Abstract—Orthogonal Frequency Division Multiplexing (OFDM) has significantly reduced the complexity of the receivers, however to allow simple equalization procedures and avoid interblock interference, a cyclic prefix (CP) has to be appended to the OFDM block. CP reduces the efficiency of the transmission, because it is discarded at the receiver. This paper proposes a receiver structure which recovers the CP and exploits it to improve data detection. The performance of this structure is then analyzed comparing it with other two structures in a simulation scenario suitable for the evolution of the current Terrestrial Digital Video Broadcasting (DVB-T). The impact of low-density parity check codes (LDPC) is also outlined.

Index Terms—cyclic prefix, OFDM, LDPC.

I. INTRODUCTION

Modulation techniques based on Orthogonal Frequency Division Multiplexing (OFDM) have been widely applied into nowadays wireless broadcasting standards such as Digital Audio Broadcasting (DAB) [1] and Digital Video Broadcasting (DVB) [2]. In the next generation DVB-T (DVB-T2) [3], OFDM is the one major candidates as modulation scheme thanks to its low complexity structure which allow simple modulation and demodulation making use of an inverse Fourier Transform (IFFT) at the transmitter and a Fourier Transform (FFT) at the receiver. Improvement of data detection exploiting the cyclic prefix (CP) has already been considered in [4]. However, due to Interblock Interference (IBI) caused by channel multipath, only part of the CP is exploited. In fact the received CP symbols which are corrupted by the previous transmitted OFDM block are discarded. In [5] IBI is taken into account, but [5] deals with a single-carrier system (SC) so it is not of practical use in our scenario. The system proposed in [5] introduces an additional latency in the OFDM block detection, as it needs the next OFDM block to perform the demodulation.

In this paper we propose a major improvement with respect to the current architecture showing how to exploit the complete CP for data detection without additional latency w.r.t. conventional receivers. Furthermore we apply this architecture to a timely application scenario i.e. the improvement of DVB-T system. Finally we provide simulation results for a receiver using the CD3 channel estimation technique [6], considering both the uncoded and coded scenarios. We allow ourselves in the proposed receiver a reasonable complexity increase, having as a priority the performance gain with respect to the conventional receiver structures. As a matter of fact this is in line with the commercial requirements for DVB-T2 [3] where the focus is on the increase of performances. For the same reason we consider in our receiver, as error correcting code, the LDPC code already standardized for the DVB-S2 system.

The paper is organized as follows. Section II describes our system model. In Section III we derive our reduced complexity Least-Square solution, generalizing the one in [4]. In Section IV simulation results are presented both for uncoded and LDPC coded case. Finally in Section V we draw the conclusions of our work.

II. SYSTEM MODEL

Let \( s^i = [s^i(1), s^i(2), \ldots, s^i(N)]^T \) be the modulated complex values of the \( i \)-th OFDM block transmitted over the channel. Applying the IFFT we obtain:

\[
s^i = F^H s^i
\]

where \( s^i \) represents the \( i \)-th OFDM block in the time domain transmitted over the channel, and \( F \) is the unitary Discrete Fourier Transform (DFT) matrix. \((\cdot)^H\) denotes the transpose conjugate of the argument. A CP of length \( G \) is appended to \( s^i \) repeating the last \( G \) values of IFFT. Then the OFDM block, as in Fig.1, is transmitted through the channel which is modelled as a FIR filter with \( L+1 \) taps: \( h^i = [h^i_0, \ldots, h^i_L]^T \). In Fig.1 we have divided the information symbols of the OFDM block in two groups: the \textit{noCP} data samples which do not fall inside the CP portion of the signal, and the CP data samples which are repeated in the CP. In the following we suppose that the CP length is at least equal to the channel length, i.e. \( L \leq G \), so the previous OFDM block leaks only into the \( i \)-th blocks CP, which normally is discarded. As in [4], the received CP samples are not discarded, but usefully exploited to improve the detection performance. According to Fig.1, we can express the relation between the transmitted signal \( s^i \) and
the received time domain samples \( \mathbf{r}^i = [\mathbf{r}^i(N + G), \mathbf{r}^i(N + G - 1), \ldots, \mathbf{r}^i(1)]^T \) as:

\[
\mathbf{r}^i = \mathbf{H} \mathbf{s}^i + \mathbf{n}^i \tag{2}
\]

where \( \mathbf{n}^i \) is the complex vector of the zero-mean Gaussian noise, and \( \mathbf{H} \) is a \( (N + G) \times (N + G + L) \) Toeplitz matrix given by:

\[
\mathbf{H} = \begin{bmatrix}
    h_0 & h_1 & \ldots & h_L & 0 & \ldots & 0 \\
    0 & h_0 & h_1 & \ldots & h_L & \ddots & \\
    \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
    0 & \ldots & 0 & h_0 & h_1 & \ldots & h_L
\end{bmatrix} \tag{3}
\]

The matrix operation in (2) is the counterpart of the convolution of the transmitted signal and the channel impulse response. The samples \([\mathbf{r}^i(G), \ldots, \mathbf{r}^i(1)]^T\) represent the receiving of the CP of the \(i\)-th OFDM block. Those samples are corrupted by the leakage of the previous OFDM block transmitted, i.e. by the convolution of \([\mathbf{s}^{i-1}(N), \ldots, \mathbf{s}^{i-1}(N - L + 1)]^T\) (which are the last \(L\) samples of the CP of the previous OFDM block) with the channel impulse response. Based on the decisions at the previous \((i-1)\)-th block, i.e. the previous decoded symbols and the estimation of the \((i-1)\)-th channel impulse response, we are able to remove the contribution of \(\mathbf{s}^{i-1}\) to the received CP. Unfortunately this procedure is not enough to be able to exploit the redundancy introduced by CP. In fact, due to channel multipath, the CP data samples have contributes given by the last \(L\) noCP data samples, \([\mathbf{s}^{i}(N-G), \ldots, \mathbf{s}^{i}(N-G-L+1)]^T\). For this reason, after the IBI removal we add to the received CP a useful component of Intersymbol Interference (ISI). This contribute is given by the convolution between the last \(L\) samples of the noCP portion of data and the current channel impulse response \(\mathbf{h}^i\). To obtain this contribution, we first decode the \(i\)-th OFDM block with the conventional receiver, that is discarding the CP and performing the Fourier Transform (FFT), then we compute the impulse channel response estimation, and finally we compute the convolution between the last \(L\) samples of noCP data and \(\mathbf{h}^i\). Supposing perfect interference cancellation and ISI recovering, we can rewrite (2) as:

\[
\mathbf{y}^i = \mathbf{H}_c \mathbf{s}^i + \mathbf{n}^i \tag{4}
\]

defining \(\mathbf{H}_c\) as the following \((N + G) \times N\) circulant matrix:

\[
\mathbf{H}_c = \begin{bmatrix}
    h_0 & h_1 & \ldots & h_L & 0 & \ldots & 0 \\
    0 & h_0 & h_1 & \ldots & h_L & \ddots & \\
    \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
    0 & \ldots & 0 & h_0 & h_1 & \ldots & h_L \\
    h_2 & \ldots & h_L & 0 & 0 & h_0 & h_1 \\
    \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    h_2 & \ldots & h_L & 0 & 0 & h_0 & h_1 \\
    \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    0 & \ldots & 0 & h_0 & h_1 & \ldots & h_L \\
    \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    0 & \ldots & 0 & h_0 & h_1 & \ldots & h_L \\
    \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    0 & \ldots & 0 & h_0 & h_1 & \ldots & h_L \\

\end{bmatrix} \tag{6}
\]

In (6) we have splitted matrix \(\mathbf{H}_c\) by a line to show that the matrix is equal to the product of a square circulant matrix \(\mathbf{H}_0\), obtained selecting the first \(N\) rows of \(\mathbf{H}_c\), by a \((N + G) \times N\) matrix as follows:

\[
\mathbf{H}_c = \mathbf{K} \mathbf{H}_0
\]

where

\[
\mathbf{K} = \begin{bmatrix}
    \mathbf{I}_N \\
    \mathbf{L}_G & \mathbf{0}_{N-G}
\end{bmatrix}
\]

where \(\mathbf{I}_n\) and \(\mathbf{0}_n\) are respectively the identity and zeros \(n \times n\) matrix. Since \(\mathbf{H}_0\) is a square circulant matrix, it can be decomposed by the well known relation between the circulant matrices and the DFT matrix \(\mathbf{F}\). Assuming to use the unitary DFT and IDFT matrix, we can express \(\mathbf{H}_0\) as:

\[
\mathbf{H}_0 = \mathbf{F}^H \Lambda \mathbf{F}
\]

where

\[
\Lambda = \text{diag} \left( \mathbf{F}^H \begin{bmatrix}
    \mathbf{h}^i \\
    \mathbf{0}_{(N-(L+1)) \times 1}
\end{bmatrix} \right)
\]

\(\Lambda\) is the diagonal matrix which contains the eigenvectors of \(\mathbf{H}_0\). Since all the terms considered refer to the current \(i\)-th OFDM block, we drop the block index \(i\) for simplicity and rewrite the system relations:

\[
\mathbf{y} = \mathbf{K} \mathbf{H}_0 \mathbf{F}^H \mathbf{s} + \mathbf{n}
\]
We remark that (9) is similar to the relation (9) of [4] but for a different number of 1s in the rows below the horizontal line in \( \mathbf{K} \) (7), i.e. \( \mathbf{K} \) considers the last \( L \) CP symbols which in [4] are discarded.

III. LEAST-SQUARE SOLUTION

Let the form \( \mathbf{y} = \mathbf{X}\mathbf{\theta} + \mathbf{w} \) be a linear model where \( \mathbf{y} \) is a known vector denoting the output signal of the system, \( \mathbf{\theta} \) and \( \mathbf{w} \) are unknown random vectors uncorrelated and \( \mathbf{w} \) is zero-mean, denoting respectively the input signal and the noise of the system, and finally \( \mathbf{X} \) is a known deterministic matrix denoting the transformation of the system. The linear Least-Square Estimator (LSE) of \( \mathbf{\theta} \) given \( \mathbf{y} \) is [7]:

\[
\hat{\mathbf{\theta}} = (\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H\mathbf{y} \tag{10}
\]

Although LSE solution is suboptimal with respect to Minimum Mean Square Error (MMSE) solution, we propose (10) as the solution of our problem because, differently from MMSE, it is implementable in a receiver with reduced complexity. Besides, [4] shows that LSE solution closely achieves MMSE optimal performance.

Applying (10) to (9) we obtain the LSE of \( \mathbf{s} \) as:

\[
\hat{\mathbf{s}} = (\mathbf{A}^H\mathbf{F}\mathbf{K}^H\mathbf{F}\mathbf{K}\mathbf{H}\mathbf{A})^{-1}\mathbf{A}^H\mathbf{F}\mathbf{K}^H\mathbf{y} \tag{11}
\]

Defining the matrix \( \mathbf{E} \) as:

\[
\mathbf{E} \triangleq \begin{bmatrix} \mathbf{I}_G & 0_{N-G} \end{bmatrix} \tag{12}
\]

and noting that \( \mathbf{K}^H\mathbf{K} = \mathbf{I}_N + \mathbf{E} \) we can rewrite (11) as:

\[
\hat{\mathbf{s}} = (\mathbf{A}^H\mathbf{A} + \mathbf{A}^H\mathbf{F}\mathbf{E}\mathbf{F}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{F}\mathbf{K}^H\mathbf{y} \tag{13}
\]

Applying the matrix inversion lemma\(^1\) to (13) and defining

\[
(\mathbf{A} + \mathbf{B}\mathbf{C}\mathbf{D})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1}
\]

we obtain:

\[
\hat{\mathbf{s}} = \mathbf{A}^H\mathbf{F}(\mathbf{I} - (\mathbf{I} - \mathbf{E})^{-1}\mathbf{E})\mathbf{K}^H\mathbf{y} \tag{14}
\]

or, since \( (\mathbf{I} - (\mathbf{I} - \mathbf{E})^{-1}\mathbf{E}) = (\mathbf{I} - \mathbf{E})^{-1} \), we obtain equivalently:

\[
\hat{\mathbf{s}} = \mathbf{A}^H\mathbf{F}(\mathbf{I} - \mathbf{E})^{-1}\mathbf{K}^H\mathbf{y} \tag{15}
\]

which, replacing \( \mathbf{\Gamma} \) with \( \mathbf{A}^H\mathbf{A} \), leads to:

\[
\hat{\mathbf{s}} = \mathbf{A}^{-1}\mathbf{F}(\mathbf{I} + \mathbf{E})^{-1}\mathbf{K}^H\mathbf{y} \tag{16}
\]

Finally, defining \( \mathbf{W} \) as \( \mathbf{W} = (\mathbf{I} + \mathbf{E})^{-1}\mathbf{K}^H \), we get:

\[
\hat{\mathbf{s}} = \mathbf{A}^{-1}\mathbf{F}\mathbf{W}\mathbf{y} \quad \mathbf{W} = \begin{bmatrix} \frac{1}{2}\mathbf{I}_G & 0_{N-G} \end{bmatrix} \tag{18}
\]

If we compare (18) with the equivalent LSE solution given in [4], we note that after the transformation of the received OFDM block, removing IBI and adding an opportune ISI contribution, the structure of the receiver remains the same; we simply extend the average computed by the structure of Fig.2, to the last \( G \) samples of the signal \( \mathbf{y} \). Since the structure of Fig.2 does not increase the complexity of the system, we note that the complexity increase reside only in the interference recovering procedure. In fact this procedure requires to compute a first conventional detection on data and channel estimation. Then the performance is increased computing an improved data detection using a structure similar to the one depicted in Fig.2. Equalization and demodulation steps of the conventional receiver are doubled for our receiver structure.

IV. SIMULATION RESULTS

Simulations have been performed as to assess the performance of the proposed receiver. In the following we will denote our proposed receiver as FR (Full Recovery), and the receiver proposed in [4] as PR (Partial Recovery). Before presenting the results, we calculate the expected performance of the system without coding. For example, we consider a conventional OFDM system with FFT size of \( N = 64 \) symbols, with CP length ratio of \( \frac{1}{8} \), i.e. \( G \) is equal to \( \frac{1}{4} \) of the FFT size, that is \( G = 16 \) symbols. The performance is measured with respect to the Signal-to-Noise Ratio (SNR). For a given value of Bit Error Rate (BER), in presence of white Gaussian noise, we suppose that the standard SNR is \( \rho \). Assuming perfect IBI cancellation and perfect reconstruction of the useful ISI components, the improvement on the conventional performance is easily calculated. With a transmitted power uniformly distributed over all carriers \( (P_{Tx}) \), taking the average of the samples of the CP, we decrease the variance of the noise \( (N_0) \) by half, so we have a portion of the received
signal with power of noise halved. Taking as reference the above example, the increased SNR is

$$\rho_{\text{new}} = \frac{P_{\text{Tx}}}{\frac{48}{64} N_0 + \frac{16}{64} N_0/2} = \frac{P_{\text{Tx}}/N_0}{\frac{5}{8} + \frac{1}{8}} = \frac{8}{7} \rho \quad (19)$$

The extent of the improvement without coding, in ideal conditions and assuming perfect recovering of the CP, is therefore $\frac{8}{7}$, or equivalently 0.58 dB. So we expect that the curve BER versus SNR, for the proposed receiver, will be shifted to the left with respect to the conventional performance curve of about 0.58 dB. In our simulations we use the CD3 channel estimation [6], that is the OFDM system organizes the transmission dividing the information bits in several frames each containing an equal number of OFDM blocks. To reduce the pilot symbols overhead, the channel estimation is computed only by the first OFDM block of the frame, which is known at the receiver, and then the channel estimation for each following block is derived from the previous block channel estimation and the received signal. In Fig.3 the performance of FR receiver with respect to a conventional and PR is shown in a Rayleigh fading channel in the following scenario. No coding is used, $T \sim 0.11 \mu s$, $N = 16384$, $G = 4096$ taps and $L = 20$ taps. The performance improvement of FR equals the expected one, moreover the curve of BER vs SNR for FR is superimposed to the curve for FR. This is due to the high number of symbols free from IBI. In fact PR receiver exploits $G - L = 4076$ CP samples, that is almost equivalent to use the whole CP. In Fig.4 the results of the simulations are plotted for the three receivers with a channel which has the same Rayleigh statistic, but longer. In particular we have used a channel which cause a leakage of the whole CP. For this type of channel, namely Long Rayleigh, the PR receiver uses $G - L = 0$ CP symbols, as the conventional receiver, instead FR exploits all the CP. With the channel having so many taps the performance improvement of our receiver is decreased to 0.2 dB, but it is still better than the other two receivers. Both

Fig.s 5 and 6 show that introducing a LDPC coding with rate 8/9, the performance improvements of the non-conventional receivers are not influenced: an improvement of about 0.5 dB and 0.2 dB respectively is maintained. This is not to be taken for granted as other types of non conventional data detection which initially appear to be promising in absence of coding, when even weak coding is used, the improvement performance decrease with respect to the conventional.

V. CONCLUSION

We have proposed a new receiver architecture which exploits the whole redundancy of the CP symbols. We have compared our solution with the conventional one and with the one proposed in [4]. Although the interference cancellation and ISI reconstruction strongly depend on the quality of the channel estimation, which in the particular scenario selected does not use any scattered pilot [6], we have shown that our
detection method keeps a performance improvement even with channels that leak into the whole CP. Using iterative interference cancellation, our educated guess is that the performance can be improved further on.

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