Cascaded approach to the path-following problem for N-trailer robots

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Outline

1. Introduction
2. Vehicle kinematics
3. Control problem
4. Cascaded controller for N-trailers
5. Results
6. Conclusions
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Examples of N-trailer vehicles
Motivation

Properties of N-trailers:

- highly nonlinear kinematics
- nonholonomic constraints
- number of control inputs $\ll$ number of variables
- structural chain-instability in backward motion
- state-constraints (joint angles)
- non-minimum-phase property due to off-axle interconnections

control of N-trailers = still challenging problem
Motivation (cont.)

- **Path-following (PF)**
  - strictly related to geometry of a motion task (most important in practice)
  - relaxes time-execution demands (in contrast to trajectory tracking)
  - desired motion geometry from a starting point to the goal (in contrast to point stabilization)
  - existing PF controllers require determination of a shortest distance to the path...

- **Cascaded control**
  - allows for control task decomposition
  - simplifies control law synthesis and its practical implementation
  - scalability of a control law

path following = task of essential practical meaning

cascaded control approach ⇒ highly scalable and *modular* solution
Kinematics of N-trailer robots

Kinematics of the $i$-th segment

$$\dot{\theta}_i = \omega_i$$
$$\dot{x}_i = v_i \cos \theta_i$$
$$\dot{y}_i = v_i \sin \theta_i$$
$$\beta_i = \theta_{i-1} - \theta_i$$

$\omega_i, v_i$ – virtual inputs of the $i$-th segment

Configuration vector

$$q \triangleq [\beta_1 \ldots \beta_N \theta_N x_N y_N]^{\top} \in \mathbb{T}^N \times \mathbb{R}^3$$

Classification of N-trailers

nSNT: $M = N$
GNT: $M \in (0, N)$
SNT: $M = 0$

$M = \text{number of off-axle joints (} L_{hi} \neq 0)$
Kinematics of N-trailer robots (cont.)

Velocity transformation

\[
\begin{bmatrix}
\omega_i \\
v_i
\end{bmatrix} =
\begin{bmatrix}
-L_{hi} \cos \beta_i \\
L_{hi} \sin \beta_i \\
L_i \sin \beta_i \\
L_i \cos \beta_i
\end{bmatrix}
\begin{bmatrix}
\omega_{i-1} \\
v_{i-1}
\end{bmatrix}
\]

Velocity propagation \((i = 1, \ldots, N)\):

\[
u_i = \prod_{j=i}^1 J_j(\beta_j)u_0
\]

\[
u_{i-1} = \prod_{j=i}^N J_j^{-1}(\beta_j)u_N
\]

\[u_0 = [\omega_0 \ v_0]^T \leftarrow \text{robot control input}
\]

Note: Formula (2) can be used only in case of nSNT robots.
Reference path and reference orientation*

Reference path on a motion plane:

\[ F(x, y) \triangleq \sigma f(x, y) = 0, \quad \sigma \in \{-1, +1\} \quad (3) \]

Assumptions – for any \((x, y) \in D \subset \mathbb{R}^2\) holds:
- \(F(x, y)\) is bounded
- \(F(x, y)\) is at least twice differentiable with respect to both arguments
- Gradient \(\nabla F(x, y) \triangleq \left[ \frac{\partial F(x, y)}{\partial x}, \frac{\partial F(x, y)}{\partial y} \right]\) is non-zero: \(\| \nabla F(x, y) \| > 0\)

Reference orientation along (3)

\[ \theta_d(x, y) \triangleq \text{Atan2c}\left(-\frac{\partial F(x, y)}{\partial x}, \frac{\partial F(x, y)}{\partial y}\right) \in \mathbb{R} \quad (4) \]

Note: \(\sigma\) included in (3) determines desired quadrant for reference orientation (4)

*Based on the paper:
Control problem formulation

Let us define the guidance-segment posture vector

\[ q_N = [\theta_N \ x_N \ y_N]^\top \in \mathbb{R}^3 \] (5)

and the path-following error

\[ e(q_N) \triangleq \begin{bmatrix} F(q_N) \\ e_{\theta}(q_N) \end{bmatrix} \triangleq \begin{bmatrix} \sigma f(x_N, y_N) \\ \theta_N - \theta_d(x_N, y_N) \end{bmatrix} \in \mathbb{R}^2 \] (6)

Definition (Control Problem)

The control problem under consideration is to find a bounded feedback control law

\[ u_0(\beta, q_N, \cdot) \]

which guarantees that error (6) is convergent in the sense that:

\[ \lim_{t \to \infty} F(q_N(t)) = 0 \quad \text{and} \quad \lim_{t \to \infty} e_{\theta}(q_N(t)) = 2\nu\pi, \ \nu \in \mathbb{Z} \] (7)

Note: \( F(q_N) \) can be treated as a signed distance to the path since \( F(q_N) = 0 \iff \) N-th trailer is on a reference path.
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Cascaded control concept for nSNT kinematics

Assumptions:

A1. \( L_{hi} \neq 0 \) for any \( i = 1, \ldots, N \)  (nSNT kinematics)

A2. \( \text{sgn}(L_{hi}) = \text{sgn}(L_{hj}) \) for any \( i \neq j \)  (sign-homogeneity)
Cascaded control concept for nSNT kinematics

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Design steps

1. According to velocity transformation formula (2) holds

\[
    u_0(\beta) = \prod_{j=1}^{N} J_j^{-1}(\beta_j) u_N
\]  
(8)

2. Suppose, one would like to force \( u_N = \Phi(q_N, t) \) with \( \Phi(q_N, t) \) being the PF control function dedicated to unicycle kinematics (guidance segment = unicycle)

3. According to (8) it seems reasonable to define

\[
    u_0(\beta, \Phi(q_N, t)) \triangleq \prod_{j=1}^{N} J_j^{-1}(\beta_j) \cdot \Phi(q_N, t)
\]  
(9)

inner-loop controller

outer-loop PF controller
Definition of the outer-loop PF controller*

\[
\Phi_{\omega}(q_N, t) \triangleq -k_1 \left( \| \nabla F(q_N) \| \Phi_v(t) \frac{k_2 F(q_N)}{\sqrt{1 + F^2(q_N)}} + |\Phi_v(t)| \nabla F(q_N) \cdot \bar{g}_2(\theta_N) \right) + \dot{\theta}_d \tag{10}
\]

\[
\Phi_v(t) \triangleq v_d(t) \tag{11}
\]

where

\[
k_1 > 0, \ k_2 \in (0, 1] \quad \text{(design parameters)}
\]

\[
v_d(t) \in C^1, \ \lim_{t \to \infty} v_d(t) \neq 0
\]

\[
\bar{g}_2(\theta_N) = [c\theta_N \ s\theta_N]^T
\]

\[
\dot{\theta}_d = \Phi_v(t) \frac{F_1(q_N)c\theta_N + F_2(q_N)s\theta_N}{\| \nabla F(q_N) \|^2}
\]

\[
F_1(q_N) = \frac{\partial F(q_N)}{\partial x} \frac{\partial^2 F(q_N)}{\partial x \partial y} - \frac{\partial F(q_N)}{\partial y} \frac{\partial^2 F(q_N)}{\partial x^2}
\]

\[
F_2(q_N) = \frac{\partial F(q_N)}{\partial x} \frac{\partial^2 F(q_N)}{\partial y^2} - \frac{\partial F(q_N)}{\partial y} \frac{\partial^2 F(q_N)}{\partial x \partial y}
\]

Note: Determination of the shortest distance to a reference path is not needed in application of the above PF controller (!)

*According to the paper:

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**Introduction**

Vehicle kinematics

Control problem

Cascaded controller for N-trailers

Results

Conclusions
Inner-loop approximation for GNT and SNT kinematics

In case $L_{hi} = 0$ (joint of on-axle type)

$$
\mathbf{J}_i(\beta_i) \triangleq \begin{bmatrix}
-\frac{L_{hi}}{L_i} c \beta_i & \frac{1}{L_i} s \beta_i \\
L_{hi} s \beta_i & c \beta_i
\end{bmatrix} = \begin{bmatrix}
0 & \frac{1}{L_i} s \beta_i \\
0 & c \beta_i
\end{bmatrix} \tag{12}
$$

may be approximated by

$$
\hat{\mathbf{J}}_i(\beta_i, \varepsilon_i) \triangleq \begin{bmatrix}
-\frac{\varepsilon_i}{L_i} c \beta_i & \frac{1}{L_i} s \beta_i \\
\varepsilon_i s \beta_i & c \beta_i
\end{bmatrix}, \quad \varepsilon_i \neq 0 \land \text{sgn}(\varepsilon_i) \overset{(A2)}{=} \text{sgn}(L_{hj}) \tag{13}
$$

The approximated inverse matrix takes the form

$$
\hat{\mathbf{J}}^{-1}_i(\beta_i, \varepsilon_i) = \begin{bmatrix}
-\frac{L_i}{\varepsilon_i} c \beta_i & \frac{1}{\varepsilon_i} s \beta_i \\
L_i s \beta_i & c \beta_i
\end{bmatrix} \tag{14}
$$

Now, singular matrices in the inner loop $\prod_{j=1}^{N} \mathbf{J}^{-1}_j(\beta_j)$ may be replaced with (14) for on-axle joints.
Corollary (Based on Theorem 2 and the proof presented in paper*)

Application of cascaded control law (9) with outer-loop PF controller (10)-(11) into nSNT kinematics solves Control Problem under consideration for any initial condition $e(q_N(0)), (x_N(0), y_N(0)) \in D$ outside the set of unstable equilibria defined as

\[ \{ F(q_N) = 0, e_\theta = (2\nu + 1)\pi | \nu \in \mathbb{Z} \} \].

Sim1: elliptical path for nS3T robot (nominal conditions)

Simulation conditions

\[ f(x, y) := \frac{x^2}{A^2} + \frac{y^2}{B^2} - 1, \quad A = 2, \ B = 1 \]

\[ v_d = -0.3 \text{ m/s} \text{ (backward motion prescribed)} \]

\[ \sigma = +1 \]

Values of parameters

\[ L_i = 0.3 \text{ m}, \quad i = 1, 2, 3 \]
\[ L_{hi} = 0.1 \text{ m} \quad i = 1, 2, 3 \]
\[ k_1 = 2 \text{ and } k_2 = 1 \]
Sim2: S-like path for nS3T robot (nominal conditions)

Simulation conditions

\[ f(x, y) := y - B \tanh(Ax), \quad A = 5, \ B = 1 \]
\[ v_d = +0.15 \text{ m/s} \text{ (forward motion prescribed) } \]
\[ \sigma = +1 \]

Parameter values

\[ L_i = 0.3 \text{ m}, \quad i = 1, 2, 3 \]
\[ L_{hi} = -0.05 \text{ m} \quad i = 1, 2, 3 \]
\[ k_1 = 2 \text{ and } k_2 = 1 \]
Sim3: elliptical path for G3T robot (non-nominal conditions)

Simulation conditions

\[ f(x, y) := \frac{x^2}{A^2} + \frac{y^2}{B^2} - 1, \quad A = 2, \quad B = 1 \]
\[ v_d = -0.3 \text{ m/s (backward motion prescribed)} \]
\[ \sigma = -1 \]

Parameter values

\[ L_i = 0.3 \text{ m}, \quad i = 1, 2, 3 \]
\[ L_{h2} = 0.1 \text{ m}, \quad L_{hi} = 0.0, \quad i = 1, 3 \]
\[ \varepsilon_1 = \varepsilon_3 = 0.01 \text{ m}, \quad k_1 = 2 \text{ and } k_2 = 1 \]
Sim4: S-like path for S3T robot (non-nominal conditions)

Simulation conditions

\[ f(x, y) := y - B \tanh(Ax), \quad A = 5, \ B = 1 \]
\[ v_d = -0.15 \text{ m/s} \text{ (backward motion prescribed)} \]
\[ \sigma = -1 \]

Parameter values

\[ L_i = 0.3 \text{ m}, \ i = 1, 2, 3 \]
\[ L_{hi} = 0.0 \text{ m}, \ i = 1, 2, 3 \]
\[ \varepsilon_i = 0.01 \text{ m}, \ i = 1, 2, 3, \ k_1 = 2 \text{ and } k_2 = 1 \]
Path-following accuracy for nominal and non-nominal conditions

Sim1, Sim2 ← nominal conditions (nSNT-type kinematics)

Sim3, Sim4 ← non-nominal conditions (GNT-type and SNT-type kinematics)
Experimental results for RMP vehicle (nSNT-type robot)

- Exp1: elliptical path for \( f(x, y) := \frac{x^2}{(0.5)^2} + \frac{y^2}{(0.3)^2} - 1 \) (movie: EllipticalPath.avi)
- Exp2: s-like path for \( f(x, y) := y - 0.5 \tanh(5x) \) (movie: SlikePath.avi)
- Selected parameters: \( L_{h1,2,3} = 0.048 \, m, \; v_d = -0.05 \, m/s, \; k_1 = 5, \; k_2 = 1 \)
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Final remarks

- Cascaded control approach $\Rightarrow$ control decomposition
  - outer-loop controller for a last trailer (unicycle kinematics!)
  - inner-loop velocity transformation (multiplication of $2 \times 2$ matrices)

- Simplicity and high scalability of the resultant control system

- Outer-loop controller by Morro et al. (novel approach to path-following control)

- Applicability of the concept
  - nSNT robots (nominal case)
  - GNT and SNT robots (approximated solution)
  - required sign-homogeneity for $L_{hi}$ (motion strategy requirement: $\text{sgn}(v_d) = -\text{sgn}(L_{hi})$)
  - only analytical paths are admissible (restriction inherited from solution of Morro et al.)
Final remarks

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  - outer-loop controller for a last trailer (unicycle kinematics!)
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Thank you for attention