Time Delay Estimator for Frequency Hopping System using Rank-Revealing Triangular Factorization

Qasaymeh M. M, Gami Hiren, Nizar Tayem†, Ravi Pendse, M.E. Sawan
EECS, Wichita State University, Wichita, KS, USA
†Engineering Technology, Miami University, Middletown, OH, USA

Abstract— In this paper, the multipath time delay estimation (TDE) problem for a slow frequency hopping (SFH) system using Rank Revealing QR Factorization method (RRQR) is considered. It gives precious information about numerical rank and null space. By applying the RRQR in association with the well-known MUSIC algorithm we achieved a highly efficient estimator. The proposed methods would generate estimates of the unknown delay parameters. Such estimates are based on the observation and/or covariance matrices. Moreover, the RRQR does not require the eigenvalue decomposition (EVD) of the cross-spectral matrix (CSM) or singular value decomposition (SVD) of the data matrix of received signals. Computer simulations are also included to demonstrate the effectiveness of the proposed method.

Index-Terms: Frequency Hopping, channel estimation, Time Delay Estimation, Rank Revealing, MUSIC, ESPRIT.

I. INTRODUCTION

Different techniques have been used to combat the impairments in the rapidly varying radio channels. Some of those are channel coding and interleaving, adaptive modulation, transmitter/ receiver antenna diversity, dynamic channel allocation (DCA) and spectrum spreading (SS). Two types of spread spectrum systems exist: The direct sequence spread spectrum (DSSS) and frequency hopping spread spectrum (FHSS), thus the frequency hopping (FH) is a spectrum spreading technique that can introduce frequency diversity and interference diversity. It is very robust and ideal for applications where data reliability is critical [1], [2]. We can achieve Code division multiple access (CDMA) via frequency hopping (FH) if we partition the bandwidth into a number of frequency sub-bands. It also is known as frequency- hopping code division multiple access (FH-CDMA). Based on the hopping duration, FH systems can be further divided into two categories: fast hopping (FH) scheme and slow hopping (SFH) scheme. In an FH system, the carrier frequency will change or hop several times during the transmission of one symbol, while in an SFH system, several symbols are transmitted during each hop. Since it is unlikely that different bands experience simultaneous fading, FH systems are robust against fast fading. At the same time, the pseudo-random hopping of frequencies during radio transmission minimizes the possibility of hostile jamming and unauthorized interception.

Typically, the FH system model is considered as a narrow band system. It was shown in [3], [4] that the flat fading model is not valid for a FH system whose bandwidth is comparable with the coherence bandwidth of the multipath channel. Therefore, the time delay estimation becomes significant when the received signal in a SFH system at a high data rate is frequency selective. The multipath time delay estimation problem for a SFH system using of estimation signal parameter via rotational invariance technique (ESPRIT) [5] and signal parameter estimation via Cayley-Hamilton constraints (SPEC) algorithms [6] were studied in [7]. Once the time delay is estimated, the complex channel gain can be estimated using maximum likelihood (ML) estimators.

In this paper, we address the problem of estimating the multipath time delay parameters through Rank-Revealing QR factorization [8], [9] in conjunction with the well-known multiple signal classification (MUSIC) method [10] without using reference symbols, pilot carriers. The RRQR is a good alternative of conventional subspace decomposition technique like SVD, EVD [11] etc. with a lower computational cost. Moreover, it is quite supportive in rank deficient least square problems. Similar technique is applies in our recent work in the joint time delay and frequency estimation problem [13], and carrier frequency offset (CFO) in orthogonal frequency division multiplexing systems (OFDM) [14].

This paper is structured as follows. In Section II, the system model and the problem formulation is presented. The development of the highly efficient method is presented in Section III. In Section IV, the development of the second method is presented. In Section V, the performance of the RRQR based algorithm is illustrated through MATLAB simulations. A comparison with the LS-ESPRIT and TLS-ESPRIT estimators [7] is made. Finally, some concluding remarks follow in Section V1.

II. CHANNEL MODEL AND PROBLEM FORMULATION

A. Channel Model

Signal multipath occurs when the transmitted signal arrives at the receiver via multiple propagation paths. Each of these paths may have a separate phase, attenuation, delay and Doppler frequency associated with it. We model the discrete multipath channel as baseband. The channel is modeled as a tapped delay line, given by

\[ h(t) = \sum_{\ell=1}^{P} \beta_\ell \delta(t - \tau_\ell(t)) \]
where $\beta_i$ and $\tau_i$ are the complex channel gain and the associated time delay of the $i$th multipath respectively. The received signal $y(t)$ when a signal $s(t)$ is transmitted through a channel is given by

$$ y(t) = \int h(t-\tau).s(\tau) d\tau $$

**B. Problem Formulation**

In this paper, we consider the baseband model of the Frequency Hopping received signal in a multipath environment. The sample version form

$$ y(n)(kT) = \sum_{i=1}^{P} \beta_i e^{-j2\pi fn_i \tau_i} s(kT - \tau_i; b_i) + w(n)(kT) \quad (1) $$

where $y(n)(kT)$ is the received signal in the $n$th hop, $T$ is the sampling period, $f_n$ is the frequency in the $n$th hop, and $b_i$ is the sequence of the transmitted bits. $\beta_i$ and $\tau_i$ are respectively the channel gain and the associated time delay of the $i$th multipath, which are assumed to be independent of the hop index. $s(kT; b_i)$ is the transmitted baseband signal and $w(n)(kT)$ is the white Gaussian noise parameter. Parameter $P$ denotes the total number of multipath considered in the model.

Channel gain, time delay, and the transmitted bit sequence are unknown. The hop frequency is known. The final goal as in any telecommunication system is to extract the transmitted bit sequence, in order to achieve that the channel gain and the time delay should be estimated first.

The problem addressed in this paper is estimation of the time delays only based on the received signal; once the time delays are estimated, two separate maximum likelihood (ML) problems can be considered to estimate the complex channel gain $\beta_i$ and the transmitted bit sequence $b_i$. Similar to [7], three assumptions were made in this paper

1. Header bits are the same for all packets. This is a practical assumption since the first few bits in each packet are usually meant to express control information. The control information could be the address of the receiver or could even be just sync and guard bits. For example, the SFH based PCS system had a total of 16 QPSK symbols or 32 bits as sync and guard bits. The access code is used for synchronization and is the same for all packets originating in the same group. The header consists of the member address, link flow bits and other control information. Thus it makes sense to assume that the First few bits of each packet received at a terminal are essentially the same.

2. Time delays remain constant or vary very slowly. The channel itself is frequency selective but that does not prevent from assuming that the delays remain constant over a range of frequencies.

3. The frequency does not change within a packet, Frequency hops from packet to packet only.

Therefore, the discrete time version of (1) is given by

$$ y(n)(k) = \sum_{i=1}^{P} \beta_i e^{-j2\pi fn_i \tau_i} s_i(k) + w(n)(k), k = 1, 2, \ldots K \quad (2) $$

where $s_i(k)$ is the delayed version of the transmitted signal through the $i$th multipath. Clearly, this part of data is constant among all hops and independent of $n$.

**III. DEVELOPMENT OF THE PROPOSED METHOD**

Let the $K$ samples of the $n$th hop in (2) given by

$$ y_n = [y^{(n)}(0), y^{(n)}(1), \ldots, y^{(n)}(K-1)] $$

We collect data from $N$ hops to form matrix $Y$ of size $N \times K$

$$ Y = [y_1, y_2, \ldots, y_N]^T $$

Matrix $Y$ will be used to form $M$ submatrices $Y_1, Y_2, Y_3, \ldots Y_M$. Partitioned the data packets into $M$ subsets $\{F_1, F_2, \ldots, F_M\}$ of received frequencies each of at least of size $N$ as

$$ f_{ij+1} = f_{ij} + \Delta_f, j = 1, 2, \ldots, N, i = 1, 2, \ldots, M - 1 \quad (3) $$

It is obvious that two successive frequency sets are differed by a constant $\Delta_f$. Let $Y_i = [y_{i1}, y_{i2}, \ldots, y_{iN}]^T$. It is easy to show that $M$ subsets can be represented as

$$ Y_1 = AS + W_1 $$

$$ Y_2 = A \Phi S + W_2 $$

$$ \vdots $$

$$ Y_M = A \Phi^{M-1} S + W_M \quad (4) $$

where

$$ A = \begin{bmatrix}
  e^{-j2\pi f_{1T1}} & e^{-j2\pi f_{1T2}} & \cdots & e^{-j2\pi f_{1Tp}} \\
  e^{-j2\pi f_{2T1}} & e^{-j2\pi f_{2T2}} & \cdots & e^{-j2\pi f_{2Tp}} \\
  \vdots & \vdots & \ddots & \vdots \\
  e^{-j2\pi f_{pTN1}} & e^{-j2\pi f_{pTN2}} & \cdots & e^{-j2\pi f_{pTNp}}
\end{bmatrix} $$

$$ S = \begin{bmatrix}
  \beta_1 \\
  \vdots \\
  \beta_p
\end{bmatrix} $$

$$ S = \begin{bmatrix}
  s_1(1) & \cdots & s_1(K) \\
  \vdots & \ddots & \vdots \\
  s_p(1) & \cdots & s_p(K)
\end{bmatrix} $$
and \( W_1, W_2, \ldots W_M \) are corresponding noise matrices, and matrix which contain the required unknown multipath delay parameters. The parameters \( \beta_i \) are the amplitudes of the respective multipath. We collect the sub-matrices calculated by (4) in the matrix \( X \) as

\[
X = [Y_1 Y_2 \ldots Y_M] + W
\]

where \( W \) is the corresponding additive white Gaussian noise matrix. We applied RRQR factorization for (5)

\[
X^T = QR = [Q_1 \quad Q_2] \begin{bmatrix} R_{11} & R_{12} \\ R_{12} & R_{22} \end{bmatrix}
\]

where the two matrices \( Q \) and \( R \) are of dimensions \( MK \times MK \) and \( MK \times (N - M + 1) \) respectively. The sub-matrix \( R_{11} \) is upper triangular full rank matrix while \( R_{12} \) is holding important information with dimensions \( P \times (N - M - P + 1) \). Because of rank-revealing QR-factorization, it is interesting to note here is that the sub-matrix \( R_{22} \) is just about null matrix. Therefore it hardly contributes in construction of either signal space or null space of a matrix.

\[
X^T \equiv Q_1 \hat{R} = Q_1[R_{11} \quad R_{12}]
\]

Clearly, any vector belongs to null space should satisfy

\[
\hat{R} G = [R_{11} \quad R_{12}] \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = 0
\]

so that \( R_{11} g_1 = -R_{12} g_2 \). Since \( R_{11} \) is an invertible matrix, \( g_1 \) can be written in terms of \( g_2 \) as

\[
g_1 = -R_{11}^{-1} R_{12} g_2
\]

Then \( G \) can be written as

\[
G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} -R_{11}^{-1} R_{12} \\ I \end{bmatrix} \begin{bmatrix} g_2 \end{bmatrix} = H g_2
\]

Therefore, \( RH = 0 \). It can be observed here is that the columns of the basis of the null space \( H \) are not orthonormal. To satisfy orthonormality we use orthogonal projection onto this subspace in order to improve the performance by making the basis of null space of \( H \) orthonormal.

\[
H_o = H (H^H H)^{-1} H^H
\]

Apply MUSIC like search algorithm [12] to estimate the frequencies using the following function

\[
\hat{P}_{MU}(e^{j\omega}) = \frac{1}{A^H(\omega) H_o A(\omega)}
\]

Instead of searching for the peaks in (11) an alternative is to use a root-MUSIC [13]. The frequency estimates may be taken to be the angles of the \( p \) roots of the polynomial \( D(z) \) that are closest to the unit circle

\[
D(z) = \sum_{i=0}^{N/2-1} V_i(z) V_i(1/z^*)
\]

where \( V_i(z) \) is the \( z \)-transform of the \( i \)th column of a projection matrix [13].

V. SIMULATION RESULTS

Extensive computer simulations using MATLAB were done to validate our proposed method. In the first experiment we considered a three multipath model \( (P=3) \). The transmission was confined to the range 1899 to 1929 MHz, the uplink frequency range for the PCS system. A total of 75 frequencies were considered with a 400 KHz frequency separation among carriers. The header part in each packet was assumed to consist of four quadrature phase-shift keying (QPSK) symbols. The symbol period for our system was considered to be 4µs. The multipath time delays were set to be 0.1, 0.4, and 0.9 µs respectively. The channel gain parameter is assumed to be a complex random and exponentially decaying with respect to the time delays. Ten packets were assumed to be available at the receiver. The hop frequencies were forming arithmetic series with nine possible maximum frequencies in each set for various concerned algorithms. We considered 1000 independent realizations with normalized mean square error (MSE) defined as

\[
MSE = E \left\{ \sum_{i=1}^{N_t} \sum_{j=1}^{P} \left( \frac{t_i - \hat{t}_i}{\tau_i} \right)^2 \right\}
\]
The first figure is showing the normalized MSE versus signal to noise ratio (SNR) at exponentially decaying random multipath. The superiority of the proposed method is apparent over all SNR range.

In the second experiment we maintained the same parameter assumptions like in the first experiment, except that the multipath gain parameters were complex Gaussian in nature. Under this environment we observed that the RRQR method in conjunction with the MUSIC is still robust against random multipath. At low SNR the superiority of the proposed method is apparent. At very high SNR of 30 dB the ESPRIT-LS algorithm performance is closely approaching to the RRQR algorithm.

In the third experiment, the estimator performance as function of the number of available packets at the receiver is considered. The algorithm behavior with respect to data acquisition at 15 dB SNR is depicted in figure 3, it is showing that with 15 data packets or more the proposed algorithm is showing a significant achievement compared to the others.

V. CONCLUSION

In this paper we proposed a novel blind channel estimation algorithm for slow frequency hopping system systems via rank revealing QR factorization instead of using EVD or SVD based complex spectral decomposition. Through MATLAB simulation the performance of the proposed estimator is showing a significant improvement compared with the LS-ESPRIT and TLS-ESPRIT estimators [7]. In terms of future work, it is an interesting to develop a closed-from parameter estimation method based on RRQR, or to show the performance of the estimator though the bit error rate (BER) and symbol error rate (SER).

REFERENCES