

e-Training and e-Assessment of Mathematical Courses by Xpress-Tutor Integrated in Moodle

Philip Slobodsky¹, Alexander Ocheretov² and Mariana Durcheva¹

¹ Halomda Educational Software, Israel

Email: halomda@netvision.net.il

² University of Ivanovo, Russia

Email: alex.oczer@gmail.com

Abstract. In the present work, we report the integration of Xpress-Tutor, comprising the assessment module of Math-Xpress into Moodle. Math-Xpress is a universal math environment used for classroom teaching, home training and assessment of math students at college and university levels. It includes linked modules of equation editor, 2D and 3D graph plotter, Computer Algebra System (CAS) expression evaluator and step-to-step solver, dynamic geometry (2D and 3D) and problem solving tutor. Using the Xpress Task Generator the courses in Elementary and High math have been developed and used for e-Evaluation at several high schools, colleges and universities in Israel regularly since 2007, involving thousands of students each academic year.

Keywords: Math-Xpress, Computer Algebra System, e-Assessment, Moodle.

1 Mathematical Assessment – the Challenges and the Existing Solutions

Assessment is a key part in the educational process, and it presents a constant challenge for educators and *teachware* developers. That is why serious attempts have been made to automatically assess students' results [1], [2]. The main problems that arise include:

1. Typing and editing formulas by a student during an exam in open questions mode and by developers of new tasks, due to the fact that most of Math-Assessment systems based on Computer Algebra [3], [4] use a string editing of math formulas, requiring knowledge of math editor' languages like LaTeX, etc. This requirement is a significant inconvenience for using the system by students (particularly elementary school pupils), as well as by developers of new content [5].
2. Comparing students' answers with the correct ones in the multiple choice questions (MCQ) mode of exam or deciding on the correctness of students' answers in open question mode [6]. The discussions on the effectiveness of the multiple-choice test format [7], [8] did not lead to cessation of its use, but rather stimulated new approaches such as SI-MCQ (*semi-intelligent* MCQ), that will be described later in this paper.

3. Integration of the LMS (Learning Management System) into Moodle and its compatibility with different Operation Systems: several Math-Assessment systems are compatible with Moodle [9], but this requires the development of a plugin, enabling an operation of the system from within Moodle and the transfer of results into Moodle LMS. In addition, since the assessment usually refers to text input, it is desirable, for the convenience of the students, to make possible running the tests on Android and iOS portable devices.
4. Additional problems are the translation and adaptation of content to the relevant curricula and providing students with opportunities of learning and training in *tutor mode*: most Math-Assessment systems offer only quiz mode of operation, and not full tutoring. Inclusion of all the options requires, in addition to content development, integration into a complete system, and the ability to recognize a mode of operation and to prevent the student from getting solution to the test problems by opening the “Learning” mode, while being in “Test” mode. On the other hand, “Learning” and “Training” modes can essentially improve students’ ability to solve test problems, offering them various exploration tools like graph plotting, algebraic calculator, interactive and dynamic geometry.

In this paper we describe the system for e-assessment of Math tests, called *XPress-Tutor*, in which many of the above-described problems are solved: input of mathematical expressions is in natural WYSIWYG form (What You See Is What You Get); checking of students’ answers is done using the advanced semi-intelligent MCQ approach, the system includes learning and training options; it runs in different LMS systems, including Moodle.

2 Math-Xpress – the Universal Math Environment

2.1. The Structure of the Package [10]

XPress-Editor. The basic module of Math-XPress is XPress-Editor – a graphical formula editor, enabling natural WYSIWYG editing of mathematical expressions that can be either pasted into the text prepared by the text editor, or recognized and evaluated by CAS-based XPress-Evaluator, XPress-Graph plotter and XPress-Tutor.

The combination of text editing and CAS evaluation of algebraic expressions allows the teacher to develop exercises in various fields of elementary and higher mathematics, and to use them in the Test mode of Math-Xpress environment.

Instead of string editing, requiring familiarity with special syntax like LaTeX, XPress-Editor uses templates (Fig.1). XPress-Editor contains templates for special symbols used in all areas of elementary mathematics and most of the subjects of symbolic algebra, making editing simple and self-obvious.

$$F_{\otimes} = \frac{1}{\sqrt{\otimes}} \left(\left(\frac{1+\sqrt{\otimes}}{\otimes} \right)^{\otimes} - \left(\frac{1-\sqrt{\otimes}}{\otimes} \right)^{\otimes} \right) \Rightarrow F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$$

Fig. 1. Editing mathematical expressions

XPress-Graph. The expressions created by XPress-Editor can be used by XPress-Graph for building and drawing graphs of functions of 2 and 3 variables; families of functions; intersections of graphs. Single graph, as well as several graphs of different functions, parametric family of graphs, graphs of relations and combinations of the graphs and geometrical drawing can be displayed simultaneously (Fig. 2).

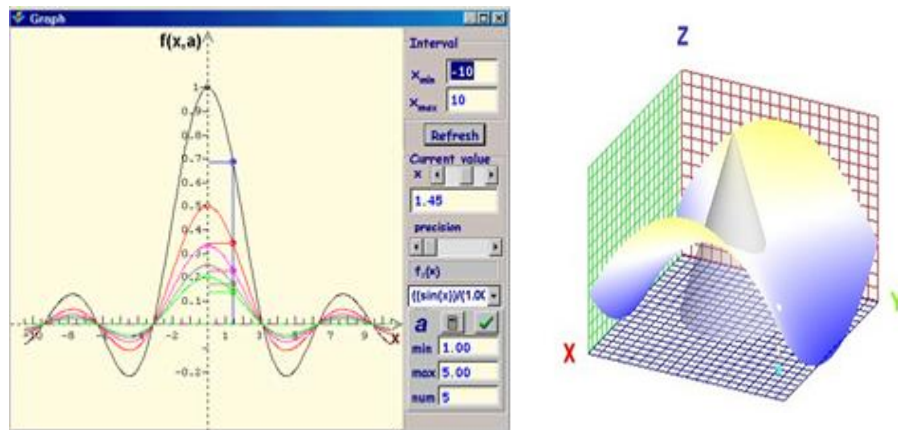


Fig. 2. Graphs of functions and surfaces

Similarly to algebraic expressions, graphs can be saved as pictures or as objects, enabling retrieving and editing in the last case. Graphs of the functions of two variables can be intersected by basic solids: *sphere*, *cylinder*, *cube* and *cone*, enabling visualization and calculation of the intersection boundaries (Fig. 3).

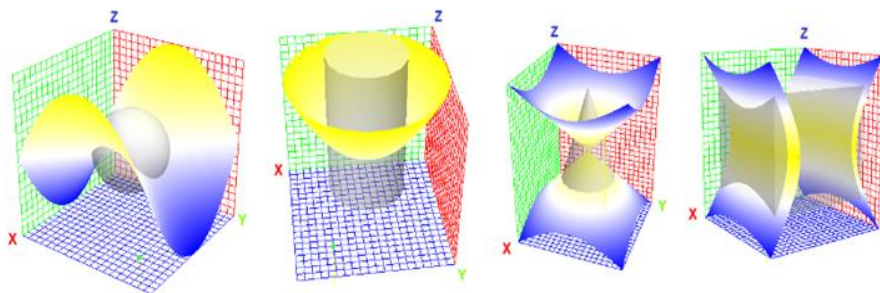


Fig. 3. Intersection of graph with solid

XPress-Geometry. Math-Xpress includes two modules of interactive geometry: 2-D and 3-D *XPress-Geometry explorer*. 2-D Geometry explorer enables drawing the figures using templates. The drawings are based on the basic figures: *triangle*, *square*, *rectangle*, *parallelogram*, *rhombus*, *general polygon*, *circle*, *ellipse* and *trapezoid*, and the elements: *point*, *line*, *segment*, *angle*, *parallel lines*, *axis*, *coordinate system*, *vector*, *arc*, *sector*, *elliptic arc* and *sector* (Fig. 4).

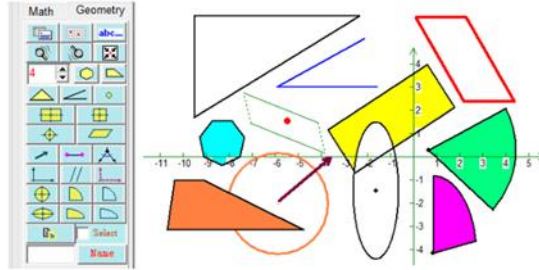


Fig. 4. Basic geometric figures

The program allows measuring *distances between points*, *lengths of line segments*, and *areas*. The measurements can be made automatically or manually, using the simulated ruler and protractor (Fig. 5). The operations: *translation*, *rotation*, *scaling up and down*, *reflection in line* and *inversion in point* can be performed on all the figures and elements, resulting in ability to draw the complicated geometrical constructions.

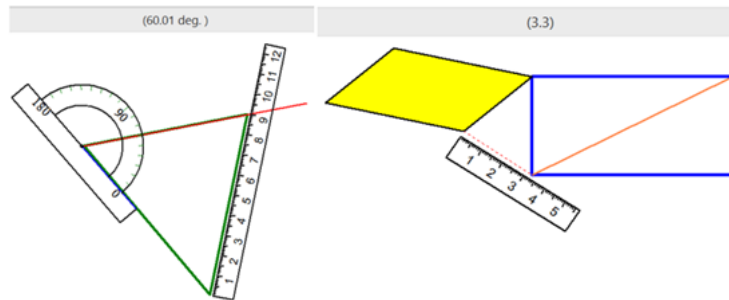


Fig. 5. Ruler and protractor

The equations of lines, circles and ellipses, as well as the coordinates of any point of the drawing are displayed, thus making *XPress-geometry Explorer* as a tool suitable for *Analytical geometry*.

Drawing of *bisectors*, *heights* and *mid-lines* in triangles, and binding of all the lines within a basic figure preserving their intrinsic properties can be used in a class for interactive demonstrations and even for the proofs of the theorems (Fig. 6).

Grouping multiple elements into one scalable figure, changing colors and line styles make *XPress-Geometry Explorer* a stand-alone *Drawing and Painting* tool, enabling the teacher and the student to produce high quality and attractive documents, posters and other printed materials containing geometric figures.

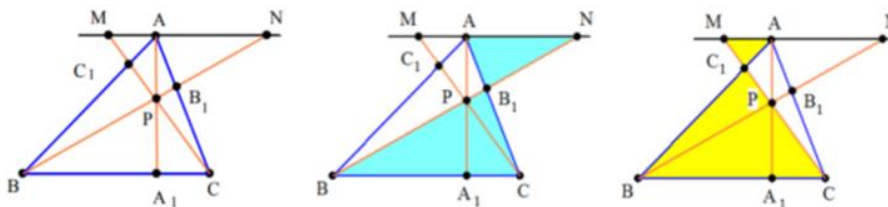


Fig. 6. Geometric constructions

XPress-Geometry Explorer accepts a graph built by *XPress-graph plotter* and displays it in the Geometry panel.

The objects created by *XPress-editor*, *Graph Plotter* and *Geometry Explorer* can be embedded into Word or PDF-pages and be called directly from them into an interactive Math-Xpress environment.

The module “Solids” enables building of basic 3-D surfaces and solids like *plane*, *cube*, *pyramid*, *parallelepiped*, *sphere*, *cone*, *cylinder* and others (Fig. 7). Different operations as *parallel translations*; *rotations*; *intersections by a plane*; *unfolding* and 3D – stereoscopic viewing can be performed interactively (Fig. 8), and the result can be copied into a Word document and in the Geometry window.

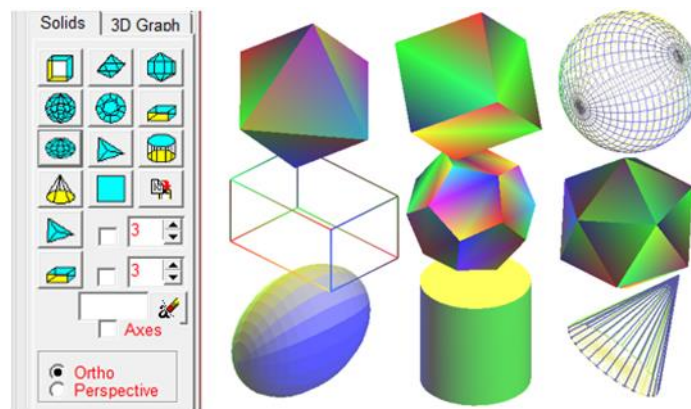


Fig. 7. Solids

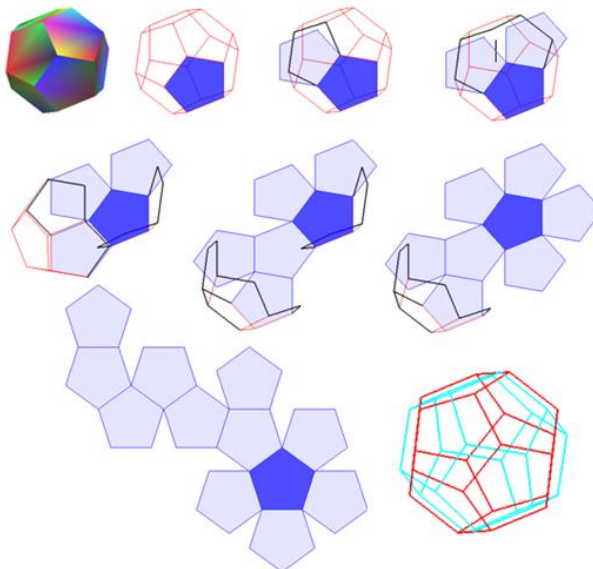


Fig. 8. Unfolding Solids

3 Using “Math-XPRESS” by a Teacher for Interactive Demonstrations of Basic Operations in Mathematics

XPRESS-evaluator enables to present many mathematical subjects in live and attractive form, demonstrating on the one hand (when possible) graphical representation of functions, equations, inequalities, complex numbers, vectors, etc., and on the other - detailed operations on mathematical objects, showing their dependence on the constants and the parameters that define them.

The subjects covered by XPRESS-evaluator include Arithmetic, Elementary Algebra, Trigonometry, Calculus, Probability and Statistics, Linear Algebra, Complex numbers. In *Elementary mathematics*, it enables to perform arithmetic operations on integers and fractions, to factorize polynomials, to solve equations and systems of equations.

The combination of XPRESS-evaluator and XPRESS-Graph plotter enables the teacher to present simultaneously different ways of solution of algebraic equations: numerically or algebraically, by factorizing (when possible) and graphically (Fig. 9).

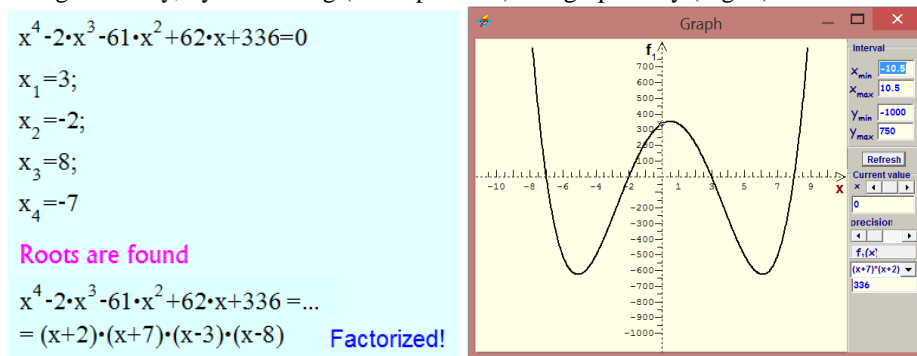


Fig. 9. Equation solutions

XPRESS-evaluator performs most of the operations in step-by-step way, giving the teacher additional possibilities to demonstrate the basic steps of the equation's solution (see Fig. 10). This option is also useful for the students in order to help them to recall what they have learned in the lesson.

$\sqrt{x-5} = 3 - \sqrt{2-x}$ $(\sqrt{x-5})^2 = (3 - \sqrt{2-x})^2$ $x-5 = 11 - 6\sqrt{2-x} - x$ $6\sqrt{2-x} = 16 - 2x$ $(6\sqrt{2-x})^2 = (16 - 2x)^2$ $72 - 36x = 4x^2 - 64x + 256$ $28x - (4x^2 + 184) = 0$ $D = 28^2 - 4 \cdot (-4) \cdot (-184) = -2160$	$x_1 = \frac{-28 - (\sqrt{2160}) \cdot i}{2 \cdot (-4)} = \frac{7}{2} + \left(\frac{3}{2} \cdot \sqrt{15}\right) \cdot i$ $x_2 = \frac{-28 + (\sqrt{2160}) \cdot i}{2 \cdot (-4)} = \frac{7}{2} - \frac{3}{2} \cdot \sqrt{15} \cdot i$ <p>final result!</p> $x_1 = \frac{7}{2} + \left(\frac{3}{2} \cdot \sqrt{15}\right) \cdot i$ $\sqrt{\frac{7}{2} + \left(\frac{3}{2} \cdot \sqrt{15}\right) \cdot i} - 5 \neq \left(3 - \sqrt{2 - \left(\frac{7}{2} + \left(\frac{3}{2} \cdot \sqrt{15}\right) \cdot i\right)}\right) \quad \text{False root!}$ $x_2 = \frac{7}{2} - \frac{3}{2} \cdot \sqrt{15} \cdot i$ $\sqrt{\frac{7}{2} - \frac{3}{2} \cdot \sqrt{15} \cdot i} - 5 \neq \left(3 - \sqrt{2 - \left(\frac{7}{2} - \frac{3}{2} \cdot \sqrt{15} \cdot i\right)}\right) \quad \text{False root!}$
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Fig. 10. Step-by-step solutions

In *Linear Algebra*, such operations include:

a) **Calculation of determinants.**

XPress-evaluator calculates determinants of an order up to 3x3 in a detailed form, where the elements of the determinant can also be expressions (Fig. 11).

$$\begin{vmatrix} t-8 & 5 & 3 \\ 2 & t^3 & 7 \\ 1 & -3 & t-4 \end{vmatrix} = (t-8) \cdot \begin{vmatrix} t^3 & 7 \\ -3 & t-4 \end{vmatrix} - 2 \cdot \begin{vmatrix} 5 & 3 \\ -3 & t-4 \end{vmatrix} + 1 \cdot \begin{vmatrix} 5 & 3 \\ t^3 & 7 \end{vmatrix} = (t^4 - 4 \cdot t^3) \cdot (t-8) + 21 \cdot (t-8) - 10 \cdot t - 3 \cdot t^3 + 57 = \dots \\ = t^5 - 12t^4 + 29t^3 + 11t - 111$$

Fig. 11. Determinant evaluation

b) **Operations with matrices**

- Sum of two matrices
- Product of a matrix with a scalar
- Product of two matrices
- Transposition of a matrix
- Inverse matrix
- Eigenvalues of a matrix 2x2 and 3x3 (Fig. 12).

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{The initial matrix}$$

$$\begin{vmatrix} 1-t & 2 & 3 \\ 4 & 5-t & 6 \\ 7 & 8 & 9-t \end{vmatrix} = -t^3 + 15t^2 + 18t = 0 \quad \text{The secular equation}$$

$$t_1 = 0;$$

$$t_2 = 16.12;$$

$$t_3 = -1.12$$

The eigen values

Fig. 12. Eigenvalues of matrix

In *Calculus*, such operations include:

a) **Calculation of limits** (Fig. 13)

$$\lim_{x \rightarrow \infty} \frac{x^3 + 1}{3 \cdot x + \sqrt{5 \cdot 2 \cdot x} + x^4} = \lim_{x \rightarrow \infty} \left(\frac{1}{x} \cdot \frac{1 + \frac{1}{x^3}}{\frac{3}{x^3} + \sqrt{\frac{2}{x^7} + \frac{5}{x^8} + 1}} \right) = 0 \cdot \frac{1+0}{0+0^{\frac{1}{2}}+1} = 0 \quad \text{Limit calculated}$$

Fig. 13. Limit calculation

b) **Calculation of derivatives** (Fig. 14)

$$\frac{d(e^{x \cdot \cos x} + \ln(2 \cdot x + 5))}{dx} = e^{x \cdot \cos x} \cdot (1 \cdot \cos x + (x \cdot (-\sin x \cdot 1))) + \frac{1}{2 \cdot x + 5} \cdot (2 \cdot 1 + 0) =$$

$$= (\cos x - \sin x \cdot x) \cdot e^{x \cdot \cos x} + \frac{2}{2 \cdot x + 5} = \cos x \cdot e^{x \cdot \cos x} - e^{x \cdot \cos x} \cdot \sin x \cdot x + \frac{2}{2 \cdot x + 5} \quad \text{Derivative calculated}$$

Fig. 14. Derivative calculation

c) Calculation of indefinite integral

d) Calculation of definite integral (Fig. 15)

$$\int_3^8 (\sin x + \sqrt{x-3} + \ln(2 \cdot x)) dx = \left(\frac{2}{3} \cdot (x-3)^{\frac{3}{2}} - \cos x + (2 \cdot \ln(2 \cdot x) \cdot x - 2 \cdot x) \cdot \frac{1}{2} \right) \Big|_3^8 =$$

$$= \frac{2}{3} \cdot (8-3)^{\frac{3}{2}} - \cos 8 + (2 \cdot \ln(2 \cdot 8) \cdot 8 - 2 \cdot 8) \cdot \frac{1}{2} - \left(\frac{2}{3} \cdot (3-3)^{\frac{3}{2}} - \cos 3 + (2 \cdot \ln(2 \cdot 3) \cdot 3 - 2 \cdot 3) \cdot \frac{1}{2} \right) = 18.41 \quad \text{Integral calculated}$$

Fig. 15. Definite integral calculation

XPress-evaluator performs operations with *Complex numbers* as well

In *Discrete Mathematics* such operations include (Fig. 16):

a) Permutations

- Permutations of n elements,
- Binomial coefficients,
- Permutations of n by k.

b) Statistics

- Calculation of mean value,
- Standard deviation, Correlation.

Math	Geometr	
\int	\sum	P(1)=1
\lim	\int^b	P(5)=120
\int_a^b	\prod	P(10)=3628800
$\frac{d}{dx}$		$C_5^2=10$
$\frac{d}{dx}$		$C_5^3=10$
$\left[\begin{smallmatrix} n \\ k \end{smallmatrix} \right]$	P(n)	$C_{10}^3=120$
C_n^k	A_n^k	$A_{10}^3=720$

Fig.16. Discrete mathematics

4 Using “Math-XPress” for e-Training and e-Assessment in Mathematical Courses

XPress-Tutor offers the student a number of problems, organized into weekly assignments, according to the course curriculum [11]. For instance, the following topics have been developed for the Linear Algebra course: *Systems of linear equations, Basics of Determinants, Basics of Matrix, Vector Spaces-1, Vector Spaces-2, Vector Spaces-3, Vector Spaces-4, Complex Numbers, Determinants – Advanced problems, Matrix – Advanced problems* – a total of 100 exercises, divided into 10 weekly tasks.

Each task is presented in three modes of operation: **Learn, Train** and **Test** (Fig. 17).

Test			Task	Train	Learn	Subject
Wrong	Correct	Problems				
3	0	10				Determinants

Fig. 17. Modes of operations

4.1 Learning Mode

During the **Learning** mode, the student are offered a series of problems on a given topic, each problem includes random parameters, so that different initial sets of parameters are displayed in different attempts. The student can solve the problem in his/her way, by typing the answer in WYSIWYG way with Editor Keys. The program checks the input expression by the computer algebra methods and responds either by question mark, if the answer does not coincide with any possible correct answer, or by remark "Wrong", if it coincides with one of the predicted (typical) wrong answers (Fig. 18).

$\begin{vmatrix} t & 0 & -1 \\ 3 & -t & 2 \\ 1 & 0 & 2t \end{vmatrix}$	(?)
$\begin{vmatrix} t+7 & 0 & 0 \\ 5 & t-7 & 1 \\ 3 & 0 & t-7 \end{vmatrix}$	Wrong...
$\begin{vmatrix} t-5 & 0 & 0 \\ 11 & t+5 & 1 \\ 3 & 0 & t+7 \end{vmatrix}$	Correct!

Fig. 18. System responses

The system allows the student to ask for help, which is presented on three levels. Let us illustrate this opportunity with the following example (Fig. 19):

Problem 3.7	Calculate the determinant of the 3-rd order:
	$\begin{vmatrix} t+2 & -1 & 1 \\ 3 & t-2 & 1 \\ 3 & -3 & t+8 \end{vmatrix}$

Fig. 19. Problem to resolve

1) **General Help**, where a method of solving the problem is described, as well as the list of intermediate steps of the solution is given (Fig. 20);

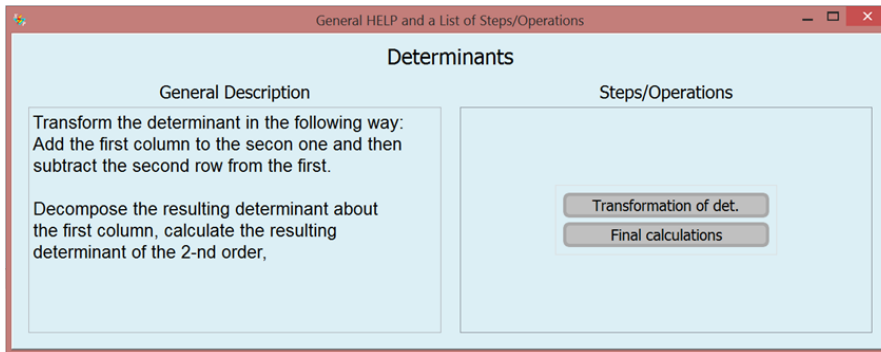


Fig. 20. General help

2) **Description of the solution steps** (see Fig. 21);

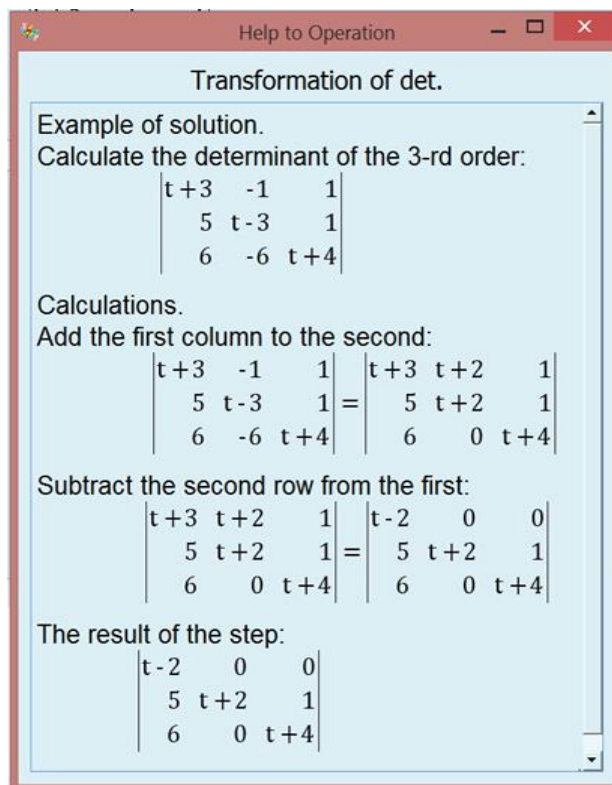


Fig. 21. Step description

3) Hint. Clicking on **Hint** button, the student will see the correct result of the nearest unresolved step.

At any moment the student can enter his/her proposal for the next step of the solution, and the program will compare it with all possible answers (both correct and typical wrong ones), and will respond correspondingly.

4.2 Training Mode

In **Training** mode, the student obtains four possible answers (one true and three false), and in order to proceed, he/she has to find the correct one (Fig. 22). In this mode, the number of trials is unlimited.

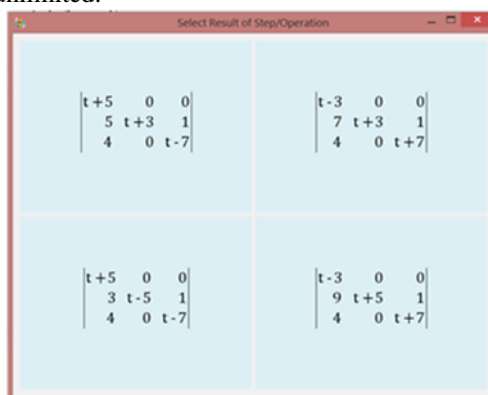


Fig. 22. Training mode

4.3 Test Mode

In **Test** mode, Help and Hint are not available, and the student has only one attempt to find the correct answer.

In **Learning** and **Training** modes, after finishing all the steps of the solution, the student can either move to the next problem, or repeat the current one with a new set of initial data. In typical student's activity scenario, he/she usually starts with a **Test** mode, looking at a set of the problems and trying to realize whether he/she can solve them without help. The system allows exit from the test, without reducing the marks.

However, for each problem only a limited number of trials are allowed, otherwise the mark for that problem will be null. While being out of the **Test** mode, the student can enter **Learning** or **Training** modes, and learn how to solve the problem, similar to that in which he/she has encountered difficulties: the difference between the problems presented in **Test** and other modes is in the values of the random parameters, defining the problem.

Generally speaking, a set of problems in **Learning**, **Training** and **Test** modes might not be identical: in **Learning** and **Training** modes, additional educational means can be included into the explanation of a problem solution, such as links to external files, sites, movies etc.

5 Principles of Semi-Intelligent e-Assessment

Math-Xpress Test uses two approaches to evaluate students' answers at exam mode of operation: advanced *Semi-Intelligent Multiple-Choice Questions* (SI-MCQ) and *Open Answer* (OA) mode. We will illustrate this with the following example (Fig. 23):

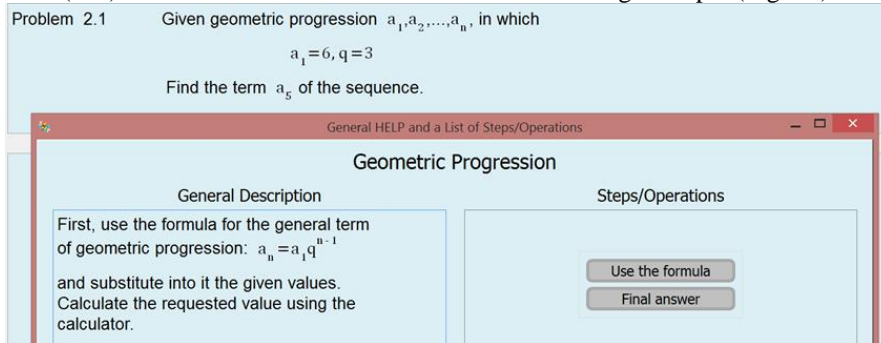


Fig. 23. Help window

During the first mode, the student can see the four possible results of each step of the problem solving, one of which is correct and the other three - wrong (Fig. 24). Like in usual *MCQ*-mode, he/she can choose an answer by clicking on the appropriate expression ($a_5 = 486$) and then confirming the selection. However, due to the possibility of free-hand typing, the student may decide to enter another expression, for example: just 486, or $a = 486$, and the system will accept them, too! (Fig. 25). This *Semi-Intelligent* ability of the system is provided by the task developer (teacher), who tries to predict the most likely options for the right answer that the student could choose (operators RESULT1 and RESULT2 in the file describing the task):

STEP1 ' Use the formula '	STEP2 'Final answer'
METH1	METH2
'Example. Given:'EXPR(a[1]=6,q=3,n=5)	'The result of the 1-st step:'EXPR(a[5]=6*3^(5-1))
'The result of the 1-st step:'EXPR(a[5]=6*3^(5-1))	'Calculations:'EXPR(a[5]=6*3^4=6*81=486)
	'The final result:'EXPR(a[5]=486)
RESULT1 EXPR(a[@n]=@a1*@q^(@n-1))	RESULT2 EXPR(a[@n]=@S(@a1*@q^(@n-1)))
EXPR(@a1*@q^(@n-1))	EXPR(@S(@a1*@q^(@n-1)))
EXPR(a=@a1*@q^(@n-1))	EXPR(a=@S(@a1*@q^(@n-1)))
FALSE1.1 EXPR(a[@n]=@a1*@q^(@n))	FALSE2.1 EXPR(a[@n]=@S(@a1*@q^(@n)))
FALSE1.2 EXPR(a[@n]=@a1+@q^(@n-1))	FALSE2.2 EXPR(a[@n]=@S(@a1+@q^(@n-1)))
FALSE1.3 EXPR(a[@n]=@a1+@q^(@n))	FALSE2.3 EXPR(a[@n]=@S(@a1+@q^(@n)))
COMM1 'Correct!'	COMM2 'Correct!'

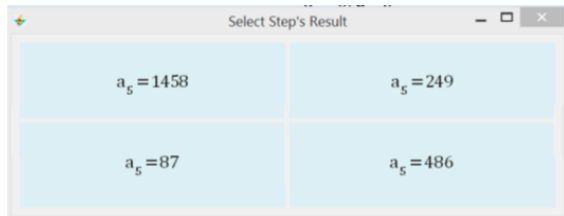


Fig. 24. Step results

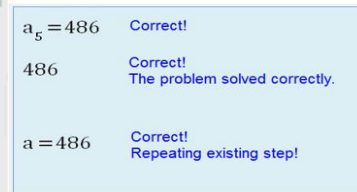


Fig. 25. System comments

In *OA* mode, the result windows are invisible, and the student is requested to enter the correct answer by typing it using the WYSIWYG interface. In this case, the task developer (teacher) can determine the way of evaluation: either to require the identity of the expressions (strict form), or their algebraic equivalent. In the first case, a comparison is made between the student's response and all possible (correct and false) answers, defined in the task. In the second case, a comparison is made using the methods of Computer Algebra, at the level of algebraic equivalence where, for instance, the following pairs of expressions are considered equal:

$$\frac{3}{\sqrt{5}}, \frac{3\sqrt{5}}{5}, 3^{2x}, 9^x, x = 2y - z, y = \frac{1}{2}(x + z)$$

The test may contain a number of problems, each of which may apply different kind of assessment. The teacher's responsibility is to decide which method of assessment implies the involved problem.

6 Development of new content

Xpress-Editor, *Xpress-Graph* and *Xpress-Geometry* offer tools rather than content, hence they do not depend on specific curriculum, nor on the language of teaching. On the contrary, the main objective of *XPress-Tutor* is training and assessment of the students studying the courses according to the national curriculum in native language. The problems are developed using *XPress-Task Editor* – a module embedded into *XPress-Tutor*, enabling compiling of new items by unexperienced in programming people [11]. A simple GUI leads the teacher through a series of windows that define a problem: text, variables, headings and expressions that determine solution' steps, etc. (Fig. 26).

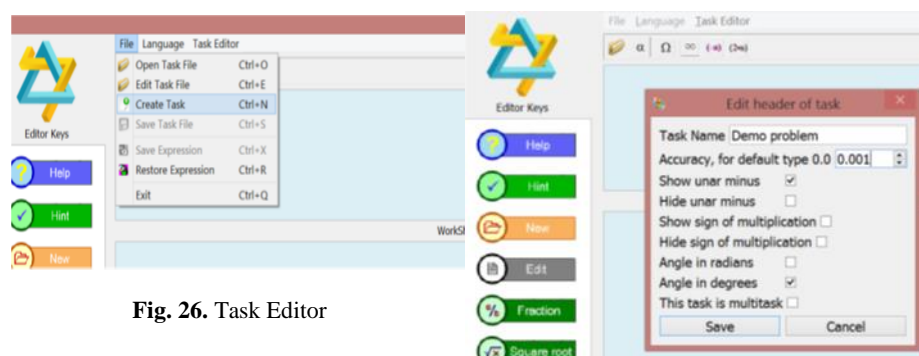


Fig. 26. Task Editor

For the Linear Algebra course, this technique is implemented in English, Hebrew and Bulgarian (Fig. 27).

<p>Problem 5.23 T is linear transformation from \mathbb{R}^3 to \mathbb{R}^3 defined by the formula:</p> $T(x, y, z) = [-x + z \quad 2x - y - z \quad x - y]$ <p>Find the basis of the region of transformation.</p>
<p>תרגיל 5.23 תהיה T - העתקה ליניארית מ \mathbb{R}^3 ל- \mathbb{R}^3 המוגדרת ע"י הנוסחה:</p> $T(x, y, z) = [-x + z \quad 2x - y - z \quad x - y]$ <p>מצאו את הבסיס של טווח ההעתקה.</p>
<p>Задача 5.23 Нека T е линеен оператор от \mathbb{R}^3 към \mathbb{R}^3, дефиниран чрез формулата:</p> $T(x, y, z) = [-x + z \quad 2x - y - z \quad x - y]$ <p>Да се намери новата база на линейния оператор.</p>

Fig. 27. Task translator

7 Integration of Math-Xpress into Moodle

Operation of any Math Assessment system with Moodle requires embedding Math Core (Computer Algebra operations, algebraic expressions evaluation and test results calculation) into Moodle code, and transferring students' data into Moodle LMS.

To do that, a plugin, enabling an operation of the system from within Moodle and moving the results into Moodle LMS was developed. The following picture shows a typical screen of the corresponding interface (Fig. 28):

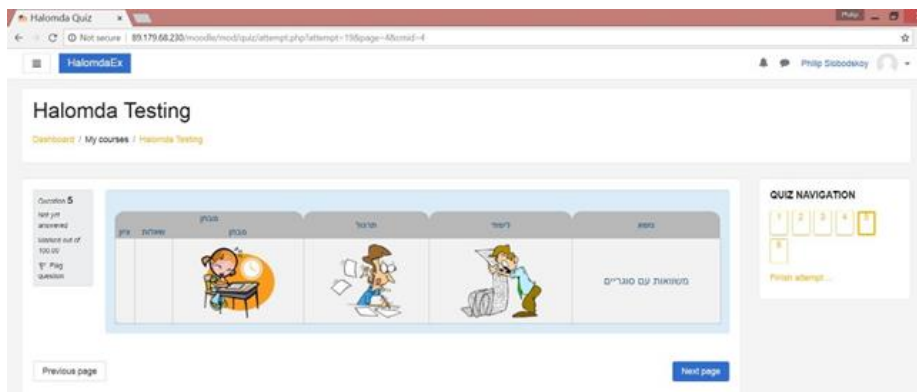


Fig. 28. Moodle interface screen

As long as the desired mode of operation – **Learning**, **Training** or **Test** – is chosen, XPress-Tutor starts working independently on Moodle, transferring to it only the test results.

8 Evaluation of Xpress-Tutor in Teaching Linear Algebra at Ariel University

During the last five academic years, several courses on Higher Mathematics used Xpress-Tutor for teaching and assessment for more than 3000 students each year at Math and Computing and Engineering faculties at Ariel University in Israel. The comparison between the tests results of two groups of students learning Linear Algebra at Ariel University is shown on the following comparative chart. The students of the first group successfully completed all the tasks of XPress-Tutor while those of the second did not work with this program (Fig. 29):

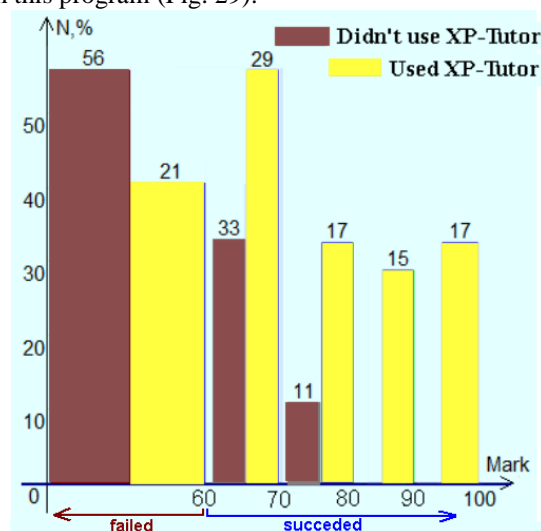


Fig. 29. Number of students who passed test

The first group consisted of 52 students, that used Xpress-Tutor during the semester, whereas 27 students of the second group did not use it. The differences in the average marks of the final exam and in the relative numbers of failures (21% and 56% correspondingly) clearly show the advantage of using the Xpress-Tutor in teaching and learning of Higher Mathematics.

In order to estimate the students' satisfaction with the use of the system, 82 students that used Xpress-Tutor were asked to complete anonymous questionnaires. The results showed that: a) most of them used Xpress-Tutor at the beginning of the course (Fig. 30); b) 87.8% of them passed successfully all the tasks (Fig. 31); c) only 7.3% of them failed the final exam (Fig. 32); d) 28% and 68.3% of the responders used **Learning** and **Training** mode accordingly, in addition to the **Test** mode (Fig. 33); e) 66% of the students rated Xpress-Tutor as very helpful (3 and higher in 1 to 5 scale) (Fig. 34); f) 67% of the students rated Xpress-Tutor as a simple and clear in operation (3 and higher in 1 to 5 scale) (Fig. 35).

Students' comments and recommendations have indicated their overall satisfaction of using Xpress-Tutor in learning of Linear Algebra (Fig. 36).

מה הציון המשוקלל של כל העבודות שהגשת?

82 תגובות

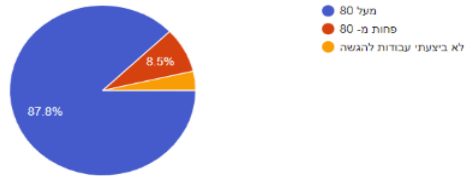


Fig. 30. Did you use Xpress-Tutor for learning Linear Algebra?

האם ניגשת בשנה שעברה ל"עבודות להגשה" באלגברה לינארית באתר Halomda?

82 תגובות

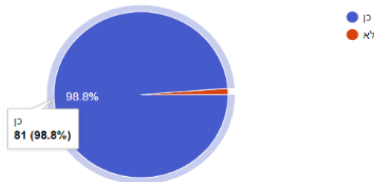


Fig. 31. What was the average mark for all the tasks of Xpress-Tutor?

מה הציון שקיבלת במבחן בסוף הסמסטר?

82 תגובות

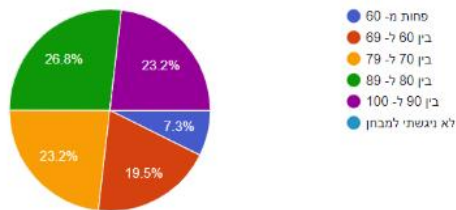


Fig. 32. What was the mark of the final class exam?

באילו אפשרויות שהתוכנה מציעה נעזרת?

82 תגובות

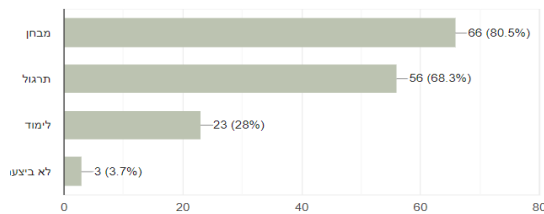


Fig. 33. What modes of operation did you use?

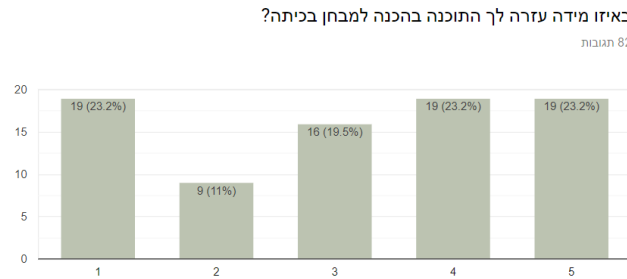


Fig. 34. To what extent did the system help you in preparing to class exam?



Fig. 35. To what extent was the operation of the system simple and clear?

To the question “What would you recommend to change or to add to the system?” 65 responses were received, some of the most common ones being the following (Fig. 36). The answers show that the students really like the system and even think that using the system should be mandatory for all of them.

-
- ... perfect system...the problems helped to understand the basics of the course...
 - ... to add exercises similar to those in exam...
 - ... to fit the problems' level to that of class exam...
 - ... to increase the problems' level ...
 - ... no doubts, it wasn't waste of time to get full explanations...
 - ... there is no need to improve anything...
 - ... to make the system the must for all the students...
-

Fig. 36. Students' responses

Similar students' responses have been obtained at Talpiot teacher's college, where the course of Quantitative Thinking has been taught for three semesters, as well as from the group of 28 students of the Technical University of Sofia [12].

9 Conclusion

In this paper we showed how technology enhanced mathematical education could help students in their studies at home and in the classroom. For that purpose, different modes of student's activity - **Learning, Training and Test** are integrated into a complete educational Math environment: Xpress -Tutor, which can run on Windows-based computers, as well as and on mobile devices. Integration of the system in LMS (Moodle) is an advantage that supports teachers in their routine test checking. The system also allows an advanced *Semi-intelligent MCQ* to be used in computer tests assessment, providing a higher level of student's freedom than in usual Multi-Choice-Questions test.

In addition to the overall satisfaction from using the system, expressed by both the teachers and the students, the quantitative estimation of usefulness of Xpress-Tutor has been performed. The results of the quantitative comparison showed an essential improvement of the exam marks of the students using Xpress-Tutor.

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