Enhancement of Flow-Like Structures in Hyperspectral Imagery using Tensor Nonlinear Anisotropic Diffusion

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ABSTRACT

Analyzing flow-like patterns in images for image understanding is an active research area but there have been much less attention paid to the process of enhancement of those structures. The completion of interrupted lines or the enhancement of flow-like structures is known as Coherence-Enhancement (CE). In this work, we are studying nonlinear anisotropic diffusion filtering for coherence enhancement. Anisotropic diffusion is commonly used for edge enhancement by inhibiting diffusion in the direction of highest spatial fluctuation. For CE, diffusion is promoted along the direction of lowest spatial fluctuation in a neighborhood thereby taking into account how strongly the local gradient of the structures in the image is biased towards that direction. Results of CE applied multispectral and hyperspectral images are presented.

Keywords: Hyperspectral Image Processing, Nonlinear Diffusion, Coherence-Enhancement, Image Processing.

1 INTRODUCTION

The most stable and reliable descriptor of the local structure of an image is the structure tensor [1]. This tensor is a symmetric positive semi definite matrix that at each pixel determines the orientation of minimum and maximum spatial fluctuation in a neighborhood of a pixel. It was presented by [2] and also by [3] in an equivalent formulation for gray level imagery. The structure tensor is derived from the gradient; it provides the main directions of the gradient in a specified neighborhood of a point. Its eigen-decomposition provides information of how strongly the gradient is biased towards a particular direction which is known as coherence [4]. Oriented flow-like structures can be found by looking to the orientation of lowest spatial fluctuation in an image, and it is determined by the eigenvector of the structure tensor with the smallest eigenvalue. This process has been studied in the context of gray and color images [5] by the pattern recognition and computer vision fields and used for automatic grading of fabrics or wood surfaces [6], segmenting two-photon laser scanning microscopy images[7], enhancing fingerprint images [1], enhancing corners [2] and 3-D medical imaging [3,6]. But few have been done with vector-valued data such as multispectral (MSI) and hyperspectral images (HSI).

This paper study a PDE-based coherence diffusion method proposed by [6] and uses a finite volume scheme proposed by [7]. The main contribution in this paper is to extend CED to MSI and HSI, and propose eigenvalues for the diffusion tensor. In addition, it will be shown that coherence enhancement can be used for remote sensing image processing problems. Results will be presented for multispectral, and hyperspectral images. This paper is organized as follows. Section 2 introduces CED, the diffusion tensor, and some properties. Section 3 presents an overview of finite volume discretization. Section 4 shows some experimental results.

2 COHERENCE ENHANCING DIFFUSION

Adaptive smoothing methods are based on the idea of applying a process which itself depends on local properties of the image. A regularized adaptive method can be extended to anisotropic processes which make use of an adapted diffusion tensor instead of scalar diffusivity. This is the case of the anisotropic diffusion model introduced by Weickert in [1,6]. Let us consider a scale image domain \( \Omega := (0,n_1) \times (0,n_2) \) with \( n_1 \) number of rows and \( n_2 \) number of columns, boundary \( \partial \Omega \) and \( I := (0,\infty) \). The following initial boundary value problem represents a class of anisotropic diffusion filters:

\[
\frac{\partial u}{\partial t} - \nabla \cdot (Du) = 0 \quad \text{in } \Omega \times I \quad \text{(1.1)}
\]

\[
u(x,0) = f(x) \quad \text{in } \Omega \quad \text{(1.2)}
\]
Hereby, \( u \), the filtered version of image \( f(x) \), is the solution of the initial boundary value problem for the diffusion equation with \( f \) as initial condition, see Equation (1.2). Equation (1.1) has a partial derivative with respect to time, in image processing there is no real evolution in time, variable \( t \) in this case is a dummy variable that represent an iterative process. The vector \( n \) denotes the outer normal unit vector to \( \partial \Omega \) and \( \langle \cdot , \cdot \rangle \) is the Euclidean scalar product on \( \mathbb{R}^2 \). Equation (1.3) is a Neumann boundary condition, which means that the flux is zero outside the boundary. Equations (1.1) to (1.3) will be called \( \text{P1} \). CED is basically a 1-D diffusion, where a minimal amount of isotropic smoothing is added for regularization purposes.

In order to make the diffusion tensor \( D \in \mathbb{R}^{2 \times 2} \) adaptive, in the sense that a strong smoothing is obtained in one direction and low smoothing along the edges, it will depend on the structure tensor that in turn depends on the edge estimator \( \nabla u_\sigma \), which is also known as the smoothed gradient. \( \nabla u_\sigma \) is obtained by finding the gradient of the smoothed image \( u_\sigma \) obtained by a Gaussian convolution \( G_\sigma \) of variance \( \sigma > 0 \)

\[
    u_\sigma(x,t) := G_\sigma * \bar{u}(\cdot,t)(x), \quad \sigma > 0
\]

\[
    G_\sigma := \frac{1}{(2\pi\sigma^2)^2} \exp\left(-\frac{|k|^2}{2\sigma^2}\right),
\]

where \( \bar{u} \) denotes the extension of \( u \) from \( \Omega \) to \( \mathbb{R}^{2 \times 2} \) that can be obtained by mirroring at \( \partial \Omega \) [8] and the * is a convolution. Figure 1 shows a block diagram of CED for a scalar image. The following sections will discuss some of these steps.

2.1 Diffusion Tensor
The diffusion tensor \( D \) was designed to generate a strong smoothing in a preferred direction and a low smoothing perpendicular to it, e.g. for images with interrupted coherence of structures, \( D \) is built upon the structure tensor \( J_\rho \). This tensor provides the main directions of the gradient in a specified neighborhood of a point. Its eigen-decomposition
provides information of how strongly the gradient is biased towards a particular direction, which is known as coherence direction. For the oriented flow-like case, we want the direction of lowest spatial fluctuation in the image. The structure tensor is defined by [6]:

\[ J_\rho = G_\rho \ast J_0 \]

where \( J_0 = \nabla u_\sigma \nabla u_\sigma^T \), with \( \sigma, \rho > 0 \), \( u \) and \( G_\sigma \) are as in Equations (1.4) and (1.5). \( J_0 \) is a \( 2 \times 2 \) symmetric, positive semi-definite matrix and its eigenvectors are parallel and orthogonal to \( \nabla u_\sigma \). Why choose the structure tensor as an outer product and do not use directly the smoothed gradient? Cottet et al [9] presented a reaction-diffusion model where the diffusion was along the eigenvector corresponding to the smallest eigenvalue, \( \sigma_2 = g \left( \left\| \nabla u_\sigma \right\|^2 \right) \) of its diffusion tensor, which, in turn, depends on the magnitude of the smoothed gradient. But this model did not produce a scale-space (see §2.3), and its diffusion tensor \( D \) had eigen-directions adapted to \( \nabla u_\sigma \), which makes them sensible to changes in \( \sigma \).

Figure 2(b)-(c) illustrate the gradient orientation using grey values. Vertical gradients are depicted in black and horizontal ones in white. It is observed that if \( \sigma \) is too small then high fluctuation of noise remains. As \( \sigma \) gets larger then it is useless, since neighboring gradients with the same orientation, but with opposite sign (direction) cancels one another. Therefore, gradient smoothing averages directions instead of orientations [8]. Figure 2(d) shows the coherence orientation for the fingerprint image and also how well the fingerprint singularity (minutiae) is described. Consequently, the matrix representation of the image gradient allows the integration of information from a local neighborhood without cancellation effects.

![Figure 2](image_url)

Figure 2. Smoothed Gradient vs. Structure Tensor in a 227×227 fingerprint image. (a) Original image. (b) Gradient orientation \( \sigma = 0.5 \). (c) Gradient orientation \( \sigma = 2.5 \). (d) Structure Tensor \( \sigma = 0.5, \rho = 4 \)

For a vector valued image with \( m \) bands, the structure tensor is defined as the average [5]:

\[ J_\rho = \frac{1}{m} \sum_{i=1}^{m} G_\rho \ast J_{0,i} \]

where the constant have been drop for clarity and all the other definitions are as in the scalar value case [10]. The idea was to have one edge descriptor for all bands, such that edge information along a given direction in one channel reinforces edge evidence in the other channels [11]. Therefore, any process following the edges founded by the structure tensor will behave the same way in each channel and have the information from all channels. \( J_\rho \), with eigenvalues \( \mu_1 \geq \mu_2 \), is the average of \( J_0 \) in a neighborhood of size \( \sim 6 \rho \). From \( \mu = \mu_1 - \mu_2 \), useful information about coherence can be obtained. \( \mu^2 \) is large for anisotropic structures and tends to zero in isotropic ones. Constant areas are characterized by \( \mu_1 = \mu_2 \), straight edges by \( \mu_1 \gg \mu_2 \) and corners by \( \mu_1 > \mu_2 \gg 0 \) [7]. Let \( (v, w) \) be the orthogonal set of eigenvectors corresponding to eigenvalues \( (\mu_1, \mu_2) \). The diffusion tensor \( D \), which depends on solution \( u \), is constructed using \( (v, w) \) producing a filtering process such that diffusion is strong along the coherence direction \( w \) and increases with \( \mu^2 \). In addition, \( D \) satisfies smoothness, symmetry and uniform positive definiteness properties. \( D \) is defined as:

\[
D = \begin{bmatrix} v & w \end{bmatrix} \begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} \lambda & \beta \\ \beta & \nu \end{bmatrix}
\]

(2.1)

For this work, the eigenvalues of \( D \) will be chosen as follow:
Note that by definition $\kappa_2 \in (0,1)$, $\alpha$ is a small number that guarantees that the process never stops and keeps the diffusion tensor uniformly positive definite. In this work, $\alpha = 1e^{-3}$ is used, otherwise it will be specified. $C = 3.31488$ (see [12] §5.1) and $\psi$ are threshold parameters. Figure 3 LEFT shows that $\psi$ separate regions in the image. If $\mu < \psi$, then $(\mu / \psi)^4 << C$ and $\kappa_2 \approx \alpha$ and there is no preferred coherence direction and isotropic diffusion $(\kappa_1 = \kappa_2)$ of size $\alpha$ will be performed. On the other hand, if $\mu > \psi$, then $\kappa_2$ will gradually increase until $(\mu / \psi)^4 >> C$ and $\kappa_2 \approx 1$, therefore, the coherence direction is given by $\omega$ (see Equation (2.1)) and anisotropic diffusion $(\kappa_1 < \kappa_2)$ along this direction is performed. In the literature, Weickert [13] has proposed to use $\kappa_2 = \alpha + (1-\alpha) \exp\left(-\frac{C}{(\mu_1 - \mu_2)^2}\right)$, for the case where $\mu_1 \neq \mu_2$, in this case $C$ is a parameter and not a fixed constant as in our case. Figure 3 RIGHTh shows that varying $C$ do not separate low contrast regions from high contrast ones. Indeed the values in the different lower elbows do not change by much. For $C = 1$, the elbow occur when $\mu_1 - \mu_2 = 0.4$ and for $C = 5$ when $\mu_1 - \mu_2 = 1$. The proposed $\kappa_2$ separates regions if the minimum value of the pixels we want to keep is known.

Figure 3. LEFT: Proposed $\kappa_2 = \alpha + (1-\alpha) \exp\left(-\frac{C}{(\mu_1 - \mu_2)^2}\right)$. CENTER: Proposed $\kappa_2$ changing the value of $\psi$. RIGHTh: Weickert’s $\kappa_2 = \alpha + (1-\alpha) \exp\left(-\frac{C}{(\mu_1 - \mu_2)^2}\right)$ changing the value of $C$ [13].

### 2.2 Discretization: Finite Volume Method

PDE methods assume that there is a continuous diffusion but images are discrete. Then it is necessary to discretize the PDE both in time (scale) and space to create the actual filter. The uniform distribution of the pixels constrains discretization meshes to be rectangular, structured, i.e., all elements and nodes have the same number of neighbors. Finite differences and finite volumes are two numerical methods suitable for meshes with those characteristics and to approximate conservative processes as in problem $P_1$ [14]. The method used in this paper is a semi-implicit finite volume using a co-volume proposed by [7]. The finite volume method is suited for differential equations describing conservative processes i.e. it assumed, as it is in our case, that the flux entering a given volume is identical to that leaving the adjacent one [14]. This method uses a uniform mesh given by the pixels but surrounding each node (pixel) there is a small volume, $K$ (in our case a square). Then it uses the integral form of Equation (1.1), which is obtained by multiplying Equation (1.1) by a weight function $\chi(A) = \begin{cases} 1 & \text{if } A \text{ is true} \\ 0 & \text{otherwise} \end{cases}$ and integrating over the volume $K$. Volume integrals in the partial differential equation are converted to surface integrals using the divergence theorem. The terms
are then evaluated as fluxes $\phi_i$ at the surfaces (edges $e_i$, $i=1,\ldots,4$) of each finite volume (square), then the fluxes are approximated using difference quotients, and in this case helped with a co-volume, then a matrix $A$ is assembled with the fluxes approximation in each volume. There are several schemes to find the solution $u$ at iteration $t = n+1$, denoted $u^{n+1}$. In the explicit scheme, the time step $n+1$ can be computed explicitly from the previous time step $n$ by the product $Au^n$. For the semi implicit, it is necessary to solve linear system of equations [15]. After the discretization in scale and space, application of the boundary condition to Equation (1.1) and assuming central-space and forward time the semi implicit scheme is given by $A(u^n)u^{n+1} = u^n$, $n = 0,\ldots,T-1$, where $A(u^n)$ is a functional notation to say that matrix $A$ depends in the actual solution $u^n$. The semi implicit finite volume scheme produces a nine diagonal linear system that for this work was solved with biconjugated gradient method [16].

\[
\begin{align*}
\text{Original Image} & \quad \rho = 1 & \quad \rho = 4 & \quad \rho = 6 \\
\end{align*}
\]

Figure 4. Impact of the integration scale on CED. $\sigma = 0.5, \psi = 1, t = 8$ with image size is 254×200.

\[
\begin{align*}
\text{Original Image} & \quad t = 10 & \quad t = 30 & \quad t = 50 \\
\end{align*}
\]

\[
\begin{align*}
\text{t = 80} & \quad \text{t = 100} & \quad \text{t = 150} & \quad \text{t = 200} \\
\end{align*}
\]

Figure 5. Scale-space produced by coherence-enhancing diffusion in a thyroid cell with $\sigma = 0.5, \rho = 4, \psi = 1, \Omega = (0, 252)^2$.

### 2.3 Scale-Space and Integration Parameter

There are two aspects about CED that are worth elaborating to fully understand this particular kind of diffusion: (i) what is the roll of the integration parameter, and (ii) the fact that this particular PDE-based CED produces a scale space.
2.3.1 Integration Parameter

Recall from Section 2.1 that the structure tensor $J_\rho$ is the average of the gradient orientations in a neighborhood of size $\rho$. Figure 4 shows an example of how the size of $\rho$ influence the result of CED. In this figure, the Van Gogh paint [17] is shown, all parameters are kept constant except the integration parameter. When $\rho = 1$, the filter will produce artifacts, while increasing it to $\rho = 4$ make flow-like structures look like completed lines since the average of orientations in this bigger neighborhood captures the coherence orientation, but further increase to $\rho = 6$ hardly makes any difference [12].

![Image](image_url)

Figure 6. Enhancement of a ship and its wake with CED using $\sigma = 1.2$, $\rho = 2.5$, $\psi = 0.01$ and $t = 10$. ROI from AVIRIS image. a) RGB of original Image with bands 29, 20, 11. b) After CED, c-d)Zoomed red box, e-f) Zoomed green box.

2.3.2 Scale-Space

Equation (1.1) has an evolution in time which simulates an iterative process instead of a real temporal evolution. In this case, $t$ is known as a scale. As $t$ is gradually increased from $t = 0$ with $u(x,0) = f(x)$ the original image, some diffusion processes produce a family $\{T_t f | t \geq 0\}$ of gradually smoother versions of $f$ which is known as scale-space, see Figure 5. Images usually contain structures at a large variety of scales. An image representation at multiple scales is useful in cases where it is not clear in advance, which the right scale for the depicted information is [18]. Moreover, a hierarchy of image structures can be obtained by comparing the structures at different scales which eases a subsequent image interpretation. Figure 5 shows a scale-space produced by CED applied to thyroid cells. In that figure, it is evident a hierarchical collection of simpler images that characterize the original image.
3 EXPERIMENTAL RESULTS

Experiments on enhancement of HSI and multispectral remote sensed data are presented in this section. In addition a discussion of the components of the diffusion tensor is provided.

![Figure 7. Effect of contrast parameter $\psi$ in the components of the diffusion tensor $D$. First row corresponds to component $\lambda$. Second row corresponds to component $\beta$. Third row corresponds to component $\nu$. Each column corresponds to a different value of parameter $\psi$.](image)

3.1 Enhancement of ships and its wakes in AVIRIS imagery [13].

Figure 6 shows the RGB composition with bands 29, 20, 11 respectively of a 335×533 region of interest (ROI) with 224 bands from an AVIRIS line taken from Deep Horizon Gulf of Mexico Oil Spill on May 17, 2010 [18]. The left column shows the original image and its two zoomed sections in red and blue boxes. The left column show the corresponding images after CED. It is notorious that the pixels belonging to the ship and its wakes are smoothed and the wake lines enhanced. In Figure 6 (d), it is shown that along the wake lines and ship pixels the oblique texture is eliminated, connecting the interrupted lines. In addition, the zoomed section corresponding to the red box shows that the wake is thickened. It is also noted that CED does not enhance the natural sea texture by much and there is no enhancement of the oblique texture. Recall from Equation (2.1) the diffusion tensor $D = \begin{bmatrix} \lambda & \beta \\ \beta & \nu \end{bmatrix}$ which in turn depends of eigenvalues $\kappa_1$ and $\kappa_2$ defined in Equation (2.2). Figure 7 shows the effect of the contrast parameter $\psi$ of eigenvalue $\kappa_2$ on the components $\lambda$, $\beta$, $\nu$ of the diffusion tensor $D$. $\lambda$ provides the changes in the horizontal direction, $\nu$ provides the changes in the vertical direction and $\beta$ changes in both directions. The ship is going approximately 17° SE, so all three components will detect it. Figure 6 was obtained using the components of diffusion tensor shown in the first column of Figure 7, with $\psi = 0.01$. Those images only show the pixels where the coherence $\mu$ is significantly larger than $\psi$. Figure 7 shows that as $\psi$ become larger fewer structures in the image are detected, to the point to almost exclusively detect the ship in the right corner of images of Figure 7, column 3. $\psi$ plays the role of a contrast parameter separating forward (low contrast) from backward (high contrast) diffusion areas, see Figure 3. With the introduction of the contrast parameter, it is possible enhance only the desired structures on the image without having to iterate many times to obtain a simpler image.
Figure 8 shows a ROI of band 7 of an image acquired using MODIS sensor [20] taken from North America between Washington state and Alaska, with spatial resolution of 500 m [19]. Also highlighted in red and green boxes there are two regions of two different kinds of clouds with observable ship contamination plumes. The rightmost column shows the enhancement of the edges of the plumes using CED. It can be observed that the edges of the plumes are smoother; the sensor texture in the blue box is not enhanced as shown in blue box in Figure 9. The red box ROI is more difficult since it is desirable to enhance the plumes edges without disturbing the cloud textures which is significant. This goal is achieved by setting $\psi$ small to capture the contrast between clouds and plumes and making $\sigma = 4$ so the process ignore the clouds as shown in red box in Figure 9.

Figure 8. ROI for Band 7 of MODIS image [19] with two smaller ROIs highlighted in red and green boxes. Blue Boxes: Plumes captured by altostratus clouds and its CED enhancement with $\sigma = 1$, $\rho = 3$, $\psi = 1$ and $t = 10$. Red Boxes: Plumes captured by altocumulus clouds and its CED with $\sigma = 4$, $\rho = 6$, $\psi = 0.35$ and $t = 10$.

Figure 9 Components $\lambda$, $\beta$, and $\nu$ of the diffusion tensor for the CED image of the red and blue boxes in Figure 8.

4 SUMMARY AND CONCLUSIONS

CED generates a scale-space evolution by means of a nonlinear anisotropic diffusion equation. Its diffusion tensor reflects the local image structure by using the same set of eigenvectors as the structure tensor. The latter is better suited to define the diffusion tensor than the smoothed gradient, since it does not produce cancellation effects and more information can be extracted from it. The eigenvalues of the diffusion tensor are chosen in such a way that diffusion acts mainly along the direction with the highest coherence, and becomes stronger when the coherence increases. The
proposed eigenvalue $\kappa_2$ for the diffusion tensor $D$ separates low contrast from high contrast diffusion areas; with this definition it is possible to customize the diffusion. It is easy to enhance desired structures since it is only necessary to know the minimum value of the pixel in the structure that will be enhanced. CED usefulness was illustrated by applying it to multispectral and hyperspectral images. In addition, this method can be used for remote sensing applications as the enhancement of ship wakes and contamination plumes in satellite data.

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6 REFERENCES

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