Entropy measures for topological approximations of uncertain concepts

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Abstract

Most real life situations have uncertain concepts, the search for approximating uncertain concepts and measuring the accuracy of approximation is an important goal for researchers in many of theoretical and application fields. The purpose of this paper is to find topological approximation for uncertain concepts in information systems, and to compute some entropy measures for these topological approximations. Entropy for some knowledge bases is computed and three sorts of lower and upper approximations are initiated. The accuracy of these approximations, as well as, corresponding entropies are obtained.

Keywords: Rough set, General relation, Entropy, Topological approximations

1. Introduction

It is possible to say that along many decades man depends in making and taking his decisions on the results of analyzing the available data about the problems of interest [1]. The current age is characterized by the revolution of communications and computer technology, these facilities imply that collected data may be incomplete, having missing values, uncertain and vague [2]. The analyses of such data need the use of intelligent computational methods such as rough sets, fuzzy sets and hybrid methods [3 – 5]. Theory of topological spaces is a well-known theory that was combined with rough set theory to get new topological approximations for uncertain concepts in information systems [6, 7].
The aim of this paper is to find entropy measures for topological approximations, to the best of our knowledge, entropy measures were only applied to approximations based on equivalence relations [8, 9]. These measures will open up the way for wide range of choices to decision makers and takers because topological approximations decompose the boundary region to a set of multi sub regions while approximations based on equivalence relations look to boundary as a single region. The rest of the paper is organized as follow; article 2 is about basic concepts of entropy measures and topological constructions on information systems. Article 3 is concerned with introducing similarity relation for each subclass of attributes in information systems. Also, to generate general topologies associated with every subclass of attributes and to use successive effect of closure and interior operators to get new approximations. This article contains six sub articles in which we construct topological approximations based on near open sets and give examples for each type. These types of topological approximations help in decreasing the boundary regions for uncertain concepts and consequently increase the accuracy of approximations for uncertain concepts in general and specially the quality of decisions [10].

2. Basic concepts

This article is devoted for presenting basic definitions of some rough entropy measures and topological approximations.

2.1 Entropy measures

The following are the entropy measures that will be computed for the topological approximations of uncertain information concepts.
Shannon’s entropy [11]
Let \(S = (U, A)\) be an information system, \(U / A = \{X_1, X_2, \ldots, X_n\}\) is a granulation on \(U\) and \(p_i = p(X_i) = \frac{|X_i|}{|U|}\).

- Shannon entropy is defined by

\[
H(A) = -\sum_{i=1}^{n} \frac{|X_i|}{|U|} \log_2 \frac{|X_i|}{|U|}
\]

Complement entropy

Complement entropy is given by

\[
E(A) = \sum_{i=1}^{n} \frac{|X_i| |X_i^c|}{|U| |U|} \quad \text{where, } X_i^c \text{ is the complement set of } X_i.
\]

The above two measures of randomness can be used to measure the significance of attributes in an information system.

\[
\text{Sig}_H(a, A) = H(A) - H(A \cup \{a\})
\]

\[
\text{Sig}_E(a, A) = E(A) - E(A \cup \{a\})
\]

Conditional entropy in decision tables

**Definition 2.1:**

Let \(S = (U, A, D)\) be an information system, \(U / A = \{X_1, X_2, \ldots, X_n\}\) and \(U / D = \{Y_1, Y_2, \ldots, Y_m\}\) are a partition on \(U\) and \(p_i = p(X_i) = \frac{|X_i|}{|U|}\), \(p_j = p(Y_j) = \frac{|Y_j|}{|U|}\).

\[
H(D/C) = -\sum_{i=1}^{n} \frac{|X_i|}{|U|} \sum_{j=1}^{m} \frac{|X_i \cap Y_j|}{|X_i|} \log_2 \frac{|X_i \cap Y_j|}{|X_i|},
\]

where \(X_i \cap Y_j \neq \emptyset\).
Complementary conditional entropy

\[ E(D/C) = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{|Y_j \cap X_i|}{|U|} \frac{|Y_j^c - X_i^c|}{|U|} \]

2.2. Topological approximations

In the original rough set model, the approximation space \((U, R)\) with equivalence relation \(R\) defines a uniquely topological space \((U, \tau_R)\) where \(\tau_R\) is the family of all clopen sets in \((U, \tau_R)\) and \(X/R\) is a base of \(\tau_R\). Also, the lower (resp. upper) approximation of any subset \(A \subseteq U\) is exactly the interior (resp. closure) of the subset \(A\). Thus the starting point of applying topological concept is the use of closure and interior in the approximation process.

In this section, we shall generalize Pawlak’s [12] concepts in the case of general relations. Hence the approximation space \((U, R)\) with general relation \(R\) defines a uniquely topological space \((U, \tau_R)\) where \(\tau_R\) is the family of all open sets in \((U, \tau_R)\) and \(X/R\) is a subbase of \(\tau_R\).

The following is an example for generating a topology from a general binary relation:

**Example 3.1:**

If \(U = \{a, b, c, d\}\), \(R\) is a general binary relation on \(X\), where

\[ aR = \{b, c\}, bR = \{a, c, d\}, cR = \{c\}, dR = \{a, c\} \& eR = \{d\} \]

\[ U/R = \{aR, bR, cR, dR, eR\} \] is the subbase and

\[ \beta = \{\phi, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{a, c, d\}\} \] is the base for the topology \(\tau_R\) generated by \(R\) and the topology is
The family of closed sets is
\[ \tau_\mathcal{R} = \{ \emptyset, U, \{ c \}, \{ d \}, \{ a, c \}, \{ b, c \}, \{ a, c, d \}, \{ c, d \}, \{ b, c, d \}, \{ a, b, c \} \} \]

The class of clopen sets with respect to this topology is
\[ \tau_\mathcal{R}^c = \{ U, \phi, \{ a, b, d \}, \{ a, b, c \}, \{ b, d \}, \{ a, d \}, \{ b \}, \{ a, b \}, \{ a \}, \{ d \} \} \]

The class of clopen sets with respect to this topology is \( \{ U, \phi, \{ d \}, \{ a, b, c \} \} \).

The construction of topological approximation space using general binary relations open the way for using closure and interior operators in the approximation process for uncertain concepts.

The following is an example for computing degree of certainty of uncertain concepts using topological approximations.

**Remark**

In General \( \overline{\mathcal{R}}(RX) \neq RX \) and \( \mathcal{R}(\overline{RX}) \neq \overline{RX} \). The following examples illustrate this idea.

**Example 3.2:**

Let \( U = \{ x_1, x_2, x_3, x_4, x_5 \} \) and \( R \) be a general relation defined as follows:

\[ R = \{(x_1,x_2),(x_1,x_4),(x_2,x_5),(x_2,x_1),(x_3,x_3),(x_3,x_2),(x_4,x_5),\]
\[ (x_5,x_5),(x_5,x_1)\} \]

\[ \frac{U}{R} = \{\{x_2,x_4\},\{x_1,x_3\}\{x_3,x_2\}\{x_5\}\} \]

\[ \tau = \{U, \phi, \{ x_2 \}, \{ x_3 \}, \{ x_2, x_4 \}, \{ x_1, x_5 \}, \{ x_3, x_2 \}, \{ x_2, x_3, x_4 \}, \]
\[ \{ x_2, x_4, x_5 \}, \{ x_1, x_2, x_5 \}, \{ x_2, x_3, x_5 \}, \{ x_1, x_2, x_4, x_5 \}, \{ x_1, x_2, x_3, x_5 \}, \{ x_2, x_3, x_4, x_5 \}\} \]

\[ \tau^c = \{U, \phi, \{ x_1 \}, \{ x_3 \}, \{ x_4 \}, \{ x_1, x_4 \}, \{ x_3, x_4 \}, \{ x_1, x_3, x_5 \}, \{ x_1, x_3, x_4 \}, \]
\[ \{ x_1, x_4, x_5 \}, \{ x_2, x_3, x_4 \}, \{ x_1, x_3, x_5 \}, \{ x_1, x_2, x_3, x_4 \}, \{ x_1, x_3, x_4, x_5 \}\} \]
Let \( X = \{x_2, x_4\} \), then

\[
RX = \{x_2, x_4\}, \quad \overline{R(RX)} = \{x_1, x_2, x_3, x_4\} \neq \{x_2, x_4\} = RX.
\]

\[
\overline{RX} = \{x_1, x_2, x_3, x_4\}, \quad \overline{R(\overline{RX})} = \{x_2, x_3, x_4\} = \overline{RX}.
\]

### 2.3 Entropy of General classifications

The use of topological structures associated with the relations resulted from available information opens up the way for applying facts and results in the theory of topological spaces in all issues of rough set theory such as classifications, approximations and accuracy of decisions.

In this paper, we aim to compute some entropy measures for some topological approximations of uncertain concepts in information systems. These approximations depend on the successive applications of closure and interior operators.

### 3. Some important definition

**Definition 3.1[12]:**

Let \((U, R)\) be an approximation space with general relation \( R \) and \( \tau_R \) is the topology generated by \( R \). Then the triple \((U, R, \tau_R)\) is called a topological approximation space.

The following definition introduces the lower and the upper approximations in a topological approximation space \((U, R, \tau_R)\).

**Definition 3.2[12]:**

Let \((U, R, \tau_R)\) be a topological approximation space. If \( A \subseteq U \), then the lower approximation (respective upper approximation) of \( A \) is defined by

\[
\overline{RA} = A^\circ (resp. \overline{RA} = A^\circ).
\]
The following definition introduces new concepts of definability for a subset $A \subseteq U$, in a topological approximation space $(U, R, \tau_R)$.

**Definition 3.3[12]:**

Let $(U, R, \tau_R)$ be a topological approximation space. If $A \subseteq U$, then

i) $A$ is totally $R$—definable (exact) set if $\overline{RA} = A = \overline{A}$,

ii) $A$ is internally $R$—definable set if $A = \overline{RA}$,

iii) $A$ is externally $R$—definable set if $A = \overline{RA}$,

iv) $A$ is $R$—indefinable (rough) set if $A \neq \overline{RA}, A \neq \overline{RA}$.

The following is an example for generating topologies using general binary relations

**Example 3.3**

Considering the following Information system:

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>86</td>
<td>83</td>
<td>77</td>
</tr>
<tr>
<td>$x_2$</td>
<td>93</td>
<td>85</td>
<td>81</td>
</tr>
<tr>
<td>$x_3$</td>
<td>89</td>
<td>60</td>
<td>78</td>
</tr>
<tr>
<td>$x_4$</td>
<td>88</td>
<td>60</td>
<td>82</td>
</tr>
<tr>
<td>$x_5$</td>
<td>91</td>
<td>85</td>
<td>87</td>
</tr>
</tbody>
</table>

The objects $\{x_1, x_2, x_3, x_4, x_5\}$ represent the ID of 5 students, the attributes $\{a_1, a_2, a_3\}$ are 3 subjects studied by the students the values are the degrees scored by the students in an examination. And consider the relation $R_i$ on the set of objects defined by:
\( xR_iy \) if \( f |a_i(x) - a_i(y)| \leq 2, i = 1, 2, 3 \)

Then we can get the following classifications corresponding to every sub class of attributes and compute the entropy of each classification using the following measure

\[
H(A) = - \sum_{i=1}^{n} \frac{|X_i|}{|U|} \log_2 \frac{|X_i|}{|U|}
\]

1. For \( a_1 \):

\[
x_1R_1 = x_4R_1 = \{x_1, x_4\}, x_2R_1 = x_5R_1 = \{x_2, x_5\}, x_3R_1 = x_4R_1 = \{x_3, x_4\}
\]

\( S_{a_1} = \{\{x_1, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}\} \) is the granulation of \( U \) its entropy is

\[
H(S_{a_1}) = -3(\frac{2}{5} \log \frac{2}{5}) = 0.48
\]

2. For \( a_2 \):

\[
x_1R_2 = x_2R_2 = x_5R_2 = \{x_1, x_2, x_5\}, x_3R_2 = x_4R_2 = \{x_3, x_4\}
\]

\( S_{a_2} = \{\{x_1, x_2, x_3\}, \{x_3, x_4\}\} \).

\[
H(S_{a_2}) = -(\frac{3}{5} \log \frac{3}{5} + \frac{2}{5} \log \frac{2}{5}) = 0.38
\]

3. For \( a_3 \):

\[
x_1R_3 = x_3R_3 = \{x_1, x_3\}, x_2R_3 = x_4R_3 = \{x_2, x_4\}, x_5R_3 = \{x_5\}
\]

\( S_{a_3} = \{\{x_1, x_3\}, \{x_2, x_4\}, \{x_5\}\} \)

\[
H(S_{a_3}) = -2(\frac{2}{5} \log \frac{2}{5}) - \frac{1}{5} \log \frac{1}{5}
\]

We can construct classes that depend on two or three attributes as follow:

i. \( S_{\{a_1,a_2\}} = \{\{x_1, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}\} \)

ii. \( S_{\{a_1,a_3\}} = \{\{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_5\}\} \)

iii. \( S_{\{a_2,a_3\}} = \{\{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_3, x_5\}, \{x_1, x_2, x_3\}, \{x_3\}\} \)
Attribute Topologies

Then the topologies and the complement topologies generated by the previous classes are:

i. **Topology associated with** \( a_1 = EL(1) \):

\[
\tau_{EL(1)} = \{U, \phi, \{x_4\}, \{x_1, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_2, x_4, x_5\}, \{x_1, x_3, x_4\}, \}
\]

\[
\tau_{EL(1)}^c = \{U, \phi, \{x_1\}, \{x_3\}, \{x_1, x_3\}, \{x_2, x_5\}, \{x_1, x_2, x_5\}, \{x_1, x_3, x_4\}, \}
\]

In the following table, we compute the entropy of topological approximations for some subsets using the following two formulas

\[
\alpha(A) = \frac{|A^0|}{|A^-|}, \quad E_\alpha(A) = -\alpha(A) \log \alpha(A)
\]

<table>
<thead>
<tr>
<th>( A )</th>
<th>( A^* )</th>
<th>( A^- )</th>
<th>( \alpha(A) )</th>
<th>( E_\alpha(A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{x_5}</td>
<td>\phi</td>
<td>{x_2, x_5}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{x_1, x_2}</td>
<td>\phi</td>
<td>{x_1, x_2, x_5}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{x_2, x_3}</td>
<td>\phi</td>
<td>{x_2, x_3, x_5}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{x_2, x_4}</td>
<td>{x_2, x_4}</td>
<td>{x_1, x_3, x_4}</td>
<td>0.66</td>
<td>0.39</td>
</tr>
<tr>
<td>{x_4, x_5}</td>
<td>{x_4}</td>
<td>U</td>
<td>0.20</td>
<td>0.46</td>
</tr>
</tbody>
</table>

In the following, we give an example for different approximations which have different accuracy measures but have the same entropy measures

ii. **Topology of** \( a_2 = Ch \) :

\[
\tau_{Ch} = \{U, \phi, \{x_1, x_2, x_5\}, \{x_3, x_4\}\}
\]

\[
\tau_{Ch}^c = \{U, \phi, \{x_1, x_2, x_5\}, \{x_3, x_4\}\}
\]
### Table (3)

<table>
<thead>
<tr>
<th>$A$</th>
<th>$A^*$</th>
<th>$A^-$</th>
<th>$\alpha(A)$</th>
<th>$E_\alpha(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${x_2}$</td>
<td>$\phi$</td>
<td>${x_1,x_2,x_5}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x_4}$</td>
<td>$\phi$</td>
<td>${x_3,x_4}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x_1,x_3}$</td>
<td>$\phi$</td>
<td>$U$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x_1,x_5}$</td>
<td>$\phi$</td>
<td>${x_1,x_2,x_5}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x_2,x_5}$</td>
<td>$\phi$</td>
<td>${x_1,x_2,x_5}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x_2,x_3}$</td>
<td>$\phi$</td>
<td>${x_2,x_3}$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table (4)

<table>
<thead>
<tr>
<th>$A$</th>
<th>$A^*$</th>
<th>$A^-$</th>
<th>$\alpha(A)$</th>
<th>$E_\alpha(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${x_1,x_4}$</td>
<td>$\phi$</td>
<td>${x_1,x_2,x_3,x_4}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x_2,x_4}$</td>
<td>${x_2,x_4}$</td>
<td>${x_2,x_4}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>${x_3,x_5}$</td>
<td>${x_5}$</td>
<td>${x_1,x_3,x_5}$</td>
<td>0.33</td>
<td>0.53</td>
</tr>
<tr>
<td>${x_4,x_5}$</td>
<td>${x_5}$</td>
<td>${x_2,x_4,x_5}$</td>
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<td>0.53</td>
</tr>
<tr>
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<td>${x_1,x_2,x_3}$</td>
<td>0.66</td>
<td>0.39</td>
</tr>
<tr>
<td>${x_1,x_2,x_5}$</td>
<td>${x_1,x_2,x_5}$</td>
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<td>0</td>
</tr>
<tr>
<td>${x_2,x_3,x_5}$</td>
<td>${x_5}$</td>
<td>${x_5}$</td>
<td>0.20</td>
<td>0.46</td>
</tr>
<tr>
<td>${x_2,x_4,x_5}$</td>
<td>${x_2,x_4,x_5}$</td>
<td>${x_2,x_4,x_5}$</td>
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</tr>
<tr>
<td>${x_3,x_4,x_5}$</td>
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<td>${x_5}$</td>
<td>0.20</td>
<td>0.46</td>
</tr>
<tr>
<td>${x_2,x_3,x_4}$</td>
<td>${x_1,x_2,x_3,x_4}$</td>
<td>${x_1,x_2,x_3,x_4}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>${x_2,x_4,x_3}$</td>
<td>${x_1,x_3,x_4}$</td>
<td>${x_1,x_3,x_4}$</td>
<td>0.60</td>
<td>0.44</td>
</tr>
<tr>
<td>${x_2,x_3,x_5}$</td>
<td>${x_2,x_3,x_5}$</td>
<td>${x_2,x_3,x_5}$</td>
<td>0.60</td>
<td>0.44</td>
</tr>
</tbody>
</table>

### iii. Topology of $a_3 = \text{AL}(1)$:

$\tau_{\text{AL}(1)} = \{U, \phi, [x_5], [x_1,x_3],[x_2,x_4],[x_1,x_3,x_5],[x_2,x_4,x_5],[x_1,x_2,x_3,x_4]\}$

$\tau_{\text{AL}(1)} = \{U, \phi, [x_5], [x_1,x_3],[x_2,x_4],[x_1,x_3,x_5],[x_2,x_4,x_5],[x_1,x_2,x_3,x_4]\}$
\[ \lambda(A) = \frac{A^0}{A^{-}} \quad \text{and} \quad E_\lambda(A) = -\lambda(A) \log \lambda(A) \]

Table (5)

<table>
<thead>
<tr>
<th>A</th>
<th>A^0</th>
<th>A^-</th>
<th>A^+</th>
<th>A^-</th>
<th>λ(A)</th>
<th>E_λ(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{x_1, x_2}</td>
<td>\phi</td>
<td>{x_1, x_2, x_3, x_4}</td>
<td>\phi</td>
<td>{x_1, x_2, x_3, x_4}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>{x_2, x_4}</td>
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<td>0</td>
<td></td>
</tr>
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<td>0</td>
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<td>{x_2, x_3, x_4}</td>
<td>{x_2, x_3, x_4}</td>
<td>0.20</td>
<td>0.46</td>
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<td>{x_2, x_3, x_4}</td>
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<td>0</td>
<td></td>
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<tr>
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<td>{x_2, x_3, x_4}</td>
<td>0.60</td>
<td>0.44</td>
<td></td>
</tr>
</tbody>
</table>

\[ \omega(A) = \frac{A^{0-q}}{A^{-q}} \quad \text{and} \quad E_\omega(A) = -\omega(A) \log \omega(A) \]

Table (6)

<table>
<thead>
<tr>
<th>A</th>
<th>A^{-} \cup A^{-}</th>
<th>A^{-}</th>
<th>A^{-}</th>
<th>A^{-}</th>
<th>\omega(A)</th>
<th>E_ω(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{x_1}</td>
<td>{x_1} \cup {x_2}</td>
<td>\phi</td>
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<td>0</td>
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<tr>
<td>{x_2}</td>
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<td>{x_2} \cup {x_2}</td>
<td>{x_2} \cup {x_2}</td>
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<td>0</td>
<td></td>
</tr>
<tr>
<td>{x_1, x_2}</td>
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<td>{x_1, x_2, x_3, x_4}</td>
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<td>0.52</td>
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</tr>
<tr>
<td>{x_2, x_3}</td>
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<td>{x_2, x_3, x_4}</td>
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</tr>
<tr>
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<td>{x_1, x_2, x_3} \cup {x_3}</td>
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</table>
iv. Topology of EL and Ch = EC

\[ \tau_{EC} = \{U, \phi, \{x_1\}, \{x_4\}, \{x_1, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_1, x_2, x_5\}, \{x_2, x_4, x_5\}\} \]

\[ \tau'_{EC} = \{U, \phi, \{x_1\}, \{x_3\}, \{x_1, x_3\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_1, x_2, x_5\}, \{x_2, x_3, x_5\}, \{x_1, x_3, x_4\}, \{x_1, x_2, x_3, x_5\}, \{x_2, x_3, x_4, x_5\}\} \]

In the following, we compute a new accuracy measure and its corresponding entropy measure

\[ \lambda(A) = \frac{A_0 - A}{A^{-1}}, \quad E_\lambda(A) = -\lambda(A) \log \lambda(A) \]

<table>
<thead>
<tr>
<th>( A_x )</th>
<th>( A^+ )</th>
<th>( A^- )</th>
<th>( A^{+ -} )</th>
<th>( A^{- +} )</th>
<th>( \lambda(A) )</th>
<th>( E_\lambda(A) )</th>
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</thead>
<tbody>
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<td>0.39</td>
</tr>
<tr>
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<tr>
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<td>{x_2, x_3, x_5}</td>
<td>{x_2, x_3, x_5}</td>
<td>{x_2, x_3, x_5}</td>
<td>0.50</td>
<td>0.39</td>
</tr>
<tr>
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</table>
In the following, we suggest a new accuracy measure for uncertain concepts and compute its corresponding Shannon entropy.

\[ \omega(A) = \frac{A^0 - \phi}{A^0} \quad \text{and} \quad E_\omega(A) = -\omega(A) \log \omega(A) \]

**Table (8)**

<table>
<thead>
<tr>
<th>( A )</th>
<th>( A^{-} \cup A^{-*} )</th>
<th>( A^{-*} )</th>
<th>( A^{-*-} )</th>
<th>( \omega(A) )</th>
<th>( E_\omega(A) )</th>
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</thead>
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<td>{x_4, x_5}</td>
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<td>{x_2, x_4, x_5}</td>
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<td>0.39</td>
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<td>0.44</td>
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</table>

Looking to this example from information point of view \( U/R \) represent information on \( U \). The topology generated by this information classifies the subset of \( U \) into certain concepts, with accuracy 1 and uncertain concepts, with accuracy different from 1 . The last two decades of 20\textsuperscript{th} century have respected the use of successive application of closure and interior operates in constructing new closure of subset and new type of closure and interior, in the following example, we use successive
application of closure and interior with respect the topology generated by relation.

**Conclusion**

The approach presented in this paper is based on using general relation in constructing topological structure from the available data tables known by information systems. The general relation used in this work is a generalization for all types of relations used in approximations, for example more general than the equivalence relation of Pawlak original theory. The topology generated by the general relation open the way for using near topological operators in the process of approximations. Moreover, many forms of lower and upper approximations increase the accuracy of approximations of uncertain concepts in information systems which consequently help in the quality of decisions by using the resulted entropy measures.

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**References**

