Modal Analysis of an Acoustic Guitar

by

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Introduction

Acoustic musical instruments are an example of dynamic structural system in which the acoustic-structural interaction is very important, but difficult to analyze[1]. Violins have been the subject of some effort including recent modal surveys using non-contacting optical methods [2,3]. In that spirit, this paper describes a modal survey of a high quality acoustic-electric guitar performed using non-contacting laser measuring equipment including a scanning laser vibrometer and a video holography system. The work was done to understand the both the mechanism by which an acoustical guitar produces sound and to give some insight into what constitutes a good guitar. The guitar used for the survey is a Yamaha acoustic model equipped with a piezo-electric pickup in the bridge.

Background

An acoustic guitar (and most other stringed instruments) make sound by using the vibration of the strings to excite a flexible soundboard which is, in tom, backed with a partially enclosed cavity. The flexible top cover of the soundbox (the soundboard on an acoustic guitar) acts as a driver which radiates both out into the room and into the cavity behind it. Time-dependent pressures inside the cavity emerge as sound from the soundhole of the instrument. A simple, but effective demonstration of the mechanics of the body and soundboard uses a small tuning fork. Strike the tuning fork and place the base against the soundboard of an acoustic guitar. The sound level produced by the tuning fork is greatly magnified by the sound board and the acoustic cavity.

Obviously, the sound produced is strongly conditioned by the response of the strings. The behavior of strings on an instrument such as guitar can be modeled very accurately with linear theory. The natural frequencies of a string under tension are

$$\omega_n = \frac{n\pi}{L}\sqrt{\frac{T}{\rho}} \qquad (1)$$

It should be noted that the fanaticism with which dynamicists pursue perfect boundary conditions is not necessarily required in this case. Related work suggests that the flexible boundary condition created by fixing one end of the string to a flexible soundboard has little effect on the resonant frequencies of the strings. The natural frequency of a given mode is an integer multiple of the fundamental mode. This fact is extremely convenient for the musician since the second string natural frequency is an octave higher than the first, the third is a perfect fifth higher than the second and the fourth mode is an octave higher than the second and two octaves higher than the first [4,5].

The soundboard of an acoustic guitar is generally designed to have a number of natural frequencies in the range of the string natural frequencies. The frequency of the string vibrations varies over several octaves. Consider a guitar tuned to concert pitch using the standard tuning. The lowest possible frequency is the low E string (string 6) at 82.4 Hz. The pitch of the high E string, (string 6) played at the 12th fret is 659.2 Hz. Note that there is no highest possible frequency since every string has (at least theoretically) an infinite number of natural frequencies.

Some knowledge of fret spacing is important to understand the relationship between frequency and pitch. Each fret on the neck corresponds to a semi-tone change in pitch. There are 12 semi-tones in an octave, so fretting a string at the twelfth fret increases its frequency by one octave. Raising a note by an octave doubles its frequency and decreasing a note by an octave halves ifs frequency. Raising any note by a semi-tone raises its frequency by a fixed ratio, no matter what the original note is. Thus, if some note with frequency, f, is to be raised by an octave and the frequency ratio corresponding to a semi-tone is r, the following relationship is true

$$2f = r^{12} f \qquad (2)$$

thus, $r = \sqrt[12]{2} \approx 1.059463$

The presence of the bridge pickup added an interesting dimension to the testing. Traditional pickups like those found on solid body electrics use the signal generated by a ferrous string vibrating in a magnetic field to form the input to the amplifier(s). Piezo-electric pickups use the time dependent forces between the bridge and saddle to generate an output signal. That signal is fed to a small, battery-powered pre-amp mounted in the body at the base of the neck. The output of the pre-amp is sent to an amplifier just as with any other electric guitar. The layout of the guitar is presented in Figure I. The basic geometry of the bridge and saddle is presented in Figure 2.

It is interesting to note that the geometry of the saddle makes the piezo-electric clement sensitive only to forces normal to the plane of the soundboard. However. the strings are plucked or picked in the plane of the soundboard. The fact that the pickup works at all (and they work well enough to be almost standard equipment on acoustic guitars) means that either there is a component to the motion of the string normal to the soundboard or that some other mechanism is at work. One possible effect is a time dependent change in tension of the string as it vibrates. Such a tension change would indeed result in a force normal to the plane of the soundboard.

Modal Testing

We removed the strings from the guitar and used both the Laser Doppler Vibrometer (LDV) and the video holography system to identify the modes of the soundboard. The laser Doppler vibrometer uses the frequency shift measured from a laser beam reflected from a target moving parallel to the axis of the beam to measure the time dependent velocity of the target (in this case the bridge or soundboard).

The video holography system uses the wave interaction of an expanded laser beam with the reflection of another expanded beam from the target to measure displacements [6,7]. We used a light coating of retroreflective glass beads held in place with lemon furniture polish to increase the reflectivity of the soundboard. The carnuba wax in the polish held the beads in place very well while being easy to remove. The mass increase was negligible and it was a great improvement over the ambient smells in the lab.

We found that clean mode shapes existed only for the lower modes. The images presented in Figures 3-10 are produced electronically by the video holography system. Node lines appear white and the dark fringe lines represent lines of constant displacement out of the plane of the soundboard.

Only the lower section of the body (called the lower bout) is presented in the figures. This does not represent a limitation in either the equipment used or the method in general. The upper part of the body (the upper bout) is extremely stiff in relation to the lower part. This is due to both its relatively small size and the overlapping section of the fretboard glued to it. Ignoring the minor contribution of the upper bout allowed us to zoo,,, in closer and increase the resolution on the lower bout.

The fundamental mode, presented in Figure 3, shows a node line around the soundboard. For purposes of this discussion, the soundboard can be considered to have a simply supported boundary Strictly, the boundary condition is between the lower bound of a simple support and the upper bound of a true clamped support The boundary condition probably also varies with location due to the change in the angle between the grain of the soundboard and the tangent to the edge.

The second mode shape, shown in Figure 4, is interesting since it is very similar to the first mode shape. However, the node line separates from the edge of the soundboard at the bottom of the body.

Modes I and 2 were the only two which appeared to be symmetric; Figure 5-10 show distinctly asymmetric modes. This is due to the asymmetric bracing glued to the back of the soundboard. The number of bracing patterns used on acoustic guitars is seemingly infinite. but they all have the same basic goal. The bracing makes the soundboard strong enough to resist the forces and moments applied to the soundboard by the strings while not raising the lower natural frequencies above those of the strings.

One reason for the complexity of the modal response of a guitar is the number of string frequencies below 600 Hz. Figure 11 shows a plot of the open string natural frequencies for a guitar at standard (EADGBE) tuning. There are 15 modes including several that are very close to having the same frequency. For instance, the first frequency of string 2 is 256.9 Hz and the third frequency of string 6 is 247.2 Hz.

The human ear is extremely sensitive to small changes in sound, so small differences in the modal response of the soundboard are important. This, combined with the fact that the idea of ‘good’ sound is entirely subjective means that there never will be a standard bracing pattern. The pattern on this guitar forms roughly an X as shown in Figure 12. Figure 13 shows the rough outline of the bracing manifested as node lines in the response of the soundboard at 890.48 Hz.

Classical guitars are more likely to have symmetric soundboard bracing patterns. Asymmetric patterns are more representative of folk guitar, which are used for a much wider range of music. Different players like different sounds, so bracing patterns reflect attempts to enhance response at different frequencies. Note also that the tension and mass per unit length varies from string to string.

Some of the modes shown were excited by pointing the speaker at the bridge or body (the shadow is visible) and the others were excited by pointing the speaker into the soundhole. This highlights the fact that the soundboard and the enclosed mass of air in the body form a system in which the acoustic response and the structural response cannot always be isolated from one another [9-11]. Indeed, as the frequencies increased, it became increasingly difficult to identify discrete normal modes.

Piezo-electric Pickup

To characterize the performance of the bridge pickup, we compared the soundboard response measured using a laser Doppler vihrometer with the output from the pickup. We placed a speaker at the soundhole and used a random signal with a frequency varying from 50-800 Hz for excitation. The vihrometer was set to interrogate a point on the bridge between strings 3 and 4. A small piece of retroreflective tape used to increase the reflectivity at this point is visible as a small white spot on the bridge in Figures 3-10.

We directed the velocity input from the vihrometer and the signal from the pickup to separate channels of a digital oscilloscope so that they could be stored simultaneously. The frequency spectrum of the two response signals are plotted on the same axes in Figure 14. It is interesting that the pickup detected a peak at about 650 Hz that was not apparent to the vibrometer. In addition, it appears that neither the pickup nor the vihrometer returned a peak corresponding to one of the soundboard modes presented above. Rather, they closely match the string natural frequencies. For comparison, the fundamental frequencies of the strings on a properly tuned guitar are presented in Table 1.

Table 1 - Ideal Fundamental Frequencies

String	Frequency (Hz)
I - E	329.6
2 - B	246.9
3 - G	196.0
4 - D	146.8
5 - A	110.0
6 - E	82.40

According to linear theory, string natural frequencies are linear multiples of the fundamental frequencies.

The first two peaks are the strongest and appear to he the first and second natural frequencies of the low E string (String 6). The third peak corresponds to the first natural frequency of the B string and the third natural frequency of the low E string. The fourth peak corresponds to the fourth natural frequency of string 6 and the first natural frequency of string I and so on.

The magnitudes of the peaks in the two frequency response plots are very different, even when the fact that they should differ by a factor of the natural frequency corresponding to each peak is considered. We took this to be a result of the fact that the pickup really measures the internal force between the saddle and the bridge. One of us (Lewis) owns an acoustic guitar with a piezo-electric pickup in the bridge and a more conventional magnetic pickup in the neck which would make an excellent subject for further tests.

Conclusions

We used high quality modal analysis equipment to characterize the dynamic response of an acoustic guitar. We found that the modes of the soundboard are a secondary component in the response of the guitar. The string response is the dominant component. There is no question that two guitars fitted with identical sets of strings might have very different sound. We conclude that the flexibility effects of the soundboard combined with the acoustic interaction in the body are subtle, but critical.

We found eight normal modes of the soundboard below 600 Hz. The first mode was symmetric and its frequency was within 5% of the natural frequency of the A string. The other seven modes were asymmetric. The natural frequencies of the soundboard generally lie close to those of one or more of the strings. Fortunately, the boundary conditions created by mounting the strings on a flexible soundboard do not significantly affect the natural frequencies of the strings.

Finally, the piezo-electric bridge pickup detects frequency peaks well, but can modify the amplitudes of those peaks compared to the velocity response measured at the bridge. The sound produced by an amplified guitar is very different than that produced by the unamplified guitar. This makes intuitive sense due to the different mechanisms at work. The unamplified guitar produces sound when the vibrating strings excite the flexible soundboard while the pickup produces a signal proportional to the internal force between the bridge and the saddle.

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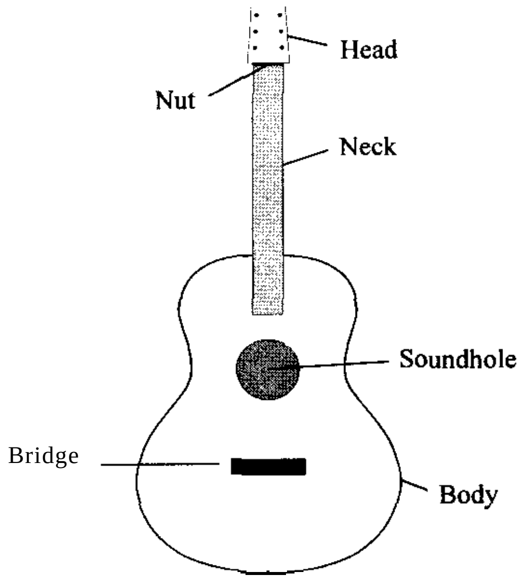

Figure 1 - Layout of Guitar

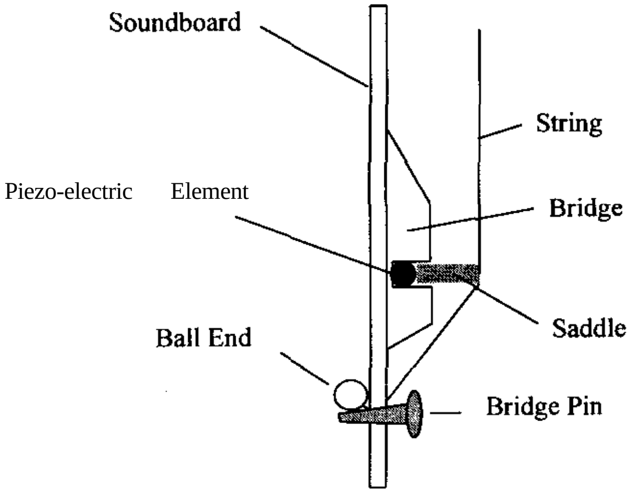

Figure 2 - Piezo-electric Bridge Pickup

Figure 3 - Fundamental Mode (I 16.5 Hz)

Figure 4 - Second Mode (224.3 Hz)

Figure 5 -Third Mode (371.0 Hz)

Figure 6 - Fourth Mode (408.0 Hz)

Figure 7 -Fifth Mode (444.0 Hz)

Figure 8 - Sixth Mode (465.9 Hz)

Figure 9 • Seventh Mode (498. I Hz)

Figure 10 • Eight Mode (580.4 Hz)

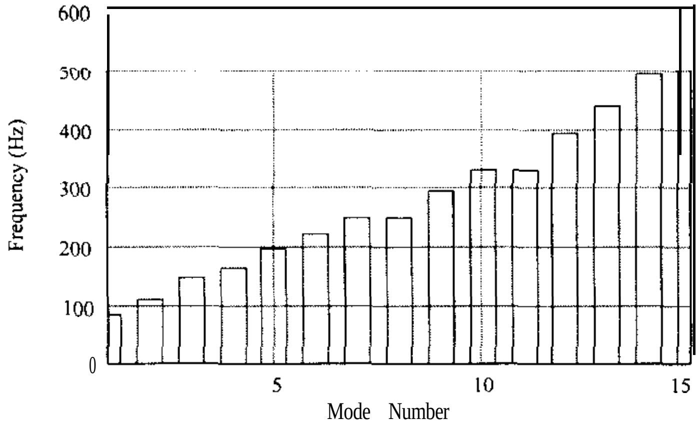

Figure 11 • String Modes Below 600 Hz

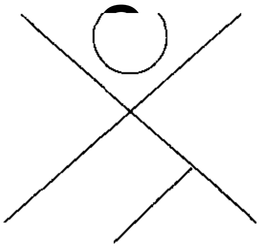

Figure If-Soundboard Bracing Pattern

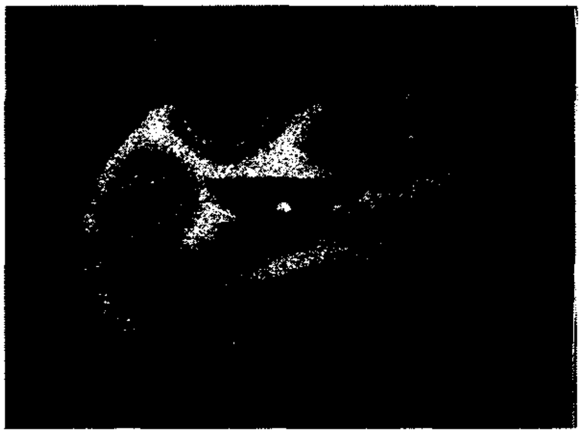

Figure 13 • Soundboard Response at 890.48 Hz

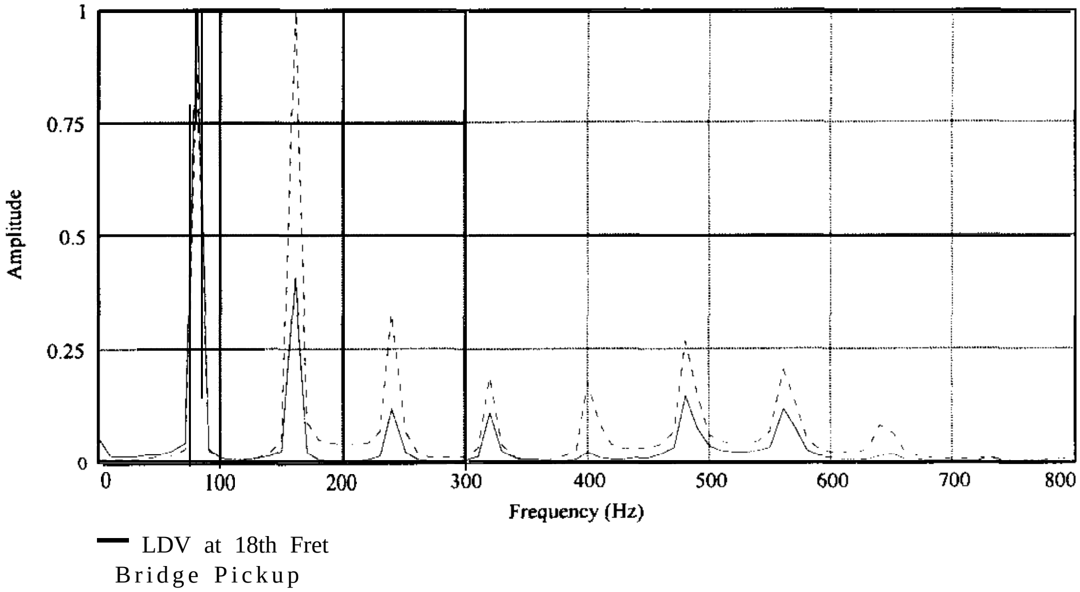

Figure 14 -Comparison of Output From Bridge Pickup With LDV Output