formal, abstract signs and traditionally taught methods, which may undermine children's ability to effectively and fully express themselves through mathematical inscriptions. This suggests that children will develop only limited lexicons of formal abstract symbols and "one way of working out", only able to apply their signs and strategies rigidly, with infrequent and superficial understanding. My personal and ample experience in a number of primary schools[87], has affirmed the difficulties children of 4-7 years have answering

[87] This experience included observing student teachers teaching mathematics, (when lecturing at a university; and observing teachers' mathematics lessons when I was a National Numeracy Consultant).

written calculations and solving written problems, due to the fact that they are not secure in their understandings of which signs and symbols, or which written strategies to use. These constraints point to the problems first identified by Hughes (1986), suggesting that in addition to an absence of the children's own informal signs, uniformity of taught signs and methods may be a contributory factor to children's identified difficulties with the abstract symbolic language of mathematics.

Hybridity 2: hybridity of language practices

In their research into young children's hybrid texts, Solsken et al. (2000) found that when young children interwove language practices of home, school and peers, their texts were "more sophisticated and situationally appropriate" than had previously been recognised (p. 179). For instance, the interplay of Shereen's graphical signs (in her two hybrid texts) revealed the relationship between her home cultural knowledge and family practices of eating out in cafés. Through what Gutiérrez et al. (2011) term "syncretic literacy practices" (p. 258), Shereen's hybrid texts can be seen to fuse both different literacy practices (of home and nursery) *and* different graphical signs into single texts. Children's own texts offer them opportunities to draw on their own linguistic (inscriptional) and cultural understandings, interweaving home, peers' and their teachers' signs, allowing social and personal agendas to be met. Focusing on such graphical texts can also reveal levels of sophistication and contextual appropriateness that may not have previously been acknowledged in their nursery or school.

Thus, two specific aspects of hybridity were identified among those children who make use of diverse graphical signs to communicate (within a single text and across contexts of home and nursery school), suggesting that symbolic diversity and hybridity are significant aspects in children's progression of grammaticisation towards the abstract symbolic language of mathematics.

Social pretend play and other open contexts have particular benefits for children's learning. In this respect, Tomasello (2016) argues that it is only cultural learning that allows individuals to learn collaboratively through another, and that this powerful learning supports cumulative cultural evolution. Moreover, recent research shows that "given a free choice" young children are more motivated to "work to solve problems collaboratively with others" than to do so alone", their motivation leading them "to work harder and persist longer" (p. 648). Collaboration "is key to cultural co-construction and creation" (p. 649). Tomasello stresses that:

> recent research has demonstrated rather convincingly that collaborative learning leads not just to the acquisition of more and

> better information, but also to skills in the construction of knowledge with others, as well as to more and better ways of thinking, and thinking about thinking, so that individual children come to respect rational norms of discourse and argumentation. (2016, p. 649)

Hughes urged teachers to "*build on children's own strategies*" […] and "*respect children's invented symbolism*" (p. 176-177), recommendations that are central to the approach espoused in *Children's Mathematical Graphics*. Our findings help resolve Hughes's concern (1986) that the "gap" between children's concrete knowledge (e.g., with physical resources to count or calculate) on the one hand, and the standard symbols of the abstract, cultural language of mathematics on the other, may be bridged.

Focal Children

Frequently and throughout the year, the focal children expanded the range of signs they used. Shereen and Elizabeth employed a comprehensive range of signs during the year: excluding standard alphanumerical symbols Elizabeth using a total of 56 different sign-types, Shereen a total of 47 and Isaac 11. Both girls freely employed standard symbols (including lower and upper-case letters), Shereen also writing her first name and surname. Using standard numerals, Shereen wrote numerals to ten, and "14", and Elizabeth numerals to ten and "100". The findings showed that the focal children continued to frequently and freely communicate through graphicacy, Elizabeth and Isaac engaging in a number of self-initiated literacy events at home. For example, Isaac engaged in pretence outside his home, play that continued for several weeks. Using his dad's van as an "ice-cream van" and with his dad's help, he wrote a sign displaying the choice of available flavours, and another with a double-headed arrow signifying the "T" junction at the end of his road. Shereen and Elizabeth also exhibited an extensive and wide range of graphical signs in their home scrapbooks, including Arabic numerals. It is notable that in the nursery Elizabeth only made use of scribble-marks once during the year, and that Shereen used none. The same two children drew on their increasingly comprehensive sign lexicons to communicate a range of meanings including those that could be understood as mathematical, confirming that children who have an extensive "vocabulary" of graphical signs appear to be at an advantage. Isaac, the third focal child, developed his continuing interest in map-making (as shown in chapters 2, 3 and 4), and his extensive knowledge of environmental signs. Isaac often spoke of numbers in meaningful contexts, but used fewer signs to represent numerals and communicate their meanings than the other two focal children, and had not yet begun to represent Arabic numerals. The non-focal children all developed their sign-use during the course of the year, but used a smaller range of iconic signs and very few standard letters or numerals[88].

Transformative Change

This study reflects what I refer to as a "*cultural-conceptual*" view of pretend play and learning activity within open social contexts, illuminating the significance for young children's learning and embracing Vygotsky's cultural-historical theory of learning. *Transformative change* signals some of the barriers to effective early childhood mathematics that is sensitive to emergent learners, and the sort of metamorphosis that is required in early childhood education in England (and in many other countries), if young children are to develop a good understanding of mathematical semiosis.

But clearly it is not enough to say that children may play: a number of features are necessary so that children engage in rich mathematical activities. For example, Wood and Atfield (2013) maintain that pedagogical approaches to play (in England) "privileges adults' provision for play and only acknowledges their interpretation of children's outcomes in line with predetermined development indicators, curriculum goals and the school readiness agenda" (p. 48). Broström (2017) proposes that, rather than a separation of play and learning, "a potential synthesis of the two might be a theoretical construction where the concept of learning includes qualities in harmony with characteristic elements of play such as imagination, construction of symbols, fantasy, creativity, social relations and communication" (p. 4). Learning happens when children are active participants, engaging and interacting with others; when their activity is meaningful and when learning "is seen as a productive and creative activity characterised by imagination" (p. 1). Broström argues for "free play" in which the children make the decisions about their play and the teacher has an active and socially-interactive role, "challenging the child and encouraging him or her to create new meanings and understandings" (p. 3). Fleer (2011) writes that:

> Conceptual play illustrates the dialectical nature of imagination and cognition […] and shows that they must act in unity. The generation of an imaginary situation […] is an important conscious act where children can rise above reality, descend to reality, connect with reality and play with reality. (p. 236)

A failure to pay attention to young children's use of graphical inscriptions "means that educators may neglect important and stimulating

[88] The non-focal children each made use of between 18 and 25 sign-types.

early events for the promotion of mathematical thinking" (van Oers, 2010, p. 32). However, a considerable lacuna in educational practice exists concerning the meaningful teaching and learning of the abstract symbolic language of mathematics. Roberts-Holmes (2015) concludes that due to pressures in England from governments:

> Complex holistic child-centred principles, sensitive pedagogies and assessments were in danger of being marginalised as early years' teachers were "burdened with the responsibility to perform" and submit to a "new' moral system" (Ball and Olmedo 2013, 88) that has the potential to reduce the rich competent child (and teacher) to a "measureable teaching subject". (p. 11)

Radford (2018) goes further, arguing:

> Our historical period can sadly be characterized as the unprecedented historical age of the most radical assault on schools and education systems at large by the economic forces of society. No school system has ever been engulfed in such a virulent manner by one of society's components. The school of today appears, indeed as an appendix to political economy, defined by global capitalism. And it is against this background that curricular contents are determined and that expectations about teachers and students are set. (p. 4)

These pressures exert multiple, confusing and contradictory demands on teachers and are the result of the neoliberal influences that have taken hold of education across most of the world and became a dominant paradigm, its impact on early childhood education (ECE) "profound"; (Moloney et al., 2019, p. 2). Rogers et al. (2020) summarise some of the ways in which neoliberalism has impacted on young children's education, diverting attention "from meeting children's rights and needs and from ECE as a principled activity" (p. 808)[89]. They identified:

> an alarming emphasis in many countries' frameworks and curriculum documents aimed at preparing young children for the labour market, focusing on literacy and numeracy to create future workforce citizens to contribute to the economy (Hunkin 2017; Sims 2017). This approach does not foster an environment where children are taught through their strengths and interests". (p. 809)

Rogers et al. (2020) identify the erosion of teachers' agency and professional identities, disempowerment and lack of spontaneity, as negatively affecting the teachers' professionalism, which of course has

[89] Whilst this article focuses on data from Australia and Canada, it characterises much that is wrong with governmental guidance for early years' education in England.

adverse repercussions on children's experiences of learning. In opposition to neoliberalism, they emphasise the value "of the researching educator (Moss, 2006), [in which teachers] are viewed as having abilities and flexibility […] where liberal and open dialogue is nurtured and flourish" (p. 817)[90].

Whilst acknowledging existing political pressures on teachers and on young children's play and learning, we must challenge the (often unquestioned) politicised dogma to reveal the potential of young minds. For example, in 2008, the government in England published the results of a two-year investigation into the teaching of mathematics in early years' settings and primary schools (DCSF, 2008a), emphasising the significance of pretend play and the approach developed by Carruthers and Worthington, asserting that "it is comparatively rare […] to find adults supporting children making marks as part of their developing abilities to extend and organise their mathematical thinking […] [missing] a valuable opportunity to encourage early experimentation" (p. 34)[91]. However, since 2009 the ever-increasing governmental constraints on the curriculum in England have led to a narrow focus on basic skills, likely to result in superficial learning and a further widening of the gap in children's understandings in this important aspect of mathematics. From the government's perspective there continues to be little room for children to use their own ways of representing, to try out, innovate or adapt others' ideas to communicate their thinking. In spite of specific recommendations by the government of the time, the early years' curriculum increasingly focuses on the "school readiness" agenda, and the findings of the 2008 the DCSF report failed to result in any noticeable changes for the teaching and learning of mathematics in the early years, regarding children's early use and understandings of graphical signs.

In 2017 the British Educational Research Association (BERA) and TACTYC, the Association for Professional Development in Early Years published their joint review, discussing Carruthers and Worthington's and Worthington and van Oers' findings in some detail. Again, in 2019 a review in England of evidence relating to the early years (and based on evidence from research)

[90] In the nursery school that is a focus of this thesis, the headteacher actively encouraged and supported all teachers and early years' professionals in research, some writing and publishing their research and presenting at English and European early childhood education conferences.

[91] An outcome of this review was that Carruthers and Worthington were commissioned by the government to write "Children thinking mathematically" (DfE, 2009).

by Pascal et al.[92] included significant sections on early years' mathematics, supporting the holistic nature of children's development, and Carruthers and Worthington's, and Worthington and van Oers' research, and making specific recommendations relating to this research. Pascal et al. (2019) assert that these studies "demonstrate the benefits of teaching that allows children to represent their mathematical understanding [in ways of their choice] permitting personal and cultural knowledge to become tools" (p. 35), emphasising:

> It is evident in recent evidence reviews (Pascal et al., 2017; Payler et al., 2017) that when a child is given freedom of expression within stimulating environments that support rich dialogues and cover a breadth of learning […] [and] abundance of graphical resources representing different modalities and materials […], emerging symbolic languages such as writing and mathematics emerge, reflecting children's growing competencies and understanding. (p. 47)

The consequence of these missed opportunities to develop the recommendations of these several reviews and reports, is that teaching seems likely to continue to be based on assessment of narrow skills and become progressively restricted, teachers afraid to innovate. Moreover, children's experiences of mathematics will seldom differ from the findings of Hughes's research (1986), which pointed to "a serious mismatch between the system of symbols which children are required to learn, and their own spontaneous conceptualisations" (p. 78). Indeed, empirical evidence since 1986 is pointing to even greater pressures on teachers and even more extreme narrowing of teaching in early childhood in England (Bubikova-Moan et al., 2019).

Recalling earlier government guidance in England, the Qualifications and Curriculum Authority [QCA] (1999), detailed the need to understand young children's beginnings with mathematical inscriptions stating:

> It is well known that some children experience great difficulty in understanding formal methods of written calculation and applying them correctly in a given situation. […] *It is essential to build on what children know, understand and can do. If more succinct and efficient abstract methods are imposed on children without ensuring their understanding, the result, all too often, is that they lose even their more primitive methods and become disabled by the "teaching" process*. (p. 3-4, emphasis added)

[92] This is a collaborative document, with contributions and endorsements from twelve professional early childhood organisations in England.

Further, QCA stated: “At first, children’s recordings may not be easy for someone else to interpret*, but they form an important stage in developing fluency*.” (p. 12, emphasis added). Yet, since 1999, the various curriculum documents in England relating to early years’ mathematics have made considerable but unsubstantiated assumptions about young children’s early ability to understand the abstract symbolic language of mathematics. This continues to result in direct teaching of written numerals and other mathematical symbols, and provision of standardised and formulaic examples of written calculations for them to copy or complete. By a process of contraction, the current *Early Years Foundation Stage* curriculum (Department for Education, [DfE], 2017) includes nothing whatsoever to guide the teaching of early graphical inscriptions for mathematics.

Changes - of the sort that can *transform* young children’s learning - are sorely needed. Moss (2014) argues that *transformative change* necessitates a distinct means of thinking about early years’ education, Lenz Taguchi (2008) encapsulating the tensions in early childhood education well:

> One way of understanding the educational arena in a wider perspective today is that there are two strong contradictory movements at work; one of complexity and diversity increase, and one of complexity and diversity reduction […] the more we seem to know about the complexity of learning, children’s diverse strategies and multiple theories of knowledge, the more we seek to impose learning strategies and curriculum goals that reduce the complexities and diversities of learning and knowledge. (p. 1)

Radford (2016) pleads for “a reconceptualization of mathematics teaching and learning based on a cultural-historical communitarian ethic of solidarity and critical stance” (p. 3), themes reflected in the aspirations of the approach exemplified in this study. Gutiérrez et al. (2011) argue that effective literate practices:

> must be part of learning ecologies that design for the social accomplishment of shared practices – those mediated by meaningful relations, historicized and expansive notions of learning, distributed expertise, technologies, multiliteracies, and multilingualism – that ratchet up expertise and agentic identities for young children. (p. 259)

This thesis views children as emergent learners, their meta-representational knowledge and proficiency increasing through their sociocultural experiences at home and in their nursery. The approach investigated in this thesis focuses *on learners*, and their sense and meaning making in relation to mathematics, particularly the sense they make of using signs and symbols to express meanings in mathematical contexts.

Contesting the dominant discourses of education in England (and in many parts of the world), Deleuze and Parnet (1989) argue that:

> If you are not allowed to invent your questions from all over the place, from never mind where, if people pour them into you, you haven't much to say. While encountering others, and while each child is bringing in her/his lot, a becoming is sketched out between the different perspectives. (p. 9)

But aspirations to enhance the learning situations in which mathematics is taught requires turning away from what Lenz Taguchi (2010) describes as "the dominant reductive and simplifying and limiting forces in education" (p. 178), brought about "by a strain of governing running through the neo-liberal project, expressing a will to standardise, manage and control" (Moss, 2014, p. 69). In contrast to "its lack of curiosity, imagination and originality", Moss proposes "democracy, experimentation and potentiality" (p. 5). For Dahlberg and Moss (2010), theoretical and philosophical perspectives can "stimulate and support "a process of collaborative invention and creation" among children and teachers" (p. xiv), where to learn "means a form of experimenting in which no method pre-exists" (p. xvi), including mathematics.

Theoretical Gains

From the outcomes of the studies presented in this thesis, we can determine a number of theoretical gains and educational implications. Together, the results ascertained in chapters 2-5 establish the distinct aspects supporting young children's advancement with mathematical signs and inscriptions as they progress towards abstract symbols. At the onset of this thesis, our submission was that, provided young children have genuine meaningful and open opportunities for collaboration within social contexts such as child-initiated pretend play, and are supported by the adults in their setting, they will freely draw on their existing cultural understandings, developing their thinking, and choosing to communicate through dialogue and graphical inscriptions. We anticipated that such meaningful contexts would often prompt children to explore diverse aspects of their cultural knowledge relating to mathematics, and that their graphical inscriptions would enable them to build on and extend that knowledge. The findings endorsed this assertion. In respect of educational implications, the four related studies highlight features that can provide personally relevant situations in which young children may explore their cultural knowledge relating to mathematics and sign-use, extending and connecting it to new knowledge.

Cultural-conceptual Play and Mathematics

Combined, the findings above reflect a *cultural-conceptual* perspective of play, by drawing on Vygotsky's (1978) theorisation of both play and "*on the very process* by which higher forms are established" (p. 64) and imagination (2004a), to help illuminate how pretence and cognition, (and other open, and holistic contexts) can together support the development of mathematical concepts and signs.

In the introduction to this thesis, I wrote, "this study takes the child's perspective through a holistic, transdisciplinary approach, drawing on research into early childhood mathematics; semiotics; pretend play; social learning; cultural knowledge; children's social literacies; multimodality; cultural evolution and language acquisition". This suggests what Wylie (1989) refers to as "cabling". This metaphor originated from Bernstein's (1983) explanation of alternatives to objectivism and relativism, hinging on a metaphor inspired by Peirce's belief that scientific explanations are more like *cables* than chains. Rather than a linear methodology, Bernstein (1983) maintained that researchers must exploit "multiple strands and diverse types of evidence, data, hunches, and arguments to support a scientific hypotheses or theory" (p. 69). It has been this interweaving of the various elements listed above and their associated evidence, that has helped achieve the insights into the beginnings and development of children's graphical signs and representations in this thesis.

On the evidence of the theories we examined, the hypotheses we developed, and our research outcomes, we resolve that understanding, valuing and supporting young children's meaning-making through graphical signs in open contexts such as social pretend play, is of evident value to them in supporting their thinking in mathematical contexts. The philosophy and culture of the studied nursery school and Carruthers and Worthington's research, have brought about *transformative change*, or in Moss's (2014) words, told "a new story about early childhood education" (p. 11). Matthews (1999) argues that an open approach (to children's representations):

> is not defined in terms of a body of knowledge, planned a priori, and simply transmitted to the learner […] The subject domain is important only insofar as it contains instruments, processes and

> experiences which will promote human development and learning (Blenkin & Kelly, 1996). *What needs to be added to our understanding of the subject discipline, is how this interacts with the learner.* (p.163 emphasis added)

Hedges and Cullen (2005b) contend that “a sociocultural perspective provides justification for strengthening the place of subject content knowledge in early childhood curriculum and pedagogy”, and suggest that the debate that has led to polarised views regarding the place of subject knowledge, “might be less acute if rethought from a sociocultural perspective” (p. 76-77).

Educational Implications

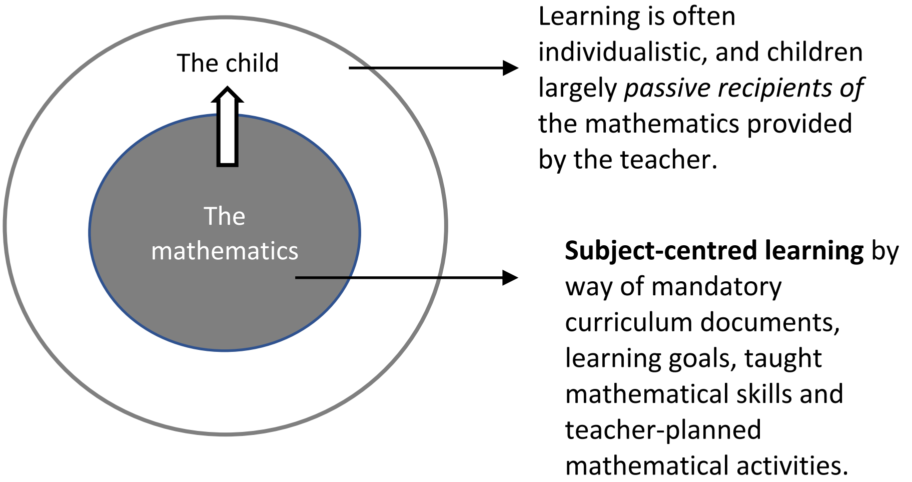
In employing a (traditional) *transmission approach*, teachers are likely to plan mathematical activities based on particular mathematical skills of the established (mandatory) curriculum for that particular age group. Whilst the children may informally develop understandings of certain aspects of mathematics in play activities (e.g., of capacity whilst playing with water or counting down while chanting number rhymes), to a great extent teaching continues to be direct instructional, subject-based knowledge. This is particularly likely in respect of mathematical symbols and (later), written calculations, solving written mathematical problems, all aspects of measuring and data handling. For curriculum writers, headteachers, teachers, other early years’ professionals and teacher educators the findings of this research calls into question a single, skills-based mathematics transmitted by the teacher, which for children often results in individualistic and passive learning. Van Oers (2010) cautions that “not paying attention to these events (related to children’s graphical markings) means that educators may neglect important and stimulating events for the promotion of mathematical thinking” (p. 32).

Valuing and supporting children’s mathematical inscriptions accentuate a notable benefit for teachers, highlighted in an interesting study by Anantharajan (2020), into teacher noticing of mathematical thinking in young children's representations of counting. Anantharajan examined the impact that young children’s representations of counting had on their teachers’ understandings of the children’s mathematics. She found that by focusing on the children’s inscriptions, the teachers noticed children’s “partial and emergent conceptions” that helped the teachers “develop a more fine-grained understanding of children’s mathematical thinking” (p. 293), that may assist them in uncovering and building a more thorough appreciation of children’s thinking. Moreover, when comparing the number of mathematical ideas the children had, Anantharajan found that the teachers noticed *from observation alone* (i.e., when observing the children engaged in counting), with observation *and* the teachers' examination of children's representations, that the teachers noticed between "two to eight additional ideas that augmented their initial observations" (p. 286). This suggests that for teachers, noticing *and* understanding children's own inscriptions has marked value in terms of their understandings of children's graphics that can guide their ensuing pedagogical practices.

The outcomes of our four studies provide a rich variety of examples of pretend play and mathematics, affirming the importance of social contexts for learning. The findings also show how together, children's diverse literacies are related, and how *Children's Mathematical Graphics* emerge and transform as children progress towards the standard symbols of mathematics. Vygotsky (1986) identified the difference between "spontaneous", "everyday" (or developmental concepts), and "scientific" concepts (which are directly instructed), declaring that "though scientific and spontaneous concepts develop in reverse directions, the two processes are closely connected. The development of a spontaneous concept *must have reached a certain level for the child to be able to absorb a related scientific concept*" (p. 194, emphasis added). Identifying a serious "threat" to effective learning in early childhood, van Oers (2013b) resolved that teachers should not "impose "scientific" answers on pupils, that do not really answer pupils' needs, [and] annihilate their freedom to develop new meanings". He argues "common classroom practice in subject matter learning is indeed not much of a playful endeavour" (p. 195). This current and widely used transmission approach is suggested in figure 1 (below), implying *scientific* concepts, where the very real problems young children can experience from such an approach originate.

Figure 6.1

Transmission approach: the mathematics is regarded as a body of knowledge to be imparted to the child



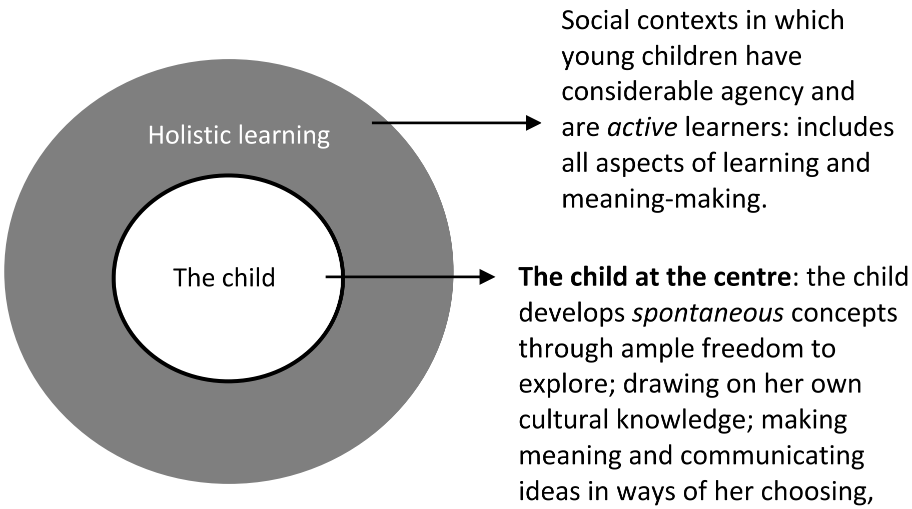
Van Oers (2015) stresses that to foster both teacher and child agency, a curriculum that establishes openness and permits children to take personal responsibility is needed.

In contrast to a transmission approach, the position advocated in this thesis is one of *holistic learning* in early childhood, children having agency in their learning, through their play and explorations in open contexts, and free to cultivate their own knowledge in partnership with their families, peers and teachers. Figure 2 suggests Vygotsky's *spontaneous* or *everyday* concepts, learnt in meaningful and holistic contexts[93]. Baxter Magolda (2000) writes that a holistic learning perspective focuses on ways in which children make meaning, "in cognitive, intrapersonal, and interpersonal dimensions" that mediate learning (p. 97). However, for teachers, such changes may necessitate a new acceptance about how knowledge is created, a shift in the idea of who has authority, and a new perspective of young learners. Accepting that learning is socially constructed, teachers then become allies and participants in children's learning (figure 2).

[93] The "Te Whāriki" curriculum in New Zealand exemplifies a holistic approach to early childhood education (see: Education Review Office, New Zealand Government, 2016).

Figure 6. 2

A holistic view of learning: the mathematics is seen within the context of all the child's meaning-making and learning



Conjecture about processes of mathematisation

The traditional view of children's "written" mathematics is that they move *from* their informal marks and signs *to* the formal abstract signs of mathematics, as a one-way journey. Hughes (1986) identified a "dangerous gap" between the two, and following their research, Carruthers and Worthington (2006) proposed a recursive, bi-directional model indicating children's free movement between their beginnings with intentional marks and the formal signs of mathematics. Vygotsky's observation that between these, "many *transitional psychological systems occur*" (1978, p. 46, emphasis in the original) appears highly relevant to this. Additionally, it must be noted here that for Vygotsky the development towards higher forms of thinking is not a case of replacement of the informal marks by formal mathematical signs. The higher forms of thinking are basically *transformations* of the spontaneously developed informal thinking. On the basis of our studies, we interpret this movement as a process of intertextuality through which formal signs (borrowed from others, and sometimes from an individual's previous text) are woven into the children's personal mathematical texts.

Tomasello emphasises that mathematics is likely to have developed over historical time by "cumulative cultural evolution" through "the ratchet effect" (1999, p. 37). He argues that in ontogeny, cultural learning creates,

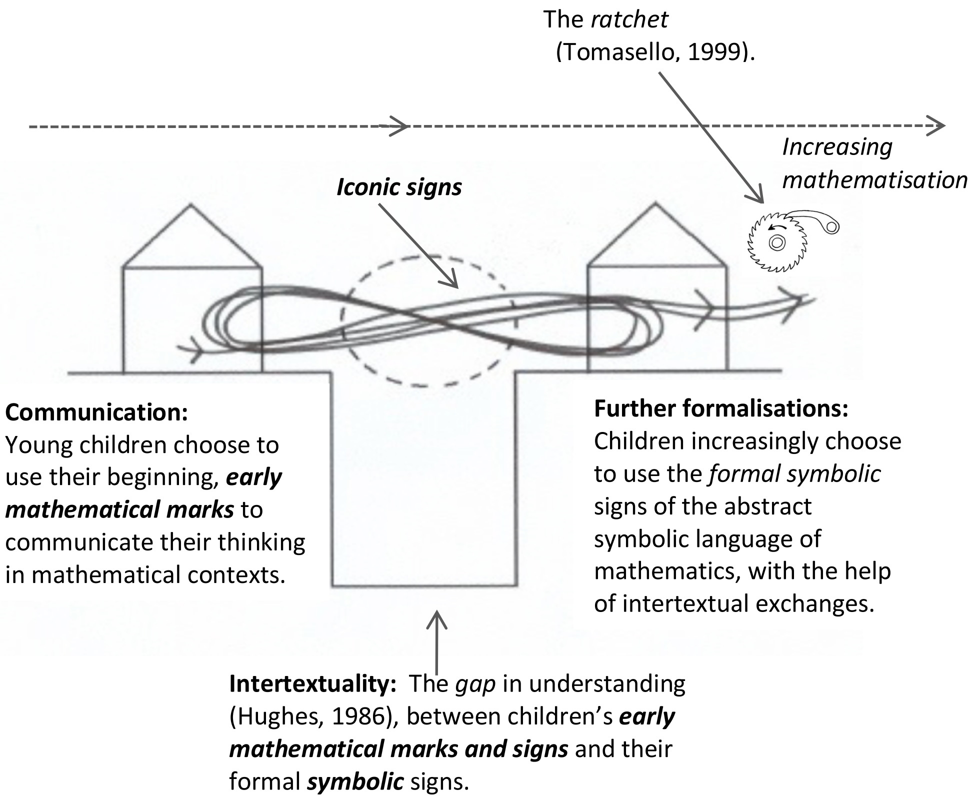
> … an especially powerful cultural ratchet […] and especially powerful forms of social-collaborative creativeness and inventiveness, that is, processes of sociogenesis in which multiple individuals create something together that no one individual could have created on its own (1999, p. 6).

In addition to this, we suppose that the embeddedness of children's new signs and texts in their shared culture minimises the chances that children will regress in their process of mathematisation. Inscriptions acquire modifications to more advanced knowledge over time as understandings become increasingly complex and integrated with children's informal (everyday) concepts and therefore, require more powerful and sophisticated communicative means to exchange and elaborate this advanced knowledge. The metaphor of a mechanical ratchet suggests that developments in children's signs do not readily regress to a previous elementary stage, although analysis showed that occasionally the children reverted to a more elementary means of communication (such as scribble-marks) when engaged in play, understood as a way to allow the flow of play to continue uninterrupted (Worthington & van Oers, 2017). Tomasello emphasises that the ratchet process involves "an especially powerful form of collaborative inventiveness" that can also be applied to "simultaneous collaboration […] in language and mathematics" (1999, p. 41/42).

Figure 6.3 combines Carruthers and Worthington's "bi-numeracy" model with Tomasello's "ratchet", suggesting cumulative changes over time from their beginnings, through integrative transformations to formal notations. The right-hand side of this model indicates that children increasingly chose to *integrate* into their textual communications the formal signs of the language of mathematics borrowed from more knowledgeable others. We propose this model as a theoretical generalisation for the process of mathematisation, as evidenced by our data. This model corresponds with Vygotsky's assertion that "embedded in the earliest stages of individual development […] transitional systems lie between the biologically given and the culturally acquired" systems (1978, 46). The appropriation of formal signs, once accomplished, cannot be undone (as in the *ratchet-effect*). We see this as an indication of growing mathematisation and development in mathematical thinking.

On the basis of our data, we can speculate further about the processes in which the child may be involved the middle of this circle. The gradual transition from spontaneous marks and signs marks to more formal signs can be presented in the behavioral descriptions as follows:

Figure 6.3: Bi-numeracy model, *showing expected processes of mathematisation to more advanced knowledge* (adapted from Carruthers & Worthington, 2006).



This is still a description of the child's behavior. Further description in terms of verbal or mental actions can be found in the previous chapters, and can be a possible topic for future research. Young children's *Mathematical Graphics* are both commonplace and remarkable, the children in this dissertation engaging *intellectually* rather than academically with mathematics and with mathematical inscriptions. Giving attention to the beginnings and development of young children's mathematical inscriptions, whilst at the same time emphasising "democracy, experimentation and potentiality" (Moss, 2014, p. 5), suggests a means by which such a reimagining of this important aspect of children's mathematical learning could favourably begin. The risk, Dahlberg et al. (2013) argue, is not in exploring what is unknown, but in retreating to the comfort of the "known" (p. 196).

Limitations and Future Research

This thesis offers a theoretical generalisation for the process of mathematisation of mathematical signs that can support teachers in creating a context in which young children may connect their concrete understandings of mathematics, with those they represent. In effect, though using their own marks and signs in personally meaningful contexts, children can “bridge the gap” of understanding of which Hughes (1986) wrote. The findings of this thesis are powerful, accentuating the value of social learning contexts and rich and spontaneous pretend play for children’s mathematics. Nevertheless, the findings of this thesis must be seen in the light of some limitations.

Issues with Sample Size and Selection

A major limitation of this research is that data were collected from only seven children in just one nursery school, thus limiting the empirical generalisability of findings from such a small sample. But at the same time, and due to the quantity and diversity of the data, the small sample size provided more extensive and detailed information than data collected from a much larger sample size could have made possible. The fact that the majority of the written observations were the teachers’ everyday documentations of children’s play and learning, provided validation of their significance for underpinning teachers’ professional understanding of children’s learning, including mathematics.

Several factors related to the small sample size could contribute to the variation between children in the findings. The children whose data is interrogated in this thesis, had attended the nursery school for contrasting amounts of time, differences that may have impacted on individuals’ level of confidence and familiarity with the nursery’s culture. Additionally, the seven children represented only a small percentage of children of this age who attended the nursery. Dissimilarities in the children’s home cultures and backgrounds are likely to have also contributed to variation in individual’s experiences and opportunities, and the home cultural knowledge they brought to the nursery. None of these differences were possible to predict prior to the onset of the study. Furthermore, Kline et al., (2018) conclude that “developmental trajectories and endpoints can vary due to the human ability to learn flexibly, acquire information from others, and to recombine socially and individually learned information in creative ways” (p. 7).

The teachers participating in this research were recommended by the headteacher, and gave their agreement to be involved in the study. These teachers have exceptional, well-developed expertise, and, whilst this was beneficial in respect of the rich data, their prowess is not necessarily typical and may limit the external validity of the outcomes. The culture and associated pedagogical strengths of staff in this nursery school, coupled with the headteachers' vision and research into *Children's Mathematical Graphics* – whilst a positive factor regarding the children's mathematical thinking – means that the children's examples are not likely to be representative of *all* nursery schools in England, of *all* children's pretend play, or their mathematics. However, the study shows that whilst it was not considered possible by previous researchers, within such a rich and democratic context, it is feasible for children to communicate their thinking with mathematical signs.

Despite these limitations, our studies have established that these children can indeed use their own marks and signs to make and communicate meanings with understanding, their signs gradually transforming into the standard abstract symbols of the culturally accepted system of mathematics. Moreover, the case studies affirmed the rich potential of spontaneous pretend play in which children may explore their personal, cultural knowledge, communicating and extending their mathematical thinking through social learning. The study of these children also produced empirical data that support a theoretical model, (abducted in the sense of Peirce), that may be seen as a powerful conjecture of conditions and processes that occur in the development of children's abstract (mathematical) thinking and use of mathematical inscriptions.

Methodological Limitations

The data collection and analyses were based on "event sampling" (Reis & Gable, 2000) from our full set of data, which could relate to our theoretical assumptions and possible answers to our research questions. Ratner (2002) observes that qualitative methodology recognises that the subjectivity of the researcher is intimately involved in scientific research, and that an important issue is how it affects objectivity. Reis and Gable (2000) write that in event sampling the data may include objective findings (such as the number of pretend play episodes in which the children engaged), and *subjective* descriptions "that are intrinsically impressionistic or that reflect mental processes (e.g., mood, sense of worth)" (p. 190), something that we attempted to avoid in producing field notes. Connor-Greene (2007) maintains that people often engage in unintentional subjectivity "when they believe they are providing factual descriptions" (p. 169). However, we took measures to reduce subjectivity, by ensuring that *all* the written observations of pretend play were used, rather than selecting only the "best". The sampling process was as robust and comprehensible as possible, by our definitions of the criteria for sampling (such as intertextuality, mathematisation, grammaticisation), and by using software (CAQDAS) for qualitative data analysis, which aided systematic coding, comparing and annotating our qualitative data. The process was also double-checked by a well-informed critical friend and colleague (Carruthers). Furthermore, we juxtaposed and compared outcomes from different types of data sources (i.e., written observations, photographs of children's mathematical inscriptions and field notes) which could be taken as a triangulation of the data.

Lack of Previous Research on this Subject

Whilst there have been several studies looking at (and for) young children exploring aspects of mathematics in pretend play, none has investigated this with children aged 3-4 years within *free and spontaneous* pretend play, meaning that we were unable to make direct comparisons with the findings of any related studies. However, the children's examples in this thesis bear comparison with examples of children of similar ages in our previous work (e.g., Carruthers & Worthington, 2006, 2011), some of these examples which were in free and spontaneous pretend play[94]. Moreover, no previous studies were found that researched the *very beginnings* of young children's signs and symbols in depth, in contexts that can be understood as mathematical, making this research unique.

Direction for Future Studies

Due to the nature of this study, children who did not readily choose to engage in graphical communication at the beginning of the year were not included: it would be worthwhile to investigate children who do not often choose to draw or write, to study how this subsequently impacts their mathematical inscriptions and investigate ways to ameliorate this. Further studies with a larger group of children should be conducted to see if the conjectured dynamics can be repeated and will enhance more children's understanding of sign-use for mathematical purposes. It would also be beneficial to investigate children's home cultural knowledge in greater depth in relation to this.

Since we were unable to follow the children into their reception classes (in the different schools to which they subsequently moved), we were powerless to ascertain their progress in mathematics over a longer period of time, particularly to gauge their success in expressing themselves though

[94] In this research, Carruthers and Worthington analysed 700 examples of *Children's Mathematical Graphics*, from children of 2-8 years of age.

their *Mathematical Graphics*. In England it seems unlikely that much can change in classrooms without a reduction in restrictions and impositions from the government, thus permitting room for teachers' agency, and, in turn, room for agency for children to engage with others in mathematics. Although there are currently considerable pressures on teachers (as reported in this thesis), it would be advantageous to do this, should government guidance become more relaxed.

In other countries where governmental pressures are not so intense as those of England, future studies should focus on teacher professionalisation to implement the described classroom culture in which play is understood as a significant "mode of activity" (van Oers, 2015, p. 23), allowing children high degrees of social learning and freedom to invent new communicative means and explore the meanings of their texts. The theoretical model that describes the main processes of this dynamic may be of help to re-create an ecologically valid scaled-up replication of our studies.

Whilst children's understanding and use of mathematical signs and inscriptions do not comprise the *whole* of mathematics, they are a highly significant aspect of children's mathematics, helping them develop understanding and confidence in mathematics as they move into school. In her study Johnsen Høines (2004) asks, "*How do we organise for the mathematics to be included into their mathematics*? (p. 1, emphasis in the original). She considered how, as a teacher and teacher educator, "the processes of children's own argumentations; their mathematising; their investigative activities" (p. 2) might be inspired. Aiming to gain insights "into children's ability in symbolisation" (p. 2), Johnsen Høines asserts that it has been argued

> that knowledge is implied within language. Through language people structure their observations; they make their categories and their hypotheses. To protect languages is about protecting knowledge. It is important to the people that own the languages, and it is also important to the world (and to the scientific field). This can be seen in the context of children's language: their knowledge is implied in their language. This supports an approach to empower the children's mathematical language. It is important to us. It also tells us that the formal mathematical language is characterised by certain ways of ordering. The content is implied in the language. This is supported by the child's voice: *It is the same, but it is not the same!* (p. 4)

In sum, this thesis furthers our knowledge of the genesis of young children's graphical inscriptions and mathematics; our understandings of social pretend play and the nature of effective learning cultures and contexts in early childhood. It appears to be *children's* mathematics, and notably, children's imaginations that make a difference, and that can transform young children's learning of the written language of mathematics. Above all, it is undeniably the socio-cultural influence of Vygotsky's ideas that can help us discern the genesis and potential of young children's graphical inscriptions made to communicate their mathematical ideas.

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Summary

The title of this dissertation: “The emergence and development of young children’s personal mathematical Inscriptions”, is chosen to affirm the significance I accord to children’s own use of graphical signs, which can assist in closing the gap between children’s concrete knowledge of mathematics and the abstract written symbolic language of mathematics. Notably, the title also reflects Vygotsky’s assertion of the importance of using a genetic approach, that traces the origins of children’s current behaviours, in order to understand the *higher forms* of behaviour (1978).

Employing a genetic approach to study the evolution of children’s signs from their early beginnings, ensured that their development during the course of one year could be determined.

The theme of this thesis is young children’s graphical signs to communicate their thinking in everyday contexts, that arouse problems that can be understood as mathematical. The principal objective of this thesis is to identify where in ontogenesis the cultural foundations of mathematical inscriptions originate. Due to the social and cultural nature of young children’s mathematics, the intention has been to investigate these origins within the children’s social pretend play and other “open” contexts, and to identify some of the factors that contribute to their success.

The headteacher of the nursery school participating in the study had co-developed *Children’s Mathematical Graphics* (i.e., children’s personal mathematical inscriptions) with the author of this dissertation, and had nurtured an open and democratic learning culture within the nursery school. Under her guidance the nursery teachers had developed exceptional understandings of pretend play, early mathematical development, graphicacy and emergent learning, and for the purposes of this research no intervention with the teachers was necessary. Seven children of 3-4 years of age were identified for case studies; three were selected because they had showed a particular interest in using early marks and graphical signs for drawing and writing, and these are termed the “focal children”; four other children were selected for comparison. Given the learning culture of the nursery school, we anticipated that all children would spontaneously and freely choose to use their graphics to communicate their thinking in the meaningful contexts of their self-initiated pretend play. We also expected that the focal children would augment the range of signs they used for drawings, maps, writing, and would communicate in contexts that could be considered mathematical, and that the non-focal children would continue to expand the range of signs they

employed. The research questions examined in the four studies were as follows:

- *What evidence of mathematics can be found in the children's free pretend play, and how does their cultural knowledge influence their thinking?* (Chapter 2)
- *What early graphical inscriptions do young children of 3-4 years of age spontaneously employ in the context of various literacies in their nursery school, and to what extent does their personal cultural knowledge strengthen their understandings?* (Chapter 3).
- *How do the children's inscriptions support their emergent abstractions?* (Chapter 4).
- *What evidence is there of intertextuality, particularly with respect to the use of graphical signs the children made in contexts that can be understood as mathematical, and how does this impact on their mathematisation?* (Chapter 5).

The methodology can be defined as observational and holistic, employing Interpretive (qualitative) analyses of the written and visual (graphical) data, and were supported by *computer assisted qualitative data analysis software* (CAQDAS), to systematically code as the data were processed, to provide answers to the research questions. These qualitative research methodologies also provided more realistic insights than statistical analysis alone could do, providing intricate and deep understandings located in specific contexts.

In chapter 2 we investigated the extent to which the children explored mathematics within their spontaneous pretend play. Due to the democratic values of the nursery school and the teachers' pedagogical practices, we expected that all children would freely and spontaneously choose to explore aspect of mathematics that arose within their pretend play. Our findings supported our beliefs, the children all demonstrating considerable enthusiasm and high levels of participation in the authentic contexts of their play, enabling them to explore and expand empirical concepts and permitting progressively complex notions relating to mathematics to develop. Moreover, the children's play was influenced by their personal and cultural knowledge. Children's spontaneous participation with aspects of mathematics (though at this age they will not understand them as "mathematics"), and their use of mathematical inscriptions appear to extend beyond consequences of biological or autonomous development. Rather, we resolved that it was the nursery school's philosophy, coupled with the teachers' advanced pedagogical understandings that promoted and sustained the children's thinking. The children's freedom to self-initiate their play, and its highly social nature, stimulated them to communicate their ideas. The findings of this study affirmed that many episodes of pretence encompassed diverse elements of mathematics, increasing throughout the year. The children drew extensively on their personal cultural knowledge, exploring and extending their mathematical knowledge and understandings within the context of their unstructured pretence and imagination. In almost half of the pretend play episodes, they freely and spontaneously communicated their thinking through their *Mathematical Graphics*. Given the many examples of children producing signs to communicate their mathematical thinking in this first study, we wanted to investigate their signs in greater depth, so in chapter 3 investigated their signs for a range of communicate purposes. This finding led to the following chapter (chapter 3), in which we investigated the same children's social and multimodal literacies.

Our purpose in chapter 3 was to establish the extent to which the children's communications in their diverse literacies (drawing, maps, writing and mathematical). contributed to their play and understanding. To achieve this, we examined the relationship between the children's cultural knowledge of literacy practices at home, and their literacy events in the nursery. Our findings demonstrated that the parents' use of literacies in meaningful contexts for authentic purposes, clearly contributed to children's understandings of the role and purposes of literacies. Particularly evident was the children's eagerness to communicate through graphical inscriptions, combining both their *interest* and anticipated *need* to communicate to further their play, their use of all literacies increasing during the year. This is notable, since all of these events were entirely child-initiated. Multimodal features were apparent across all the children's inscriptions. All the children developed their graphical repertoires from their beginning scribble-marks, some children also including standard alphanumerical symbols. Furthermore, the relationship between the various inscriptions and the frequency of their use across all the children's literacies, related to their growing use of emergent signs and symbols in contexts that could be understood as mathematical. Building on the findings of chapter 3, we interrogated the data in order to determine how the children's graphics conveyed their emerging mathematical understandings, and how this supports their emergent abstractions.

Chapter 4 explored how the children's inscriptions supported their emergent abstraction. The evolution of mathematical abstraction is viewed here as *from a specific point of view*, early in children's advancement towards the fully abstract symbolic language of mathematics. The children's own words, drawings, marks and signs help them focus on relationships they see as appropriate for their representation, assisting them to express their (abstract) view on their world.

We examined the range of signs the children used to communicate their mathematical thinking, identifying how their intention-reading and pattern-finding supported increasing grammaticisation. Our findings highlighted the various strategies the children employed, beginning with their *early explorations with marks* to which they ascribed mathematical meanings, to using a wide range of iconic signs and some that were indexical. Iconic signs have some resemblance to the object signified and, in this study, included tallies, wavy or zigzag lines writing-like lines, letter and numeral-like signs, crosses (to signify none or nothing), and dots (used to refer to uncounted quantities). The iconic signs the children used suggest these serve as elements of their protolanguage, their use anticipating and increasingly transforming into standard symbols. The importance of symbolic number knowledge demonstrated the significance of their ability to *write* standard symbols in acquiring the abstract graphical language of mathematics. Once again, we determined that the social nature of their shared play was significant. Frequent graphical inscriptions (including standard letters and numerals) across all the children's literacies, seems an important factor in developing *mathematical* abstractions for communication, and it was the children with the most extensive lexicons of graphical signs whose graphical texts to communicate mathematical ideas were most advanced. These findings alerted us to the need to appreciate how young children come to know the various graphical signs. To do this, in chapter 5 we examined examples of children's use of different inscriptions and their intertextual use of signs signifying both tallies and numerals.

Chapter 5 provided evidence of intertextuality with respect to the children's use of mathematical signs in their nursery school. The findings determined how signs flowed between each other's (and their teacher's) texts. This sharing of signs establishing that the children made use of each other's representations of quantities and number, and affirmed mathematising *in statu nascendi*. The children's intertextual sharing of signs showed that they made use of quantities, number, order and relationships, realising their emerging mathematising. Iterated signs permitted the children to appreciate the disparate contexts in which signs are used, extending their understandings of sign-use and augmenting their sign lexicons. Through their pretend play children gain access to any cultural activity (including mathematising) by emulating this activity: rather than imitating directly taught mathematical activity, it is everyday *cultural* activity that they imitate. We resolved that the beginnings of mathematisation are founded in the cultural (adult) sign-use and through collaborative activity with others, and that mathematisation is accomplished through negotiating meanings. Intertextuality also contributed to symbolic diversity within the social group, providing multiple perspectives of signs. Intertextual transactions of others' graphical signs and texts provide critical conditions for children's mathematisation. The findings of this study (chapter 5) revealed that like the signs they employed in their drawings, maps and writing, *all* of their graphical signs are embedded in meaningful social practices.

Each of these four research studies (i.e., chapters 2-5), establishes the significance of the *cultural* and *social* nature of pretend play and other open learning situations, beneficial in promoting collaborative learning and encouraging imitation, emulation and intertextuality of peers' and adults' graphical signs-use, in personally meaningful ways. The children's pretend play episodes and their associated inscriptions were clearly underpinned by the social practices of their play, and rooted in their funds of knowledge, their cultural knowledge from home. Intertextuality contributes to symbolic diversity within the social group, providing multiple perspectives of signs and enriching children's expanding sign-lexicons with the help of adult signs and texts. Our findings concerning intertextuality and mathematisation also reaffirm the importance of children's social learning.

Social learning empowers children to develop rich and diverse repertoires of graphical signs, helping them build on their own strategies, teachers respecting their spontaneous signs. Combined with the theoretical and philosophical discussions, the research results determined that young children's early sign-use begins in authentic contexts at home and develops in social contexts such as pretend play in their nursery school. In this pretence problems arise in the course of their narratives, and with regard to the roles the children want to play, their cultural knowledge ensures that their learning is purposeful and relevant. These findings also challenge any separation of signs for the various literacies, including those they used to communicate their mathematical thinking. This is especially important for children's mathematical understandings, helping them build on and connect their existing understandings with those of their new experiences and knowledge of the nursery.

The positive outcomes of these combined studies reveal a number of distinct advantages for young children's mathematical understandings and development. Children who had developed a rich lexicon of graphic signs were at a particular advantage in representing and communicating their thinking, two of the focal children freely using *spontaneous written Arabic number symbols* (or SWANS). We believe that the social nature of pretend play and other open learning contexts provide potentially rich opportunities for multiple perspectives of signs and for cultural cooperation, helping them to solve mathematics problems with understanding and contributing to mathematisation.

Throughout the year, two of the focal children in particular expanded the range of signs they used, frequently communicating through their *Mathematical Graphics.* At home, the children sometimes self-initiated literacy events (the focal children appearing to do so most often): the more they freely did so at home, the more this enabled them to experiment with graphical signs and symbols and use them spontaneously in their nursery school. Shereen and Elizabeth (two of the focal children) drew on their increasingly extensive repertoires of signs to communicate a range of meanings, including those that could be understood as mathematical: this confirms that children who have an extensive “vocabulary” of graphical signs appear to be at an advantage. Isaac, the third focal child, developed his continuing interest in making maps and extended his knowledge of environmental signs. He often spoke of numbers in meaningful contexts, but used fewer graphical signs to communicate through literacies or to represent numerals. The non-focal children all developed their sign-use during the course of the year, but used fewer iconic signs than the focal children and very few standard symbols, such as letters or numerals.

Given the favourable findings of these studies it would be of benefit to further investigate issues identified in this research. This could focus on the nature of free and spontaneous pretend play to further determine its value for the development of young children, and to establish any “subject” learning that ensues. It would also be advantageous to study children’s progression in mathematics (especially their use of mathematical inscriptions) over an extended period, from children aged 3-4 years in their nursery school, to children of 6 years of age in school.

Our research findings have provided evidence of the significance of *Children’s Mathematical Graphics* in helping “bridge the gap” between young children’s concrete knowledge of mathematics, and their understandings of the abstract written language of mathematics; between their intuitive early marks and signs and the standard symbols of the accepted cultural system of mathematics. It seems reasonable to consider that further expansion of this approach can deepen teachers’ appreciation of children’s beginnings with mathematical signs and their powerful mathematical thinking, and enhance their pedagogy. Extending this research is likely to bring about greater benefits for young children that are more age-appropriate, especially regarding free and spontaneous pretend play, and subsequently impact on their attitudes, understandings and confidence with mathematics in school.

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My special thanks go to the two teachers Emma and Hugo at Redcliffe, who so generously shared your insights and documentations of the children's play. I learnt such a lot from reading these, and my own observations of the children's rich play and graphical communications have considerably deepened my understandings of young children's thinking and pretence. Sincere thanks to the children of Redcliffe, whose play so nourished their thinking and understandings, and provided me with a window into their thinking and graphical communications. My heartfelt thanks and appreciation are of course extended to the children's families.

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Topsham, England, July 2021

Curriculum Vitae

Maulfry Worthington was born on the 26th September 1946 in Salisbury, England. She studied teacher education at St Gabriel's College, University of London, gaining her Certificate of Education in 1968. From 1968-2002 she taught children of 4-7 years of age in city, town and village schools. Studying at the University of Exeter, Maulfry was awarded her Master's (M.Ed.) in early years' education in 1996.

In 1995 Maulfry taught two early years' literacy courses at the University of Exeter. In 1992-1993, and again in 2003, Maulfry lectured at the University College of St Mark & St John in Plymouth, on early years' education, and primary and early years' mathematics. In 2000-2001 she worked in Dorset as a National Numeracy Consultant (Primary), and as a Mathematics Consultant for early years. During more than 30 years, she also led numerous professional development courses for early years' teachers throughout the United Kingdom, including many on early years' mathematics. From 2003-2007 Maulfry worked as an e-learning facilitator, managing early years' online courses for headteachers and other school leaders at the National College for School Leadership (NCSL) Nottingham. Between 2013 and 2017 Maulfry taught Master's courses on early years' mathematics in Bristol and Birmingham.

Maulfry has been a key speaker and presented at many conferences in in the UK and in Europe, and has published extensively, including *Children's mathematics: Making marks, making meanings* (2006) with Elizabeth Carruthers. Maulfry and Elizabeth's work has been featured in a number of independent, and government-commissioned research reports and reviews relating to early years' mathematics, and they were commissioned by the government in England to write guidance for teachers, *Children thinking mathematically* (DCSF, 2009).

From 2008 Maulfry has been a PhD candidate at the Vrije Universiteit, Amsterdam, of which this dissertation is the final result.