Inversion Charge Linearization in MOSFET Modeling and a Rigorous Derivation of the EKV Compact Model

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ABSTRACT

In this paper, the implications of inversion charge linearization in compact MOS transistor modeling are discussed. The charge-sheet model provides the basic relation among inversion charge and applied potentials, via the implicit surface potential. A rigorous derivation of simpler relations among inversion charge and applied external potentials is provided, using the technique of inversion charge linearization versus surface potential. The new concept of the pinch-off surface potential and a new definition of the inversion charge linearization factor are introduced. In particular, we show that the EKV charge-based model can be considered as an approximation to the more general approach presented here. An improvement to the EKV charge-based model is proposed in the form of a more accurate charge-voltage relationship. This model is analyzed in detail and shows an excellent agreement with the charge sheet model. The normalization of voltages, current and charges, as motivated by the inversion charge linearization, results in a major simplification in compact modeling in static as well as non-quasi-static derivations.

I. INTRODUCTION.

The “charge-sheet” modeling approach [1-3] for the MOS transistor is the usual basis for physics-based compact MOSFET models. Inversion charge is expressed in terms of an implicit variable, the surface potential, as a continuous function from weak to moderate and
strong inversion. The inversion charge is usually evaluated at the source end of the MOS channel and at a saturation point close to the drain, and is then used to build all quantities in the MOS structure (drain current, total charges, transcapacitances etc.). The surface potential is obtained from the physical variables of the device and the bias conditions, either via numerical iteration (e.g. [4-6]) based on suitable initial conditions, or via explicit analytical approximations (e.g. [7,8]) of varying degrees of accuracy.

The implicit nature of the surface potential however is not conceptually helpful for circuit design. Simpler relationships among charge, current, transcapacitances, transconductances and applied biases are desirable. Therefore, beyond the surface potential model, inversion charge linearization versus surface potential has proven to be a powerful method to achieve simpler analytical models, which retain the essential physics of the MOS device while offering adequate accuracy for compact MOSFET modeling. Linearization of inversion charge versus channel voltage was exploited in the EKV MOSFET model approach [9], leading to the first publicly available model combining substrate reference and symmetric device operation with continuous modeling from weak to strong inversion [9-12]. The incrementally linear relationship between inversion charge and surface potential was considered e.g. by Bagheri et al. [2]. Inversion charge linearization versus surface potential was pioneered by Maher et al. [13]. Iñiguez et al. [14,15] derived an approximate explicit relation between inversion charges and surface potential by including a fitting parameter. While their first linearization was done at the source [14], they later obtained a substrate referenced model [15] based on the original EKV MOSFET model approach [9], that also included some short channel effects). A similar approach was also proposed by Cunha et al. [16] who obtained an interpolate expression of the charges versus the potentials that used the basic EKV model definitions [9] and that had the correct asymptotic behavior in weak and strong inversion. Later, these authors proposed a different expression that however required the evaluation of a pinch-off current $i_p$ that could not be explicitly formulated [17]. Following these developments, a coherent approach based on the EKV model formalism was finally proposed where a simple relation between normalized charges and potentials was obtained [18,19], which we call here the charge-based EKV model. However, as will be evidenced in the present work, such a formulation is an approximation of a more general relationship between inversion charge and potentials. Based on the inversion charge linearization method, accurate DC and non-quasi static analytical expressions of MOS characteristics, including submicron technology effects could be achieved [19-26]. Independently of the present work, Gummel et al. [28] recently obtained a very similar
expression while following an empirical formulation. Yet another recent noteworthy approach by Chen et al. [29] focused on keeping source/drain symmetry in transcapacitances while linearizing the bulk charge.

In the present paper, we propose, for the first time, a complete derivation of the relationship among inversion charge and potentials in long-channel MOS transistors, assuming a linear dependence of the inversion charge versus surface potential when the gate voltage is kept constant. In particular, we will show how such a relationship can be considered as a generalization of the EKV charge-based model. Basic definitions, such as the pinch-off surface potential, and normalization factors, are a direct consequence of the inversion charge linearization concept.

Note that the general approach presented here does not take into account 2D effects, which would be beyond the scope of this work. For the latter, the reader is referred to [21,26] and references therein.

II. THE LINEARIZATION OF THE INVERSION CHARGE DENSITY

In the present derivation, we assume an N-channel MOSFET. According to the classical charge sheet approximation in a MOS transistor, the gate and channel potentials are related to the inversion charge density by the following equations [3]:

\[
V_G - V_{FB} = \Psi_S + \Gamma \cdot \sqrt{\frac{\Psi_S}{C_{ox}^'}} - \frac{Q^'}{C_{ox}^'}
\]

(1)

\[
Q^' = -\Gamma \cdot C_{ox}^' \cdot \sqrt{U_T} \left[ \frac{\Psi_S}{U_T} + \exp \left( \frac{\Psi_S - 2 \cdot \Phi_F - V_{CH}}{U_T} \right) - \frac{\Psi_S}{U_T} \right]
\]

(2)

where \( \Gamma \) is the substrate body factor, \( \Phi_F \) is the substrate Fermi potential, \( U_T \) is the thermal potential, \( C_{ox}^' \) is the gate oxide capacitance per unit area, \( \Psi_S \) and \( V_{ch} \) are the surface potential and channel potential (or electron quasi-Fermi potential), respectively. As shown in Fig. 1, the inversion charge density calculated according to (1) suggests that its variation is almost linear with the surface potential when the gate voltage is kept constant (the deviation to linearity is closely related to the substrate body factor through the dependence of the depletion charges versus \( \Psi_S \)). Defining the pinch-off surface potential \( \Psi_p \) as the surface potential at which the inversion charge density is zero, we obtain:
\[ \Psi_p = V_G - V_{FB} - \Gamma^2 \left( \frac{V_G - V_{FB}}{\Gamma^2 + 1} - \frac{1}{2} \right) \]  \hspace{1cm} (3)

We then propose to approximate the inversion charge density by its secant between two \((Q_i', \Psi_S)\) points corresponding respectively to \((0, \Psi_p)\) and \((Q_{i0}', \Psi_{S0})\), where \(Q_{i0}'\) verifies relation (1) and \(\Psi_{S0}\) is an arbitrary value of the surface potential that has to be defined. Then, the linearized inversion charge density can be approximated as:

\[ \frac{Q_i}{C_{ox}} = n_q \cdot (\Psi_S - \Psi_p) \]  \hspace{1cm} (4)

where \(n_q\) is called the inversion charge linearization factor and is given by:

\[ n_q = 1 + \frac{\Gamma}{\sqrt{\Psi_{S0}} + \sqrt{\Psi_p}} \]  \hspace{1cm} (5)

In principle, \(\Psi_{S0}\) could be a function of the channel potential. However, for the sake of simplicity, we propose to set \(\Psi_{S0}\) to a constant value equal to \(2 \cdot \Phi_F\):

\[ n_q = 1 + \frac{\Gamma}{\sqrt{2 \cdot \Phi_F} + \sqrt{\Psi_p}} \]  \hspace{1cm} (6)

A different definition of the charge linearization factor \(n_q\) was previously used in [22]. Note that the present derivation of the charge linearization factor can also be applied in case of poly-depletion.

The three equations (3), (4) and (6) constitute the approximate charge sheet model using charge linearization; this is illustrated in Fig. 1 for three values of the gate voltage, showing a good agreement with the exact charge-sheet model. We intentionally used a relatively high value of the substrate body factor \((\Gamma = 0.7 \sqrt{V})\) in order to illustrate that the linear dependence of the inversion charge density can still be considered as a good approximation.

Then, using relations (2) and (4) with (6), the electron quasi-Fermi potential \(V_{ch}\), the inversion charge density \(Q_i\) and the pinch-off surface potential \(\Psi_p\) are related through:

\[ \ln \left[ \frac{-Q_i'}{\Gamma \cdot C_{ox} \cdot \sqrt{U_T}} \left( \frac{-Q_i'}{\Gamma \cdot C_{ox} \cdot \sqrt{U_T}} + 2 \cdot \frac{Q_i}{n_q \cdot C_{ox} \cdot U_T} + \frac{\Psi_p}{U_T} \right) \right] - \frac{Q_i}{n_q \cdot C_{ox} \cdot U_T} = \frac{\Psi_p - 2 \Phi_F}{U_T} - \frac{V_{ch}}{U_T} \]  \hspace{1cm} (7)

The important equation (7) constitutes the general relation among the inversion charge density and the external potentials in the context of the inversion charge linearization.
approach. Figure 2 shows the dependence of the inversion charge density with what we call the reduced gate voltage \( V_G - V_{FB} \). The accuracy of relation (7) as compared with numerical simulations is excellent from weak to strong inversion, even for a relatively high value of the substrate body factor used here. The absolute value of the relative error between the inversion charge density as obtained from relation (7) and the exact evaluation is also plotted in Fig. 3, where the relative error is below 1%, confirming the accuracy of our approach.

Normalization of currents and charges

The normalization of current and charges is a direct consequence of the linearization approximation [19,21,23]. The current expression in a MOS transistor is simply given by the well known relation [3]:

\[
I = \mu \cdot W \left( -Q_i \cdot \frac{d\Psi_s}{dx} + U_T \cdot \frac{dQ_i}{dx} \right)
\]

(8)

where \( \mu \) is the channel carrier mobility and \( W \) the MOSFET width. Replacing the inversion charge density \( Q_i \) by relation (3) and integrating from source \( x = 0 \) to drain \( x = L \), we can define a normalized current \( i \) and a normalized inversion charge density \( q_i \) as follows:

\[
i = \frac{I}{I_0} = \left( q_s^2 + q_s \right) - \left( q_d^2 + q_d \right)
\]

(9)

\[
q_i = \frac{Q_i}{Q_0}
\]

(10)

where \( I_0 \) and \( Q_0 \) are the specific current and charge (per unit surface) given by [19, 21]:

\[
I_0 = 2 \cdot n_q \cdot \mu \cdot C_{OX} \cdot U_T^2 \cdot W/L
\]

(11)

\[
Q_0 = -2 \cdot n_q \cdot C_{OX} \cdot U_T
\]

(12)

Defining normalized potentials as lower case letters, i.e. \( v = V/U_T \), normalized body factor as \( \gamma = \Gamma/\sqrt{U_T} \) and normalized pinch-off surface potential as \( \phi_p = \Psi_p/U_T \), relation (7) can be written in terms of normalized quantities:

\[
\ln(q_i) + \ln \left[ \frac{2 \cdot n_q}{\gamma} \left( q_i - 2 \cdot \sqrt{\phi_p - 2 \cdot q_i} \right) \right] = 2 \cdot q_i = \phi_p - 2 \cdot \varphi_f - \nu_{ch}
\]

(13)
Henceforth, knowing gate, source and drain potentials, the linearization and normalization factors as well as the normalized charges and currents can be calculated. Asymptotic expressions in weak and strong inversion are obtained from relation (13). In strong inversion, corresponding to $q_i \gg 1$, relation (13) simplifies to:

$$q_i \approx \frac{\varphi_p - 2 \cdot \varphi_f - v_{ch}}{2}$$

(14)

In weak inversion ($q_i \ll 1$), relation (13) becomes:

$$q_i \approx \frac{\gamma}{4 \cdot n_q \cdot \sqrt{\varphi_p}} \cdot \exp(\varphi_p - 2 \cdot \varphi_f - v_{ch})$$

(15)

Note that relation (14) suggests that, in strong inversion, $\varphi_p - 2 \cdot \varphi_f$ and $-v_{ch}$ play the same role from the point of view of inversion charge density (this is still true in weak inversion since $\sqrt{\varphi_p}$ varies slowly compared to the exponential term). Then $\sqrt{\varphi_p}$ can be simply considered as the effect of the gate potential referred to the channel.

III. AN APPROXIMATE SOLUTION: THE EKV MODEL

Simulation of analog integrated circuits requires accuracy and continuity of the large- and small-signal characteristics even for advanced technologies. The original EKV model [9] was initially developed for low voltage-low power applications, for which the transitions between the different modes of operation of the MOS transistor must be especially well described. The EKV model introduced new concepts [9] among which the pinch-off voltage plays a major role. Based on the particular structure of the model, effects in advanced CMOS technology can be considered as a generalization of its basic definitions [9], still keeping a minimum number of parameters [12,18-26].

In this section, we will show how the concepts introduced in the EKV formalism can be simply considered as a special case of the present derivation. Previous work on inversion charge linearization has led to a simple relation between normalized inversion charges and potentials that satisfy [18-20]:

$$\ln(q_i) + 2 \cdot q_i = v_p - v_{ch}$$

(16)

where $v_p$ is to the so called normalized pinch-off voltage [9,20] corresponding to the value of the channel potential at which the inversion charge density extrapolated from strong inversion is zero.
In recent work in the context of inversion charge linearization, the pinch-off voltage was simply considered as the limit condition to a first-order differential equation [19-21,23]. However, its definition was still somewhat controversial since it was obtained from an asymptotic form of (16) corresponding to strong inversion, whereas the condition imposed was a vanishingly small inversion charge, implicitly assuming weak inversion. Nevertheless, relation (16) constitutes the basic relation of the EKV model that can be considered as a major improvement to the relation proposed by [16,17].

According to its definition [22], the normalized pinch-off voltage $v_p$ and pinch-off surface potentials $\phi_p$ are related simply by,

$$v_p = \phi_p - \phi_0$$

where $\phi_0 = 2 \cdot \phi_i + m$ and $m$ is an adjustable parameter, typically ranging from 1 to 4. The origin of $\phi_0$ is related to the asymptotic value of the surface potential (non-normalized) in strong inversion that cannot exceed the channel potential by more than $2 \cdot \Phi_F$ plus a few $U_T$ [3,9,21].

As a consequence, the parameter $m$ is somewhat arbitrary and can be considered as a fitting parameter whose optimal value in terms of model accuracy is slightly dependent on MOSFET process parameters.

Identifying relations (13) and (16), we find that they are equivalent provided that $m$ verifies the following relation:

$$m = \ln\left[\frac{2 \cdot n_q}{\gamma} \left(\frac{2 \cdot n_q}{q_i} + 2 \cdot \sqrt{\phi_p - 2 \cdot q_i}\right)\right]\quad (18)$$

According to (18), $m$ should depend on the inversion charge density and on the gate potential (or equivalently on the channel and gate potentials), which is not the case in the EKV model where its value is assumed constant. However, assuming $m$ constant (the optimal value may slightly depend on the technological MOSFET parameters), is not satisfying in terms of accuracy in weak and moderate inversion. This point is illustrated in Fig. 3 where the relative errors between the exact model and the approximated relations (13) and (16) are plotted as a function of the normalized inversion charge density. Using $m=2$ in relations (16) and (17) (corresponding to a best fit in terms of electrical characteristics) confirms that the relative error increases dramatically in weak inversion. On the contrary, the relative error never exceeds 1% when using relation (13), in particular in weak inversion where the agreement is
nearly perfect. However, even though equation (13) represents the most accurate relation between charge and potential, it is still desirable, in terms of implementation, to obtain a general relation such that:

$$\ln(q_i) + 2 \cdot q_i = G(\varphi_p, \nu_{ch})$$  \hspace{1cm} (19)$$

where $G(\varphi_p, \nu_{ch})$ is a general function only depending on the normalized pinch-off surface potential and channel voltage. As will be discussed later, such formulation is very interesting from the point of view of numerical resolution in simulators [30]. Unfortunately, relation (13) cannot be put in this form. However, it is interesting to note that in weak inversion ($q_i \ll 1$), the asymptotic value of $m$ simplifies to:

$$m = \ln \left[ \frac{4 \cdot n_q \cdot \sqrt{\varphi_p \gamma}}{\gamma} \right]$$  \hspace{1cm} (20)$$

Since in strong inversion the logarithmic terms are negligible in relation (13), we expect that we can still use $m$ as defined from relation (20) even in strong inversion.

This lets us propose an approximate expression of relation (13) that still satisfies (19):

$$\ln(q_i) + 2 \cdot q_i = \varphi_p - 2 \cdot \varphi_i - \nu_{ch} - \ln \left[ \frac{4 \cdot n_q \cdot \sqrt{\varphi_p \gamma}}{\gamma} \right]$$  \hspace{1cm} (21)$$

The accuracy in using relation (21) is further confirmed in Fig. 3, where the relative error in the inversion charge density is strongly reduced, especially in moderate and weak inversion, as compared to the case where $m$ is constant (in this case, the error becomes very important in weak inversion, not represented). The result obtained with (13), i.e. linearization without any further assumptions, remains the best approximation.

As a consequence, we propose relation (21) as a generalization of the charge-based EKV model, with a considerably improved accuracy of the expression of inversion charges versus external potential. This form can be considered as a trade-off between simplicity obtained from relation (16), and accuracy as given by relation (13).

The results obtained for the drain current are represented in Fig. 4 where the current and the corresponding relative error are depicted as a function of the gate voltage, in saturation mode. At first glance, the drain current is very well fitted by the different approximations discussed above, even when $m$ is set to a constant value. However, when looking at the relative error, we conclude that using a constant value of the parameter $m$ still generates important errors in weak and moderate inversion. As for the charges, the improvement in
using relation (21) is very important, leading to a relative error below 4%, and negligible in weak inversion.

Concerning second-order effects, such as mobility reduction and short/narrow channel effects that are ignored in this work, they can be simply included in the model since they do not affect the charge-voltage relation (21). However they may impact the pinch-off voltage (or surface potential), the charge linearization factor and normalizing factors. Further modeling of these effects within the framework of the EKV approach can be found in literature [21-24,26].

It is also interesting to see how transcapacitances can be affected by the different degree of approximation. In the following example, we will only focus on the self-gate capacitance $C_{GG}$. Figure 5 shows that the total gate capacitance is well modeled by the various approximations, showing an error less than 2.5% from weak to strong inversion. According to the results obtained on the inversion charge density, one might have expected a stronger impact of the various approximations on $C_{GG}$.

The last point to be developed concerns small-signal analysis. Differentiating relation (21) with respect to $\varphi_p$ and $v_{ch}$ leads to:

$$\delta q_i = \delta \varphi_p \left(1 - \frac{1}{\varphi_p}\right) \delta v_{ch} \quad (22)$$

Furthermore, from relation (3), $\delta \varphi_p$ can be expressed as a function of the gate variation $\delta v_g$:

$$\delta \varphi_p = \delta v_g \cdot \left(1 + \frac{\gamma}{2 \sqrt{\varphi_p}}\right)^{-1} \quad (23)$$

Replacing (23) in (22), the incremental function of the inversion charge density can be put in a compact formulation:

$$\delta q_i = \frac{\delta v_g}{n_v} - \delta v_{ch} \quad (24)$$

where the slope factor $n_v$ is given by:

$$n_v = \left(1 + \frac{\gamma}{2 \sqrt{\varphi_p}}\right) \left(\frac{\varphi_p}{\varphi_p - 1}\right) \quad (25)$$
The relation (24) has many implications in terms of small-signal analysis since it suggests that the effect of a channel variation \( \delta v_{ch} \) is equivalent to a variation in the gate potential of \( n_v \cdot \delta v_{ch} \). As a consequence, the gate, source and drain transconductances are related by [19]:

\[
g_{mg} \cdot n_v = g_{md} - g_{ms}
\]  

(26)

Note that relation (25) can be considered as a correction of the slope factor defined in the EKV model [9,22] where the parameter \( m \) was set to a constant. It can be shown that in common cases, the correction to this original definition is relatively small since the normalized pinch-off surface potential is usually above 10. In addition, it can be also shown that the complex matrix transadmittances that is composed of 16 transadmittances [19], can be expressed in terms of a linear combinations of 4 independent transadmittances provided the charge linearization factor (relation (6)) is known, leading to a simple equivalent circuit valid from weak to strong inversion that includes non-quasi-static effects [25].

**IV. CONCLUSIONS**

In summary, the inversion charge linearization vs. surface potential is a powerful method to obtain simple analytical expressions for the MOS transistor physics in all regions of operation. A rigorous, complete and coherent derivation of the fundamental relationship between charges and potentials has been proposed within the context of the inversion charge linearization concept. The linearization procedure maintains the physical basis of the modeling, while the accuracy of the obtained relationships has been demonstrated and further improved. New definitions of the pinch-off surface potential and charge linearization factor have been introduced. The pinch-off surface potential, a clearly defined notion, replaces the formerly used concept of pinch-off voltage, while the definition of the charge linearization factor as a secant is a further clarification of the inversion charge linearization concept. We have shown that the EKV charge-based model can be considered as an approximation to the charge-voltage relationship resulting from the charge sheet model. A more accurate charge-voltage relationship in the EKV charge-based model was proposed, maintaining the simplicity of its formulation. The systematic normalization of MOS physical variables coupled with the linearization scheme leads to very simple relationships among the physical quantities in the MOSFET. Finally, the analytical framework developed leads to an elegant and compact small-signal analysis.
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REFERENCES


Figure 1. Inversion charge density as a function of the surface potential for three values of the gate voltage (curve a: $V_G=2$ V, curve b: $V_G=2.5$ V, curve c: $V_G=3$ V). Full line: exact model. Square dots: linearisation with the slope factor calculated from relation (5). Dash-dotted: linearisation with the slope factor calculated from relation (6).

Figure 2. Inversion charge density as a function of the gate voltage in linear and logarithmic scales. Circles: exact model. Triangles: linearized model from relation (7).
**Figure 3.** Relative errors in the inversion charge as a function of the normalized inversion charge density. Full line: linearized model given by relation (7). Dash-dotted: EKV model given by relation (16) with $m=2$. Square dots: improved EKV model given by relation (21).

**Figure 4.** Drain current and relative error as a function of the normalized reduced gate voltage, in saturation. Full line: exact model. Dash-dotted: EKV model given by relation (16) with $m=2$. Square dots: improved EKV model given by relation (21). Dashes: linearized model given by relation (7).
Figure 5. Gate capacitance $C_{GG}$ and corresponding relative error as a function of the normalized reduced gate voltage. Full line: exact model. Dash-dotted: EKV model given by relation (16) with $m=2$. Square dots: improved EKV model given by relation (21). Dashes: linearized model given by relation (7).