Ultrasonic beam radiation from a piston source through a water-immersed steel plate using ASM. Pressure-to-pressure transfer function and numerical challenges

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Extended abstract

Sound beams reflected from and transmitted through an isotropic elastic plate immersed in a fluid have been subject to numerous studies [1–18]. The interaction of the beam with the plate may involve excitation of leaky Lamb waves in the plate. Several near-field effects have been reported for sound beams transmitted through the plate: (i) increased axial sound pressure level (SPL) of the transmitted field as compared to the incident field [7–10,12,14–18]; (ii) downshifted plate resonances as compared to plane wave thickness-mode resonance frequencies [4,6,8–10,12–19]; and (iii) widening or narrowing of the transmitted beam as compared to the incident beam [12,14–17].

If plane wave theory is used for modeling the plate transmission, such phenomena are not described. To describe the mentioned effects observed in acoustic measurements involving ultrasonic transducers and leaky Lamb waves, three-dimensional (3D) finite element modelling (FEM) [9,12,14,16,19] angular spectrum modelling (ASM) [4,5,8,13,18], and a combination of these methods [9,10,12,14–17], have been succesfully used.

To study transmission of leaky Lamb waves in a solid plate, the pressure-to-pressure transfer function through the plate, $H_{pp} \equiv p/p_i$, has typically been used [7–12,14–18]. p_i is the the incident free-field pressure at the plate's upper surface (without the plate present) and p is the in-water transmitted pressure through the plate. In prior work, p_i and p have been calculated with a high degree of accuracy [4,5,8–10,12–19]. Error estimates in calculation of p_i and p would be useful. This is the topic of this work.

To enable a transducer independent and thus a relatively general analysis of phenomena related to leaky Lamb waves, an ASM piston type model [18] is used in the present work, where “piston” refers to a planar circular piston vibrating with uniform particle velocity and mounted in a rigid baffle of infinite extent. This 3D ASM model is hereby referred to as the "BPASM" model. The objective of an ongoing work is to obtain an error estimate of the incident and transmitted pressures calculated with the BPASM model using the Simpson's adaptive integration method [20]. For this purpose the integrand of the incident pressure is reformulated using integration by parts, enabling use of the Simpson's adaptive method. Preliminary results are reported here.

In the BPASM model implemented in this work, polar coordinates (r, θ, z) are used with origin at the center of the piston's front surface. A plate of thickness d is located at distance z_0 from the piston front. For normal beam incidence to the plate p_i and p are given as [18]

ISBN 978-82-8123-021-7

$$p_{\mathrm{i}}(r,z,f)=\frac{1}{2\pi}\int_0^{\infty}P_i(\eta,z,f)J_0(\eta r)\eta d\eta, \tag{1}$$

$$p(r,z,f)=\frac{1}{2\pi}\int_0^{\infty}P(\eta,z,f)J_0(\eta r)\eta d\eta \tag{2}$$

where J_0 is the zeroth order Bessel function of the first kind and

$$P_i(\eta,z,f)=P(\eta,0,f)e^{-ih_{f,z}z}, \tag{3}$$

$$P(\eta,z,f)=P(\eta,0,f)T(\eta,d,f)e^{-ih_{f,z}(z-d)}. \tag{4}$$

η is the r - component of the fluid wave vector, $h_f=c_f/2\pi f$, c_f is the sound velocity in the fluid, and f is the frequency. $P(\eta,0,f)$ is the pressure wave number spectrum at the piston's surface, the exponential is the plane wave propagation term through the fluid, $h_{f,z}$ is the z component of h_f, and T is the plane wave pressure transmission coefficient of the plate [5],

$$T(\eta,d,f)\equiv P(\eta,z_0+d,f)/P_i(\eta,z_0,f). \tag{5}$$

$P_i(\eta,z_0,f)$ and $P(\eta,z_0+d,f)$ are the sound pressure wave number spectra in the fluid at the lower and upper surface of the plate respectively.

p_i and p are obtained by conducting the wave number integrations in Eqs. (1) and (2), respectively, where η is the integration variable. The complex-valued integrands have oscillatory natures and their magnitudes tend to approach zero for some η typically equal to 500 rad/m higher than h_f. Thus, the wave number integrations can be evaluated using a numerical integration method and truncating the integrands for higher η [5,8,18].

In the Simpson's adaptive integration method [20], two segments with the same length $\Delta\eta$ are defined to conduct the wave number integration for each of Eqs. (1) and (2). One of these $\Delta\eta$-segments has twice the wave number sampling interval as the other, resulting in a more accurate integration estimate for this high resolution $\Delta\eta$-segment. The length of the $\Delta\eta$-segments are adjusted until the difference between the integration result of these two segments is less than a certain threshold, 15ϵ, where ϵ is the tolerance. This difference gives an error estimate of the integrated segment, and can be used to give a total error estimate in the numerical integration over the relevant η -range.

The Simpson's method may be applied directly to the integral of the transmitted pressure in Eq. (2). For the incident pressure, however, there is a very sharp peak in the inverse Hankel transform integrand when η approaches h_f. For the reference case of a lossless fluid, this peak goes to infinity, and in this case, the Simpson's adaptive method is not applicable. In the ongoing work, it can be shown that the integrand may be reformulated using integration by parts to avoid this infinite peak, enabling the use of Simpson's method. The incident pressure given in Eq. (1), is the free-field radiated by a baffled piston in an infinite fluid.

Fig. 1 shows preliminary results for the magnitude (a) and phase (b) of the axial p_i , calculated using the Simpsons adaptive method applied to the reformulated integrand of Eq. (1). These result are presented as "BPASM Simpson adapt" (blue curve), together with the error bound estimates for the method (red dashed curves), and calculations using an exact analytical model of the axial pressure [21], radiated by a baffled piston, denoted "Piston KF" (black dashed curve). The pressure is calculated at a distance z = 270 mm using a tolerance $\epsilon=10^{-11}$ for use in the Simpson's adaptive method. The deviation between the Piston KF analytical and the BPASM model is the true error, which is much smaller than the error bound estimates. The error bound values are in the fifth decimal on a dB scale for the magnitude and in the sixth decimal of a radian for the phase. A small frequency region close to 478 kHz is shown to emphasize the narrow error bound estimates. The narrow error bounds provided by the Simpson's method indicates that the BPASM simulations are highly converged.

The Simpson's adaptive method is of especial interest when calculating the off-axis incident piston field, for which no analytical model exists to compare with. The approach presented here, for which the integrand of the baffled piston field integral is reformulated using integration by parts, might also be useful applying other adaptive numerical integration methods, such as Filon type methods [22], which are also suited for use with integrands having an oscillatory nature.

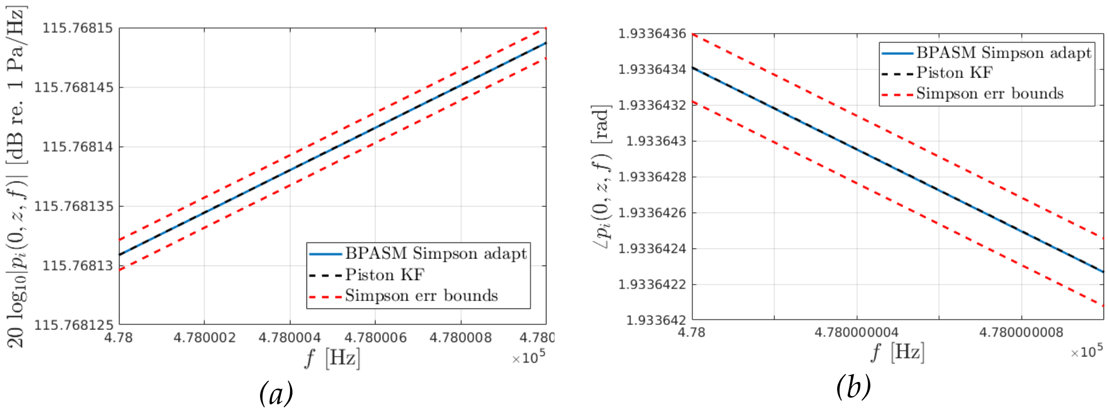

Figure 1: Incident axial pressure frequency spectrum at z = 270 mm, calculated using the BPASM method and Simpson's adaptive method (blue curve) with error bounds (red dashed curves), and the exact analytical model for the axial pressure [21] (black dashed curve). (a) Magnitude and (b) phase.

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