Efficient Power and Subcarrier Allocation for OFDMA-Based Relay Networks

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Abstract—This paper investigates OFDMA downlink resource (spectrum and power) sharing algorithms for fixed relay stations. Iterative waterfilling is used in the power allocation process and shown to result in an optimal power allocation solution for multiple-relay networks. Iterative waterfilling is then developed with a power reallocation process to achieve more efficient spectrum utilization. Furthermore, joint subcarrier and power allocation algorithms are proposed with and without consideration for multiuser fairness. Simulation results show that iterative waterfilling can significantly improve the system capacity. The proposed joint allocation algorithm achieves gains of up to 21% in system capacity compared to iterative waterfilling. The joint allocation algorithm with fairness is shown to degrade system capacity, however fairness is important in a multiuser network.

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) and relaying are regarded as leading candidates for future generation cellular networks [1], [2]. OFDMA serves as a promising multiple-access technique for high-data-rate transmissions, while relaying helps to increase system capacity, transmission reliability, as well as coverage. In this paper, the downlink of a cell in an OFDMA-based fixed relay network is studied. The base station first broadcasts OFDMA symbols, and the relay stations retransmit these data symbols using their own OFDMA subcarriers to the end users to exploit multiuser diversity and to utilize the frequency selective fading channel. By knowing the channel state information (CSI) at the transmitters, efficient spectrum sharing and resource allocation techniques for fixed relays can be used to increase system capacity [3].

OFDMA-based resource allocation schemes for non-relaying networks have been extensively studied in the literature [4], [5], [6], while for relaying networks they have only been considered in a limited number of scenarios due to their high complexity [7], [8]. Our aim here is to jointly optimize the relaying strategies and resource allocation process in an OFDMA system. Existing suboptimal techniques use either fixed power allocation (and perform only subcarrier allocation), or handle subcarrier and power allocation separately. This work considers joint subcarrier and power allocation to ensure efficient use of power. In [3], algorithms for resource allocation are proposed based on the assumption that each relay station uses exclusive spectrum to communicate with one another, however this is not practical due to spectrum inefficiency. The iterative waterfilling power allocation algorithm (IWF) [9] is optimal for a Gaussian vector multiple access channel. This strategy is utilized in our OFDMA-based multiple relay transmissions scenario.

This paper makes the following contributions. (1) We identify iterative waterfilling as the optimal power allocation strategy and study its performance and convergence property. (2) We propose iterative waterfilling with a power reallocation algorithm for near-optimal power allocation under the assumption that all RSs share the same bandwidth. (3) We further investigate a joint subcarrier and power allocation scheme and extend it to address the multiuser scenario. Simulations are evaluated using 3GPP LTE standard parameters and channels [10], [11]. Results show the effectiveness of the proposed techniques.

The paper is organized as follows. Section II describes the relay network model. In Section III our iterative waterfilling algorithm for multiple-relay cells is investigated. In Section IV we propose iterative waterfilling with power reallocation. In Section V we propose a suboptimal joint subcarrier and power allocation scheme. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

This paper investigates the problem of resource allocation in an OFDMA-based relaying wireless network. OFDMA downlink transmissions (with $N$ data subcarriers) in a single cell are studied. The system consists of one base station (BS), $K$ mobile users or user equipment (UE), and $L$ relay stations (RS). Each RS is equipped with a single antenna (see Figure 1(a)). The downlink paths are assumed to have equal average power. The impact of inter-cell interference is not considered in this analysis.

The relaying scenario works as follows. The BS broadcasts OFDMA symbols, and the RSs decode the symbols and retransmit them to the end users (decode-and-forward relaying) in a separate frequency band to the BS. Two-hop transmission is considered in this scenario. We assume that the transmission spectrum of all the relaying links is the same size as that used at the BS ($N$ data subcarriers in one OFDM symbol). We assume that the channel gains between the BS and the RSs are high enough (e.g., selective decode-and-forwarding is...
Noise at each subcarrier is modelled as a zero-mean circular
UE. Similarly, we use
or RS
signal-to-noise ratio). This allows the BS to exploit adaptive
back information on the combined channels (e.g. combined
sent by the BS for decoding. We also assume that UEs send
with the UEs after subcarrier allocation such that the UEs
are able to combine the relayed symbols with the symbol
sent by the BS to decoding. Perfect CSI at the receivers is assumed, and this is fed
back to the transmitters (BS or RSs). The RSs communicate
with the UEs after subcarrier allocation such that the UEs
are able to combine the relayed symbols with the symbol
sent by the BS to decoding. We also assume that UEs send
back information on the combined channels (e.g. combined
signal-to-noise ratio). This allows the BS to exploit adaptive
modulation and coding to improve system capacity.

In this paper, we use \( h_{l,k}^{(n)} \) to represent the channel gain
at subcarrier \( n \) from BS \( l \) to UE \( k \), where \( n \in [1, N], \ l \in [1, L] \) and \( k \in [1, K] \). To simplify the notation, we use \( h_{0,k}^{(n)} \)
\( l = 0 \) to represent the channel gain from the BS to the
UE. Similarly, we use \( p_{l,k}^{(n)} \) to represent the power allocated
to subcarrier \( n \) at the BS or RS \( l \) to UE \( k \). We use \( \Omega_{l,k} \)
to represent the ordered subcarrier set allocated to UE \( k \) at BS
or RS \( l \). \( \Omega_{l,k}(j) \) is the \( j \)th element in \( \Omega_{l,k} \), which refers to
the subcarrier used to transmit the \( j \)th symbol to UE \( k \) at BS
or RS \( l \). We assume the total bandwidth for the \( N \) subcarriers
is \( B \), such that the bandwidth for each subcarrier is \( B/N \).
Noise at each subcarrier is modelled as a zero-mean circular
symmetric complex Gaussian process with variance \( N_0 B/N \).

Throughout this paper we adopt the following simulation
parameters unless specified otherwise. In the downlink
transmissions, the bandwidths of the BS transmission link
and all the relaying links are 5MHz, and the number of
data subcarriers in each OFDMA symbol is 300, as used
in the LTE OFDMA downlink [10]. The wireless channel
used in this simulation is modelled using the 3GPP typical
urban parameters (TUx) [11], which represent a frequency-
selective fading channel consisting of 12 taps. To make a fair
comparison, we constrain the transmit power of each RS or
BS to be \( P_{\text{total}}/(L+1) \). Performance is evaluated by way of
Monte Carlo simulations.

III. ITERATIVE WATERFILLING FOR MULTIPLE RELAY
TRANSMISSIONS

We first investigate the optimal power allocation problem at
different relaying links, given the assumption that subcarrier
allocation has already been determined. To simplify the prob-
lem, we further assume that each RS uses different spectrum.
This ensures that RSs do not interfere with one another. Under
this assumption, maximal ratio combining (MRC) is assumed
at the UE for optimal signal combination. This assumption
requires a large amount of bandwidth for all the RSs, and is
not acceptable since wireless radio spectrum is a finite and
precious resource. We show in the next section that removing
this assumption has almost no effect on the optimal result.

The optimal power allocation problem can be formulated to
find the maximum capacity, which is the capacity sum of each
user \( R_k \) is the capacity of user \( k, k \in [1, K] \).

\[
\max_{\Omega_{l,k}, p_{l,k}^{(n)}} \sum_{k=1}^{K} R_k, \quad (1)
\]

subject to,

\[
\sum_{k=1}^{K} \sum_{n \in \Omega_{l,k}} p_{l,k}^{(n)} - P_l = 0, \quad \text{for all } l
\]

\[
-\sum_{l,k,n} p_{l,k}^{(n)} \leq 0, \quad \text{for all } l, k, n \quad (2)
\]

where

\[
R_k = \frac{C_k}{N} \log(1 + \sum_{l=0}^{L} P_{l,k}^{(\Omega_{l,k}(j))} |h_{l,k}^{(\Omega_{l,k}(j))}|^2 N_0 B / N). \quad (3)
\]

The problem can be solved by using its Lagrangian dual
and applying Karush-Kuhn-Tucker (KKT) conditions [12]. The optimal power allocation is thus,

\[
p_{l,k}^{(\Omega_{l,k}(j))} = \max \left( \frac{B}{\mu_{l,k} N} - \frac{\gamma}{|h_{l,k}^{(\Omega_{l,k}(j))}|^2} \right), \quad (4)
\]

where \( \gamma = \frac{N_0 B}{N} + \sum_{m \neq l} P_{m,k}^{(\Omega_{m,k}(j))} |h_{m,k}^{(\Omega_{m,k}(j))}|^2 \). Such a problem
can be solved by an iterative waterfilling algorithm [9], where
each RS performs waterfilling in turn by treating all other RSs
as “interferers” until the final solution is found. The algorithm
is described in Figure 2. The initialization step in Figure 2
is not compulsory because iterative waterfilling can guarantee
reaching the optimal solution from any point. Convergence has
also been proved to be very fast [9], i.e., the optimal solution
can be found after just a few iterations. It should be noted
that the algorithm proposed here is a centralized algorithm.
However, it can be easily modified to produce a distributed
solution.

We now run a set of simulations to observe the performance
of iterative waterfilling. Figure 3(a) shows that iterative
waterfilling significantly improves system capacity compared to the
equal power allocation case. For a single user in the cell, 6 RSs
achieves more capacity gain than 3 RSs due to the increasing
degree of diversity. Figure 3(b) shows that the convergence
of the iterative waterfilling algorithm is very fast regardless of the
number of RSs. The first two iterations reduce the percentage
difference of capacity (compared to the capacity result in the
previous iteration) to \( 10^{-2} \) even for large numbers of RS. A
Iterative Waterfilling for Multiple Relay Transmissions

**assumption** assume that the power of the BS transmissions has been allocated

**initialization** let the power allocated to each subcarrier at RS \( l \) be \( P_l / N \) (equal power allocation)

**repeat**

for \( l = 1 \) to \( L \)

- do waterfilling according to Equation 4

**until** the improvement in capacity compared to that of the previous iteration is less than some threshold (e.g., \( 10^{-4} \))

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**IV. ITERATIVE WATERFILLING WITH POWER REALLOCATION**

In the previous section we assumed that each RS is assigned exclusive spectrum. This is not practical because the total bandwidth required is then much larger than that without relaying \((L + 1) \times B\) including the spectrum for the BS). Fortunately, the optimal solution has a very simple form and is presented in the following theorem. Due to the lack of space here, a detailed proof is not provided, however this can be found in [3].

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**Theorem 1:** In general situations, for the optimal power allocation of a relay network with \( L \) RSs, there are at most \((\frac{L}{2})\) data symbols (out of \( N \)) that can be allocated power by more than one RS. When the number of subcarriers \( N \gg L \), and \( L \) is a small integer value, for each data symbol, among all \( L \) RSs, there is at most one RS that allocates power to relay the symbol.

Theorem 1 can be explained by using the simple example shown in Figure 4. For illustration purposes we set the number of data subcarriers \( N = 32 \), and there are 1 UE and 2 RSs (\( K = 1 \), \( L = 2 \)). We assume that the BS and the two RSs occupy completely different spectrum. For simplicity, we further assume that the subcarrier allocation follows a one-to-one correspondence strategy: i.e., the data symbol transmitted by the first subcarrier of the BS is relayed by the first subcarrier of RS 1 and RS 2, and so on. The upper graph of Figure 4 shows the channel responses of the BS-UE and RS-UE links (in the frequency domain), and the lower graph shows the optimal power allocation result. From Figure 4 it can be observed that for most data symbols either RS 1 or RS 2 allocates power to relay to the end user. Although neither RS 1 nor RS 2 may allocate power, it is unlikely RS 1 and RS 2 both allocate significant power on the same subcarrier. The only exception in this example is subcarrier 16, where both RS 1 and RS 2 allocate relay power. We say that RS 1 and RS 2 collide at subcarrier number 16. Theorem 1 states that the number of such collisions is at most \((\frac{L}{2}) = 1\) for \( L = 2 \).

There is another way to understand the theorem. Although we assume that all RSs occupy a total bandwidth of \( L \times B \), for the optimal power allocation, they only allocate power on subcarriers with a total bandwidth of at most \((1 + (\frac{L}{2}) / N)B\), which approximates to \( B \) when \( N \gg (\frac{L}{2}) \). In fact, the number of collisions in practice is much smaller than \((\frac{L}{2})\). This is shown in Figure 5. For \( L = 6 \) and \((\frac{L}{2}) = 15\), the average number of collisions is 3.8, which is quite small compared to \( N = 300 \).

Based on the observations in Theorem 1, we propose a simple but effective algorithm to allocate power to the RSs with the assumption that all RSs share the same bandwidth \( B \). The algorithm is shown in Figure 6. The main idea is to first apply iterative waterfilling, and then for each collision to reallocate the power at the appropriate RSs (except for the RS with the maximum allocated RS power). By doing this, the...
assumption 1 assume that all RSs share the same bandwidth $B$

assumption 2 assume that the power at the BS has been allocated

apply the iterative waterfilling algorithm by assuming that all RSs use different spectrum

for each collision (at most $L$ collisions)

for each RS that collides, except for the RS with maximum allocated power

reallocate the allocated power to the other active subcarriers in each RS

Greedy Joint Subcarrier and Power Allocation Algorithm

sort the subcarriers at the BS by $p_0^{(n)} h_{0,k}^{(n)}$ in an ascent order

for each subcarrier $n_b$ of the BS (in the above order)

let $k$ be the UE that subcarrier $n_b$ at the BS is transmitting to

for each $RS \in [1, L]$

find $s_{t,k} = \arg\max_{n} |h_{t,k}^{(n)}|^2$ (from all unallocated subcarriers)

find $t = \arg\max_{l} |h_{l,k}^{(s_{t,k})}|^2 / \mu_l$

allocate subcarrier $s_{t,k}$ at RS $t$ to relay for subcarrier $n_b$ at the BS

update $\mu_l$

calculate power allocation at each RS according to $\mu_l$

V. JOINT SUBCARRIER AND POWER ALLOCATION ALGORITHM

In the previous section, a simple power allocation algorithm was proposed for multiple relay transmissions based on iterative waterfilling and power reallocation. In this section the problem of subcarrier allocation is also taken into consideration by exploiting the joint allocation of subcarriers and power. The problem is hard because even subcarrier allocation (with an equal power assumption) can be viewed as a multidimensional weighted matching problem and is NP-complete.

Traditional solutions for resource allocation [4], [5], [6] split the problem into two independent steps. For the relaying scenario, firstly the subcarriers can be allocated to the BS and RSs with the assumption of equal power, and then power can be allocated based on the subcarrier allocation in the first step. However, traditional solutions do not work well because even when a subcarrier at an RS is allocated to relay for a data symbol, the optimal power allocation scheme may not allocate power on this subcarrier. For efficient spectrum and power usage, it is necessary to design a joint subcarrier and power allocation algorithm.

To find out the good subcarriers for the RS-UE links we can make use of the theorem presented earlier and proved in our previous work [3]. We simply state the theorem below.

Theorem 2: $P_{l,k}^{(s_{l,k})}$ is not equal to 0 only when $|h_{l,k}^{(s_{l,k})}|^2 / \mu_l \geq |h_{l',k}^{(s_{l',k})}|^2 / \mu_{l'}$ for all $l' \neq l$, where $1/\mu_l$ and $1/\mu_{l'}$ are proportional to the waterlevel of RS $l$ and $l'$ respectively.

Based on Theorem 2 we propose a joint subcarrier and power allocation algorithm as shown in Figure 8. The intuition of the proposed algorithm is that for each subcarrier at the BS, we greedily select the RS and the subcarrier that maximizes $|h|^2 / \mu_l$ to relay for it. Note, the algorithm is based on the assumption made in the previous section, i.e., that all RSs share a total bandwidth $B$.

So far power allocation, as well as joint subcarrier and power allocation, has been investigated to maximize the system performance. For a multiuser scenario, fairness is also a very important factor in the system. The algorithm in Figure 8 can be easily adapted to consider user fairness, which is shown in Figure 9. The basic idea is to greedily help the UE with the lowest data rate in each round.

A set of simulations were performed to compare the performance of the different algorithms. Cells with 3 RSs and
Greedy Joint Subcarrier and Power Allocation Algorithm with Fairness

repeat
    select a subcarrier $n_b$ at the BS that is transmitting a data symbol to UE $k$ who has the least proportional fairness
    for each $RS_t \in [1, L]$
        find $s_{t,k} = \arg\max_{s} \left| h_{t,k}^{(n)} \right|^2$ (from all the unallocated subcarriers)
        find $t = \arg\max_{t} \left| h_{t,k}^{(n)} \right|^2 / \mu_{t,k}$
        allocate subcarrier $s_{t,k}$ at RS $t$ to relay for subcarrier $n_b$ at the BS
    update $\mu_{t,k}$
until all subcarriers at the BS have been enumerated
calculate power allocation at each RS according to $\mu_t$

Fig. 9. Greedy Joint Subcarrier and Power Allocation Algorithm with Fairness

6 RSs are simulated for SNR values from 2dB to 20dB respectively. The SNR is combined as the average received SNR of 3 UEs from all transmitters in the cell (assuming all links have the same path loss). Figure 10(a) shows the capacity improvement of the proposed algorithms and Figure 10(b) shows their fairness index.

The results show that the joint subcarrier and power allocation algorithms outperform the iterative waterfilling approach for both cells with 3 RSs and 6 RSs. The curves correspond to the trends expected since the joint allocation of subcarriers and power provides a more efficient utilization of the resources. For the cell with 3 RSs, the joint algorithm without fairness has a gain of approximately 21% at SNR = 2dB, and 5% at SNR = 20dB. For the cell with 6 RSs, the gains are approximately 15% at SNR = 2dB, and 4% at SNR = 20dB. The degradation of gain with increasing RS number occurs because the total transmission power remains the same for both scenarios, which limits the algorithmic improvement.

The joint algorithm with fairness is slightly better than the iterative waterfilling approach. However, system fairness is vastly improved as shown in Figure 10(b). To evaluate the fairness level, we use the fairness index as defined in [13], with the maximum value of 1 representing the fairest case. The algorithm with fairness satisfies the quality of service (QoS) for 3 UEs in this scenario.

VI. CONCLUSION

In this paper, resource allocation strategies have been investigated for OFDMA-based relaying downlink transmissions. Iterative waterfilling was adopted to perform power allocation among the relaying links. To make efficient use of spectrum, a power reallocation process is exploited which assumed that all relay stations shared the same spectrum. Simulation results showed that the reallocation process did not degrade performance. We further proposed a joint subcarrier and power allocation algorithm both with and without fairness for a multiuser scenario. Simulation results showed that the proposed joint allocation algorithm can achieve up to 21% more system capacity compared to the iterative waterfilling algorithm. The joint allocation algorithm with fairness degrades system capacity compared to the non-fairness algorithm, however fairness is required to maintain QoS in the system.

REFERENCES