Minimum Energy and Minimum Time Control of Electrical Drive Systems

CORNELIU BOTAN, VASILE HORGA, FLORIN OSTAFI, MARCEL RATOI
Department of Automatic Control
Technical University of Iasi
Blvb. D. Mangeron 53A, 700050 Iasi
ROMANIA
e-mail: cbotan@ac.tuiasi.ro, horga@tuiasi.ro, fostafi@ac.tuiasi.ro, ratoi@tuiasi.ro

Abstract: - A comparison between time optimal control and minimum energy control for electrical drives is performed. The drives with DC, PMSM or induction motors are considered. A simple strategy using the currents as control variables is proposed. A unified theoretical approach for optimal control of electrical drives with different motor types is indicated. Simulation and experimental results are presented.

Key-Words: - Electrical drives, Optimal control, Energy, Time, AC, DC motors

1 Introduction

The paper presents certain advantageous control strategies for the transient state of the electrical drives. There are different requirements for the transient period and especially for the start of the drives. The main demands refer to the productivity and to the energy consumption. If one of these aspects has a dominant importance for the technological process, the optimal control from the corresponding point of view can be adopted. If the interest is for a great productivity, the minimum time control of the electrical drive can be introduced, taking also into account that this control can be easy achieved. The optimal control from the energetic point of view is important because in the transient period the energy losses (especially the Joule losses) are very high.

Although the optimal control [1], [2] of the electrical drives represents an important way for energy saving, the number of applications is nowadays very small. We appreciate that a cause of the reluctance in this direction is the complexity of the algorithm, but certain easy implementable procedures were proposed in the last years [3] ,[4], [5] and they show that the optimal control of the electrical drives is not only necessary, but it is possible in the context of the modern control technologies. Moreover, the optimization is appreciated as a main direction of the developing of the drive systems in the future [6].

There are numerous studies dedicated to the optimal control of the electrical drive systems, for different types of motors, criteria, or used methods. For instance, we mention [7],..., [19], but many other papers can be indicated.

The minimum energy control must be performed so that an acceptable behaviour in the transient state to be obtained. Some differences in the adopted criteria can occur for different applications. For instance, the minimization of the global losses is usually a goal for the steady state operation and the establishing of an adequate flux level is the most approached technique in this direction [10],[16],[19]. Although a similar criterion can be also adopted for the transient period, we have to take into account that the Joule losses significantly overcame all other losses, since the start currents have very great values. Therefore, only a criterion based on Joule losses for the transient period (eventually accompanied by a minimum error condition [4], [8], [9], [13], [17]) can be adopted. This is in fact the main direction of the present study.

Other demands are introduced in different papers as specific criteria or in combination with the energetic aspects. For instance, the increase or obtaining of a convenient form of the electromagnetic torque [10], [11], the decrease of the harmonics content [12], the obtaining of a good robustness [12], the consideration of a some especial operating conditions [14] etc. Of course, such demands or others can be introduced in the design of electrical drives for different applications, but the main considerations remain in the most cases the energetic and productivity considerations and the behaviour in the transient state and these aspects are taken into account in the sequel.

It is known that in many cases the demands of energy and time conditions are contradictory [1]. Therefore, it is useful in certain applications to consider a combined energy-time criterion. Some particular cases of such problems for linear systems are presented in [1] and a general study for an extended linear quadratic problem is performed in [20].

It should be also noticed that the optimal control is useful not only for energy saving, but in many cases offers the possibility to reduce the motor rated power and therefore the weight and volume. Indeed, the motor rated power is chosen from heat consideration and the optimal control leads just to a diminished Joule losses.

Unfortunately, the design of electrical drives rarely take into account these aspects and does not offer users the possibility of choice of an adequate operating mode, in concordance with the needs of the technological process and with the economical advantages.

The main approach of the paper is to consider the currents (and not voltages) as control variables. This fact allows a significant simplification of the algorithm and of the implementation by comparison with all above mentioned papers (the currents are also considered as control variable in [11], but the problem is formulated in other terms – to ensure a desired form for electromagnetic torque, being imposed the terminal values of the flux) and it is justified because the motor currents are usually controlled in the drives control structures (one uses current control loops or current source inverter). The optimal control obtained by this way is an *ideal* one, since the currents cannot have instantaneous variations, because of the inductances influences, and therefore, the solution is suboptimal. However, it is possible to implement a current control loop imposing as reference the ideal optimal value. This approach leads to a simple structure of the optimal system, comparable with a usual cascade one.

The mentioned simplification allows performing the study for a general case, for different motor types, while other works are dedicated only to a single motor type, for instance [4], [7], [8], [9] to the drive system with DC motor, [10], [11], [13] for induction motors, [16],..,[19], for permanent synchronous motors.

2 Problems Formulation

The mechanical equilibrium equation for a drive system is

$$\dot{\Omega}(t) = \frac{1}{J}[M_e(I_1(t), I_2(t)) - M(t)] \quad (1)$$

where Ω is the angular speed, J is the inertia, M_e is the electromagnetic torque (depending on the stator and rotor currents I_1 and I_2 or on certain components of these currents) and M is the load torque (we suppose that its dependence on Ω can be neglected).

Since I_1 and I_2 are control variables, no other equation is necessary for the drive system model, which is a simple first order system. The dependence of the electromagnetic torque on currents has specific forms for different motor types. In all cases, we suppose that the iron is non-saturated and therefore, the flux is proportional with the corresponding current. Also, the transient electromagnetic process will be neglected; in this situation, the small delay of the flux is neglected, but the main conclusions remain valid.

- *For DC motors,*

$$M_e = cI_1 I_2, \quad (2)$$

where I_1 and I_2 are the rotor and stator currents, respectively, and *c* is a constant of the machine (in all below equations, *c* has a similar significance).

- *For permanent magnet synchronous machines (PMSM),*

$$M_e = cI_1, \quad (3)$$

where $I_1 = I_{1q}$ is the stator active current in a d-q frame (usually, $I_{1d} = 0$)

A similar formula is used for *brushless DC* motors.

- *For induction motors*

$$M_e = c(I_{1q} I_{2d} - I_{1d} I_{2q}), \quad (4)$$

where I_1 and I_2 correspond to the stator and rotor currents and the indices *q* and *d* denote the active and reactive components in the *d*-*q* frame. The choice of the currents depends on the used control technique. We shall consider in the sequel that the indirect rotor flux orientation is applied and in this case $I_{2d}=0$. Since the transient electromagnetic process is neglected, $I_{2q} = -\beta I_{1q}, \beta = L_m / L_r$ (L_m - magnetizing inductance, L_r - rotor self inductance) and thus the electromagnetic torque is

$$M_e = c\beta I_{1q} I_{1d}. \quad (5)$$

For *synchronous motors with excitation winding*, the electromagnetic torque can be expressed as a sum of terms with the above forms and therefore, the conclusions regarding the optimal control can be extended to this case [5], but this application will not be presented in the paper.

Summarising, the electromagnetic torque has the forms (2) or (3) for different motor types. We shall adopt in the sequel for I_1 the significance of the current creating the torque and for I_2 the current creating the flux. Therefore, the control can be performed using one control variable (for PMSM, brushless DC motors and DC motors with constant excitation flux), or two control variables (for induction motors and for DC motors with variable excitation current).

In the sequel we shall use the normalized (relative) values of the variables. In this respect, we introduce the normalized variables

$$i_1 = I_1 / I_{1N},\ i_2 = I_2 / I_{2N},\ m = M / M_N, \quad \omega = \Omega / \Omega_N,\ \tau = t / T_N,\ m_e = M_e / M_N \tag{6}$$

where the subscript N refers to the rated corresponding values, and

$$T_N = J\Omega_N / M_N \tag{7}$$

is a nominal time of the system. We shall use however in the sequel the notation t for the normalized time (instead τ).

In the steady state with rated values,

$$cI_N = M_N . \tag{8}$$

Taking into account (6), (7) and (8), the equation (1) becomes

$$\dot{\omega}(t) = i_1(t)i_2(t) - m(t) \tag{9}$$

(Of course, the term m corresponds to the load torque and the term $m_e = i_1 i_2$ to the electromagnetic torque.)

The solution to the equation (9) is

$$\omega(t) - \omega(0) = \int_0^t m_e(\theta)d\theta - \int_0^t m(\theta)d\theta. \tag{10}$$

If the final desired value ω_d of the speed is imposed, and if we consider $\omega(0) = 0$, the equation (10) leads to

$$\omega_d = \omega(T) = \int_0^T i(\theta)d\theta - \int_0^T m(\theta)d\theta . \tag{11}$$

Remark 1: In many control problems, the adopted value for current depends on the load torque $m(t)$.

The equation (11) indicates that in certain cases it is necessary to know especially the mean value

$$m_m = \frac{1}{T}\int_0^T m(t)dt \tag{12}$$

on the interval $[0,T]$. This fact implies to beforehand know at least the shape of $m(t), t \in [0, T]$ and to measure or estimate the magnitude of the load torque at the beginning of the control process. This aspect occurs in the optimal energy control problem (independent of the used method), but not in the minimum time problem.

In many applications, the load torque can be considered constant on the short interval of the start, and in this case, of course $m_m = m$. For simplicity, in the sequel, we shall consider $m(t) = \text{const}$. If this condition is not satisfied, the above remark referring to the mean value has to be considered and m will be replaced with m_m.

From the energetic point of view, the interest is to minimize the Joule losses on the interval $[0, t_f]$, tacking into account that these ones are significantly greater than other losses in the transient period. For this reason, the criterion is

$$\bar{I} = \int_0^{t_f} r_1 I^2(t)dt \tag{13}$$

if only a variable current is used, or

$$\bar{I} = \int_0^{t_f} [r_1 I^2(t) + r_2 I'^2(t)]dt \tag{14}$$

for two variable currents. Of course, if I'=const., the second term in (14) can be neglected, since it does not influence the minimum conditions and the form (13) is obtained.

The normalized energy loss is introduced dividing (13) through nominal winding losses $W_N = r_1 I_N^2 T_N$. Using the normalized current value, the performance index from the energetic point of view becomes

$$I_E = \int_0^T i^2(t)dt,\ T = t_f / T_N \tag{15}$$

If one performs a similar normalization for the two variables case, the criterion is

$$I_E = \int_0^T [i_1^2(t) + \gamma^2 i_2^2(t)]dt \tag{16}$$

where γ is a constant parameter. Details referring this index are presented in the Section 5.

One can now formulate the minimum energy problem:

P1: Find i(t) and ω(t) which minimize the criterion (15) (or $i_1(t)$, $i_2(t)$ and ω(t) associated with (16)) and satisfy the equation (9) and the terminal conditions $\omega(0)=0$, $\omega(T)= \omega_d$.

On the other hand, from the productivity point of view, the interest is to minimize the transfer time T, or the index

$$I_T = T = \int_0^T dt . \tag{17}$$

(In all equations, the subscript T is adopted for minimum time problem and the subscript E refers to the optimal energy consumption problem).

It is obviously know [1] that certain restrictions have to be introduced in minimum time problem, that is

$$|i(t)| \le i_M , \quad (18)$$

where i_M is a maximum acceptable value of the current. The minimum time problem is

P2: Find the control variable $i(t)$ (or $i_1(t)$ and $i_2(t)$) and the state variable $\omega(t)$ so that the criterion (17) to be minimized, subject to the constraints (9) and (18).

3 Optimal control using one control variable

As it was mentioned in the Section 2, in this case the current I_2 does not appears in the formula of the electromagnetic torque and therefore the system equation (9) can be written in the form $\dot{\omega} = i - m$.

We shall firstly present the solution to the **P2** problem, which can easy be find from the Pontryagin minimum principle [1] and it is

$$i_T^*(\tau) = i_M , \quad \tau \in [0, T] . \quad (19)$$

The minimum time is

$$T = \tau_T^* = \frac{\omega_d}{i_M - m} \quad (20)$$

and the normalized dissipated energy is

$$w_T^* = \frac{\omega_d}{i_M - m} i_M^2 . \quad (21)$$

In (21), $w^* = W / W_N$, where W is the Joule losses on the interval $[0,T]$ and W_N was defined above. The control variable does not depend on the load torque (it is always i_M), but the transfer time τ_T^* depends on m.

The solution for the **P1** problem can be established starting from the Hamitonian [1] $H = i^2 + \lambda(i - m)$; using Hamilton conditions, yields

$$\lambda(\tau) = \lambda = const. \text{ and } \quad i(\tau) = i = const. \quad (22)$$

In this case, the index (15) becomes $I_E = i^2 T = i^2 \omega_d/(i-m)$, where $T = \omega_d/(i-m)$ was replaced from (11). Further, the condition $\partial I_E / \partial i = 0$ leads to

$$i_E^* = 2m , \quad (23)$$

the minimum Joule losses

$$w_E^* = 4m\omega_d \quad (24)$$

and the corresponding transfer time

$$\tau_E^* = \omega_d / m . \quad (25)$$

Remark 2: We conclude from (23) that the optimal current must ensure an electromagnetic torque being twice the mean value of the load torque. This is a general feature of the electrical machine, valid for any motor type, using one control variable; with a good approximation, (23) is also valid for the two control variable case.

Remark 3: If the load torque has a small value, the transfer time (25) increases very much. This time must be limited in many applications to a maximum value τ_M and in this case $i_E = m + \frac{\omega_d}{\tau_M}$ and variation of the current is

$$\Delta i_E = i_E - i_E^* = \frac{\omega_d}{\tau_M} - m > 0 , \text{ since } \tau_M < \tau_E^* .$$

Corresponding, the copper losses increase with

$$\Delta w_E = w_E - w_E^* = \tau_M i_E^2 - \tau_E^* i_e^{*2} = \\ = \tau_M \left(m - \frac{\omega_d}{\tau_M} \right) = \tau_M \Delta i_E^2 . \quad (26)$$

The comparison between **P1** and **P2** problems can be performed referring to the time transfer and the energy losses in the two cases.

Firstly, it is easy to remark that the solutions for both problems coincide if it is chosen $i_M = 2$.

The differences between the two cases depend on the ratio $\mu = m / i_M$. One obtains from (20) and (25)

$$\frac{\tau_E^*}{\tau_T^*} = \frac{i_M - m}{m} = \frac{1}{\mu} - 1 , \quad (27)$$

with $\mu < 1$, since the motor cannot start if $i < m$.

This variation is presented in Fig. 1.

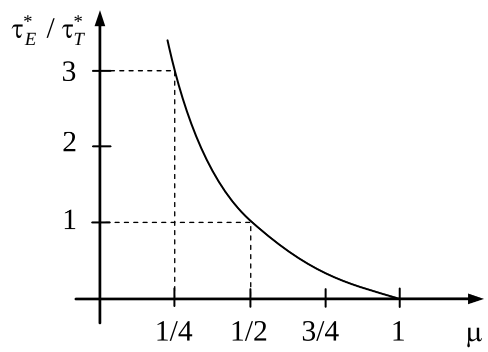

Fig. 1

Similarly, from (21) and (24), yields

$$\frac{w_E^*}{w_T^*} = \frac{4m(i_M - m)}{i_M^2} = 4\mu(1 - \mu)$$

and this variation is indicated in Fig. 2.

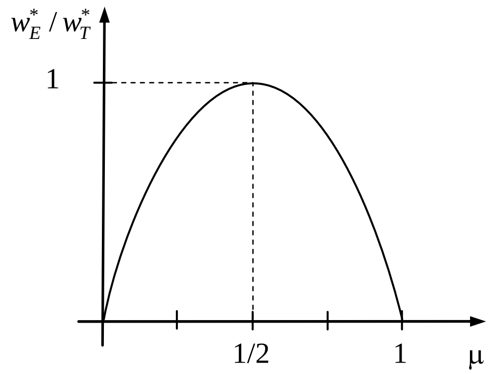

Fig. 2

The Fig. 1 and 2 show again the coincidence of the solutions for **P1** and **P2** problems if $i_M=2m$. From energetic point of view, any other choice of i_M is unfavourable. The choice $1/2 < \mu < 1$ (or $m < i_M < 2m$) is not recommended from both point of view. If the productivity considerations have priority, one can adopt $i_M > 2m$ $(\mu < 1/2)$, but the energy losses increase in this case.

4 Optimal control with combined criterion

Taking into account the contradictions between time and energy demands, it is not lack of interest to consider an optimal control with combined criterion and to formulate the following problem:

P3: Find $i(\tau)$ and $\omega(t)$ which minimize

$$I_c = \int_0^T \left[\alpha + i^2(\tau)\right] d\tau \,, \tag{28}$$

subject to (9) and $\omega(0)=0,\ \omega(T)=\omega_d$.

The coefficient α represents in (28) the weight for minimum time control problem: for $\alpha = 0$, one obtains the **P1** problem and for α very great, the problem becomes the minimum time one (a restriction for *i* have to be introduced in this case).

The Hamiltonian of the problem is $H = \alpha + i^2(\tau) + \lambda\left[i(\tau) - m(\tau)\right]$. Using a similar way as in Section 3, it results an optimal constant current and from the condition $\partial I / \partial i = 0$, one obtain the optimal control current

$$i_c^* = m + \sqrt{m^2 + \alpha} \tag{29}$$

and then, the optimal transfer time

$$\tau_c^* = \omega_d / \sqrt{m^2 + \alpha} \tag{30}$$

and minimal losses

$$w_c^* = i_c^{*2} \tau_c^* = \omega_d \left(m + \sqrt{m^2 + \alpha}\right). \tag{31}$$

These energy losses are greater then in the **P1** case (16):

$$w_c^* - w_E^* = \omega_d \left(m - \sqrt{m^2 + \alpha}\right)^2 \Big/ \sqrt{m^2 + \alpha}$$

and $w_c^* = w_E^*$ if $\alpha = 0$.

Also, from (25) and (30), it results

$$\tau_E^* / \tau_c^* = \sqrt{m^2 + \alpha} \Big/ m > 1 .$$

Of course, for **P3** problem, the transfer time is improved, but the energy losses increase by comparison with **P1** case. Similarly, the transfer time increases and energy loss decreases in comparison with **P2** problem.

5 Optimal control using two control variables

As it was exemplified in the Section 2, this problem arises in the cases of drive systems with separately excited DC motors or with induction motors.

There are no differences for the **P2** problem, because the currents must be maintained at their maximal values as in the Section 3.

For the *unconstrained* **P1** problem, we shall firstly establish in this Section the expression of the performance index for normalized variables.

For the both cases, the total Joule losses are expressed by (14), where the two terms express the stator and rotor losses.

For DC motor, we denote with $I_1=I$ the rotor current and with $I_2=I'$ the stator one and we refer the losses to the nominal rotor losses $W_{1N} = r_1 I_{1N}^2 T_N$. In this case, the performance index can be written in the form (16), with

$$\gamma^2 = \frac{r_2}{r_1} \frac{I_{2N}^2}{I_{1N}^2} \tag{32}$$

For an induction motor, we denote in (14) with $I_1=I$ and $I_2=I'$ the stator and rotor currents, respectively and then we express (14) in the form

$$I = \int_0^{t_f} \{r_1[I_{1q}^2(t) + I_{1d}^2(t)] + r_2[I_{2q}^2(t) + I_{2d}^2(t)]\} dt$$

where the *d-q* components of the currents were introduced. Taking into account the conditions $I_{2d}=0$ and $I_{2q} = -\beta I_{1q}, \beta = L_m / L_r$, introduced in the Section 2, one can write

$$I = \int_0^{t_f} [(r_1 + r_2\beta) I_{1q}^2(t) + r_1 I_{1d}^2(t)] dt \tag{33}$$

The normalization of the energy is defined dividing (33) to the nominal losses caused by the active stator current $W_{1qN} = r_1 I_{1qN}^2 T_N$ and the same form (16) for the performance index is obtained, with

$$i_1 = I_{1q} / I_{1qN},\ i_2 = I_{1d} / I_{1dN},\ \gamma = \mu / \rho, \\ \mu = I_{1dN} / I_{1qN},\ \rho^2 = (r_1 + r_2\beta^2) / r_1 \quad (34)$$

The presented transformations lead to a multiplying factor ρ^2 in the performance index, which is not considered in (16), because its presence does not affect the minimum conditions.

Therefore, for the both cases (DC and induction motor) we refer to an optimal control from energetic point of view with the same equation (9) of the system and the same criterion (16) and these cases can be simultaneously discussed.

The Hamiltonian of the problem is

$$H = i_1^2(t) + \gamma^2 i_2^2(t) + \lambda(t)[i_1(t) i_2(t) - m(t)] \quad (35)$$

and the necessary optimality conditions $\partial H / \partial i_1 = 0$, $\partial H / \partial i_2 = 0$, $\partial H / \partial \omega = -\dot{\lambda}(t)$ lead to

$$\lambda(t) = \ const.\ \text{and}\ i_1(t) = \pm\gamma i_2(t) \quad (36)$$

Only the sign plus will be considered in the sequel, corresponding to the motor operating mode of the machine. The equation of the drive system is in this condition

$$\gamma i_2^2(t) = \dot{\omega}(t) + m(t)\ \text{ or }\ i_1^2(t) = \gamma[\dot{\omega}(t) + m(t)] \quad (37)$$

and the solution to this system, for ω(0)=0 and ω(*T*)= ω_d, is

$$\gamma \int_0^T i_2^2(t) dt = \omega_d + mT \quad (38)$$

We remember that *m* is the mean value of the normalized torque on the interval [*0,T*].

The minimum value of the criterion is

$$I^* = 2\int_0^T i_1^2(t) dt = 2\gamma^2 \int_0^T i_2^2(t) dt \quad (39)$$

and, using (38), it can be expressed as

$$I^* = 2(\omega_d + mT) \quad (40)$$

This relation shows that the minimum energy losses have two components: one corresponding to the acceleration torque and one corresponding to the load torque. For given ω_d and *m*, the both components are constant if the final time *T* is imposed. If *T* is free, the Joule losses can be diminished if *T* decreases. Of course, the decrease of *T* implies to increase the charge of the motor and the currents will have great values. Therefore an inferior limit has to be imposed ($T \geq T_{\lim}$). These aspects will be discussed below.

The variation form for currents is not imposed by necessary condition (14); only a constant ratio between the currents must be ensured. But it results from (21) that *T* must have the smallest possible value and this condition can be achieved if the currents have the maximal values; in other words, the currents must be constant and in this case

$$I = T(i_1^2 + \gamma i_2^2) \quad (41)$$

and the minimum value is

$$I^* = 2T i_1^2 = 2T\gamma^2 i_2^2 \quad (42)$$

The transfer time results from (11)

$$T = \frac{\omega_d}{i_1 i_2 - m}\,. \quad (43)$$

Replacing this value in (42), one deduce from the condition $\partial I^* / \partial i_1 = 0$

$$i_1^* i_2^* = 2m \quad (44),$$

that is a similar condition as for one control variable: the optimal electromagnetic torque must have a double value as the mean load torque. The corresponding optimal values for optimal currents are

$$i_1^* = \sqrt{2m\gamma}\ \text{ and }\ i_2^* = \sqrt{2m / \gamma} \quad (45)$$

The above obtained optimal control can be used only when the load torque has small values or the final time is not too big, otherwise the problems become *constrained*. Two main types of restrictions can be introduced: referring to the currents and to the time, respectively.

Current restrictions must be introduced for the component i_2 (in order to avoid the saturation of the machine) and for the component i_1 (depending especially on the inverter capabilities). Usually, the restriction for i_2 firstly becomes active and only this case is presented below ($|i_2| \leq i_{2M}$); the case when the both restrictions are active is usually avoided, since it corresponds to an overloaded machine.

If i_2 is fixed, it results from the optimality condition that the i_1 component must be constant. In this case, the performance criterion has the form (41), but the relation (36) cannot be applied and the condition $\partial I / \partial i_1 = 0$ leads to

$$i_1^2 i_2 - 2 i_1 m - \gamma i_2^3 = 0 \quad (46)$$

The electromagnetic torque is in this case

$$i_1 i_2 = 2m + \gamma \frac{i_2^3}{i_1} \tag{47}$$

The last term can be usually neglected for load torque greater than 1/2 and then (44) is satisfied with a good approximation.

The positive solution to the equation (46) is

$$i_1 = (m + \sqrt{m^2 + \gamma i_2^4}) / i_2 \tag{48}$$

The limit value m_l for m when the maximal allowed value for i_2 is reached results from the second expression (45).

For $m \geq m_l$, the component i_2 has to be restricted and the relations (47) and (48) hold (with $i_2 = i_{2M}$).

If $m < m_l$, the established values (45) for currents in the unconstrained case can be used. But the simulation and experimental results show that there are small differences in this case by comparison with the situation when the i_2 component is maintained at its maximal value i_{2M} and the i_1 component is obtained from (36). The explanations is based on the fact that the electromagnetic torque has a greater value in the last variant and therefore, the transient time T decreases (see, for instance (40)), that is, the increase of currents is compensated by the decrease of the transfer time.

Moreover, if one adopts $i_2 = i_{2M}$, the i_1 component can be chosen from (48) in all situations, because the condition (36) is verified with a good approximation even for small values of m (unconstrained case). This alternative has the advantage of an easier implementation, because the control law algorithm must not be changed depending on the value of m.

Time restrictions: one can remark from (40) that the decrease of transition time T leads to a smaller energy losses. But the decrease of the time implies the increase of currents. On the other hand, if the currents are adopted as it is indicated above, the time transfer can have a very large value, not satisfactory from technological point of view, in order to obtain a greater productivity. In such situations a superior limit T_s for T can be imposed, but it is preferable to specify the superior and inferior limits for acceleration $\varepsilon = \omega_d / T$. Of course, the decrease of T leads to the increase of the energy losses, by comparison with the optimal control.

6 System implementation and experimental results

As it was mentioned in the Section 1, the above established control is an *ideal optimal control*. The fact that the ideal optimal values of the currents are known offers a possibility for an easy implementation: the controller must ensure the desired value of the currents with a great accuracy and a better accuracy ensures a better proximity to the ideal optimal control. Of course, a task for load torque estimation and for computing of the desired current values has to be introduced in the minimum energy problem. This task is not necessary in the minimum time problem. Our attention will be focused in the sequel on the proper optimal controller, described above. There are different ways in this direction and we shall present only a structure based on a cascade control with PI controllers. This specific part (the internal current loop of the cascade) is presented in Fig.3: the output i^0 of the speed controller is transmitted to the input of the current controller via a saturation bloc *S*. The level of the saturation is variable and corresponds to the desired value of the current, depending on the estimated value of the torque *m*. The control loop contains the controller *C* and the controlled plant *P*. If it is necessary, a similar structure is introduced for the current i_2. The level of saturation of the block *S* is i_M for minimum time problem (in fact, in this case a usual cascade structure is adopted) or it is established in dependence of the load torque *m*, in the minimum energy problem.

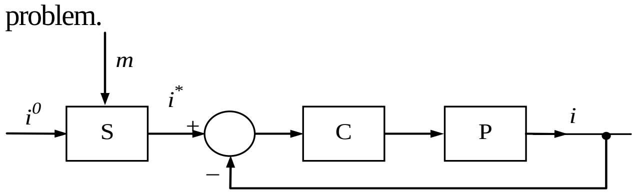

Fig. 3

Since the block *S* is saturated in the most part of the transient state, the (energy or time) optimal control is ensured in all this period. Only in the final part of the transient state the block *S* and all system becomes linear and it is ensured a smooth achieving of the desired value ω_d of the speed.

The next figures present a selection of simulation and experimental tests, performed for the proposed structure, for DC, PMSM or induction motors and for different operation conditions. We have supposed in all cases that the load torque is known.

For a drive system with DC motor (having rated data 1.7kW, 110V, 20A, 1500rpm) and for ω_d=125 rad/s, J=0.05Nms^2/rad and normalized load torque *m*=0.2, the simulation results are presented in Fig.4.

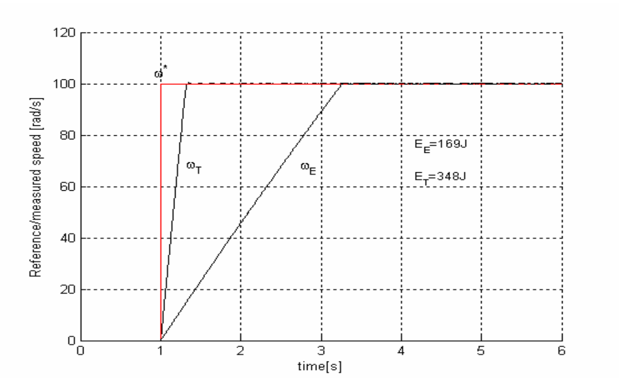

Fig. 4

The prescribed value of the speed ω^* and the obtained variation for the minimum energy system (ω_E) and for the minimum time system (ω_T) (in the case i_M=2) are presented. For both cases, the energy losses (E_E and E_T) are indicated on the figure. The expected result is obtained: the minimum time control ensure a smaller transient time, but an increase of the energy losses, because the current is forced too much.

The Fig. 5 presents the experimental results for energy optimal control for the same conditions as in previous case, being indicated the speed and active current variations.

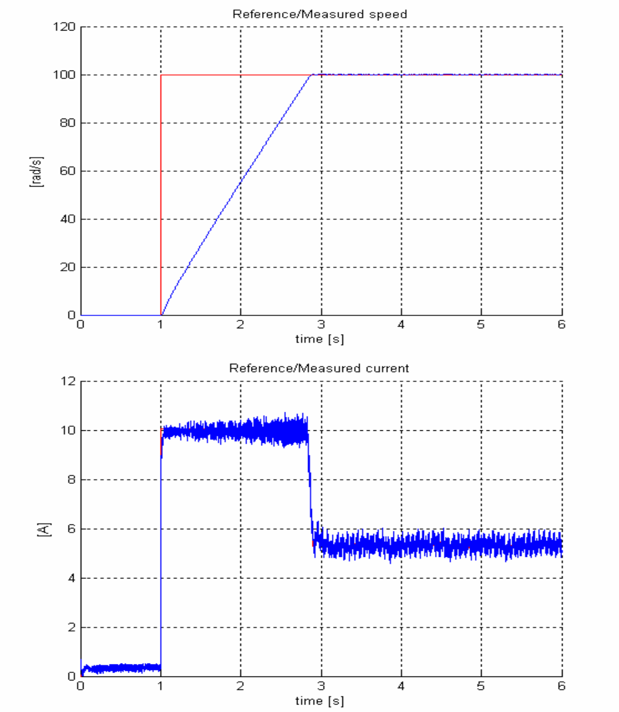

Fig. 5

Fig. 6 illustrates the important aspect underlined in the paper: the optimal control of the drive system from the energetic point of view depends on the mean value of the load torque and not on their instantaneous values.

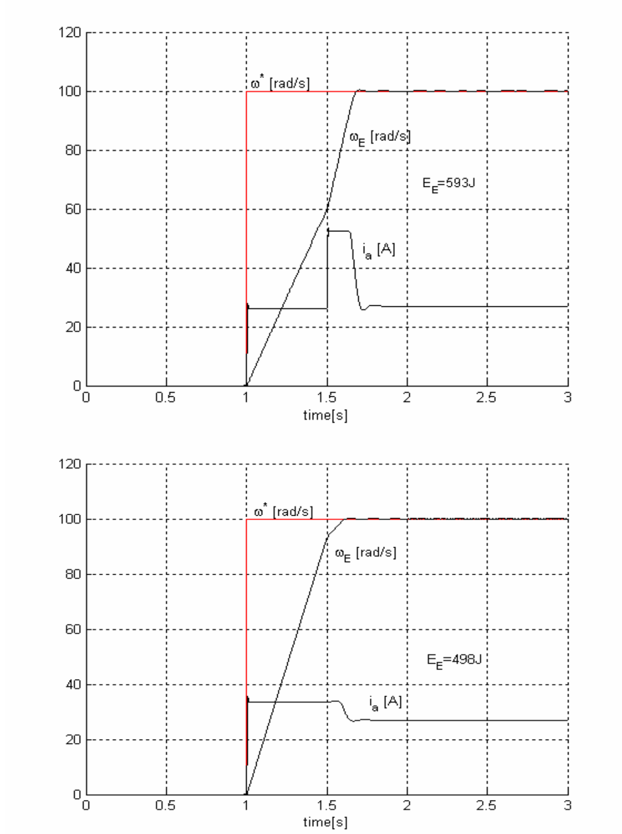

Fig 6

A step variation of the load torque (from m=0.5 to m=1) was introduced at the moment t = 1.5 s. The first figure presents the variations of the speed and of the active current when the control variable is computed with formula (48) for optimal control, but using the instantaneous values of the load torque. The second figure presents the same variations, but for true optimal control, depending on the mean value of the load torque. A small decrease of the transient time was obtained in the last case, but it is more important the significant decrease of the energy losses E_E. This difference in energy losses is higher when the step variation of the load torque is bigger (this fact can be theoretical proved and was experimentally verified).

Fig. 7 indicates the simulation and experimental results (similarly with Fig. 4 and 5) for a PMSM with rated data 2kW, 330V, 4.4A, 4500rpm, 225Hz and for normalized values m= 0.25 and i_M = 2.5.

The last figures show the experimental results referring to the variations for speed, active current and phase currents for the energetic optimal control of an induction motor with rated data 4kW, 380V, 8.64A, 1430rpm, f_N = 50Hz. The behaviours of the minimum time and minimum energy systems are presented in Fig. 8 and Fig. 9, respectively. The load torque was $0.2m_N$ in both cases. The stator losses were 29J for the minimum energy control and 42J for the minimum time control.

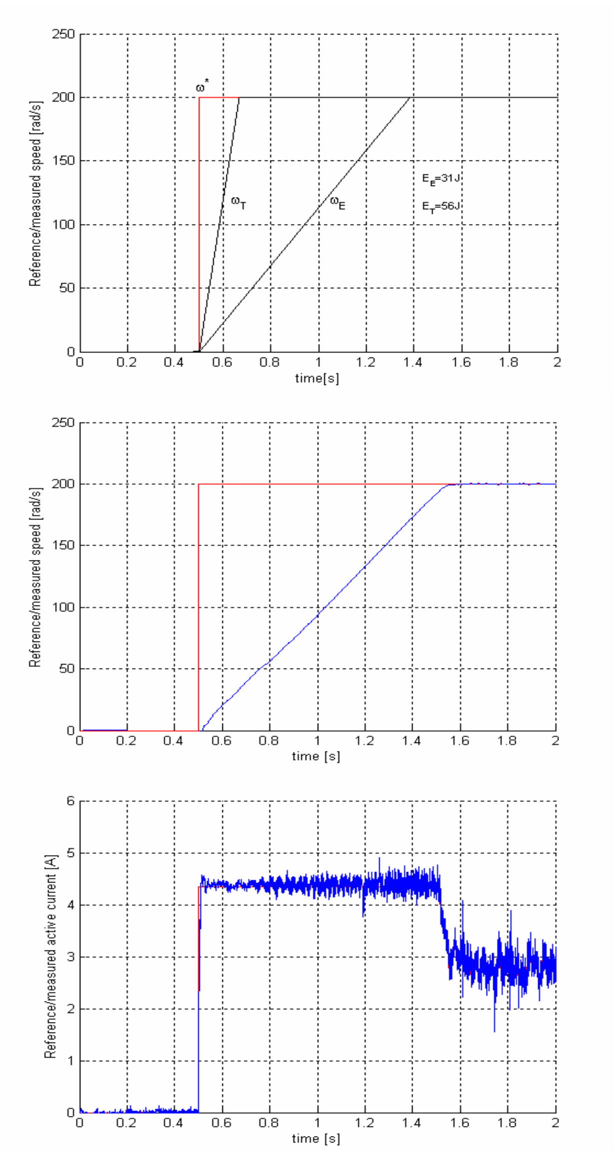

Fig. 7

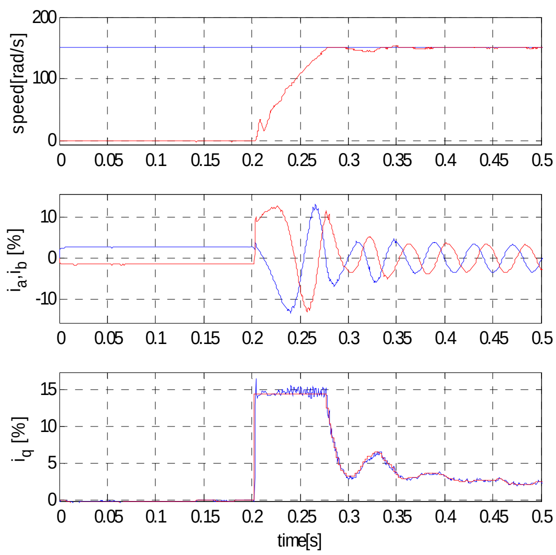

Fig. 8

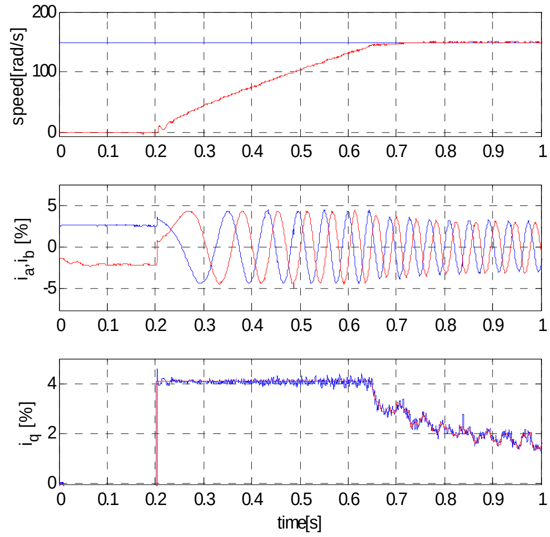

Fig. 9

The experimental tests were performed in each case on a corresponding set-up containing two electrical machines: a drive motor and a loading machine. The last one was also used for emulating the inertia of the drive system. It should be noticed that it was not possible to achieve a step variation of the load torque because of the inertia of the load machine. A good concordance among the theoretical, simulation and experimental results was obtained for all motor type.

7 Conclusion

The minimum energy control and minimum time control of the electrical drives have in many cases contradictory demands. Generally, the endeavour of the decrease of the transfer time leads to the increase of the energy losses.

In all cases, the minimum time control can be achieved using only one variable control. For optimal energy control, the use of two variables presents interest only for very small load torque.

The structure of the optimal control is not complicated and is based on a usual cascade one, if we adopt the currents as control variable. This choice has a practical justification, based on the fact that the motor currents are frequently controlled in the variable speed drives.

There is a great similitude (theoretical and experimental) in optimal control of different motor types.

The performed simulation and experimental tests show a very good behaviour of the optimal control systems which represents an opportunity for energy consumption decrease in the transient period of an electrical drive.

References:

[1] M. Athans and P.L. Falb, *Optimal Control*, Mc Graw Hill, New York, 1966.

[2] B.D.O. Anderson, J.B. Moore, *Optimal Control. Linear Quadratic Methods*, Prentice Hall, 1990.

[3] C. Boțan, F. Ostafi, A. Onea, A Solution to the Optimal Tracking Problem for Linear Systems, *WSEAS Transactions on Systems*, Vol. 1, ISSN 1109-2777, 2002, pp. 185-189.

[4] A. Onea and C. Botan, Real Time LQ-Optimal Control of a D.C. Drive System, *Proceedings of the 24th Annual Conference of the IEEE Industrial Electronics Society, IECON'98*, Aachen, 1998, pp.1417-1421.

[5] C. Botan, V. Horga, F. Ostafi, M. Albu, M Ratoi, General Aspects of the Electrical Drive Systems Optimal Control, *12th European Conference on Power Electronics and Applications* - EPE 2007, September 2007, Aalborg, Denmark, CD-ROM.

[6] R.D. Lorentz, Future Motor Drive Technology Issues and their Evolution, *12th Int. Power Electronics and Motion Conference (EPE-PEMC)*, Portoroz, Slovenia, 2006, CD-ROM.

[7] C. De Capua and C. Landi, Measurement Station Performance Optimization for Testing on High Efficiency Variable Speed Drives, *IEEE Trans. on Instrumentation and Measurement*, vol. 48, no. 6, 1999, pp. 1149-1154.

[8] M. Ghribi, Y. Dube and K. Al-Haddad, Linear Quadratic Control of a DC Motor, *Proceedings of the 3rd European Conference on Power Electronics and Applications, EPE'89*, Aachen 1989, pp. 261-265.

[9] T. Egami, H. Morita and T. Tsuchiya, Efficiency Optimized Model Reference Adaptive Control System for a DC Motor, *IEEE Transactions on Industrial Electronics*, vol. 37, no. 1, Feb. 1990, pp. 28-33.

[10] I.T. Wallace, D.W. Novotny, R.D. Lorenz and D.M. Divan, Verification of Enhanced Dynamic Torque per Ampere Capability in Saturated Induction Machines, *IEEE Transactions on Industry Applications*, vol. 30, no. 5, Sept./Oct. 1994, pp. 1193-1201.

[11] C. Canudas de Wit, J. Ramirez, Optimal Torque Control for Current - Fed Induction Motors, *IEEE Trans. on Autom. Control*, vol. 44, No.5, 1999, pp. 1084 – 1089.

[12] J. A. Ga'eb, Optimal Real-Time Digital Control for DC-Motor Proposed for Minimum Generation of Harmonics, WSEAS Transactions on Power Systems, Issue 5, Vol. 1, May 2006, pp. 878-884.

[13] S.N. Vukosavic, E. Levi, Robust DSP-based Efficiency Optimization of a Variable Speed Induction Motor Drive, *IEEE Trans. on Ind. Electronics*, vol. 50, No.3, 2003, pp. 560 – 570.

[14] M. F. Stan, M. Ionel, O.M. Ionel, Contribution to the Optimization of the Electrical Drives of the Pronounced Dynamic Regime Working Machines, Y. Birbir, *WSEAS Transactions on Power Systems*, Issue 9, Vol. 1, Sept. 2006, pp. 1654-1658.

[15] Y. Birbir, M. Yilmaz, Optimal Speed Application on the Universal Motor by Means of Microcontroller, *WSEAS Transactions on Power Systems*, Issue 11, Vol.1, Nov. 2006, pp. 1903-1910.

[16] T. Yamakawa, S. Wakao, K. Kondo, T.Yoneyama, A New Flux Weakening Operation of Interior Permanent Magnet Synchronous Motors for Railway Vehicle Traction, *European Power Electronics EPE*, Dresden, Germany, 2005, CD-ROM.

[17] C. Mademlis, N. Margaris, Loss Minimization in Vector-Controlled Interior Permanent-Magnet Synchronous Motor Drives, *IEEE Trans. on Ind. Electronics*, Vol. 49, No. 6, 2002, pp. 1344-1347.

[18] C. Cavallaro, A.O. DiTommaso, R. Miceli, A. Raciti, G.R. Galluzzo, M. Trapanese, Efficiency Enhancement of Permanent-Magnet Synchronous Motor Drives by Online Loss Minimization Approaches, *IEEE Trans. on Ind. Electronics*, Vol. 52, No. 4, Aug. 2002, pp. 1153- 1160.

[19] Z. Q. Zhu, Y.S. Chen, D. Howe, Online Optimal Flux-Weakening Control of Permanent-Magnet Brushless DC Drives, *IEEE Trans. on Industrial Application*, Vol. 36, No. 6, 2000, pp.1661- 1668.

[20] E.I. Verriest, F.L. Levis, On the Linear Quadratic Minimum-Time Problem, *IEEE Trans. on Autom. Control*, Vol. 36, No. 7, 1991, pp. 859 – 863.