Experimental evidence of hysteresis in the head-discharge relationship for a leak in a polyethylene pipe

Marco Ferrante[1,*], Christian Massari[2], Bruno Brunone, M.ASCE[3], Silvia Meniconi[4]

Abstract

The relationship between leak outflow from a damaged pipe and flow condition inside the pipe plays a crucial role in the pressurized pipe systems management. As an example, this relationship is used in leakage reduction techniques based on pressure control and in leak detection techniques based on inverse analysis.

To explore the relationship between total head inside the pipe and leak outflow for a single leak in a polyethylene pipe, tests were carried out at the Water Engineering Laboratory of the University of Perugia. These tests point out that the viscoelastic nature of the pipe material gives rise to a hysteretical behavior of the investigated relationship, i.e. the outflow dependence not only on the synchronous total head but also on the total head time-history and variation rate.

Keywords: Leakage, Water distribution systems, Viscoelasticity, Plastic pipes.

*Corresponding author: ferrante@unipg.it

[1]Associate Professor, Dipartimento di Ingegneria Civile ed Ambientale, University of Perugia, Via G. Duranti, 93 - 06125 Perugia, Italy

[2]Ph.D. Student, Dipartimento di Ingegneria Civile ed Ambientale, University of Perugia, Via G. Duranti, 93 - 06125 Perugia, Italy

[3]Professor, Dipartimento di Ingegneria Civile ed Ambientale, University of Perugia, Via G. Duranti, 93 - 06125 Perugia, Italy

[4]Assistant Professor, Dipartimento di Ingegneria Civile ed Ambientale, University of Perugia, Via G. Duranti, 93 - 06125 Perugia, Italy

Introduction

In the last decades several papers have addressed the issue of leakage reduction in pressurized pipe systems, contributing to qualify leak detection and control as two important research topics. The correct definition of the relationship between leakage and functioning conditions in a damaged pipe is crucial for both these topics. In leak detection techniques based on inverse analysis such a relationship determines the system response and hence its incorrect definition may impact the diagnosis reliability (Brunone and Ferrante 2004; Covas et al. 2004; Ferrante et al. 2009). In leakage control techniques based on pressure reduction, it affects the reliability of the forecasted leakage reduction and hence of the estimated cost-effectiveness of a system improvement (e.g. Thornton and Lambert 2005).

Based on literature, the leak outflow, Q_L, depends mainly on the leak effective area $C_L A_L$ — defined as the product of the discharge coefficient, C_L, and of the leak area, A_L — and on the total head inside the pipe, H, or the pressure head, h. Other quantities can also be considered, as pipe thickness, discharge conditions (in air/submerged), ratio Q_L/Q_U, with Q_U being the discharge upstream the leak, and leak shape (van Zyl and Clayton 2007; Brunone and Ferrante 2001; Osterwalder and Wirth 1985).

In steady-state conditions the general equation

$$Q_L = aH^b \tag{1}$$

is often used (e.g.: Greyvenstein and van Zyl 2007; van Zyl and Clayton 2007; Thornton and Lambert 2005) that includes the Torricelli's equation

$$Q_L = C_L A_L \sqrt{2gH} \tag{2}$$

when $a = C_L A_L \sqrt{2g}$ and $b = 1/2$, with H referred to the leak elevation. The Torricelli's Equation (2) can be derived analytically for an orifice on the thin horizontal bottom wall of a constant head reservoir. Its extension to the orifice on the side of a pipe by means of the Bernoulli theorem is not trivial. For the special case of a streamtube linking a gradually varied flow section upstream the leak and the vena contracta downstream the leak, assuming that all the upstream flow exits from the leak (no downstream flow) Eq. (2)

holds with H coincident with the total head at the upstream section. While some theoretical derivations seem to push towards the use of Eq.(2), an empirical approach in interpreting the experimental evidence led to the more general Eq.(1) when both a local and a global scale is considered (e.g. AWWA 2009).

In many papers, especially those addressing leakage control by means of pressure management, Eq.(1) is used at a global - district area scale (e.g. Thornton and Lambert 2005), with H being a "mean pressure" over the district and Q_L is the flow entering the district. In other papers, Eq.(1) is also used to interpret experimental results at a local scale, considering a single leak (e.g. Greyvenstein and van Zyl 2007; van Zyl and Clayton 2007). In both cases the variation of $C_L A_L$ with H can be used to explain the increase of the b exponent with respect to Torricelli's formula (May 1994; van Zyl and Clayton 2007).

Cassa et al. (2010) show that for an elastic pipe the increase in the leak area depends linearly on H and hence propose the relationship

$$Q_L = cH^{0.5} + dH^{1.5} \tag{3}$$

where $c = A_0 C_L \sqrt{2g}$, $d = mC_L\sqrt{2g}$, A_0 is the leak area for $H = 0$, and m is a parameter.

For practical applications and in the carried out tests, the use of the pressure head, h, instead of H in Eqs. (1), (2), and (3) is irrelevant, since the kinetic head is negligible when compared to the total head — actually the original formulation of Eq. (3) in (Cassa et al. 2010) is in terms of h. Although our choice does not affect the results, for the sake of generality in this paper the effects of H on Q_L are analysed.

To explore the $H - Q_L$ relationship on a local scale in a polyethylene pipe, tests were carried out at the Water Engineering Laboratory (WEL) of the University of Perugia. Eqs.(1), (2) and (3) are applied on a local scale, considering a single leak discharging into the atmosphere. In the following, some results of these tests will be shown, suggesting that the well-known viscoelastic behavior of polyethylene pipes (e.g. Pezzinga 2002; Covas et al. 2004) has an impact on the definition of the relationship between Q_L and H.

Experimental set-up

Tests were carried out at WEL on a high density polyethylene pipe DN110 PN10, about 20 m long, with an internal diameter 93.3 mm and a wall thickness 8.1 mm (Fig. 1). The pump (P) supplied the needed discharge from the recycling reservoir (R) to the upstream air vessel (AV), which behaves as a constant head tank. At the downstream end section of the pipe, there was a hand-operated ball valve (DV) discharging into the air. An automatically controlled butterfly valve (MV) was placed immediately upstream DV. At a distance of about 10.25 m from the air vessel there was a trunk of 2.12 m with a longitudinal leak of about 1.5x92 mm (L) discharging into atmosphere. Since a crack took place before the first test, the irregular geometry of such a leak did not allow to have a precise measure of A_0. This situation, of an "irregular" leak with an unknown geometry, is common when leaks coming from functioning systems are considered. The leak length was measured before and after the tests to ensure that the crack did not propagate further. Another "regular" leak of 2x92 mm was also machined, with rounded ends to prevent the same crack enlargement, on another trunk of 2.71 m. The carried out tests can be grouped in two different series: Series 1 refers to the first set-up, with irregular leak, while Series 2 refers to second set-up, with the regular leak.

Two electromagnetic flowmeters were used to measure the discharge upstream (UD) and downstream (DD) the leak, with an accuracy of 0.05% of the measured value. Two piezoresistive pressure transducers, with a 7 bar full scale (f.s.) and an accuracy of 0.1% f.s., measured the pressure upstream (UP) and downstream the leak (DP). The pipe between UD and DD was horizontal.

[Figure 1 about here.]

Flow and pressure signals were acquired at a 1 Hz sampling frequency and down-sampled to 1/60 Hz by a lowpass Chebyshev Type I filter, to simplify representation and data analysis. The total head, H, is evaluated by adding the kinetic head corresponding to the upstream measured discharge to the pressure head at the transducer UP.

As shown in Fig. 2, strain gages were placed close to the leak (SGA2) and to DP (SGT) to measure radial strains. Simultaneous pressure, discharge and strain signals were acquired for some of the experiments.

[Figure 2 about here.]

Experimental results

During the tests the pipe mean head was changed by maneuvering the two valves MV and DV. While the hand-operated end valve DV was kept at a constant opening degree during the tests, the opening and closure maneuvers were automatically controlled by the butterfly valve MV. The order of magnitude of the water hammer characteristic time is 10^{-1} s. As a consequence, since the duration of the closure and opening maneuvers is much larger than such a characteristic time, the inertial effects can be neglected and in the shown tests the system can be regarded as passing through a sequence of steady-states. Using the language of the pressurized pipe network modelers, this condition is typical of an extended period simulation (EPS).

Series 1: the irregular leak

In this first series the system was kept at the same functioning conditions for at least 12 h, with the butterfly valve MV totally opened (i.e. with an opening degree $\alpha = 0^o$). Then the MV valve was partially closed in a few seconds to a given open degree, α_c, at a constant angular velocity. Different valve open degrees were considered, with α_c ranging from 55.84^o to 66.15^o. After the closure maneuver, the MV valve was kept at $\alpha = \alpha_c$ for 12 h and then it was completely opened ($\alpha = 0^o$).

Fig. 3 shows the time-history of H and Q_L for one of the tests of Series 1 ($\alpha_c = 66.15^o$). The closure maneuver took place at $t = 1.0$ h. In about 15 min after the end of the maneuver the system reached new functioning conditions with Q_L increasing with H (hollow circles). For the remaining hours preceding the opening maneuver (black circles), the outflow Q_L increased in time while H decreased. This behavior cannot be explained by Eq. (2), which assumes a direct proportionality of Q_L and $\sqrt{H}$, and it contradicts the physical experience that suggests an increase of Q_L with H, with positive values of a and b in Eq. (1). At $t = 13$ h the total opening of MV took place in a few seconds and, as a result, H and Q_L showed a sudden synchronous decrease (hollow squares). In about 25 min the system moved toward new functioning conditions but in the following hours until the test ended, H increased in time while Q_L decreased, again contradicting Eqs. (1), (2), and (3) (black squares).

[Figure 3 about here.]

To enhance the different trends of the couples of H and Q_L values, they have been plotted in the (H, Q_L) plane in Fig. 4.

[Figure 4 about here.]

Considering the different behavior of the data, in the short and in the long period following the maneuver, distinct fittings are proposed. The fitting by Eq. (1) of the "loading" curve immediately succeeding the closure maneuver (short period, hollow circles) yields $a = 3.48\ 10^{-4}$, $b = 0.862$ and $R^2 = 0.9930$. The fitting by Eq. (3) of the same data yields $c = 6.49\ 10^{-4}$, $d = 0.029$ and $R^2 = 0.9925$. Since the two fittings are very similar, only the curves referring to Eq. (1) are shown in Fig. 4. Excluding the short period following the maneuver and considering the data coming from the remaining period of time before the opening maneuver (long period, black circles), the fitting is relatively poor ($R^2 = 0.3511$) and yields negative values of b. Besides the R^2 value, the qualitative analysis of the figure clearly confirms that the data displacement in the considered plane cannot be properly explained by means of curves defined by Eqs (1) and (3), passing through the origin.

The opening maneuver originates an "unloading" curve similar to the loading one, with $b = 0.799$ and close to that of the loading curve, $a = 4.71\ 10^{-4}$ (hollow squares, $R^2 = 0.9956$), $c = 7.80\ 10^{-4}$, $d = 0.023$ ($R^2 = 0.9956$). In the following hours (black squares) the system reached the initial conditions moving along a curve parallel to the one defined by the black circles, i.e. the period of time after the closure maneuver. Again, the fitting of this data by means of Eq. (1) has a low R^2 and gives a negative value of b. It is worth noting that when a satisfactory goodness of fitting is obtained, both for the loading and unloading, the values of b are greater than 1/2; hence, these data can not be properly fitted by Eq. (2) where $b = 1/2$.

All the carried out tests of Series 1, with different opening degrees α_c, show the same behavior of Figs. 3 and 4. Fig. 5 summarizes the measured values of H and Q_L obtained in several tests of Series 1, for different values of α_c.

[Figure 5 about here.]

The loading and unloading curves are almost parallel to each other and their position is defined by the starting values of H and Q_L.

The "loop" shown for every test in the considered plane suggests a "hysteretic" behavior of the leak. This behavior could be ascribed to the viscoelastic properties of the pipe material and this is why we designated as "loading" and "unloading" the different curves, borrowing these terms from the analysis of the constitutive laws of the viscoelastic materials in the stress-strain plane.

Such a viscoleastic behavior can explain different curves in the (H, Q_L) plain depending on the total head increase rate and time-history.

Series 2: the regular leak

Since the leak geometry of the previous shown series was partially machined and partially obtained by a spontaneous enlargement, to asses if the shown results were affected by this circumstance, a second longitudinal leak was machined of 92 mm, with a constant width of 2 mm and with rounded edges to prevent the spontaneous enlargement. The machined total area was 183.1 mm^2. In these tests, the MV valve was programmed to close slowly, with respect to Series 1, from $\alpha = 0^o$ to $\alpha_c = 66.15^o$, with a constant angular velocity and for a given duration of time, $t_M = 24h$, and then to open at the same rate back to $\alpha = 0^o$.

The data are plotted in Fig. 6 in the (H, Q_L) plane.

[Figure 6 about here.]

Data from Series 2, as well as those from Series 1, show a hysteretical loop in the (H, Q_L) plane and hence we can conclude that the presence of the crack and the irregular geometry cannot explain this behavior by themselves. The couples of data (H, Q_L) and (h, Q_L) shown in Fig. 6 , almost coincident, clearly demonstrates that the differences between H and h are negligible. For Series 2 the estimated leak Reynolds numbers are greater than 10^4.

The pipe strains

The strain gages collected data give another, independent, point of view of the depicted phenomenon.

Fig. 7 shows the variation of the pipe radial strain, ϵ, measured at SGA2 with H, for test of Series 1 plotted in Fig. 3.

[Figure 7 about here.]

Due to the viscoelastic behavior of the pipe, some differences can be observed at SGA2 for loading and unloading. The strain gage close to the pressure transducer DP gives very similar results (not shown). Comparison of Figs. 7 and 4 shows the same general behavior.

Discussion of the results

The shown results give an interesting and original impulse to consider a possible effect of the viscoelastic nature of the polyethylene pipe in leak characterization. In fact, neither the simple Torricelli's Equation (2) nor the more general Eq. (1), is able to explain completely the variation of the leak discharge with the total head when polyethylene pipes are considered. Even Eq. (3), that represents an improvement to the Torricelli's equation when the increase in the leak area due to the elastic deformation of the pipe is considered, is not able to explain the shown results. While Eqs. (1), (2), and (3) define a single curve in the (H,Q_L) plane, different loading and unloading curve are obtained experimentally depending on the H time-history and rate of variation, giving rise to a hysteresis in the considered plane.

When single loading and unloading curves are fitted by Eqs. (1) or (3), they show different values of the parameters.

Sets of data from Series 1 show completely different results during the short and long period following the maneuver. During the short period, the data show high goodness of fitting with Eqs. (1) and (3), and values of $b > 1/2$. On the long period, keeping the valve at the same opening degree for several hours after a maneuver of a few seconds, data cannot be properly fitted by the considered equations.

It is worthwhile noting that in Eq. (1), variations of both coefficients, a and b and not only b, are relevant. As an example, according to the analysis of Thornton and Lambert (2005) on the effect of the variation of b on the leakage reduction, for a given b the variation of a from 3.48 to 4.71 10^{-4} in the shown test of Series 1 would produce a variation of Q_L of about 35%.

Considering that the shown data cannot be fitted by Eq. (2) since it does not take into account the variation of the effective area $C_L A_L$ with H, in Fig. 8 the estimated effective area $Q_L/\sqrt{2gH}$ is plotted for tests of Fig. 6. Assuming that Torricelli's equation holds, meaning that the mean velocity of the leak outflow is almost equal to $\sqrt{2gH}$, the variation of the effective area with H could be used as an attempt to explain the viscoelastic behavior of the leak, although further studies are needed. In a previous work (Ferrante et al. 2009) tests carried out on a smaller leak have shown an almost constant effective area. Since the "irregular" leak of this paper was machined on the same trunk by enlarging a smaller leak, the comparison with these data are relevant. The length of the smaller leak, of about 36 mm, likely hides the effective area variation and hence the viscoelastic effects that are evident in the data shown in this paper.

[Figure 8 about here.]

Conclusions

In this paper the effects of the pressure on the leak discharge in a high density polyethylene pipe are investigated. Based on the shown results, pipe material could play a crucial role in determining the relationship between leak discharge, Q_L, and total head inside the pipe, H. In fact, the relationship linking these quantities clearly shows a hysteresis in the (H,Q_L) plane, i.e. an area delimited by two non coincident loading and unloading curves.

The dependence of the leak characteristics on the total head time-history and rate variation can be relevant for modeling the leakage in water distribution systems. The steady-state sequence of the considered tests is typical of the extended period simulations used in pressurized pipe network models. The considered test durations, of about one day or more, also confirm that the time scale of the shown hysteretical phenomenon is of the same magnitude of the time scale of the functioning conditions changes in real systems. Tests carried out over three days with the typical variation in the functioning conditions of real systems (not shown) confirm this result. The measured strains seem to confirm that leak area deformation, more evident in a plastic pipe and for a longitudinal leak, can be responsible for differences in fitting parameters. The definition of a complete formula linking Q_L with all the parameters it depends on, for any kind of pipe material, is still an open issue.

Notation

a	=	leak coefficient;
A_L	=	area of the leak;
A_0	=	area of the leak corresponding to $h = 0$;
AV	=	air vessel;
b	=	leak exponent;
c	=	leak parameter;
C_L	=	discharge coefficient;
d	=	leak parameter;
DD	=	downstream electromagnetic flowmeter;
DP	=	downstream pressure transducer;
DV	=	downstream hand-operated ball valve;
g	=	gravitational acceleration;
H	=	total head;
h	=	pressure head;
L	=	leak;
m	=	leak parameter;
MV	=	automatically controlled butterfly valve;
p	=	pressure;
P	=	pump;
Q_L	=	leak discharge;
Q_U	=	upstream discharge;
R	=	reservoir;
Re	=	Reynolds number;
SGA2	=	radial strain-gage close to the leak;
SGT	=	radial strain-gage close to the downstream pressure transducer;
t	=	time;
t_M	=	manoeuvre duration;

UD	=	upstream electromagnetic flowmeter;
UP	=	upstream pressure transducer;
α	=	opening degree of the automatically controlled butterfly valve;
α_c	=	characteristic opening degree of the automatically controlled butterfly valve;
ε	=	radial strain;

ACKNOWLEDGMENTS: This research has been supported by the Italian Ministry of Education, University and Research (MIUR) under the Project of Relevant National Interest "Innovative criteria for the sustainable management of water resources in the water distribution systems". The help of Federico Cluni, Massimiliano Gioffre' and Vittorio Gusella for the strain gage measurements is acknowledged.

References

AWWA (2009). *Water Audits and Loss Control Programs*. Manual of Water Supply Practices. American Water Works Association.

Brunone, B. and Ferrante, M. (2001). "Detecting leaks in pressurised pipes by means of transients." *Journal of Hydraulic Research*, 39(5), 539–547.

Brunone, B. and Ferrante, M. (2004). "Closure on "detecting leaks in pressurised pipes by means of transients", by b. brunone and m. ferrante." *Journal of Hydraulic Research*, 42(1), 105–109.

Cassa, A. M., Zyl, J. V., and Laubscher, R. (2010). "A numerical investigation into the effect of pressure on holes and cracks in water supply pipes." *Urban Water Journal*.

Covas, D., Ramos, H., and Almeida, A. D. (2004). "Detecting leaks in pressurised pipes by means of transients - discussion." *Journal of Hydraulic Research*, 42(1), 105–108.

Covas, D., Stoianov, I., Mano, J., Ramos, H., Graham, N., and Maksimovic, C. (2004). "The dynamic effect of pipe-wall viscoelasticity in hydraulic transients. part i - experimental analysis and creep characterization." *Journal of Hydraulic Research*, 42(5), 516–530.

Ferrante, M., Brunone, B., and Meniconi, S. (2009). "Leak detection in branched pipe systems coupling wavelet analysis and a lagrangian model." *J Water Supply Res T*, 58(2), 95–106.

Ferrante, M., Massari, C., Cluni, F., Brunone, B., and Meniconi, S. (2009). "Leak discharge and strains in a polyethylene pipe." *Integrating Water Systems, Proc. of the 10^{th} Int. Conf. Computing and Control for the Water Industry, CCWI2009*, 203–208.

Greyvenstein, B. and van Zyl, J. (2007). "An experimental investigation into the pressure - leakage relationship of some failed water pipes." *J Water Supply Res T*, 56(2), 117–124.

May, J. (1994). "Pressure dependent leakage." *Worl water environmental engineering*, (October).

Osterwalder, J. and Wirth, C. (1985). "Experimental investigations of discharge behaviour of crack-like fractures in pipes." *Journal of Hydraulic Research*, 23(3), 255–272.

Pezzinga, G. (2002). "Unsteady flow in hydraulic networks with polymeric additional pipe." *Journal of Hydraulic Engineering*, 128(2), 238–244.

Thornton, J. and Lambert, A. (2005). "Progress in practical prediction of pressure: leakage, pressure: burst frequency and pressure: ...." *Proceedings of IWA Special Conference 'Leakage2005'*.

van Zyl, J. and Clayton, C. (2007). "The effect of pressure on leakage in water distribution systems." *Proceedings of the Institution of Civil Engineers: Water Management*, 160(2), 109–114.

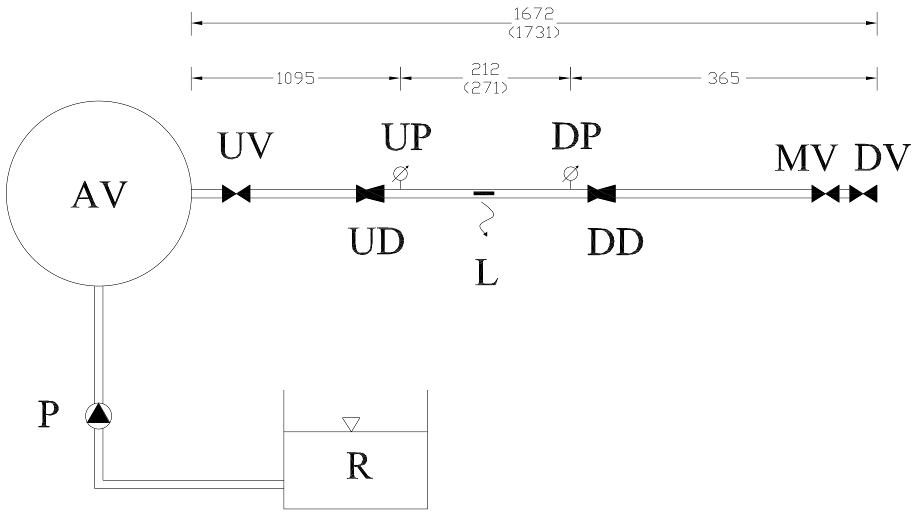

Figure 1: Layout of the experimental set-up. R=recycling reservoir; P=pump; AV=air vessel; UV=upstream valve; UD (DD)=upstream (downstream) flowmeter; UP(DP)=upstream (downstream) pressure transducer; L=leak; MV=butterfly valve; EV=end valve. Measures are in millimeters. Measures in brackets refer to the system with the "regular" leak (Series 2).

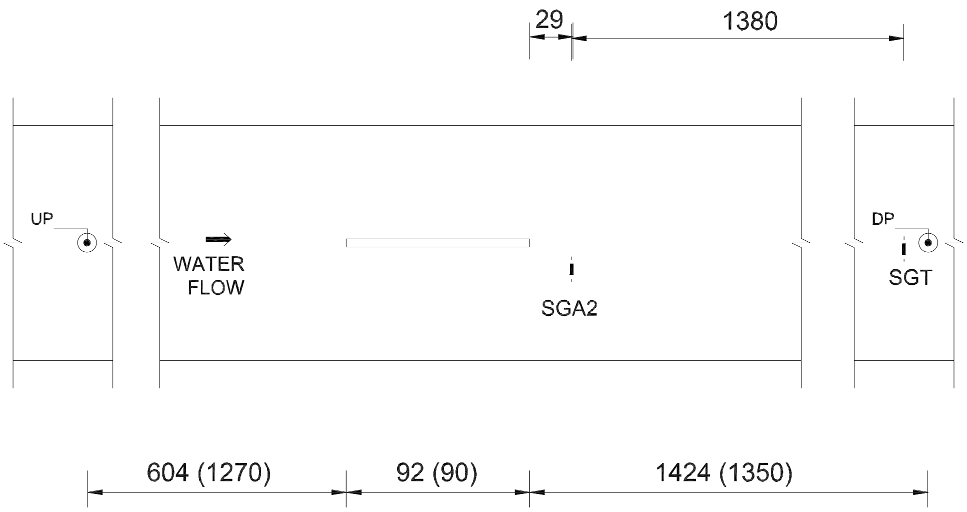

Figure 2: Location of the strain gages close to the leak (SGA2) and to the DP pressure transducer (SGT). Distances are in millimeters. Measures in brackets refer to the system with the “regular” leak (Series 2).

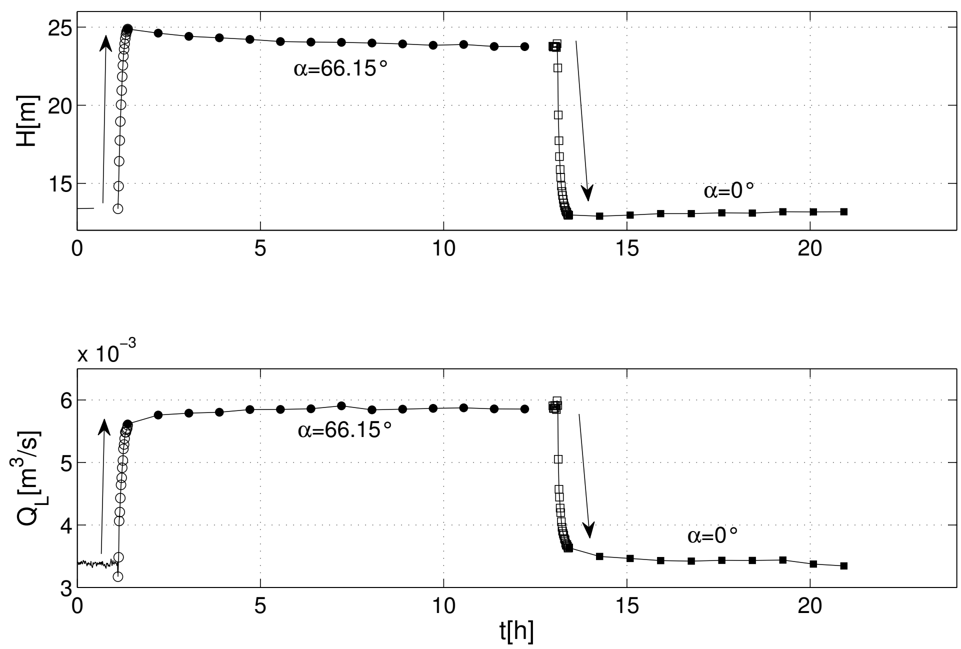

Figure 3: Variation of H and Q_L with time t for a test of Series 1 ($\alpha_c = 66.15^o$)

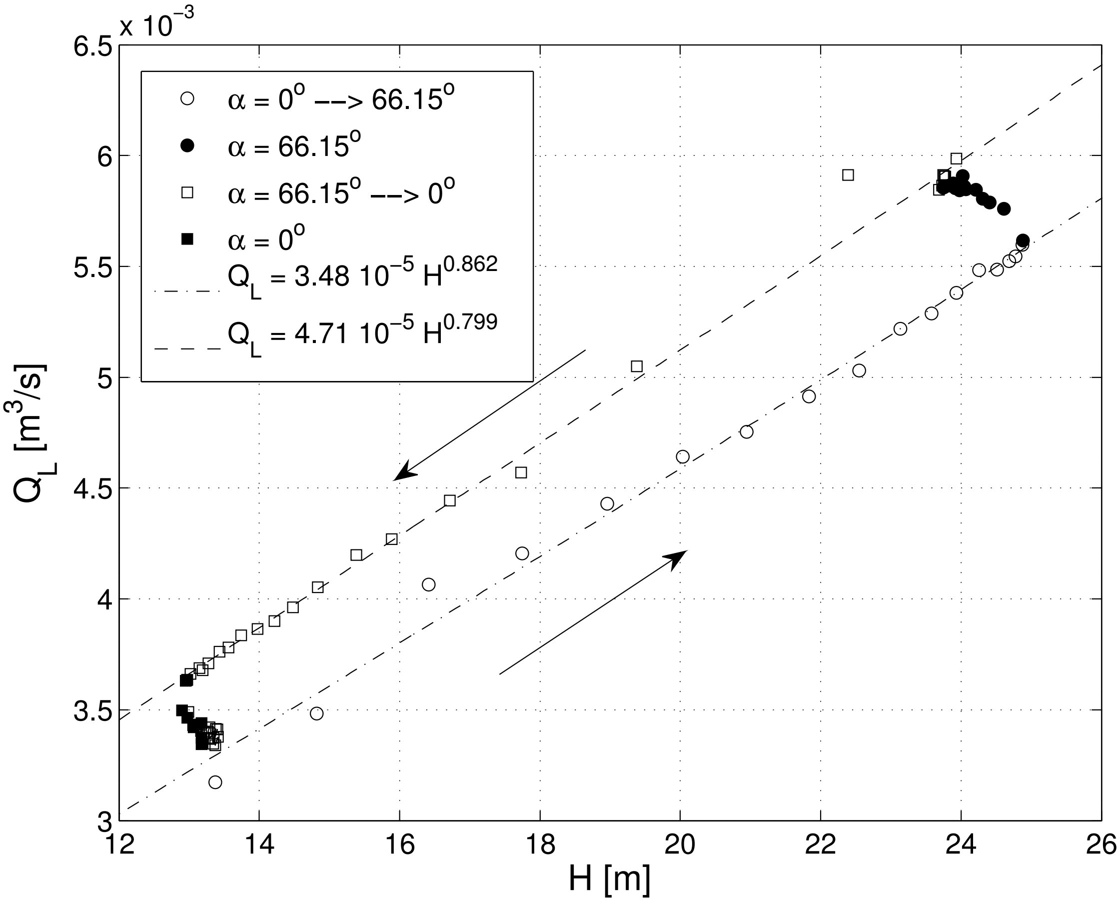

Figure 4: Variation of Q_L with H for the same test of Fig. 3 ($\alpha_c = 66.15^o$)

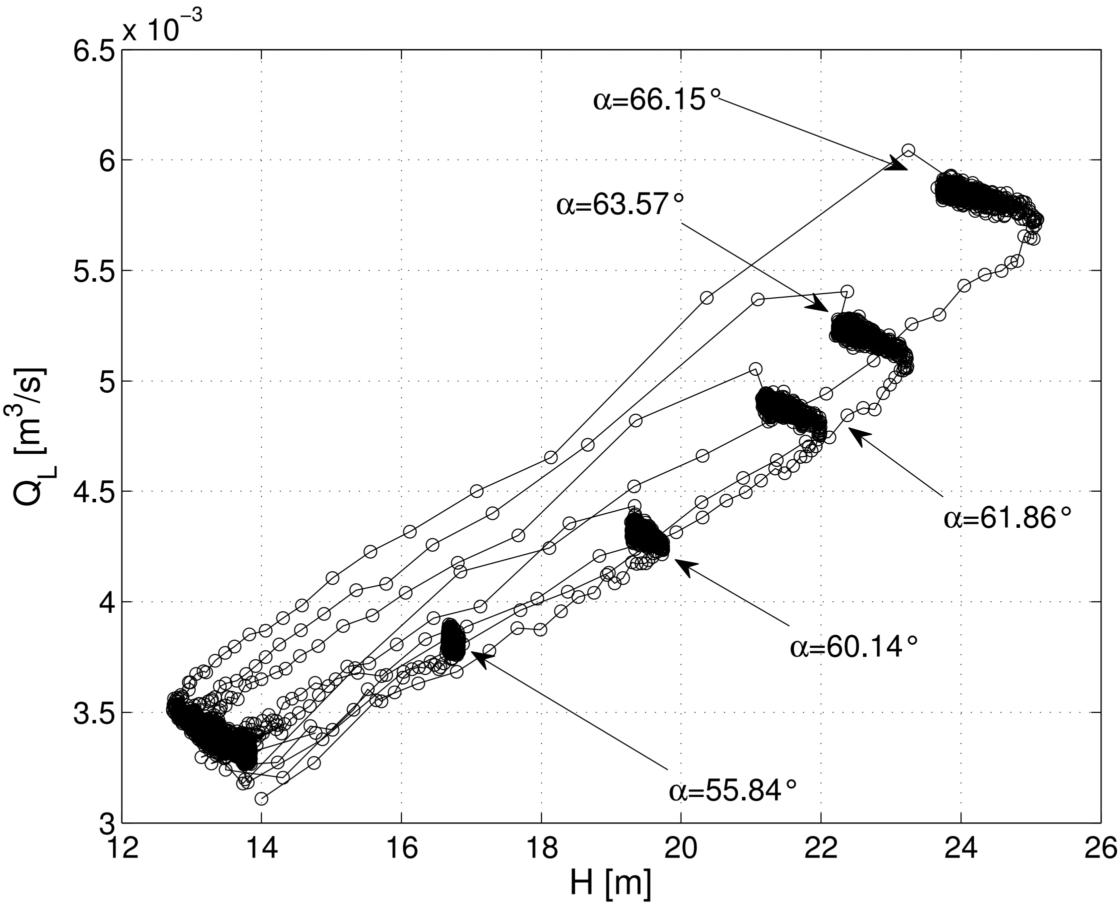

Figure 5: Variation of Q_L with H for the tests of Series 1 (hollow circles, α_c =55.84, 60.14, 61.86, 63.57, and 66.15^o).

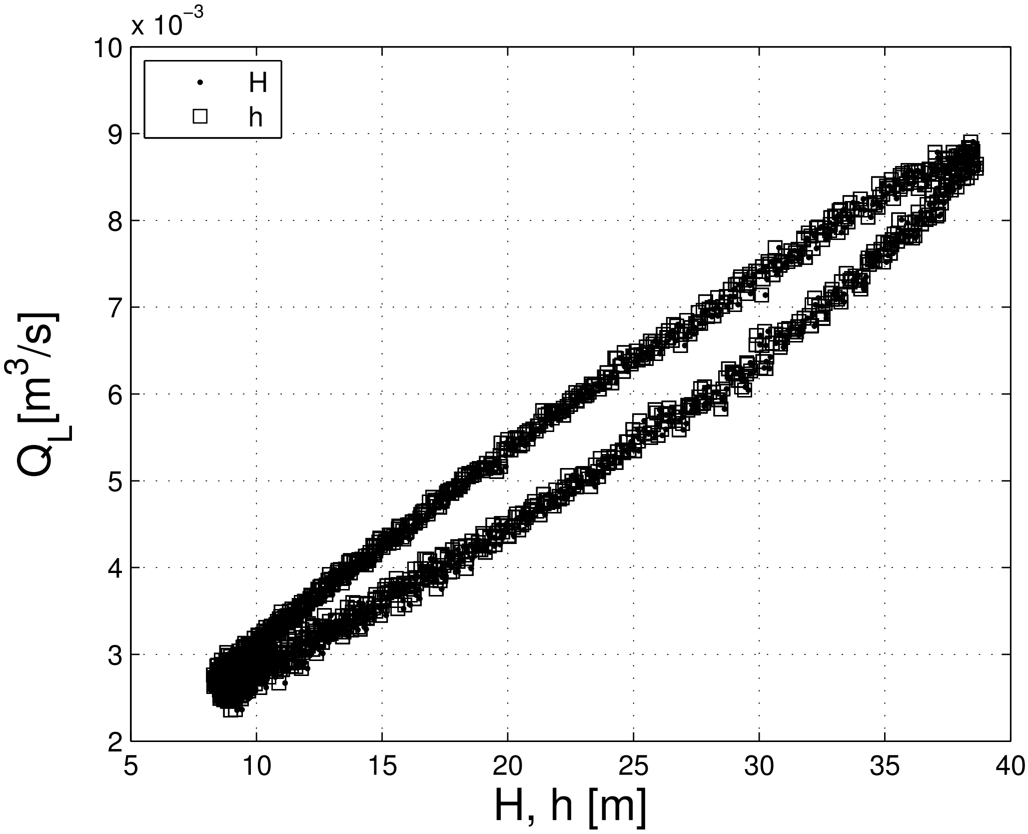

Figure 6: Variation of Q_L with H and h for of Series 2

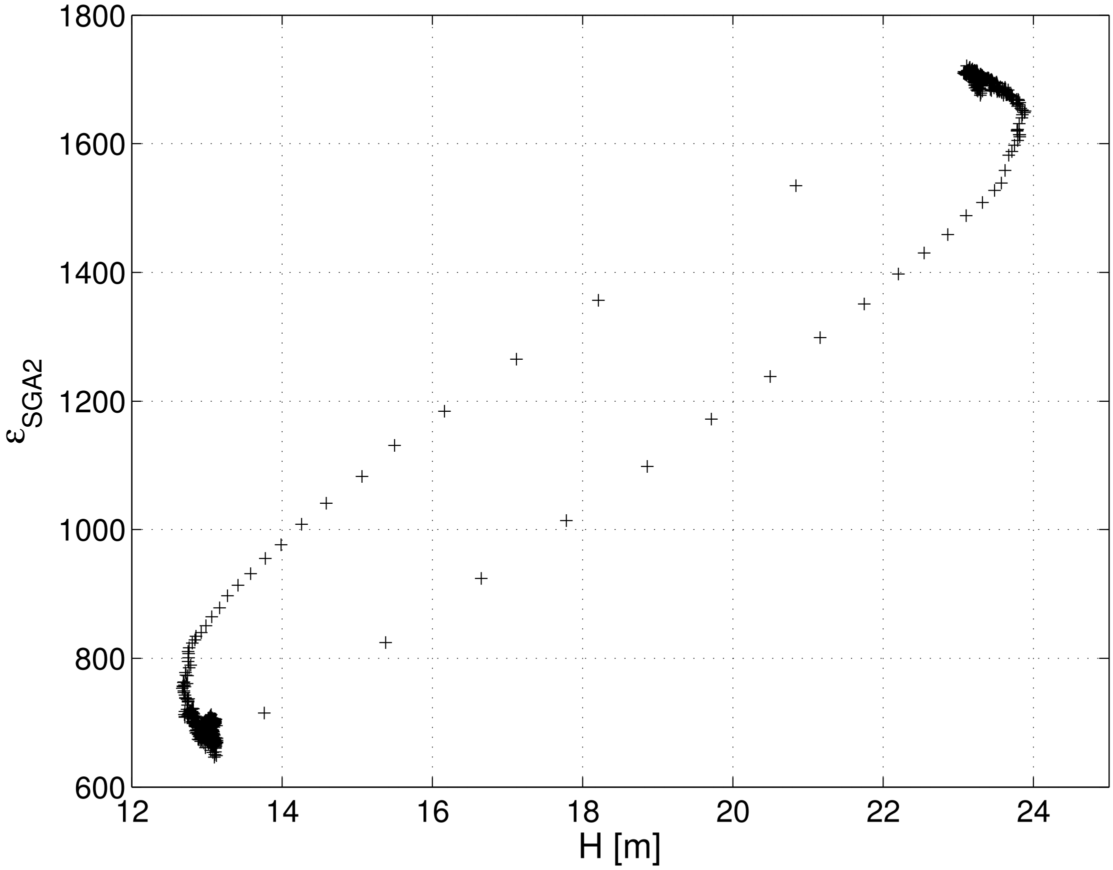

Figure 7: Variation of the radial strain at SGA2 with H for data of Fig. 3

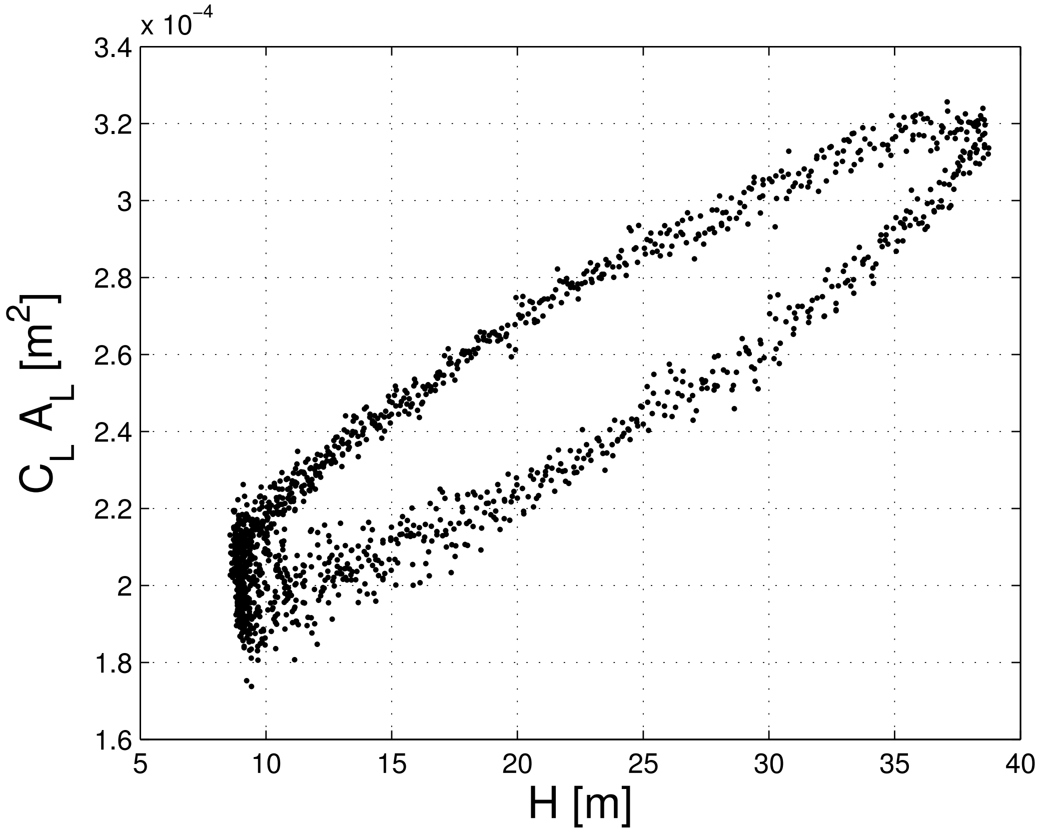

Figure 8: Variation of $C_L A_L$ for data of Fig. 6