Speculative production and anticipative reservation of reactive capacity
by a multi-product newsvendor
by
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Abstract

In this paper we study the optimal sourcing decisions of a multi-product newsvendor prior to the selling season of the products. To satisfy the uncertain demands, the newsvendor can either utilize speculative production, or anticipatively reserve capacity. During the selling season when demand has become known, the newsvendor can utilize its reserved capacity and reactively satisfy demand uncovered by its speculative production. For the case where capacity for speculative production may be limited, but potential reservation of reactive capacity is unlimited we analyze and compare two capacity reservation settings, namely the case where capacity for each product has to be reserved separately and the case where one joint capacity reservation for all products is permitted which can then be allocated to the different products optimally during the selling season. For the case of separate individual reservations we analytically derive the optimal strategies and present structural insights concerning their existence. As the model allowing for joint reservation can not be tackled analytically in general we present an approximation based on an LP formulation. Using a numerical example we present insights on the value of the increased flexibility induced by joint reservation, the cost-premium acceptable for joint reservation and the relative levels of capacity reservation in the two settings.
1 Introduction and related work

To survive on markets becoming ever more competitive firms need to effectively match their supply with the demand for their products. Shorter life cycles and increased diversification due to growing product proliferation magnify the uncertainty about actual demands in many markets such as the apparel, PC or automotive industry. To handle increasing volatility and product variety firms need to define their core competencies more narrowly than before (see [12]) and outsourcing and the possibility to supply a particular product from several internal and external sources are important supply-side strategies for firms to mitigate risks arising from demand (and other supply chain) uncertainties. According to [7], strategic sourcing may be the biggest competitive advantage for a company.

In this paper we focus on a manufacturer who is faced with uncertain demands for its product portfolio. The manufacturer can allocate limited capacity for the speculative production of its products. To avoid shortages the possibility/necessity to anticipatively reserve some reactive capacity exists in which case it needs to be decided for which products to produce speculatively and for which products to reserve reactive capacity. Possible reservation of reactive capacity is unlimited, but the total of reservation and utilization cost may exceed the cost of speculative production. We model this situation as an extension of a multi-product newsvendor problem to study the implications of two different reservation settings. In the first setting anticipative reservation has to take place for each product individually, while in the second setting a joint reservation is possible. Clearly the second case induces more flexibility as the manufacturer can allocate the reserved capacity upon demand realization as it sees fit. The specific research questions addressed are:

1. How do the optimal speculative production and anticipative capacity reservation decisions differ in the two capacity reservation settings?

2. What is the effect of the two capacity reservation settings on the manufacturers’ profitability? How does the value of flexibility arising from a joint capacity reservation depend on the available capacity for speculative production?

3. What premium should the manufacturer be willing to pay for a joint capacity reservation and how does this premium depend on the available capacity for speculative production?

Our model combines aspects of multiple sourcing (including outsourcing) and capacity reservation with issues of demand uncertainty and focuses on the relationship between supply cost and lead times. It is thus related to different streams of previous work discussed below.
There is a significant body of literature dealing with single and multiple sourcing strategies in a single product context. For a general review of multiple-supplier inventory models we refer to [11]. In the light of uncertain demand fluctuations for a single product, the value of short term capacity adjustments through outsourcing is studied in [16]. Particularly, a manufacturer can increase its capacity by exercising a capacity option bought from a subcontractor. In [17] this model is extended to deal with backlog-dependent demand, where customers may decide not to buy if the lead-time quoted is too long. Using dynamic programming feedback policies for the manufacturer’s internal capacity and the subcontracting capacity are derived. Several analytical studies have provided justifications for multiple sourcing on the basis of uncertainty in supply prices ([8]), random supply yield ([6]), stochastic lead times ([9]), capacity and supply quality ([1]) and the relationship between supply cost and lead times (see e.g., [13] and [18]).

Clearly these single product models can not be used to analyze the additional challenges arising from the above mentioned increased product proliferation. Thus, a different stream of literature has focused on multi-item newsvendor models. For a review of several variants of newsvendor models, see [10]. In [4] a multi-item newsvendor problem with a single capacity constraint is studied to analyze the optimal allocation of the scarce resource to the different products. In [15] and [19] the issue of multiple (dual) sourcing has been combined with the consideration of product portfolios. Under the assumption that both capacity sources have to be utilized under demand uncertainty it is shown that at most one product will be dual sourced.

The utilization of sources having differing characteristics with respect to the supply cost and lead time tradeoff is analyzed in a number of papers where outsourcing/dual sourcing is proposed as a reactive strategy to counter potential shortages of a product portfolio (see e.g., [2], [3], [5] and [14]). In [5] a policy termed accurate response was introduced, where low-cost speculative sourcing prior to a selling season can be supplemented by fast, reactive supply during the selling season upon obtaining first signals about the true total demand through early orders. The main assumption is that early demand and total demand are correlated. A two-stage stochastic optimization model is presented. However, this model is difficult to solve even for small instances and some approximations based on relaxing the coupling of products in the reactive stage are proposed. Through a numerical example it is shown that the proposed approach significantly improves upon the actual practice of the industrial partner from the fashion industry. A slightly different problem is studied in [3]. They also consider a firm with uncertain demands for a portfolio of products and two periods of production. The demand becomes fully known at the end of period 1. Thus, period 1 production is under uncertainty, while period 2
production is reactive to the observed demands. However, for each product the maximum level of period 2 capacity has to be determined at the beginning of period 1 and total period 2 capacity is limited. This assumption makes the problem much easier to solve and allows for an explicit characterization of the optimal policy. In the two papers mentioned above, capacities of the speculative and reactive phase are assumed to be fixed and thus constraints. For the industrial context of specialty chemicals a more complex model, including setups and operational scheduling aspects is presented in [14]. In this model speculative and reactive capacities are interpreted as operational and contractual flexibilities, respectively. Due to the complexity of the model an LP approximation is presented and solved to analyze the tradeoff between these operational and contractual flexibilities. More precisely, the value of increased contractual flexibility arising from increased lead-times and operationalized through the level of reactive capacity is measured and compared with the value of operational flexibility. Further, the composition of the product portfolio and contract acceptance/rejection criteria are analyzed. Finally, differing from these three papers, the optimal choice of speculative and reactive capacity levels is analyzed in [2] for single product and two product cases. However, as in [5] the resulting problem is in general difficult to solve analytically and the authors show numerically for the simplified two product case - with identical products - that pooling of the reactive capacity may yield benefits. Studying varying cost settings for the products, the authors mimick different industries and characterize the optimal use of reactive capacity.

Summarizing, while our model derives part of its characteristics from these different literatures it simplifies some aspects (e.g. product demands are independent, supply yield is deterministic, potential (outsourcing) capacity reservation is unlimited, no setups) to (partly) allow for analytical tractability and to obtain clearer insights into the question of how to optimally utilize capacity reservation/dual sourcing for a product portfolio. Specifically, our approach follows the last stream of research discussed above most closely, extending it in the following aspects:

1. We present analytical results for the case of limited capacity for speculative production but unlimited individual reservation of reactive capacity for the different products.

2. We present an LP formulation for the case of joint capacity reservation allowing for more than two, structurally different products.

3. We compare the two reservation settings and present structural insights into the value and utilization of capacity reservation.

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Our main findings are that (i) the level of joint capacity reservation may be larger than the total individual capacity reservations for all products, (ii) the premium acceptable for joint reservation increases with an increasing level of capacity for speculative production and (iii) the different reservation settings may imply structurally very different sourcing decisions. The first result is somewhat counter-intuitive as from a pooling perspective we would have expected that joint capacity reservation is more efficient and thus can be smaller. While this holds true in our model for some scenarios, we show that it is not generally true. The second result highlights the flexibility enhancing/risk reducing effect of joint capacity reservation. Under high levels of speculative production potential overproduction is larger and an additional individual reservation of reactive capacity for a given product even magnifies this effect. Thus, the manufacturer is willing to invest more in the flexibility to reactively decide how to allocate the joint capacity reservation. Finally, the third result shows the importance of considering product portfolios rather than individual products when analyzing sourcing strategies and decisions.

The remainder of the paper is organized as follows. Section 2 is dedicated to the development and analysis of our general model and its two variants. In Section 3 we present results for the two model settings based on a numerical study. Finally, Section 4 summarizes our main findings and gives some directions for further research. The appendix contains the theoretical results and proofs.

2 The model

In this paper we consider a manufacturer offering a portfolio of \( n \) demand-wise independent products to the market. The per-unit selling price of a product \( i \) is given by \( p_i \). Each product \( i \) is characterized by a demand \( d_i \) which occurs and has to be satisfied in a single selling season. To cater for these demands the manufacturer can produce speculatively using a single, limited capacity source. The total capacity available is \( C \), the per-unit capacity consumption of product \( i \) is given by \( k_i \) and the per-unit production cost of product \( i \) are \( c_i \). Due to lead-time considerations production has to take place prior to the selling season, when the actual demand \( d_i \) of a product \( i \) is still unknown.

Alternatively, the manufacturer can anticipatively reserve capacity for some or all products prior to the selling season when demands are uncertain. However, this reservation offers some flexibility to the manufacturer, as when demands unfold, the manufacturer can request production up to the reserved capacity if necessary, i.e. if demand exceeds speculative production. The per-unit cost of this production request for a product \( i \) is given by \( z_i \). Concerning the reservation we will analyze
two different settings. In the first case, the manufacturer has to make an individual capacity reservation $r_i$ for each product $i$ at the per-unit cost of $v_i$. In the second case, the manufacturer has even more flexibility as a single, joint capacity reservation $r$ for all products at a per-unit cost $v$ is possible. As demands for the different products $i$ unfold, the manufacturer can then allocate this reserved capacity $r$ as it sees fit. In either case the total capacity reservation is unlimited.

For determining the optimal speculative production quantities $q_i$ and capacity reservation $r_i$ (or $r$) we assume that the manufacturer is risk neutral and hence maximizes its expected profit. Further, for each product the manufacturer knows the distribution of the uncertain demand $D_i$, which is given by the density $f_i(D_i)$ and cumulative distribution function $F_i(D_i)$. Throughout we assume that the distributions are continuous and twice differentiable. Finally, for notational convenience we will refer to the vectors of decision variables and uncertain demands as $q$, $r$ and $D$.

2.1 Individual capacity reservation $r_i$ for each product $i$

To eliminate trivial cases in this setting we will require that the relationships $p_i > v_i + z_i > c_i > v_i > 0$ hold. Given the problem structure discussed above the model formulation is given by

$$\max_{q,r} \quad \pi_{r_i} = \mathbb{E}[g(q, r, D)]$$

$$= \sum_{i=1}^{n} \max_{q_i, r_i} \mathbb{E}[g_i(q_i, r_i, D_i)]$$

$$\text{subject to} \quad \sum_{i=1}^{n} k_i q_i \leq C$$

$$q_i \geq 0, \quad i = 1, \ldots, n$$

$$r_i \geq 0, \quad i = 1, \ldots, n$$

The objective is to maximize the sum of the expected profits of all products which are given by

$$\mathbb{E}[g_i(q_i, r_i, D_i)] = -c_i q_i - v_i r_i + p_i \int_{0}^{q_i + r_i} x f_i(x) dx - z_i \int_{q_i}^{q_i + r_i} (x - q_i) f_i(x) dx$$

$$+ p_i (q_i + r_i)(1 - F_i(q_i + r_i)) - z_i r_i (1 - F_i(q_i + r_i))$$

$$+ p_i (q_i + r_i)(1 - F_i(q_i + r_i)) - z_i r_i (1 - F_i(q_i + r_i))$$
Besides the capacity constraint on speculative production (2) we need the non-negativity constraints for \( q_i \) and \( r_i \) as otherwise excess speculative production could be used as a negative capacity reservation at a per-unit yield of \( v_i + z_i - c_i \).

**Proposition 1.** The objective function (1) is jointly concave in \((q, r)\) and the constraints (2)-(4) are convex.

All proofs are given in the Appendix. As a consequence of Proposition 1, a globally optimal solution can be found easily using the system of Karush-Kuhn-Tucker (KKT) conditions. The structure of the optimal strategy for each individual product is characterized in Proposition 2.

**Proposition 2.** The optimal speculative production quantity \( q_i^* \) and capacity reservation \( r_i^* \) are as follows:

a) Exclusive speculative production: If \( \lambda C \leq \frac{v_i p_i - z_i - c_i}{k_i} \), then

\[
q_i^* = F_i^{-1}\left(\frac{p_i - c_i - \lambda C k_i}{p_i}\right) \text{ and } r_i^* = 0 \tag{5}
\]

b) Dual sourcing: If \( \frac{v_i p_i - z_i - c_i}{k_i} < \lambda C < \frac{z_i + v_i - c_i}{k_i} \), then

\[
q_i^* = F_i^{-1}\left(\frac{z_i + v_i - c_i - \lambda C k_i}{z_i}\right) \text{ and } r_i^* = F_i^{-1}\left(\frac{p_i - z_i - v_i}{p_i - z_i}\right) - q_i \tag{6}
\]

c) Exclusive anticipative reservation of reactive capacity: If \( \lambda C \geq \frac{z_i + v_i - c_i}{k_i} \), then

\[
q_i^* = 0 \text{ and } r_i^* = F_i^{-1}\left(\frac{p_i - z_i - v_i}{p_i - z_i}\right) \tag{7}
\]

Let us now focus on a couple of interesting facts to be observed from Proposition 2. First, as soon as some capacity reservation for a product takes place its total anticipative supply is constant at \( q_i^* + r_i^* = F_i^{-1}\left(\frac{p_i - z_i - v_i}{p_i - z_i}\right) \). A similar result was presented in [15] and [19] for the case where both unlimited outsourcing and capacitated speculative production occur under uncertainty, a special case of our present model where \( z_i = 0 \) and \( v_i > c_i \). Second, consider the requirement \( \lambda C \leq \frac{v_i p_i - z_i - c_i}{k_i} \)
for exclusive speculative production and a situation where capacity \( C \) is unlimited. This gives rise to \( \lambda^C = 0 \) and by simple manipulation we obtain

\[
v_i \geq c_i \frac{p_i - z_i}{p_i}.
\] (8)

Thus, even under unlimited capacity \( C \), exclusive speculative production only occurs when \( v_i \) is not too small. Third, assume the extreme case \( v_i = 0 \). This case is equivalent to a situation where capacity reservation is not necessary and unlimited reactive production is possible upon demand realization. Clearly, the optimal strategy in this case depends on the relationship between \( c_i + \lambda^C k_i \) and \( z_i \). Whenever \( c_i + \lambda^C k_i \geq z_i \) no speculative production will take place \((q_i = 0)\) and the complete demand is satisfied reactively. On the other hand, if \( c_i + \lambda^C k_i < z_i \), the optimal speculative production quantity is given by

\[
q^*_i = F_i^{-1} \left( \frac{z_i - c_i - \lambda^C k_i}{z_i} \right).
\] (9)

Considering again unlimited capacity \( C \), i.e. \( \lambda^C = 0 \) we get that speculative production only takes place if \( z_i > c_i \). Note that this situation corresponds to the ZO strategy used as a benchmark in [19].

By linking the results for the individual products shown in Proposition 2 we obtain 7 structurally different strategies which depend on \( \lambda^C \). To formally describe this relationship let us first assume that products are indexed in a way that

\[
\frac{v_i p_i}{p_i - z_i - c_i} \leq \frac{v_j p_j}{p_j - z_j - c_j} \quad \text{if} \quad i < j.
\]

Let us further introduce the identifiers \( N^S \), \( N^D \) and \( N^R \) corresponding to the subsets of products with exclusively speculative production \((S)\), dual sourcing \((D)\) and exclusively anticipative capacity reservation \((R)\), respectively. Which of these subsets a product \( i \) belongs to follows directly from Proposition 2. Then the seven strategies are given by:

1. Exclusive speculative production of all products:

\[
\lambda^C \leq \min_{i=1,\ldots,n} \frac{v_i}{p_i - z_i - c_i} \frac{p_i}{k_i}.
\]

2. Dual sourcing for some products \( i \in N^D \), exclusive speculative production for the remaining products \( j \notin N^D \):

\[
\max_{i \in N^D} \frac{v_i}{p_i - z_i - c_i} \frac{p_i}{k_i} < \lambda^C \leq \min \left[ \min_{i \in N^D} \frac{z_i + v_i - c_i}{k_i}, \min_{j \notin N^D} \frac{v_j}{p_j - z_j - c_j} \frac{p_j}{k_j} \right].
\]

3. Dual sourcing for all products:

\[
\max_{i=1,\ldots,n} \frac{v_i}{p_i - z_i - c_i} \frac{p_i}{k_i} < \lambda^C \leq \min_{i=1,\ldots,n} \frac{z_i + v_i - c_i}{k_i}.
\]
4. Exclusive anticipative capacity reservation for some products \( i \in N^R \), exclusive speculative production for all remaining products \( j \notin N^R \):

\[
\max_{i \in N^R} \frac{z_i + v_i - c_i}{k_i} < \lambda^C \leq \min_{j \notin N^R} \frac{v_j}{p_j} \frac{p_j}{z_j} - c_j
\]

5. Exclusive anticipative capacity reservation for some products \( i \in N^R \), dual sourcing for some other products \( j \in N^D \), exclusive speculative production for the remaining products \( l \in N^S \):

\[
\max \left[ \max_{i \in N^R} \frac{z_i + v_i - c_i}{k_i} ; \max_{j \in N^D} \frac{v_j}{p_j} \frac{p_j}{z_j} - c_j \right] < \lambda^C \leq \min \left[ \min_{j \in N^D} \frac{z_j + v_j - c_j}{k_j} ; \min_{l \in N^S} \frac{v_l}{p_l} \frac{p_l}{z_l} - c_l \right]
\]

6. Exclusive anticipative capacity reservation for some products \( i \in N^R \), dual sourcing for the remaining products \( j \in N^D \):

\[
\max \left[ \max_{i \in N^R} \frac{v_i}{p_i} \frac{p_i}{z_i} - c_i ; \max_{j \in N^D} \frac{z_j + v_j - c_j}{k_j} \right] < \lambda^C \leq \min_{i \in N^R} \frac{z_i + v_i - c_i}{k_i}
\]

7. Exclusive anticipative capacity reservation for all products:

\[
\lambda^C = \max_{i=1,...,n} \frac{z_i + v_i - c_i}{k_i}
\]

Note that only Strategies 6 and 7 will occur in any case. The existence of all other strategies depends on the problem data. For Strategies 1 and 2 this follows directly from the discussion leading to equation (8) above. Moreover Strategy 3 and Strategies 4 and 5 are mutually exclusive and the latter two strategies can only occur if Strategy 2 exists. Algorithm 1 can be used to find the optimal strategy and the associated speculative production quantities and anticipative capacity reservation levels for a given capacity \( C \). The algorithm is based on bisection over the shadow price \( \lambda^C \) of the capacity.

### 2.2 Joint capacity reservation \( r \) for all products

In this setting additional flexibility is granted to the manufacturer in the form of a joint capacity reservation \( r \) for all products at a per-unit cost of \( v \). Unfortunately this setting is much more difficult mathematically than the one presented in the previous section. While the profit function is well behaved (as shown for a similar setting in [2]) the difficulty arises from the fact, that whenever demand exceeds speculative production \( q_i \) of a product \( i \) the actual amount of this residual demand that is satisfied not only depends on product \( i \) but on the residual demands of all other products as well. This leads to the necessity of integrating over the \( n \)-dimensional space of demands for all products and the number of regions to be considered grows exponentially in \( n \). For the example with two identical products the number of
Algorithm 1: Multi-product newsvendor with speculative production and anticipative reservation of reactive capacity

\begin{algorithm}
\textbf{begin} \\
\hspace{1em} Let $\lambda_{\text{min}} = 0$, $\lambda_{\text{max}} = \max_{i=1,...,n} \frac{x_i + v_i - c_i}{\xi_i}$ and $\lambda^C = \lambda_{\text{min}}$. Let $\text{opt} = \text{FALSE}$. \\
\hspace{1em} \textbf{while} ($\text{opt} \neq \text{TRUE}$) \textbf{do} \\
\hspace{2em} Determine the sets $N^S$, $N^D$ and $N^R$. \\
\hspace{2em} Compute the speculative production quantities for each product $i$ as \\
\hspace{3em} $q_i = \begin{cases} \\ F_i^{-1} \left( \frac{x_i + v_i - c_i - \lambda^C k_i}{\xi_i} \right) & i \in N^S \\ F_i^{-1} \left( \frac{x_i + v_i - c_i - \lambda^C k_i}{\xi_i} \right) & i \in N^D \\ 0 & i \in N^R \end{cases}$ \\
\hspace{2em} if $(\lambda^C = 0 \&\& \sum_{i=1}^{n} k_i q_i \leq C) \; \| \; \sum_{i=1}^{n} k_i q_i = C$ then \\
\hspace{3em} $\text{opt} = \text{TRUE}$ \\
\hspace{2em} else if $(\sum_{i=1}^{n} k_i q_i < C)$ then \\
\hspace{3em} $\begin{cases} \\ \lambda_{\text{max}} = \frac{\lambda^C}{\lambda_{\text{min}} + \lambda_{\text{max}}} \\ \lambda^C = \frac{\lambda_{\text{min}} \lambda_{\text{max}}}{2} \end{cases}$ \\
\hspace{2em} else if $(\sum_{i=1}^{n} k_i q_i > C)$ then \\
\hspace{3em} $\begin{cases} \\ \lambda_{\text{min}} = \frac{\lambda^C}{\lambda_{\text{min}} + \lambda_{\text{max}}} \\ \lambda^C = \frac{\lambda_{\text{min}} \lambda_{\text{max}}}{2} \end{cases}$ \\
\hspace{2em} $q_i^* = q_i, \forall i = 1,...,n$ \\
\hspace{2em} $r_i^* = \begin{cases} \\ 0 & i \in N^S \\ F_i^{-1} \left( \frac{z_i + v_i - c_i}{\xi_i} \right) - q_i^* & i \in N^D \\ F_i^{-1} \left( \frac{x_i + v_i - c_i}{\xi_i} \right) & i \in N^R \end{cases}$ \\
\hspace{1em} \textbf{end}
\end{algorithm}
regions was already 10 (see [2]). Thus, an analytical approach is not tractable for larger values of $n$ and we will approximate the optimum by sampling the demand distributions and solving a linear programming formulation of the model. More precisely we will consider a sample of $j = 1, ..., m$ scenarios, where each scenario $j$ is characterized by a tuple $d_j = (d_{ij}, ..., d_{nj})$ of the demands for the $n$ products drawn from the associated demand distributions.

Before we can turn to the model formulation, we need to define some auxiliary variables. Let $y_{ij} \geq 0$ denote the amount of residual capacity utilized for product $i$ in scenario $j$. Note, that w.l.o.g. we assume that reactive capacity consumption is identical for all products, i.e. $k_i = k_j, \forall i, j = 1, ..., n$ in this setting. Thus, capacity and quantity can be used interchangeably. Further, let $x_{ij} \geq 0$ denote the sales quantity of product $i$ in scenario $j$. Clearly, $x_{ij} = \min\{d_{ij}; q_i + y_{ij}\}$.

The maximization of the expected profit can then be approximated by

$$\max \pi_r = \frac{1}{M} \sum_{j=1,\ldots,m} \left[ \sum_{i=1,\ldots,n} (p_i x_{ij} - c_i q_i - z_i y_{ij}) - v \right]$$

subject to

$$\sum_{i=1,\ldots,n} k_i q_i \leq C \quad (11)$$

$$\sum_{i=1,\ldots,n} y_{ij} \leq r \quad j = 1, ..., m \quad (12)$$

$$q_i + y_{ij} \geq x_{ij} \quad i = 1, ..., n \quad \text{and} \quad j = 1, ..., m \quad (13)$$

$$d_{ij} \geq x_{ij} \quad i = 1, ..., n \quad \text{and} \quad j = 1, ..., m \quad (14)$$

$$q_i \geq 0 \quad i = 1, ..., n \quad (15)$$

$$x_{ij}, y_{ij} \geq 0 \quad i = 1, ..., n \quad \text{and} \quad j = 1, ..., m \quad (16)$$

$$r \geq 0 \quad (17)$$

The objective function (10) maximizes the sample average of the profits over all scenarios $j = 1, ..., m$. Constraint (11) ensures that the capacity for speculative production is not exceeded, while constraints (12) ensure that the reserved capacity is not violated in any scenario $j$. Constraints (13) and (14) model the fact that sales quantity can neither exceed supply nor demand. The remaining constraints guarantee the non-negativity of the decision variables.
3 Numerical Analysis

In this section we will compare the two models in order to answer the following questions:

1. How do the optimal speculative production and anticipative capacity reservation decisions differ in the two capacity reservation settings?

2. What is the effect of the two capacity reservation settings on the manufacturers’ profitability? How does the value of flexibility arising from a joint capacity reservation depend on the available capacity for speculative production?

3. What premium should the manufacturer be willing to pay for a joint capacity reservation and how does this premium depend on the available capacity for speculative production?

In order to analyze these questions we consider a portfolio of 4 different products. In the base case each product is characterized by a demand following a normal distribution with $\mu = 30$ and $\sigma = 6$. In the case of individual capacity reservations $r_i$, the reservation cost $v_i = \{8, 9, 12, 11\}$ while in the setting with joint capacity reservation $v = 10$, i.e. the cost of the joint reservation corresponds to the average reservation cost in case of individual reservations. Operational data of the four products are given in Table 1.

<table>
<thead>
<tr>
<th>Product &amp;</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i$</td>
<td>20</td>
<td>36</td>
<td>36</td>
<td>26</td>
</tr>
<tr>
<td>$c_i$</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
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<td>4</td>
<td>9</td>
<td>20</td>
<td>13</td>
</tr>
<tr>
<td>$k_i$</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1: Operational data of the four product example

Let us first turn to the structural results concerning the optimal decisions and the resulting profits. Figures 1 and 2, show these results for varying levels of capacity for speculative production ranging from $C = 0$ to $C = 302$. For reasons of comparability with the case of joint reservation we plot $\sum_{i=1}^{n} r_i$ for the setting with individual capacity reservation and refer to this value as total individual capacity reservation.
Figure 1: Optimal speculative production quantities under individual (bold lines) and joint (fine lines) capacity reservation

Figure 2: Optimal capacity reservation (dotted lines) and associated profits (solid lines) under individual (bold lines) and joint (fine lines) capacity reservation
From Figures 1 and 2 we first of all observe that under sufficiently large capacity, i.e. when speculative production is hardly constrained, reservation is larger in the joint setting and accordingly the speculative production is smaller than under individual capacity reservation. This is surprising as from a capacity pooling perspective we would have expected that - under the same price - in the joint capacity reservation setting \( r \) should be smaller than the total individual capacity reservation \( \sum_{i=1}^{n} r_i \) in the individual reservation setting. However, this expected behavior only holds for small to medium values of capacity \( C \). The reason for this is that under relatively large values of capacity \( C \) most of the demand is satisfied through speculative production and the risk of individual capacity reservation is comparably large as this reserved capacity may not be needed after all. When capacity for speculative production is small, individual capacity reservation of each product needs to be high to ensure sufficient levels of demand satisfaction. On the other hand, the joint reservation can be utilized more effectively upon demand realization and this flexibility reduces the risk of oversupply in case of large capacity for speculative production and the risk of undersupply in case of small capacity for speculative production. Together these facts imply the observed results.

Apart from that we observed different switching behavior from dual sourcing to exclusive reservation of reactive capacity for some products in the two settings. Particularly, while under joint capacity reservation product 1 is the last product dual sourced when capacity for speculative production is very small, in case of individual capacity reservation this is product 3. Again this effect can be easily understood. For the setting with individual capacity reservation the relevant condition is given by \( \frac{v_i + z_i - c_i}{k_i} \), i.e. the relative additional cost of utilizing reserved capacity, which is largest for product 3 in our example. On the other hand, in case of joint capacity reservation, the relevant condition is \( \frac{v + z_i - c_i}{k} \) which is largest for product 1. Further, \( v = 10 \) is greater than \( c_1 = 9 \) implying that even the reservation of capacity (without actually utilizing it) is more expensive than speculative production for product 1.

Finally with respect to expected profits we make two observations. First, expected profits increase with increasing capacity \( C \) for speculative production in both reservation settings which is straightforward. Second, expected profits are always larger in case of joint capacity reservation as compared to individual capacity reservation. This is due to the fact that the manufacturer does not have to pay any premium for joint capacity reservation in our base case. While the cost of joint reservation \( v \) is larger than the individual reservation cost \( r_1 \) and \( r_2 \) for products 1

\(^{1}\)A capacity level \( C = 302 \) is sufficient to produce the independent newsvendor quantities of all products, i.e. the capacity no longer constrains the optimal decisions and \( \lambda^C = 0 \). Thus any further increase of capacity has no additional value.
and 2 it is smaller than the individual reservation costs \( r_3 \) and \( r_4 \) for products 3 and 4. As mentioned earlier it corresponds exactly to the average individual reservation costs. Below we will analyze the effects of increases in \( v \) on profitability and solution structure to quantify the premium to be paid by the manufacturer for the increased sourcing flexibility. Before we do that let us briefly conclude our discussion of the base case by looking at the value of flexibility induced by the joint reservation for different levels of capacity \( C \) for speculative production. In our case this value of flexibility is simply given by the profit differential between the two settings and is shown in Figure 3 alongside the shadow prices of the capacity for speculative production for both settings.

![Figure 3: Value of flexibility induced by joint capacity reservation, and shadow prices of capacity for speculative production](image)

In line with the findings from Figure 2, the value of flexibility is always greater than zero in this base case. However, it shows no smooth behavior but rather increases and falls as capacity for speculative production increases. This is due to the differential in the shadow prices also shown in Figure 3 which arises from the different switching points between exclusive anticipative reservation of reactive capacity and dual sourcing of the products in the two settings. Summarizing, these results suggest that the value of the flexibility associated with joint capacity reservation depends in a nontrivial way on the production decisions. While in the current setting it is always positive we will now examine under what conditions it may be beneficial to stick with individual capacity reservation.
More precisely, in Figure 4 we analyze for different values of capacity reservation cost $v$ and different levels of capacity for speculative production $C$ whether joint capacity reservation $r$ is larger than total individual capacity reservation $\sum_{i=1}^{n} r_i$ and whether joint capacity reservation leads to increased profits when compared to individual capacity reservation.

From Figure 4 we observe three different regions when capacity for speculative production is limited. First, when the cost of joint capacity reservation is rather high, joint capacity reservation is less profitable than individual capacity reservations and total individual capacity reservation exceeds joint capacity reservation. Here joint capacity reservation is too expensive and thus for any given level of capacity for speculative production the total supply of products is too small leading to lost sales and consequently reduced profits. Second, under medium capacity reservation cost total individual capacity reservation still exceeds joint capacity reservation but profits are now larger under joint capacity reservation. Here the pooling effect of joint capacity reservation strikes, as the fluctuations between demands of different products can be dealt with more efficiently with the joint reservation leading to increased sales figures for the products and thus to increased profits when compared to individual capacity reservation. The third region occurs under rather small joint capacity reservation cost and is characterized by larger profits and excess capacity.
reservation under joint capacity reservation as compared to individual capacity reservation. Here most of the sales are realized by utilizing the less expensive reserved capacity and the more costly speculative production is reduced.

Further, note that when capacity for speculative production is unlimited (right border of the graph) only the third region exists and profits and capacity reservation are larger under joint capacity reservation than under individual capacity reservations. In the latter case total individual capacity reservation is zero and the unconstrained newsvendor quantities for each product are produced speculatively. Under joint capacity reservation at a high cost $v$ the policy will be identical, i.e. there will be no capacity reservation. When joint capacity reservation is possible at a low cost $v$, the manufacturer can reduce the more costly speculative production and utilize the capacity reservation thus increasing its profits.

Besides the existence of these regions the interesting observation is also when the switching between these regions occurs. The solid line visualizes the maximum premium the manufacturer should be willing to pay for joint capacity reservation. Interestingly this premium increases as capacity for speculative production increases. For a level of capacity for speculative production equal to zero, the premium must not exceed zero (remember that the average reservation cost in case of individual reservation is equal to 10). On the other hand, when capacity for speculative production is almost unlimited, the premium can be 8. Here two effects are at work. As capacity for speculative production increases the necessity of (more costly) capacity reservation decreases. Consequently reservation levels are smaller and their impact on profits is reduced. This is true for both capacity reservation settings. Additionally, due to the pooling effect joint capacity reservation can be smaller than total individual capacity reservation and this difference is large enough to offset the premium of up to 8. Finally, the dotted line in Figure 4 shows the switching points between increased and reduced joint capacity reservation compared to total individual capacity reservation. Here we observe that the maximum admissible joint reservation cost $v$ first falls and only gradually increases as capacity for speculative production increases. This is due to the structurally different optimal production policies. Let us first consider the case of small capacity for speculative production. While under individual capacity reservation, the limited capacity for speculative production is initially used only for product 3, under joint capacity reservation the available capacity for speculative production is first used for product 1. From Table 1 we know that capacity consumption of product 3 is four times larger than capacity consumption of product 1. Consequently, a fixed level of capacity for speculative production allows a larger speculative production quantity of product 1 than of product 3 and as a result a larger reduction of necessary capacity reservation in the former case. Thus, under joint capacity reservation the level of capacity reser-
vation falls faster than under individual capacity reservation and the threshold cost of the joint capacity reservation needs to decrease to counteract this effect.

On the other hand as capacity for speculative production gets sufficiently large (greater approx. 200) we observe joint capacity reservation levels greater than total individual capacity reservation levels even for a cost of joint reservation clearly exceeding the average individual reservation cost of 10. As mentioned earlier, the additional flexibility associated with joint capacity reservation balances the risks of over- and undersupply and justifies (some) additional cost of joint capacity reservation.

4 Conclusion

In this paper we have analyzed a manufacturers' optimal sourcing decisions for a portfolio of products under uncertain demands. Prior to a selling season the manufacturer can allocate limited capacity for speculative production of its products as well as anticipatively reserve capacity for reactive production. With respect to this capacity reservation we analyze and compare two settings. In the first setting the manufacturer can reserve capacity for each product individually, which can then - upon demand realization - be utilized to satisfy any demand not covered by speculative production. In the second setting additional flexibility is available to the manufacturer as a single reservation is possible, which can then - upon demand realization - be allocated to the products as the manufacturer sees fit.

Our analysis focuses on the interplay between the cost of the joint reservation and the available level of capacity for speculative production and our main findings are:

1. The level of capacity reservation in the joint reservation setting may be higher or lower than under individual reservations for each product depending on its cost but also on the level of available capacity for speculative production. Interestingly, under higher levels of capacity for speculative production joint reservation is larger than individual reservations for a larger range of reservation cost. This is due to the flexibility associated with the joint reservation which reduces the risk of undersupply when capacity for speculative production is small and the risk of oversupply when capacity for speculative production is large.

2. The price premium acceptable for the additional flexibility included by a joint reservation also increases as capacity for speculative production increases. In our numerical case markups of up to 80% on the reservation cost still lead
to profit gains when capacity for speculative production was high. Moreover, increased profits under joint reservation may result from joint reservation levels larger or smaller than total individual capacity reservation. While the former situation is explained by the effect explained above, the latter case is a result of the capacity pooling effect which is predominant under high (but not too high) joint reservation cost.

3. The sourcing decisions for any given product may differ significantly in the two settings, e.g. a product dual sourced under individual capacity reservation may not be produced speculatively under joint reservation and vice versa. These differences also lead to non-trivial implications for the value of joint reservation and highlight the importance of considering the whole product portfolio when deciding the capacity reservation policy.

In closing this paper we want to outline two interesting directions of future research. The first one is concerned with a more realistic modelling of the supply process by including e.g. fixed (setup) costs or constraints on the possible capacity reservation. This should provide more detailed insights into which products to produce speculatively and for which products to anticipatively reserve reactive capacity. Second, the incorporation of more demand side complexity into the model seems promising. More precisely, an interesting question would be to analyze the relationship between the sourcing decisions and make-to-order vs. make-to-stock decisions. In terms of our model, the reactive use of reserved capacity could be viewed as a possibility to customize an otherwise standardized product. The potential premium on the price then has to be balanced against the additional cost of sourcing. Moreover, given the portfolio of products the interaction between the different products should lead to interesting insights concerning the optimal supply of MTO and MTS products.

Appendix

Proof of Proposition 1: The convexity of the constraints is trivial to see. With respect to the objective function we get

$$\frac{\partial E_i[q,r,D]}{\partial q_i} = -(p_1 - z_i) f_i(q_i + r_i) - z_i f(q_i) \leq 0 \quad i = 1, ..., n$$

$$\frac{\partial E_i[q,r,D]}{\partial r_i} = \frac{\partial E_i[q,r,D]}{\partial q_i} \frac{\partial E_i[q,r,D]}{\partial r_i} = -(p_i - z_i) f_i(q_i + r_i) \leq 0 \quad i = 1, ..., n$$

and

$$\frac{\partial^2 E_i[q,r,D]}{\partial q_i \partial q_j} = \frac{\partial^2 E_i[q,r,D]}{\partial r_i \partial q_j} = \frac{\partial^2 E_i[q,r,D]}{\partial q_i \partial r_j} = \frac{\partial^2 E_i[q,r,D]}{\partial r_i \partial r_j} = 0 \quad i \neq j \quad i = 1, ..., n$$

such that its Hessian is negative semi-definite. ☐
Proof of Proposition 2: Let \( \lambda^C \) correspond to the lagrangian multiplier of the capacity constraint, while \( \lambda^q \) and \( \lambda^r \) correspond to the lagrangian multipliers of the non-negativity constraints for \( q_i \) and \( r_i \), respectively. Then the system of Karush-Kuhn-Tucker (KKT) conditions is given by

\[
-c_i + p_i - (p_i - z_i) F_i(q_i + r_i) - z_i F_i(q_i) - \lambda^C k_i + \lambda^q = 0, \quad i = 1, ..., n \tag{18}
\]

\[
-v_i - z_i + p_i - (p_i - z_i) F_i(q_i + r_i) + \lambda^r = 0, \quad i = 1, ..., n \tag{19}
\]

\[
\lambda^C \left( \sum_{i=1}^{n} k_i q_i - C \right) = 0 \tag{20}
\]

\[
\sum_{i=1}^{n} k_i q_i - C \leq 0 \tag{21}
\]

\[
q_i \lambda^q = 0, \quad i = 1, ..., n \tag{22}
\]

\[-q_i \leq 0, \quad i = 1, 2, ..., n \tag{23}
\]

\[r_i \lambda^r = 0, \quad i = 1, ..., n \tag{24}
\]

\[-r_i \leq 0, \quad i = 1, ..., n \tag{25}
\]

\[\lambda^C \geq 0 \tag{26}
\]

\[\lambda^q, \lambda^r \geq 0 \quad i = 1, ..., n \tag{27}
\]

Case a) Exclusive speculative production

This case implies that \( \lambda^C \geq 0 \) as well as \( \lambda^q = 0 \) and \( \lambda^r \geq 0 \ \forall i = 1, ..., n \). Consequently, equation (18) leads to

\[
F_i(q_i + r_i) = \frac{p_i - c_i - \lambda^C k_i - z_i F_i(q_i)}{p_i - z_i} \tag{28}
\]

while equation (19) leads to

\[
F_i(q_i + r_i) = \frac{p_i - v_i - z_i + \lambda^r}{p_i - z_i}. \tag{29}
\]

To get zero capacity reservation \( r_i^* = 0 \) we need \( F_i(q_i) = F_i(q_i + r_i) \) and from (28) it follows that

\[
F_i(q_i) = \frac{p_i - c_i - \lambda^C k_i}{p_i} \tag{30}
\]

leading directly to (5).
Using (29), (30) and the fact that \( \lambda_{ri} \geq 0 \) we obtain

\[
\lambda^C \leq \frac{v_i \frac{p_i}{p_i - z_i} - c_i}{k_i}.
\]

which completes the proof of case a).

Case b) Dual sourcing

In this case \( \lambda^C \geq 0 \) whereas \( \lambda^q = \lambda^{ri} = 0 \ \forall i = 1, \ldots, n \). While equation (18) again gives rise to (28), equation (19) now yields

\[
F_i(q_i + r_i) = \frac{p_i - v_i - z_i}{p_i - z_i}.
\]

Substituting (32) into (18) we obtain

\[
F_i(q_i) = \frac{z_i + v_i - c_i - \lambda^C k_i}{z_i}
\]

and together (32) and (33) lead directly to (6), and - in combination with the requirement \( F_i(q_i + r_i) > F_i(q_i) \) - to the lower boundary condition \( \lambda^C > \frac{v_i \frac{p_i}{p_i - z_i} - c_i}{k_i} \).

The condition \( q_i^* > 0 \) gives rise to the requirement \( F_i(q_i^*) > 0 \) and from (33) we can readily derive the upper boundary condition

\[
\lambda^C < \frac{z_i + v_i - c_i}{k_i}
\]

thus completing the proof of case b).

Case c) Exclusive anticipative reservation of reactive capacity

This case is induced by \( \lambda^C \geq 0 \) as well as \( \lambda^q \geq 0 \) and \( \lambda^{ri} = 0 \ \forall i = 1, \ldots, n \). Further, \( q_i^* = 0 \) implies that \( F_i(q_i^*) = 0 \) which yields

\[
F_i(q_i + r_i) = \frac{p_i - c_i - \lambda^C k_i + \lambda^q}{p_i - z_i}
\]

from equation (18). As in case b) equation (19) gives rise to (32), readily leading to (7). Finally, using (32), (35) and \( \lambda^q \geq 0 \) we obtain the lower boundary condition \( \lambda^C \geq \frac{z_i + v_i - c_i}{k_i} \) and this concludes the proof of case c).
References


