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A study on fuzzy irresoulte topological Vector spaces

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Abstract

In this paper, our focus is to investigate the notion of fuzzy irresolute topological vector spaces. Fuzzy irresolute topological vector spaces are defined by using fuzzy ρ-open sets and fuzzy irresolute functions. Some of their properties and characterizations are also discussed.

Keywords: Fuzzy irresolute topological vector space, Fuzzy left translation, Fuzzy right translation, Fuzzy ρ-irresolute homeomorphism, Fuzzy ρ-irresolute homogenous space. Rigid, anti-rigid, homogeneous, anti-homogeneous.
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1. Introduction

The concept of a fuzzy group was introduced by Rosenfeld [8]. This notion was extended to that of a fuzzy vector space by Katsaras and Liu [5] . The transition from a fuzzy group to a fuzzy vector space is similar to that from a group to a vector space. The extension consists of a fuzzification of the vector addition and the scalar multiplication in ordinary vector spaces. D. Mevamanoharan, S. Pious Missier and S. Jafari [4] initiated and explored a new class of open sets in a topological space called ρ-open sets.

The motivation behind the study of this paper is to investigate structures in which the topology is endowed upon a vector space which fails to satisfy the continuity condition for vector addition and scalar multiplication or either. The notion of irresolute topological vector spaces was definitioned by Moizud Din Khan and Muhammad Asad Iqbal [6] in 2016. The concept of irresolute function was further developed in [1]. We are interested to study such structures for fuzzy irresolute mappings. The concept of fuzzy irresolute was introduced by M.N. Mukherjee and S.P. Sinha [7]. In this paper, several new facts concerning fuzzy topologies of fuzzy irresolute topological vector spaces are established. The notion is definitioned although analogously but is independent of fuzzy linear topological space.

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2. Preliminaries

Let us go through some definitioninitions and theorems which are used in this paper.

Definition 2.1. [10] Let X be a non-empty set and I be the unit interval $[0, 1]$. A fuzzy set in X is an element of the I^X of all functions from X to I.

Definition 2.2. [3] Let X be a set and τ be a family of fuzzy subsets of X. Then τ is called fuzzy topology on X if satisfies the following conditions:

(1) $0_X, 1_X \in \tau$.

(2) If $\lambda, \mu \in \tau$, then $\lambda \wedge \mu \in \tau$.

(3) If $\lambda_i \in \tau$ for each $i \in I$, then $\vee\lambda_i \in \tau$.

The ordered pair (X, τ) is said to be a fuzzy topological space (in short, FTS). Moreover, the members of τ are said to be the fuzzy open sets and their complements are said to be the fuzzy closed sets.

Definition 2.3. [4] Let (X, τ) be a topological space. A subset $A \subseteq X$ is said to be

(1) $\hat{g}$-closed if if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

(2) ${}^*g$-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in X.

(3) ${}^\sharp g$-semi closed if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is ${}^*g$-open in X.

(4) $\tilde{g}$-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ${}^\sharp g$-semi open in X.

(5) ρ-closed if $pcl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is $\tilde{g}$-open in X.

Definition 2.4. [9] A space X is nearly (w) compact and (w) Hausdorff iff it is almost (w) compact and (w) Urysohn.

Definition 2.5. [6] A space $(X_{(F)}, \tau)$ is said to be an irresolute topological vector space over the field F if the following two conditions are satisfied:

(1) for each $x, y \in X$ and for each semi open neighbourhood W of $x + y$ in X, there exist semi open neighbourhoods U and V in X of x and y respectively, such that $U + V \subseteq W$.

(2) for each $x \in X$, $\lambda \in F$ and for each semi open neighbourhood W of λx in X, there exist semi open neighbourhoods U of λ in F and V of x in X, such that $U \cdot V \subseteq W$.

Definition 2.6. [5] A fuzzy topological vector space is a vector space E equipped with a fuzzy topology such that the two maps

(1) $\phi : E \times E \to E, (x, y) \to x + y$.

(2) $\psi : K \times E \to E, (\lambda, x) \to \lambda x$.

are continuous when K has the usual topology and $E \times E$, $K \times E$ are given the product fuzzy topologies.

Definition 2.7. [2, 3] A fuzzy set U in a fts (X, T) is a neighborhood, or nbhd for short, of a fuzzy set A if and only if there exists an open fuzzy set O such that $A \subset O \subset U$.

Definition 2.8. [6] If $X_{(F)}$ is a vector space then for fixed $x \in X$, ${}_xT : X \to X$, $y \mapsto x + y$ and $T_x : X \to X$, $y \mapsto y + x$ denote the left and right translation by x, respectively.

3. Fuzzy Irresolute Topological Vector Spaces

Throughout this paper, V is a vector space over the field K, the field of real or complex numbers. In this section, the concept of fuzzy irresolute topological vector space is introduced and some of their properties are discussed.

Definition 3.1. Let (X, τ) be a fuzzy topological space. Any fuzzy set $\lambda \in I^X$ is said to be fuzzy ρ-closed in (X, τ) if $Fpcl(\lambda) \leq Fint(\mu)$ whenever $\lambda \leq \mu$ and μ is a fuzzy $\tilde{g}$-open set. The complement of a fuzzy ρ-closed set is said to be a fuzzy ρ-open set.

Definition 3.2. Let (X_1, τ_1) and (X_2, τ_2) be any two fuzzy topological spaces and let $f : (X_1, \tau_1) \to (X_2, \tau_2)$. Then f is said to be fuzzy ρ-irresoulte at $\lambda \in I^{X_1}$, if for each fuzzy ρ-open $\mu \in I^{X_2}$ with $f(\lambda) \leq \mu$, there exists a fuzzy ρ-open set $\gamma \in I^{X_1}$ with $\lambda \leq \gamma$ such that $f(\gamma) \leq \mu$.

Definition 3.3. Let (X_1, τ_1) and (X_2, τ_2) be any two fuzzy topological spaces and let $f : (X_1, \tau_1) \to (X_2, \tau_2)$. Then f is said to be fuzzy ρ-irresoulte function if for each fuzzy ρ-open set $\mu \in I^{X_2}$, $f^{-1}(\mu) \in I^{X_1}$ is fuzzy ρ-open set in (X_1, τ_1).

Definition 3.4. Let (X, τ) be fuzzy topological space and $\lambda, \mu \in I^X$. Then λ is said to be fuzzy ρ-open neighborhood of μ if there exists a fuzzy ρ-open set $\delta \in I^X$ such that $\mu \leq \delta \leq \lambda$.

Definition 3.5. Let V be a vector space over the field F where F is endowed with usual fuzzy topology $\mathfrak{F}$. Then τ is said to be fuzzy irresolute topological vector space if

(1) for each $\alpha, \beta \in I^V$ and for each fuzzy ρ-open neighbourhood λ of $\alpha + \beta$ in (V, τ), there exist fuzzy ρ-open neighbourhoods $\gamma, \delta \in I^V$ of α and β respectively, such that $\gamma + \delta \leq \lambda$.

(2) for each $\alpha \in I^V$, $\xi \in I^F$ and for each fuzzy ρ-open neighbourhood λ of $\xi\alpha$ in (V, τ), there exist fuzzy ρ-open neighbourhoods $\gamma \in I^F$ of ξ in $(F, \mathfrak{F})$ and $\delta \in I^V$ of α in (V, τ) such that $\gamma \cdot \delta \leq \lambda$.

Definition 3.6. Let (V, τ) be any fuzzy topological vector space. Then

(1) If ${}_\lambda T : (V, \tau) \to (V, \tau)$ is defined by ${}_\lambda T(\mu) = \lambda + \mu$, for all $\lambda, \mu \in I^V$, then ${}_\lambda T$ is fuzzy left translation.

(2) If $T_\lambda : (V, \tau) \to (V, \tau)$ is defined by $T_\lambda(\mu) = \mu + \lambda$, for all $\lambda, \mu \in I^V$, then T_λ is fuzzy right translation.

Proposition 3.7. Let (V, τ) be any fuzzy irresolute topological vector space over the field F. Then

(1) The fuzzy (left) right translation $T_\lambda : (V, \tau) \to (V, \tau)$ be defined by $T_\lambda(\mu) = \mu + \lambda$, for all $\lambda, \mu \in I^V$ is fuzzy ρ-irresolute function.

(2) The fuzzy translation $T_\xi : (V, \tau) \to (V, \tau)$ be defined by $T_\xi(\mu) = \xi\mu$, for all $\xi \in I^F, \mu \in I^V$ is fuzzy ρ-irresolute function.

Proof. (1) Let $T_\lambda : (V, \tau) \to (V, \tau)$ be defined by $T_\lambda(\mu) = \mu + \lambda$, for all $\lambda, \mu \in I^V$. Let $\alpha \in I^V$ be fuzzy ρ-open neighborhood of $\mu + \lambda$. Then by Definition 3.5, there exist fuzzy ρ-open neighborhoods $\gamma, \delta \in I^V$ of μ and λ respectively such that $\gamma + \delta \leq \alpha$. Therefore, $T_\lambda(\gamma) = \gamma + \lambda \leq \gamma + \delta \leq \alpha$. Hence T_λ is fuzzy ρ-irresolute function.

(2) Let $T_\xi : (V, \tau) \to (V, \tau)$ be defined by $T_\xi(\mu) = \xi\mu$ for all $\xi \in I^F, \mu \in I^V$. Let $\beta \in I^V$ be any fuzzy ρ-open neighborhood of $\xi\mu$. Then by Definition 3.5, there exist fuzzy ρ-open neighborhoods $\gamma \in I^F$ of ξ and $\delta \in I^V$ of μ such that $\gamma \cdot \delta \leq \beta$. Therefore $T_\xi(\delta) = \xi\delta \leq \gamma \cdot \delta \leq \beta$. Hence T_ξ is fuzzy ρ-irresolute function. □

Notation 3.1. Let (V, τ) be any fuzzy topological space. Then the collection of all fuzzy ρ-open sets in (V, τ) is denoted by $F\rho O(V, \tau)$.

Proposition 3.8. Let (V,τ) be any fuzzy irresolute topological vector space over the field F. If $\mathfrak{R} \in F\rho O(V,\tau)$, then

(1) $\mathfrak{R} + \mu \in F\rho O(V,\tau)$, for every $\mu \in I^V$.

(2) $\xi\mathfrak{R} \in F\rho O(V,\tau)$, for every $\xi \in I^F$.

Proof. (1) Let $\gamma, \delta \in I^V$ and let $\delta \in \mathfrak{R} + \gamma$. This means that $\delta = \mu + \gamma$ where $\mu \in I^V$ is some fuzzy set in $\mathfrak{R}$. Also $\mu = \delta - \gamma \in \mathfrak{R} + \gamma + (-\gamma) = \mathfrak{R}$. Therefore $\mu = \delta - \gamma \in \mathfrak{R}$. By fuzzy right translation, $T_{(-\gamma)} : (V,\tau) \to (V,\tau)$ defined by $T_{(-\gamma)}(\delta) = \delta - \gamma = \mu$. Since (V,τ) is fuzzy irresolute topological vector space and by Proposition 3.7(i), $T_{(-\gamma)}$ is fuzzy ρ-irresolute. Therefore for any fuzzy ρ-open neighborhood $\mathfrak{R}$ containing $T_{(-\gamma)}(\delta) = \mu$, there exist fuzzy ρ-open neighborhood $\lambda \in I^V$ of δ such that $T_{(-\gamma)}(\lambda) = \lambda - \gamma \in \mathfrak{R}$ which implies that $\lambda \in \mathfrak{R} + \gamma$. Thus for any $\delta \in \mathfrak{R} + \gamma$, there exist fuzzy ρ-open neighborhood λ such that $\lambda \in \mathfrak{R} + \gamma$. Hence $\mathfrak{R} + \gamma \in F\rho O(V,\tau)$.

(2) Let $\xi \in I^F (\xi \neq 0)$ and $\delta \in I^V$. Let $\delta \in \xi\mathfrak{R}$. This means that for some fuzzy set $\mu \in I^V$ in $\mathfrak{R}$, $\delta = \xi\mu$. Since

$$\begin{aligned} &\delta \in \xi\mathfrak{R} \\ &\xi\mu \in \xi\,\mathfrak{R}\ \{\because \delta = \xi\mu\} \\ &\mu \in \xi^{-1}\xi\,\mathfrak{R} \\ &\mu \in \mathfrak{R} \end{aligned}$$

which implies $\mu = \xi^{-1}\delta$. Therefore By fuzzy right translation, $T_{(\xi^{-1})} : (V,\tau) \to (V,\tau)$ defined by $T_{(\xi^{-1})}(\delta) = \xi^{-1}\delta = \mu$. Since (V,τ) is fuzzy irresolute topological vector space and by Proposition 3.7(ii), $T_{(\xi^{-1})}$ is fuzzy ρ-irresolute. Therefore for any fuzzy ρ-open neighborhood $\mathfrak{R}$ containing $T_{(\xi^{-1})}(\delta) = \mu$, there exist fuzzy ρ-open neighborhood $\lambda \in I^V$ of δ such that $T_{(\xi^{-1})}(\lambda) = \xi^{-1}\lambda \in \mathfrak{R}$ which implies that $\lambda \in \xi\mathfrak{R}$. Thus for any $\delta \in \xi\mathfrak{R}$, there exists a fuzzy ρ-open neighborhood λ such that $\lambda \in \xi\mathfrak{R}$. Hence $\xi\mathfrak{R} \in F\rho O(V,\tau)$. □

Proposition 3.9. Let (V,τ) be any fuzzy irresolute topological vector space over the field F where F is endowed with usual fuzzy topology $\mathfrak{F}$. Then $\pi : (F,\mathfrak{F}) \times (V,\tau) \to (V,\tau)$ defined by $\pi((\xi,\lambda)) = \xi\lambda$ where $\xi \in I^F$ and $\lambda \in I^V$, is fuzzy ρ-irresolute function.

Proof. Let $\pi : (F,\mathfrak{F}) \times (V,\tau) \to (V,\tau)$. Let $\delta \in I^V$ be a fuzzy ρ-open neighborhood of $\xi\lambda$ in (V,τ). Since (V,τ) is fuzzy irresolute topological vector space, there exist fuzzy ρ-open neighborhoods $\alpha \in I^F$ of ξ and $\beta \in I^V$ of λ such that $\alpha \cdot \beta \leq \delta$ (i.e,) $\pi((\alpha,\beta)) = \pi(\alpha \times \beta) = \alpha \cdot \beta \leq \delta$. Since α is fuzzy ρ-open neighborhood of ξ in $(F,\mathfrak{F})$ and β is fuzzy ρ-open neighborhood of λ in (V,τ), therefore $\alpha \times \beta$ is also fuzzy ρ-open neighborhood of $\xi \times \lambda$ in $(F,\mathfrak{F}) \times (V,\tau)$. Thus $\pi : (F,\mathfrak{F}) \times (V,\tau) \to (V,\tau)$ is fuzzy ρ-irresolute. □

Proposition 3.10. Let (V,τ) be any fuzzy irresolute topological vector space over the field F. Then $\phi : (V,\tau) \times (V,\tau) \to (V,\tau)$ defined by $\phi((\lambda,\mu)) = \lambda + \mu$ where $\lambda, \mu \in I^V$, is fuzzy ρ- irresolute.

Proof. Let $\lambda, \mu \in I^V$ and $\phi((\lambda,\mu)) = \lambda + \mu$. Let $\delta \in I^V$ be a fuzzy ρ-open neighborhood of $\lambda + \mu$ in (V,τ). Since (V,τ) is fuzzy irresolute topological vector space, there exist fuzzy ρ-open neighborhoods $\eta, \theta \in I^V$ of λ and μ respectively such that $\eta + \theta \leq \delta$ (i.e,) $\phi((\eta,\theta)) = \phi(\eta \times \theta) = \eta + \theta \leq \delta$. Since η is fuzzy ρ-open neighborhood of λ in (V,τ) and θ is fuzzy ρ-open neighborhood of μ in (V,τ), therefore $\eta \times \theta$ is also fuzzy ρ-open neighborhood of $\lambda \times \mu$ in $(V,\tau) \times (V,\tau)$. Thus $\phi : (V,\tau) \times (V,\tau) \to (V,\tau)$ is fuzzy ρ-irresolute. □

Definition 3.11. Let (X_1,τ_1) and (X_2,τ_2) be any two fuzzy topological spaces. Then the function $f : (X_1,\tau_1) \to (X_2,\tau_2)$ is said to be fuzzy ρ-open function if for all fuzzy ρ-open set $\lambda \in I^{X_1}$ in (X_1,τ_1), $f(\lambda) \in I^{X_2}$ is fuzzy ρ-open in (X_2,τ_2).

Definition 3.12. Let (X,τ) be a fuzzy topological space. Then the mapping $f : (X,\tau) \to (X,\tau)$ is said to be fuzzy ρ-irresolute homeomorphism if it is bijective, fuzzy ρ-irresolute and fuzzy ρ-open function.

Proposition 3.13. Let (V, τ) be a fuzzy irresolute topological vector space over the field F. For given $\lambda \in I^V$ and $\gamma \in I^F$ with $\gamma \neq 0$, each fuzzy translation mapping $\phi_\lambda(\sigma) = \sigma + \lambda$ and $\pi_\gamma(\sigma) = \gamma\sigma$ where $\sigma \in I^V$, is fuzzy ρ-irresolute homeomorphism onto itself.

Proof. To prove $\phi_\lambda : (V, \tau) \to (V, \tau)$ is fuzzy ρ-irresolute homeomorphism. It is obvious that ϕ_λ is bijective. By Proposition 3.7, ϕ_λ is fuzzy ρ-irresolute. Also for any fuzzy ρ-open set $\delta \in I^V$ and for any fuzzy set $\lambda \in I^V$, by Proposition 3.8(i), $\phi_\lambda(\delta) = \delta + \lambda$, where $\delta + \lambda$ is fuzzy ρ-open set in (V, τ). Thus ϕ_λ is fuzzy ρ-open function. Therefore ϕ_λ is fuzzy ρ-irresolute homeomorphism.

Similarly $\pi_\gamma : (V, \tau) \to (V, \tau)$ is fuzzy ρ-irresolute homeomorphism. □

Definition 3.14. A fuzzy irresolute topological vector space (V, τ) over the field F is said to be fuzzy ρ-irresolute homogenous space, if for each $\lambda, \sigma \in I^V$, there exists a fuzzy ρ-irresolute homeomorphism $f : (V, \tau) \to (V, \tau)$ such that $f(\lambda) = \sigma$.

Proposition 3.15. Every fuzzy irresolute topological vector space is fuzzy ρ-irresolute homogenous space.

Proof. Let (V, τ) be a fuzzy irresolute topological vector space over the field F. Let $\lambda, \sigma \in I^V$ and also let $\sigma = \gamma + \lambda$ where $\gamma \in I^V$. Define, fuzzy left translation $\phi_\gamma : (V, \tau) \to (V, \tau)$ such that $\phi_\gamma(\lambda) = \lambda + \gamma = \sigma$. By Proposition 3.13, $\phi_\gamma : (V, \tau) \to (V, \tau)$ is fuzzy ρ-irresolute homeomorphism. Therefore (V, τ) is fuzzy ρ-irresolute homogenous space. □

Proposition 3.16. Let (X, τ) be any fuzzy topological space and $Y \subset X$. Let $\lambda, \mu \in I^X$ with $\mu \leq \lambda$. If $(Y, \tau_{/Y})$ is fuzzy topological subspace and $\lambda \in F\rho O(X, \tau)$, then $\mu \in F\rho O(V, \tau)$ if and only if $\mu \in F\rho O(Y, \tau_{/Y})$.

Proposition 3.17. Let (V, τ) be a fuzzy irresolute topological vector space over the field F, $Y \subset X$. Let χ_Y be fuzzy characteristic function of Y and fuzzy ρ-open. If $(Y, \tau_{/Y})$ is fuzzy ρ-open subspace of (V, τ), then $(Y, \tau_{/Y})$ is also fuzzy irresolute topological vector subspace over the field F.

Proof. Let (V, τ) be a fuzzy irresolute topological vector space over the field F and $\lambda \in I^V$ be a fuzzy open. Let (λ, τ_λ) is fuzzy topological subspace of (V, τ). Then it satisfies the following two properties:

(1) For all $\mu, \gamma \in I^V$ with $\mu \leq \lambda, \gamma \leq \lambda$, then $\mu + \gamma \leq \lambda$.
(2) For any $\sigma \in I^F$ and $\mu \in I^V$ with $\mu \leq \lambda$, then $\sigma\mu \leq \lambda$.

To prove (λ, τ_λ) is fuzzy irresolute topological vector space over the field F:

Let μ, γ be any two fuzzy sets in (λ, τ_λ). Let $\delta_1 \in I^V$ be a fuzzy ρ-open neighborhood of $\mu + \gamma$ in (λ, τ_λ). Then δ_1 is fuzzy ρ-open neighborhood of $\mu + \gamma$ in (V, τ). As (V, τ) is a fuzzy irresolute topological vector space over the field F, therefore there exist fuzzy ρ-open neighborhoods $\alpha_1, \beta_1 \in I^V$ in (V, τ) of μ and γ respectively such that $\alpha_1 + \beta_1 \leq \delta_1$. Now $\eta_1 = \alpha_1 \wedge \chi_\lambda$ and $\theta_1 = \beta_1 \wedge \chi_\lambda$ where $\eta_1, \theta_1 \in I^V$, are fuzzy ρ-open sets in (V, τ). Hence by Proposition 3.16, $\eta_1, \theta_1 \in F\rho O(\lambda, \tau_\lambda)$ and $\eta_1 + \theta_1 \leq \delta_1$.

For $\sigma \in I^F$ and $\mu \in I^V$, let $\delta_2 \in I^V$ be fuzzy ρ-open neighborhood of $\sigma\mu$ in (λ, τ_λ). Then δ_2 is fuzzy ρ-open neighborhood of $\sigma\mu$ in (V, τ). As (V, τ) is a fuzzy irresolute topological vector space over the field F, therefore there exist fuzzy ρ-open neighborhoods $\alpha_2 \in I^F$ of σ and $\beta_2 \in I^V$ of μ such that $\alpha_2 \cdot \beta_2 \leq \delta_2$. Then fuzzy ρ-open sets $\eta_2 \in I^F$ and $\theta_2 \in I^V$ where $\eta_2 = \alpha_2 \wedge 1_F$ and $\theta_2 = \beta_2 \wedge \chi_\lambda$ are fuzzy ρ-open in $(F, \mathfrak{F})$ and (V, τ). Since θ is fuzzy ρ-open in (V, τ), by Proposition 3.16, θ is fuzzy ρ-open in (λ, τ_λ). Hence for each fuzzy ρ-open neighborhood δ_2 of $\sigma\mu$ in (λ, τ_λ), there exist fuzzy ρ-open neighborhoods $\eta_2 \in I^F$ of σ and $\theta_2 \in I^V$ of μ such that $\eta_2 \cdot \theta_2 \leq \delta_2$. Hence (λ, τ_λ) is fuzzy irresolute topological vector space over the field F. □

Proposition 3.18. Let (V, τ) be a fuzzy irresolute topological vector space over the field F and $\lambda, \gamma \in I^V$. Then $F\rho cl(\lambda) + F\rho cl(\gamma) \leq F\rho cl(\lambda + \gamma)$.

Proof. Suppose $\alpha, \beta, \delta \in I^V$ where $\alpha = F\rho cl(\lambda)$, $\beta = F\rho cl(\gamma)$ and δ be a fuzzy ρ-open neighborhood of $\alpha + \beta$. Then there exist fuzzy ρ-open neighborhoods $\eta, \theta \in I^V$ of α and β respectively, such that $\eta + \theta \leq \delta$. Since $\alpha = F\rho cl(\lambda)$ and $\beta = F\rho cl(\gamma)$, there are $\sigma_1, \sigma_2 \in I^V$ such that $\alpha \leq \sigma_1 \leq \lambda \wedge \eta$ and $\beta \leq \sigma_2 \leq \gamma \wedge \theta$. Then,

$$\begin{aligned}
\alpha + \beta &\leq \sigma_1 + \sigma_2 \\
&\leq (\lambda \wedge \eta) + (\gamma \wedge \theta) \\
&\leq (\lambda + \gamma) \wedge (\eta + \theta) \\
\alpha + \beta &\leq (\lambda + \gamma) \wedge (\delta) \\
\alpha + \beta &\leq (\lambda + \gamma), \alpha + \beta \leq \delta \\
F\rho cl(\alpha + \beta) &\leq F\rho cl(\lambda + \gamma) \\
\therefore\ \alpha + \beta \leq F\rho cl(\alpha + \beta) &\leq F\rho cl(\lambda + \gamma).
\end{aligned}$$

This implies that $\alpha + \beta \leq F\rho cl(\lambda + \gamma)$. Hence $F\rho cl(\lambda) + F\rho cl(\gamma) \leq F\rho cl(\lambda + \gamma)$. □

Proposition 3.19. Let (V, τ) be a fuzzy irresolute topological vector space over the field F and $\xi \in I^F$. Let $T_\xi : (V, \tau) \to (V, \tau)$ be a fuzzy translation. Then the scalar multiplication of fuzzy ρ-closed set is fuzzy ρ-closed.

Proof. Let (V, τ) be a fuzzy irresolute topological vector space over the field F. By Proposition 3.7, $T_\xi : (V, \tau) \to (V, \tau)$ is fuzzy ρ-irresolute. Let $\lambda \in I^V$ be a fuzzy ρ-closed set. Also $1_X - \lambda$ is fuzzy ρ-open in (V, τ). Then $T_\xi(1_X - \lambda) = \xi(1_X - \lambda) = \xi 1_X - \xi\lambda = 1_X - \xi\lambda$ where $1_X - \xi\lambda$ is fuzzy ρ-open in (V, τ). Therefore $\xi\lambda$ is fuzzy ρ-closed in (V, τ). □

Definition 3.20. Let (X, τ) ba a fuzzy topological space. If a family $\{\lambda_i : i \in J$, where J is an index set$\}$ of fuzzy open sets in (X, τ) satisfies the condition $\bigvee_{i \in J} \lambda_i = 1_X$, then it is called a fuzzy automata open cover of (X, τ).

Definition 3.21. A fuzzy topological space (X, τ) is said to be fuzzy compact if for every fuzzy open cover $\{\lambda_i : i \in J$, where J is an index set$\}$ of fuzzy open sets of (X, τ) (i.e.,) $\bigvee_{i \in J} \lambda_i = 1_X$, there exists a finite subset J_0 of J such that $\bigvee_{i \in J_0} \lambda_i = 1_Q$.

Definition 3.22. Let (X, τ) be a fuzzy topological space and $\mu \in I^X$. A fuzzy set μ is said to be fuzzy compact if for every collection $\{\lambda_i : i \in J$, where J is an index set$\}$ of fuzzy open sets of (X, τ) with $\mu \leq \bigvee_{i \in J} \lambda_i$, there exists a finite subset J_0 of J such that $\mu \leq \bigvee_{i \in J_0} \lambda_i$.

Proposition 3.23. Let (V, τ) be a fuzzy irresolute topological vector space over the field F and $\xi \in I^F$. Then the scalar multiplication of fuzzy ρ-compact set is fuzzy ρ-compact.

Proof. Let $\lambda \in I^V$ be a fuzzy ρ-compact set in (V, τ). Let $\{\mu_i : i \in J\}$ be a fuzzy ρ-open cover of $\xi\lambda$ for some $\xi \in I^F$ with $\xi \neq 0$, then $\xi\lambda \leq \vee_{i \in J} \mu_i$. Thus $\lambda \leq (1/\xi) \vee_{i \in J} \mu_i \leq \vee_{i \in J}((1/\xi)\mu_i)$. Since μ_i is fuzzy ρ-open and (V, τ) be a fuzzy irresolute topological vector space over the field F, by Proposition 3.19, $(1/\xi)\mu_i$ is fuzzy ρ-open for each $i \in J$. Since λ is fuzzy ρ-compact, there exists a finite subset J_0 of J such that $\lambda \leq \vee_{i \in J_0}((1/\xi)\mu_i)$. This implies that $\xi\lambda \leq \vee_{i \in J_0}(\mu_i)$. Hence $\xi\lambda$ is fuzzy ρ-compact in (V, τ). □

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