DS/CDMA with Two Sets of Orthogonal Spreading Sequences and Iterative Detection

Frederic Vanhaverbeke, Marc Moeneclaey, Senior Member, IEEE, and Hikmet Sari, Fellow, IEEE

Abstract—In this letter we introduce a direct-sequence code-division multiple access (DS/CDMA) concept which accommodates a higher number of users than the spreading factor \( N \). Each of the available orthogonal spreading sequences of length \( N \) is assigned to one of the first \( N \) users which employ a common pseudonoise (PN) scrambling sequence. When the number of users \( K \) exceeds \( N \), say \( K = N + M \) with \( M < N \), the \( M \) additional users reuse \( M \) of those orthogonal spreading sequences but in combination with another PN scrambling sequence. An iterative multistage detection technique is used to cancel interference between the two sets of users when \( K > N \). The proposed technique thus accommodates \( N \) users without any mutual interference and a number of additional users at the expense of a small signal-to-noise ratio penalty.

Index Terms—CDMA, cell capacity, multiple access.

I. INTRODUCTION

In a recent paper [1], the present authors introduced a general multiple access concept which makes it possible to accommodate a number of users that is significantly higher than achievable with presently available multiple access techniques. The idea was to use two sets of orthogonal signal waveforms and to iteratively cancel the interference between users which use resources from different signal sets. More specifically, assuming that the total bit rate on the multiple access channel is \( R \) bit/s and that each user requires a bit rate of \( R/N \) bit/s, the presented technique made it possible to accommodate a number of users \( K > N \) at the expense of a small degradation of the signal-to-noise ratio (SNR).

Note that \( N \) is a hard limit to the number of users in orthogonal waveform multiple access (OWMA) which includes classic frequency-division multiple access (FDMA), time-division multiple access (TDMA), orthogonal code-division multiple access (OCDMA) [2], orthogonal frequency-division multiple access (OFDMA) [3], and any other multiple access scheme which assigns orthogonal signal waveforms. On the other hand, code-division multiple access (CDMA) with pseudonoise (PN) spreading sequences which we refer to as PN-CDMA does not exhibit a hard limit on user capacity, but it suffers from multiuser interference (MUI) which grows linearly with the number of users. As a consequence, PN-CDMA is inherently interference-limited, and the number of users that it can support is significantly lower than that achievable in OWMA techniques.

The beauty of the multiple access concept presented in [1] is that it combines the advantages of OWMA and PN-CDMA while avoiding their shortcomings and undesirable features. Stated clearly, this method makes it possible to accommodate a number of users in excess of \( N \) while ensuring zero MUI when the number of users \( K \) is less than or equal to \( N \). The only problem is that it involves two different types of multiple access techniques which implies two types of transmitters and receivers. The example developed in [1] is based on a particular combination of TDMA and OCDMA.

The purpose of this paper is to generalize the concept introduced in [1] to full DS/CDMA. By defining two sets of orthogonal spreading sequences and using an iterative detection technique to cancel interference between the two sets of users, we devise a DS/CDMA scheme which can accommodate \( N \) users (where \( N \) is the spreading factor) without any mutual interference, while also accommodating a number of additional users at the expense of some SNR penalty. The proposed technique consists of assigning orthogonal Walsh–Hadamard (WH) spreading sequences, overlaid by the same PN scrambling sequence, to the first \( N \) users, and reusing the same set of WH spreading sequences, but overlaying them with a different PN sequence, for all additional users. We therefore have two sets of users when \( K > N \). The signals transmitted by users from the same set are mutually orthogonal, but there is no orthogonality between users from different sets. Detection is performed iteratively, each iteration consisting of two steps, one to detect the signals transmitted by the first set of users and the other to detect the signals transmitted by the second set of users.

II. BASIC PRINCIPLE

Consider a DS/CDMA system with a spreading factor of \( N \), and assume that we want to accommodate a number of users \( K = N + M \), where \( M < N \). It is well known that the number of orthogonal sequences of length \( N \) is exactly \( N \), and therefore orthogonal sequences of this length can be assigned to \( N \) users only. Accordingly, we assign the \( N \) WH sequences to the first \( N \) users which form the first set of users, and overlay them by a common PN sequence for scrambling. Throughout the paper, we assume that the channel is an additive white Gaussian noise (AWGN) channel and that different user signals are in perfect time synchronism. Then, there is obviously no mutual interference between these users, and the only interference for them is that of the \( M \) additional users to which we assign \( M \) of those WH sequences and a different PN sequence for scrambling. In
the sequel, we refer to the first set of \( N \) users as set1-users and to the second set of \( M \) users as set2-users.

The interference power (in-phase plus quadrature) from each set2-user (assuming that the useful signal power is normalized by 1) is \( 1/N \), and therefore the total interference power that affects set1-users is \( M/N \). As long as \( M \) remains small compared to \( N \), preliminary decisions can be made on the symbols transmitted by the set1-users with some good reliability. But each of the set2-users gets an interference power of \( N(1/N) = 1 \) from set1-users. Clearly, the bit error rate (BER) performance will be poor for this set of users if detection is made prior to interference cancellation. The idea we exploit here is that once preliminary decisions are made for the symbols transmitted by set1-users, their interference can be synthesized and subtracted from the set2-user signals. But note that performance will be limited for both sets of users, particularly if the excess number of users \( M \) is large. Therefore, the first iteration decisions can only be regarded as preliminary decisions to be used for interference synthesis and cancellation, and the detection process must be reiterated to get more reliable final decisions. The detection process is described below.

### III. Iterative Multistage Detection

To begin with, suppose \( \{W_i\} \), \( i = 1, 2, \ldots, N \) designate the \( N \) binary WH sequences. We write \( W_i = (w_{i,1}, w_{i,2}, \ldots, w_{i,N}) \). That is, \( w_{i,m} \) designates the \( m \)th chip of the sequence \( W_i \). Note that the \( W_i \) sequences are independent of the symbol index, because they repeat from one symbol to the next. Next, suppose that \( \{P_i\} \), \( i = 1, 2 \) designate the two PN sequences that overlay the WH sequences. We write \( P_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,N}) \). The resulting composite sequences for user \( i = 1, 2, \ldots, N \) and user \( N + k = 0, 1, \ldots, M \) are denoted \( (\alpha_{i,1}, \ldots, \alpha_{i,N}) \) and \( (\beta_{k,1}, \ldots, \beta_{k,N}) \), respectively, with \( \alpha_{i,m} = w_{i,m}p_{i,m} \) and \( \beta_{k,m} = w_{k,m}p_{2,m} \) for \( m = 1, 2, \ldots, N \). In order to split the interference power evenly over the in-phase and quadrature components of the useful signal (irrespective of the carrier phases), we consider complex-valued PN sequences: The chips \( p_{i,m} \) randomly take values from the set \( \{\exp(j\pi/2), \exp(-j\pi/2), \exp(j3\pi/2), \exp(-j3\pi/2)\} \).

**Step 1:** As mentioned earlier, the interference affecting set1-users is limited, and therefore, the received signals for this set of users can be passed to a threshold detector right after despeaking by the corresponding composite chip sequences. This first step of the detection process gives a set of decisions which we denote \( \hat{a}_{1}, \hat{a}_{2}, \ldots, \hat{a}_{N} \).

**Step 2:** The receiver decisions available from step 1 are used to synthesize the interference of set1-users on set2-users and subtract it from the signals of the latter set of users. Let us examine how interference cancellation is performed for the user with index \( N + k \), \( k = 1, 2, \ldots, M \). The total interference from the set1-users at the input of the decision device is

\[
I_{N+k} = \frac{1}{N} \sum_{i=1}^{N} a_i \left( \sum_{j=1}^{N} \alpha_{i,j} \beta_{k,j}^* \right)
\]

where \( a_i \) is the data symbol transmitted by the \( i \)th user during the current symbol interval. Each term in the outer sum in (1) represents the interference from one user. Since the chip sequences \( (\alpha_{i,1}, \ldots, \alpha_{i,N}) \) and \( (\beta_{k,1}, \ldots, \beta_{k,N}) \) are known to the receiver, \( I_{N+k} \) can be estimated once the symbol decisions corresponding to users 1 to \( N \) of user-set 1 are made. This estimate of \( I_{N+k} \) is subtracted from the corresponding signal at the correlator output before sending this signal to the threshold detector. If all decisions are correct, interference cancellation is perfect, and there is no mutual interference left. But every wrong decision in the first step will cause the corresponding term in \( I_{N+k} \) to double, worsening detection for set2-users.

**Second Iteration:** The symbol decisions made for set2-users in the first iteration are used to synthesize and subtract the interference of these users from the set1-users signals. The interference from set2-users in the \( k \)th user signal \( k = 1, 2, \ldots, N \) is given by

\[
I_k = \frac{1}{N} \sum_{i=1}^{N} a_{N+i} \left( \sum_{j=1}^{N} \alpha_{N+i,j}^* \beta_{k,j} \right).
\]

This interference is synthesized by substituting \( \hat{a}_{N+i} \) for \( a_{N+i} \) in (2) for \( i = 1, 2, \ldots, M \). Since \( \hat{a}_{N+i} = a_{N+i} \) with a probability close to 1, the synthesized replica is virtually identical to the actual interference. The synthesized interference is subtracted from the \( k \)th set1-user signal at the correlator output, and the resulting signal is passed to a threshold detector. This process is repeated for all set1-users. Since interference cancellation is close to perfect, the BER performance of the second-iteration decisions for set1-users will be close to the ideal curve corresponding to zero MUI.

The second-iteration decisions for set2-users are made after subtracting the interference of set1-users based on the second-iteration decisions. The total interference corrupting the \( k \)th set2-user (user with index \( N + k \)) is given by (1). After subtracting the best available estimate of this interference, the correlator output for the \( k \)th set2-user is sent to the threshold detector which makes the second-iteration decision for this user. The results indicate that if the number of excess users \( M \) is not too large, the second or third iteration gives sufficiently good performance and the detection process stops at this iteration. But for larger values of \( M \), further improvements are still possible from additional iterations. Note that operation of the iterative multistage detector used has some similarities with the iterative decoding used in turbo codes [4].
Performance of the presented DS/CDMA concept was investigated by means of computer simulations using binary phase-shift keying (BPSK) modulation, a spreading factor of $N = 64$, and $M = 12$ excess users. The simulated BER results are given as a function of the transmitted energy per bit to the noise spectral density ratio $E_b/N_0$ in Fig. 1 for set1-users and in Fig. 2 for set2-users. The figures also show the BER curve when symbol decision errors are ignored in the interference synthesis and cancellation stages. This corresponds to ideal BPSK performance over an AWGN channel. Note that in the absence of decision errors, steady-state performance is always reached at the second iteration, as confirmed by Figs. 1 and 2 which show both the theoretical curve as well as that obtained after the second iteration in the computer simulations. The results show that the third iteration gives a BER performance for set1-users that is extremely close to the ideal BPSK curve. As for set2-users, the SNR degradation at the BER of $10^{-4}$ that is achieved at the second iteration is 0.5 dB. The performance difference between the two sets of users is due to the higher interference affecting the latter set.

V. CONCLUSIONS

We have introduced a new DS/CDMA scheme which significantly increases the number of users that can be accommodated in a given bandwidth. With a spreading factor of $N$, it assigns orthogonal spreading sequences to the first $N$ users and another set of orthogonal sequences to the additional users when the number of users $K$ exceeds $N$. An iterative multistage detection technique is used to cancel interference between the two sets of users. At each stage, preliminary symbol decisions from the previous stages are used to synthesize and subtract interference from the received signal before passing it again to a threshold detector and make more reliable decisions. Our simulation results using BPSK modulation, a spreading factor $N = 64$, and $M = 12$ excess users indicate that already at the second iteration the BER curve is close to the ideal BPSK curve.

REFERENCES