INDUCTION MOTOR PARAMETER ESTIMATION BASED ON THE NONLINEAR STATE SPACE MODEL

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Abstract: This paper presents a method for estimation of induction motor parameters. The proposed approach uses analytical relations to identify the 3rd order model parameters. The method transforms the 3rd order nonlinear model to a linear regression equation. In this study, the field voltage is considered as the input and the active and reactive output power and the rotor speed are considered as the outputs of the induction motor.

Using the nonlinear state space relations of the motor, a simple procedure, based on the known least squares method, is generated, through which, it is possible to estimate all parameters of motor. The estimated parameters are tested with different conditions. The simulation results show the accuracy of this approach.

Keywords: System identification, induction motor, nonlinear state space model, analytical identification, linear identification.

1. Introduction

An induction motor is a type of alternating current motor where electromagnetic induction supply power to the rotor. In recent years, due to the importance of induction motors in industries, induction motor parameter identification has become one of the most interesting fields for academic researchers.

Researches in this field are divided in two categories: on-line and off-line estimation. In on-line approaches like [1,2,3,4], some parameters are estimated while the motor is working in real time. These methods mainly identify motor parameters by assuming some parameters to be known. In [1], the rotor mechanical speed and the stator currents/voltages are assumed to be measured. A particular nonminimal state representation of the electrical model of the machine is derived to build an adaptive observer. Lyapunov design is used to develop the adaptation law of the proposed solution. Like many other on-line methods, the proposed solution just estimates the electromagnetic state of the induction motor. Furthermore, the Lyapunov design makes it a bit hard for implementation.

Estimation of the rotor resistance & self inductance as well as the stator leakage inductance of a three-phase induction machine is investigated in [2]. Obviously, all the parameters of the motor are not estimated in [2].

In [3], authors use artificial neural networks (ANNs) for online identification of rotor resistance and mutual inductance. Although, some complex equations are generated due to use of ANNs, just two parameters of the motor are obtained.

In some papers like [4], we can see the usage of transfer functions to obtain a linear parametric model for estimation of the parameters. An important problem about this method is that it is highly dependent on the operating point and can not be valid in a wide range of operating points. Moreover, that method cannot estimate all parameters.

The on-line estimation of the stator resistance, stator inductance, transient inductance, and rotor resistance is discussed in [5, 6, 7]. The algorithms that are used in the above mentioned papers are somehow complex and difficult for implementation.

Some authors present off-line methods, which rely on measurements carried out in the sinusoidal steady state mode and parameters rely on some information from motor nameplate [8, 9]. The main problem in off-line estimation is that the parameters are determined individually using off-line tests and the final model may not be accurate when used in an online process.

In this paper, an analytical on-line method that can estimate all parameters of the 3rd order model of induction motors is presented. The approach uses the mathematical relations among motor parameters and variables to generate a framework for parameter estimation. The main advantage of the proposed method is that we can identify all motor parameters through a simple experiment.

The paper is organized as follows: Section 2 introduces the induction motor model. The identification algorithm is described in Section 3. The simulations and results are discussed in section 4. Finally, concluding remarks are presented in Section 5.
2. The Induction Motor Model

An induction motor, as it is seen in Fig.1, is considered as the study system. We assume that the rotor speed, the active electrical power and the reactive electrical power are measurable.

![Image 1: An induction motor connected to an infinite bus bar](image)

In this paper, we try to estimate the motor parameters based on the 3rd order model (Fig. 2). The model is described by the following equations [10]:

**Dynamic equations:**

\[
\begin{align*}
\sigma &= \frac{1}{2H_w} (T_a - T_r) \\
\epsilon' &= \Omega_w \sigma' - \left(\epsilon' + (X_a - X')i_a\right) \frac{T_a}{T_r} \\
\epsilon'' &= -\Omega_w \sigma' - \left(\epsilon' - (X_a - X')i_a\right) \frac{T_a}{T_r}
\end{align*}
\]

**Algebraic equations:**

\[
\begin{align*}
\sigma &= 1 - \omega \\
\epsilon' &= \nu_1 + X_1i_a - r_1i_a \\
\epsilon'' &= \nu_a - X'i_a - r'i_a \\
T_a(\sigma) &= \alpha + \beta \sigma + \gamma \sigma^3 \\
T_r &= \epsilon'i_a + \epsilon''i_a
\end{align*}
\]

And for active & reactive electrical power we have:

\[
\begin{align*}
P_r &= v_1i_a + v_2i_a \\
Q_r &= v_1i_a - v_2i_a
\end{align*}
\]

Where:

\[
\begin{align*}
v_1 &= V \sin \theta \\
v_2 &= V \cos \theta \\
i_a &= I \sin(\theta + \phi) \\
i_a &= I \cos(\theta + \phi)
\end{align*}
\]

![Image 2: Order III induction motor (electrical circuit)](image)

The definition of parameters is given in the appendix.

3. Identification Method

In this section, we divide our identification procedure in two parts. In the first one, we introduce the state variable, the inputs and the outputs and then discretize the model. In second part, we substitute some helpful equation in the discrete state space equation and obtain a linear regression equation.

3.1 Discrete Model

The third order model is introduced in section 2. The system states and inputs are defined as follows:

\[
X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\]

\[
V = \begin{bmatrix} v_r \\ v_a \end{bmatrix}
\]

The nonlinear state space model of the system is:

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{2H_w} (\alpha + \beta i_1 + \gamma i_1^2 - x_1i_a - x_1i_a) \\
\dot{x}_2 &= \Omega_w x_1 x_3 - (x_2 + (X_a - X')i_a) \frac{T_a}{T_r} \\
\dot{x}_3 &= -\Omega_w x_1 x_3 - (x_3 - (X_a - X')i_a) \frac{T_a}{T_r}
\end{align*}
\]

In the above model, \(i_1\) and \(i_a\) are the real & imaginary parts of the rotor current, which cannot be measured explicitly. If we use (2), we can write them as a function of \(P_r\), \(Q_r\), \(v_r\) and \(v_a\) which are measurable signals. This means that we can estimate these two signals using the measurable variables. We will have:

\[
\begin{align*}
i_1 &= \frac{1}{(v_r^2 + v_a^2)(v_1 Q_r + v_2 P_r)} \\
i_a &= \frac{1}{(v_r^2 + v_a^2)(-v_1 Q_r + v_2 P_r)}
\end{align*}
\]

The system outputs in this study are the rotor speed, the active electrical power and the reactive electrical power. The output vector is given in the following formula:

\[
Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} P_r \\ Q_r \\ \omega \end{bmatrix}
\]

In this paper, it is assumed that we can measure all the above variables simultaneously during a suitable test. The gathered data are used for parameter identification as discussed in the introduction.

First of all, we replace output and input parameters and state variables in equations (1) and (3) and then discretize them. We will acquire the following relations:

\[
\begin{align*}
x_1(k) &= 1 - y_1(k) \\
x_2(k) &= u_1(k) + X_1i_a(k) - r_1i_a(k) \\
x_3(k) &= u_a(k) - X'i_a(k) - r'i_a(k)
\end{align*}
\]

Where:

\[
\begin{align*}
i_1(k) &= \frac{1}{(u_1(k) + u_1(k))(u_2(k)y_1(k) + u_1(k)y_2(k))} \\
i_a(k) &= \frac{1}{(u_1(k) + u_1(k))(-u_1(k)y_1(k) + u_1(k)y_2(k))}
\end{align*}
\]

In this step, we discretize the state space model with a sample time ‘\(h\)’ [11]. The discretized equations are as follows:
\[
x_{i}(k+1) - x_{i}(k) = \frac{1}{h} \left[ (\alpha + \beta \xi(k) + \gamma x_{i}(k) - x_{i}(k)u_{i}(k) + x_{i}(k)i_{i}(k) \right]
\]

The goal is to convert the above equations to a linear regression form to estimate the unknown parameters.

### 3.2 Deriving the Regression Equation

In this part, we substitute (4) in (5) in order to find the regression equation (Since the mechanical torque cannot change so rapidly, we assume that it is constant during the identification procedure).

\[
c(k) = \frac{\alpha}{2H_{u}} \frac{1}{2H_{u}} m(k) + \frac{1}{2H_{u}} r, n(k) \text{ or}
\]

\[
m(k) = \alpha - 2H_{u} c(k) + r, n(k)
\]

Where:

\[
c(k) = \frac{y_{i}(k) - y_{i}(k+1)}{h}
\]

\[
m(k) = i_{i}(k)u_{i}(k) + i_{i}(k)u_{i}(k)
\]

\[
n(k) = i_{i}(k) + i_{i}(k)
\]

Equation (7) is linear in parameters. If we apply N different time instants to the above formula, we will have:

\[
\begin{bmatrix}
m(1) \\
m(k) \\
m(N)
\end{bmatrix} = 
\begin{bmatrix}
1 & -2c(1) & n(1) \\
\vdots & \vdots & \vdots \\
1 & -2c(N) & n(N)
\end{bmatrix} \begin{bmatrix}
\alpha \\
H_{u} \\
n
\end{bmatrix}
\]

(8)

It is obvious that: \( \theta = (U_{i}^{T}U_{i})^{-1}U_{i}^{T}H_{i} \)

The same as the above procedure, we will find the other parameters, too. If we substitute \( x_{i}(k), x_{i}(k) \) and \( x_{i}(k) \) from (4) to (6), we will obtain the regression equation that can be used for estimating the remaining parameters \( X' \), \( T'_{o} \) and \( X'_{o} \).

In this part, we use estimated \( r, \) from (8). Furthermore, we know that \( \Omega_{i} \) equals \( 2\pi f \) with frequency of 60 Hz (50 Hz). With these assumptions, (6) can be simplified. The regression equation is given as follows:

\[
f(k) = \frac{1}{T_{o}} a(k) + X' b(k) + \frac{X'_{o}}{T_{o}} i_{u}(k)
\]

Where:

\[
f(k) = f_{1}(k) + f_{2}(k)
\]

\[
f_{1}(k) = \frac{u_{i}(k) - u_{i}(k+1) - r(i_{i}(k) - i_{i}(k+1))}{h}
\]

\[
f_{2}(k) = -\Omega_{i}(1 - y_{i}(k))(u_{i}(k) - r(i_{i}(k) - i_{i}(k+1))
\]

\[
a(k) = r(i_{i}(k) - u_{i}(k)
\]

\[
b(k) = -\Omega_{i}(i_{i}(k)(1 - y_{i}(k))) + (i_{i}(k) - i_{i}(k+1))
\]

If we apply N different time instants to the above formula, we will have:

\[
\begin{bmatrix}
f(1) \\
f(k) \\
f(N)
\end{bmatrix} = 
\begin{bmatrix}
a(1) \\
\vdots \\
a(N)
\end{bmatrix} \begin{bmatrix}
b(1) \\
\vdots \\
b(N)
\end{bmatrix} \begin{bmatrix}
i_{u}(1) \\
i_{u}(k) \\
i_{u}(N)
\end{bmatrix} \begin{bmatrix}
\frac{1}{T_{o}} \\
\frac{X'_{o}}{T_{o}}
\end{bmatrix}
\]

We have: \( \Theta = (U_{i}^{T}U_{i})^{-1}U_{i}^{T}H_{i} \)

In this step, we can calculate \( T'_{o} \) and \( X' \). Then, with the estimated amount of \( \frac{X'_{o}}{T_{o}} \), we can find \( X'_{o} \).

We summarize the method as follows: we’ve firstly selected appropriate input signal that can be applied to the system. Then we measure the input and output, simultaneously. We use input, output and some helpful algebraic equations to change the state space equations into linear regression forms to estimate the unknown parameters. Finally, by use of the least square method, it could be possible to acquire the parameters.

### 4. Simulation Results

In this section, we should use a proper signal to execute the system and gather suitable input/output data. The signal has been applied at the input terminal voltage. The motor start-up time is at the start of simulation. The electrical active and reactive power and the motor speed are sampled with the sample time of 1 millisecond (h=0.001).

We implemented the method in two different conditions. In the first condition, the input signal does not change rapidly. The transient response of the motor at startup is captured. In the second condition, we have changed the input signal randomly and rapidly. In order to make different conditions for exciting the system, we suddenly change the input voltage of the motor terminal from the nominal value. Since we want the input signal to be safe, we choose its variation from the nominal point about 5-10 per cent.

We have examined the method in three different cases. The results are shown in Fig. 3-5.
In TABLE I, the simulated and the estimated values of the 3rd order induction motor parameters are summarized. It is seen that the method is successful in identifying the parameters.

**TABLE I.** Comparison between third order models in different approaches

<table>
<thead>
<tr>
<th>parameter</th>
<th>Real values</th>
<th>Fig. 3</th>
<th>Fig. 4</th>
<th>Fig. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.25</td>
<td>0.2507</td>
<td>0.2505</td>
<td>0.2507</td>
</tr>
<tr>
<td>$H_\omega$</td>
<td>3</td>
<td>2.9616</td>
<td>2.9642</td>
<td>2.9606</td>
</tr>
<tr>
<td>$r_s$</td>
<td>0.01</td>
<td>0.0103</td>
<td>0.0103</td>
<td>0.0104</td>
</tr>
<tr>
<td>$\lambda'$</td>
<td>0.3279</td>
<td>0.3299</td>
<td>0.3302</td>
<td>0.3325</td>
</tr>
<tr>
<td>$X'$</td>
<td>0.2956</td>
<td>0.2961</td>
<td>0.2957</td>
<td>0.2942</td>
</tr>
<tr>
<td>$X_c$</td>
<td>5.15</td>
<td>5.1612</td>
<td>5.1608</td>
<td>5.1673</td>
</tr>
</tbody>
</table>

As it is seen from Fig. 3-5, by using the input signal which contains rapid changes, outputs can better follow the input than when we do not use these changes in the inputs. By more precisely looking at results in TABLE I, it shows a good accuracy for all results of all figures. However the results of Fig. 4 are a bit more accurate in comparison with other two ones.

5. **Conclusion**

In this paper, we identify the induction motor parameters with an analytical approach. The most important point of this method is the usage of least squares (LS) to estimate the nonlinear model parameters.
The method requires applying a suitable input voltage signal to the terminals of the motor. The stator real and imaginary voltages are the inputs. The rotor speed and the electrical active and reactive powers are assumed to be measurable outputs. Simulation results show that the proposed method has good accuracy for estimation of state space induction motor parameters.

APPENDIX
Definition of the variables used in this paper, are shown in the following table:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s$</td>
<td>Stator resistance</td>
<td>p.u.</td>
</tr>
<tr>
<td>$x_s$</td>
<td>Stator reactance</td>
<td>p.u.</td>
</tr>
<tr>
<td>$r_{x1}$</td>
<td>1st cage rotor resistance</td>
<td>p.u.</td>
</tr>
<tr>
<td>$x_{x1}$</td>
<td>1st cage rotor reactance</td>
<td>p.u.</td>
</tr>
<tr>
<td>$r_{x2}$</td>
<td>2nd cage rotor resistance</td>
<td>p.u.</td>
</tr>
<tr>
<td>$x_{x2}$</td>
<td>2nd cage rotor reactance</td>
<td>p.u.</td>
</tr>
<tr>
<td>$X_m$</td>
<td>Magnetization reactance</td>
<td>p.u.</td>
</tr>
<tr>
<td>$H_m$</td>
<td>Inertia constant</td>
<td>kWkVA</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1st coef. Of $T_{m}(\sigma)$</td>
<td>p.u.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2nd coef. Of $T_{m}(\sigma)$</td>
<td>p.u.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3rd coef. Of $T_{m}(\sigma)$</td>
<td>p.u.</td>
</tr>
<tr>
<td>$X'_x$</td>
<td>$X_a + x_s$</td>
<td>-</td>
</tr>
<tr>
<td>$X''_x$</td>
<td>$x_s (X_a + x_{x1})$</td>
<td>-</td>
</tr>
<tr>
<td>$T'_m$</td>
<td>$X_a + x_{x1}$</td>
<td>-</td>
</tr>
</tbody>
</table>

References