On the Optimality of Handbrake Cornering

Davide Tavernini, Efstathios Velenis, Roberto Lot, Matteo Massaro

Abstract—The aim of this paper is to investigate the optimality of the handbrake cornering technique for a Front Wheel Drive vehicle. Nonlinear Optimal Control theory is used to formulate the problem of optimal cornering and to simulate manoeuvres used by race drivers. Handbrake cornering is optimal with an appropriate selection of the minimization cost. The optimal solution is validated against data collected during the execution of the technique by an expert race driver on a loose off-road surface. Further optimization results considering high adhesion road surface are obtained to show that the optimality of the technique is not affected by the road conditions.

I. INTRODUCTION

In recent years the importance of investigating the nonlinear region of operation of tyres has grown with the idea that the so-called aggressive driving techniques could be more efficient than manoeuvres at low vehicle-slip-angle under some particular conditions of road geometry and tyre-ground adhesion [1]. Furthermore, increasing the knowledge on these techniques can lead to autonomous vehicles able to perform aggressive manoeuvres [2]. Experience suggests that drifting manoeuvring is not a peculiarity of a particular kind of vehicle and transmission layout, i.e. FWD (Front-Wheel-Drive), AWD (All-Wheel-Drive) or RWD. Some of the well known rally driving techniques, used by professional drivers in the competitions, e.g. pendulum-turn and trail-braking, have been studied recently [3], [4], [5] using minimum time nonlinear optimal control.

In the present paper handbrake cornering, another typical rally driving technique, is considered. This manoeuvre basically consists of braking the rear axle in order to produce a reduction of the lateral force, due to the lateral-longitudinal coupling in tyre-ground interactions [6], and rotating the steering wheel to trigger the skidding of the rear end of the vehicle. This technique is mostly used to approach very tight low speed corners, e.g. low radius ‘hairpin’ turns, on different road surfaces.

The aim of the present research is to reproduce such a manoeuvre through nonlinear optimal control simulations. The control problem is defined as follows: find the optimal manoeuvre (in terms of vehicle states, trajectory and control inputs) given the car characteristics, tyre road characteristics, road geometry and driver limitations (control bandwidth and magnitude). To reproduce the handbrake cornering technique, the selected cost function compromises between minimum time and deviation from the inner limit of the road. The resulting nonlinear optimal control problem is solved using the indirect method detailed in [7]. Driver’s input is limited in frequency and magnitude to reproduce real drivers limitations [8]. The method has been used successfully in the past for minimum time manoeuvring of car [1] and motorcycles [9],[10]. Similar approaches on minimum time manoeuvring are reported in [11],[12],[13].

A single-track vehicle model with nonlinear tyre force characteristics is used for simulations. Recent papers [3], [4] have shown the capability of this simple model to reproduce very complex and aggressive manoeuvres while matching experimental data. The results of simulations have been validated against real experimental data including an handbrake corner performed by a professional driver on off-road. Finally, a solution including the handbrake technique for the same vehicle on a paved road has been calculated.

II. DATA COLLECTION

The data collection took place at the facilities of the Bill Gwynne Rally School in Brackley, UK, using a front wheel drive rally vehicle represented in Fig.1. The vehicle velocity and sideslip angle were measured using a Racelogic VBox twin GPS antenna sensor. An inertial measurement unit (IMU) was used to measure 3-axis body accelerations and 3-axis body rotation rates. The rotational speed of each individual wheel and the percent application of throttle were measured using the existing vehicle sensors and a CAN-bus interface to collect the data from the vehicle’s ECU. A potentiometer was used to measure the steering angle at the steering wheel. The vehicle was fitted with two brake pressure sensors to measure the brake pressure at the front and rear wheels. The handbrake was integrated in the hydraulic brakes circuit and engaged the rear brakes only.

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Fig. 1. FWD Ford Fiesta test vehicle during data collection.
The problem is posed as the constrained minimization of an integral cost function:

$$\min \int_0^T \varphi(x, u, t)dt$$  \hspace{1cm} (1)

The constraints include: the equations of motion of the vehicle system given by:

$$f(x, \dot{x}, u) = 0$$  \hspace{1cm} (2)

where \(u\) are driver controls and \(x\) is the mechanical system state vector. Initial and final conditions (vehicle velocities and positions on the road at the start and finish line) are given by:

$$b(x(0), x(T)) = 0$$  \hspace{1cm} (3)

Furthermore, the constraints include some inequalities:

$$g(x, u) \leq 0$$  \hspace{1cm} (4)

accounting for the vehicle physical limitations (e.g. maximum engine torque), the road geometry, and the tyre-road friction. One of the main advantages of the method is that no driving rules have to be predefined. In particular, we do not impose the application of handbrake in the solution. It rather emerges from the optimality criterion and from the solution of the control problem itself.

A. Vehicle model

The single-track model employed is depicted in Fig. 3 and includes nonlinear tyres and front/rear load transfer. For each axle the wheel includes the contribution of the left and the right wheel of the real vehicle. Pitch and roll rotations are neglected so the motion is planar. The equations of motion in the longitudinal, lateral and yaw directions are:

$$\begin{align*}
( & -\Omega V \sin \beta + \dot{V} \cos \beta - V \sin \beta \dot{\beta} - g s_F \cos \delta \\
& + g \frac{f_F}{\rho C_a A} \sin \delta - s_R g) M + k_D V^2 \cos^2 \beta = 0 \\
- M(-\Omega \cos \beta - \dot{V} \sin \beta - V \cos \beta \dot{\beta} & + g s_F \sin \delta + g \frac{f_F}{\rho C_a A} \cos \delta + g f_R) = 0
\end{align*}$$  \hspace{1cm} (5)

$$I_G \dot{\Omega} = M g a \cos \delta s_F - M g a \sin \delta f_F + b f_R M g = 0$$  \hspace{1cm} (6)

$$I_M \dot{\omega}_R + (r s_R - t_R) M g = 0$$  \hspace{1cm} (7)

where \(V\) is the velocity, \(\beta\) is the vehicle slip angle, \(\Omega\) is the yaw rate, \(f_F\) and \(f_R\) are lateral loads, \(s_R\) and \(s_F\) are longitudinal loads, \(\delta\) is the steer angle. Vertical, lateral and longitudinal loads are normalized on the weight \(M g\) throughout the model formulation. Vehicle parameters are those reported in [16] with the addition of the drag coefficient \(1/2 \rho C_a A = 0.21 N s^2 m^{-2}\), relaxation length \(\sigma_F = 0.5 m\), suspension lag \(\tau_N = 0.25 s\) and maximum engine power \(P_{max} = 110 kW\).

For the two axles the equilibrium is expressed as follows:

$$I_M \dot{\omega}_R + (r s_R - t_R) M g = 0$$  \hspace{1cm} (8)

Hence, the two brake sensors allowed to distinguish between application of foot brake and handbrake. Fig. 2 reports data including the handbrake cornering technique. The handbrake command (that engages the rear wheels only) is actuated at 0.7 s while the driver is steering inward the hairpin in order to decelerate the vehicle and to reduce the rear lateral forces. The handbrake rear braking pressure is applied between 0.7 < \(t\) < 2 s, while the front pressure remains null. This braking force causes a reduction in the spin velocities of the wheels until their complete locking (1 < \(t\) < 2 s). The coupling between the steering input and a reduced total rear lateral force triggers the skidding of the rear end of the vehicle and a rapid growth of the vehicle slip (\(t > 1 s\)). For the whole manoeuvre the steer angle is inward the corner and at its maximum value for \(t > 1 s\). Counter-steer and foot brake are not present at all during this test.

III. MATHEMATICAL FORMULATION

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$$\begin{align*}
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- M(-\Omega \cos \beta - \dot{V} \sin \beta - V \cos \beta \dot{\beta} & + g s_F \sin \delta + g \frac{f_F}{\rho C_a A} \cos \delta + g f_R) = 0
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where \(u\) are driver controls and \(x\) is the mechanical system state vector. Initial and final conditions (vehicle velocities and positions on the road at the start and finish line) are given by:

$$b(x(0), x(T)) = 0$$  \hspace{1cm} (3)
\[ I_M \dot{\omega}_F + (r_s f - t_F)Mg = 0 \] (9)
in which \( t_r \) and \( t_f \) are torques at rear and front axle respectively. They are defined as a particular function that allows to allocate different fraction of \( s_i \) (i.e. total traction/braking force) on the two axles due to its own sign, i.e. driving or braking conditions, and of \( h_0 \), i.e. handbrake braking force. \( \omega_R \) and \( \omega_F \) are wheel spin velocities.

Practical longitudinal slip quantities \( \kappa_R \) and \( \kappa_F \) are defined as follows:
\[ \kappa_R = \frac{\omega_R r - V \cos \beta}{V \cos \beta} \] (10)
\[ \kappa_F = \frac{\omega_F r - V \cos(\beta - \delta) - \Omega a \sin \delta}{V \cos(\beta - \delta) + \Omega a \sin \delta} \] (11)

Practical lateral slip quantities \( \lambda_R \) and \( \lambda_F \) are:
\[ \lambda_R = -\arctan\left(\frac{-\Omega b + V \sin \beta}{V \cos \beta}\right) \] (12)
\[ \lambda_F = -\arctan\left(\frac{V \sin(-\beta + \delta) + \cos \delta \Omega a}{V \cos(-\beta + \delta) + \sin \delta \Omega a}\right) \] (13)

The theoretical longitudinal and lateral slip quantities are:
\[ \sigma_x = \frac{\kappa}{1 + \kappa} \quad \sigma_y = \frac{\tan \lambda}{1 + \kappa} \] (14)
in which \( \kappa \) is the practical longitudinal slip for each axle defined in (10) and (11). \( \tan \lambda \) is the practical side-slip angle as calculated in (12) (13).

The equivalent slip quantity is expressed as:
\[ \sigma = \sqrt{\sigma_x^2 + \sigma_y^2} \] (15)

Using these definitions it is possible to calculate lateral \( \mu_f \) and longitudinal \( \mu_s \) friction coefficients for both the axles:
\[ \mu_f = \frac{\sigma_x}{\sigma} D \lambda \sin\left[C \lambda \arctan\left(\sigma B \lambda - E \lambda (\sigma B \lambda - \arctan \sigma B \lambda )\right)\right] \] (16)
\[ \mu_s = \frac{\sigma_y}{\sigma} D \lambda \sin\left[C \lambda \arctan\left(\sigma B \kappa - E \lambda (\sigma B \kappa - \arctan \sigma B \kappa )\right)\right] \] (17)

with the \( B, C, D, E \) coefficients reported in Tab.I.

Since the collected data refer to an off-road manoeuvre, it is necessary to adjust the traditional diagram of the tyre-ground interaction. In this paper we consider an effect of the soil consistency on lateral interactions only. The adjustments are in the direction of reducing the cornering stiffness and, at the same time, moving the peak of the tyre force outside of the working range as shown in [14] and [15].

In section V we consider a tyre model without such adjustments to show that the optimality of the manoeuvre remains independent of the type of road surface. The resulted

<table>
<thead>
<tr>
<th>Longitudinal</th>
<th>( B_\lambda = 7.0 )</th>
<th>( C_\lambda = 1.8 )</th>
<th>( D_\lambda = 0.6 )</th>
<th>( E_\lambda = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral</td>
<td>( B_\lambda = 7.0 )</td>
<td>( C_\lambda = 1.0 )</td>
<td>( D_\lambda = 0.6 )</td>
<td>( E_\lambda = 0 )</td>
</tr>
</tbody>
</table>

![Fig. 4. Longitudinal \( \mu_s \) and Lateral \( \mu_f \) tyre-ground adhesion coefficients, off-road conditions.](image)

diagrams for longitudinal and lateral force interactions are shown in Fig.4.

Longitudinal forces are simply calculated as:
\[ s_R = n_R \mu_{sr}(\lambda_R, \kappa_R) \] (18)
\[ s_F = n_F \mu_{sf}(\lambda_F, \kappa_F) \] (19)
in which \( n_R \) and \( n_F \) are vertical loads at each wheel. These include longitudinal load transfers that arise from the longitudinal acceleration. Vertical loads include a time-lag \( \tau_N \) to account for suspension dynamics:
\[ \tau_N \dot{n}_R + n_R + \frac{-h \mu_{sr}(\lambda_R, \kappa_R) \cos \delta + h f_f \sin \delta - a}{h \mu_{sf}(\lambda_F, \kappa_F) \cos \delta + a - h \mu_{sr}(\lambda_R, \kappa_R) + b} = 0 \] (20)
\[ \tau_N \dot{n}_F + n_F + \frac{-h \mu_{sr}(\lambda_R, \kappa_R) + b + h f_f \sin \delta}{h \mu_{sf}(\lambda_F, \kappa_F) \cos \delta + a - h \mu_{sr}(\lambda_R, \kappa_R) + b} = 0 \] (21)

where \( h \) is the height of the centre of mass from the ground. The relaxation properties of the lateral forces are described using a first-order differential equation:
\[ \frac{\sigma_f}{V \cos \beta} \dot{f}_R + f_R - n_R \mu_f(\lambda_R, \kappa_R) = 0 \] (22)
\[ \frac{\sigma_f}{V \cos \beta} \dot{f}_F + f_F - n_F \mu_f(\lambda_F, \kappa_F) = 0 \] (23)

where \( \sigma_r \) is the relaxation length for lateral forces. Summarizing, the model has 12 state variables:
\[ x = \{ s_s, s_n, \alpha, V, \beta, \Omega, f_R, f_F, n_R, n_F, \mu_{sr}(\lambda_R, \kappa_R), \mu_{sf}(\lambda_F, \kappa_F) \}^T \] (24)

where \( s_s \) is the curvilinear abscissa, \( s_n \) is the lateral displacement from the middle lane, \( \alpha \) is the angle of the vehicle orientation with respect to road.

The vehicle trajectory is described by three equations:
\[ s_n K(s_s) \dot{s}_s - s_n + V \cos(\alpha + \beta) = 0 \] (25)
\[ -s_n + V \sin(\alpha + \beta) = 0 \] (26)
\[ K(s_s) \dot{s}_s + \dot{\alpha} - \Omega = 0 \] (27)
Fig. 5. (a) Distribution of the two terms of the cost function, on the “hairpin” path (b).

where \( K \) is the curvature of the road for the correspondent curvilinear abscissa \( s_n \).

The controls vector consists of 3 elements:

\[
u = \{s_t, h_b, \delta\}^T
\]

(28)

where \( s_t \) is the total driving/braking force, \( h_b \) is the handbrake force, \( \delta \) is the steer angle.

B. Optimality criteria

The collected data are in agreement with empirical guidelines from the expert driver in that the handbrake cornering technique is used to negotiate very tight corners with quick entry and exit. To this end we propose an optimization cost function which considers the lateral distance from the inner boundary of the road as well as the time of travel. This additional cost on the lateral deviation from the inner limit is only applied near the corner, whereas the minimum time cost is applied throughout the trajectory including the initial straight segment leading to the corner (Fig.5).

The combined cost is given by:

\[
\varphi = C_t + C_s \frac{\max(s - s_{target}, 0)}{s_{target}} s_n
\]

(29)

where \( C_t \) and \( C_s \) are weights for the two terms of the cost function, \( s_{target} \) is the value of the curvilinear abscissa where the cost on the lateral distance is applied, \( s_n \) is the lateral displacement from the middle lane.

As it will be shown later on, depending on the available tyre friction force the vehicle may approach the corner near the inner limit of the road. In this formulation the initial position of the vehicle is not affected by the cost function.

C. Constraints

In this specific case of rally car simulations the constraints described by (4) include boundary limits of the road for both tyre-ground contact points:

\[
-L_w + b \sin \alpha \leq s_n \leq R_w + b \sin \alpha
\]

(30)

\[
-L_w - a \sin \alpha \leq s_n \leq R_w - a \sin \alpha
\]

(31)

where \( R_w \) and \( L_w \) are right and left road widths respectively.

Maximum value for the steer angle and maximum available engine power:

\[
|\delta| \leq \delta_{\text{max}} \quad (t_F \omega_F) Mg \leq P_{\text{max}}
\]

(32)

IV. OPTIMAL HANDBRAKE CORNERING

The handbrake manoeuvre discussed in section II has been reproduced by an optimal control problem simulation and results are reported in Fig.6. Tab.II reports boundary conditions for this set of simulations.

Results show that as the handbrake force command is applied and reaches its maximum at \( t = 0.5 \) s, the rear wheel spin velocity drastically decreases (Fig.6a). The wheel’s spin inertia decreases and hence a lower force is required to maintain wheel lock. The velocity continues decreasing until \( t = 1 \) s. The steer angle is applied during the handbrake braking between \( 0 < t < 1 \) s (Fig.6b), and then again since the locking of the rear wheel is reached at \( t = 1 \) s, its maximum value is maintained during \( t > 1 \) s. In the first part of the simulation between \( 0 < t < 1 \) s, the longitudinal force generated by the handbrake input causes a reduction of the rear lateral force (Fig.6c) that remains small during the first application of the steer. In this condition the rear tyre loses adhesion and drift starts to appear with a vehicle slip angle that grows almost linearly until the middle of the turn.

A comparison between collected data and simulation is reported in Fig.7. The absolute velocity \( V \) of the vehicle does not present relevant differences. The steering command is applied in a more aggressive way in the simulation but the sign is always respected and counter-steer does not appear. This is in accordance also with [16] that shows that for very

### TABLE II

<table>
<thead>
<tr>
<th>Variables</th>
<th>( V )</th>
<th>( \beta )</th>
<th>( \Omega )</th>
<th>( \alpha )</th>
<th>( s_n )</th>
<th>others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial B.C.</td>
<td>37 kph</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>free</td>
<td>free</td>
</tr>
<tr>
<td>Final B.C.</td>
<td>free</td>
<td>free</td>
<td>free</td>
<td>0</td>
<td>free</td>
<td>free</td>
</tr>
</tbody>
</table>

Final B.C. free free free free free free

Initial B.C. 37 kph 0 0 0 free free

Fig. 6. Handbrake cornering simulation results. Longitudinal, lateral and vertical forces are normalized by the vehicle weight \( Mg \).
tight corners there is no counter-steering for the same FWD car. The vehicle slip angle in simulation reaches high values although slightly lower than the data. Matching of slip angle on the time scale is still relevant. The yaw rate exhibits the same maximum value, but it is reached in different instants. The last relevant observation can be drawn by means of Fig.8 where the 3D representations allows to appreciate the main difference between the real (Fig.8b) and the optimal driver (Fig.8a) for this particular technique. The real driver applies the handbrake command (i.e. braking pressure) essentially as an on-off input, to lock the rear wheel for the whole first part of the manoeuvre. In the optimal control formulation the handbrake command corresponds to a rear wheel braking force and it’s modulated to achieve the optimal manoeuvre. Moreover we observe that the optimal driver uses both handbrake and pedal brake to reach his target as shown by the second peak of the rear longitudinal force in Fig.8a.

**V. GENERALIZATION OF THE HANDBRAKE TECHNIQUE**

A new set of simulations have been performed assuming the same vehicle, same boundary conditions (II), but changing the operation conditions, i.e. asphalt ground and high performance road tyres. In practice, the adjustment with respect to the previous tyre parameters regards the peak values ($D_\lambda, D_\kappa$) and the shape factor ($C_\Lambda$) as reported in Tab.III and Fig.9. Simulation results reported in Fig.10 show that the calculated manoeuvre is once more an aggressive high-slip one (Fig.10b). The handbrake is still actuated in the first phase between 0.5 < $t$ < 1.7s (Fig.10a) and the deceleration of the rear wheel with respect to the front one starts to appear at $t = 1$s. In this case the ideal driver couples the handbrake and the foot brake generating an extra braking force between 0.5 < $t$ < 2.5s (Fig.10d), this last allows to continue decelerating the rear wheel until its locking, even if the handbrake has been released. The steering command is applied first during the braking phase between 1 < $t$ < 2.4s (Fig.10b) and then just before the release of all braking forces at $t = 2.4$s. Afterwards its value remains constant and maximum (0.35 rad) until $t = 4$s. Fig.11 presents a comparison between the current case (Fig.11a) and two different simulations under the same ground-tyre contact conditions (Fig.11b and 11c). The figure shows trajectory, vehicle drift and handbrake usage (through coloured markers) for each case. Fig.11a deserves an important observation on the vehicle trajectory: the handbrake manoeuvre in this case is coupled with another typical rally technique, the pendulum-turn. The vehicle starts from the inner boundary of the road and turns towards the outer limit while still braking. The deceleration causes a significant load transfer from the rear to the front axle of the vehicle. At this point the steering is applied towards the inner limit. Due to the

<table>
<thead>
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<th>TABLE III</th>
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<tbody>
<tr>
<td><strong>PACEJKA ASPHALT TYRE PARAMETERS.</strong></td>
</tr>
<tr>
<td>Longitudinal</td>
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<tr>
<td>Lateral</td>
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yaw momentum and the low rear lateral force, caused by a reduction in the rear vertical load, the rear part of the vehicle starts to skid. The handbrake is then used to amplify this phenomenon, increasing yaw acceleration in order to reach the correct yaw rate to approach the turn and to exit as close as possible to the inner limit. If the handbrake input is neglected (Fig. 11b) the pendulum is once more the optimal manoeuvre, but more lateral space is necessary in order to generate yaw momentum and reach the same yaw rate at the apex of the turn. This fact highlights once more the optimality of handbrake with respect to the lateral deviation from the inner limit. The last case in Fig. 11c is the minimum time solution, again with the same boundary conditions. This shows how removing the cost on the lateral displacement, the resulted trajectory is the classic one, in which the vehicle start from the outer limit, reach the apex of the turn on the inner boundary and then exit on the outer limit again using the whole available width of the road.

VI. CONCLUSIONS
Nonlinear optimal control techniques have been used to reproduce numerically the handbrake cornering. A cost function which compromises between manoeuvre time and lateral displacement from the road inner limit proved effective to trigger the handbrake usage in the optimal manoeuvre simulations. The numerical model has been validated against experimental off-road test. Additional simulations on paved road showed that the optimality of the handbrake cornering technique is more related to the road geometry (i.e. tight hairpin) rather than to road surface adhesion properties (i.e. off-road vs paved).

VII. ACKNOWLEDGEMENTS
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