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Design of an Oscillating Coil Pendulum Energy Generating System

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Abstract

The purpose of this paper is to study the theoretical work for a new type of wind power system. This wind system, called Wing oscillating coil rod pendulum system. This is a small prototype that generates low current and voltage. This system is a new type of wind turbine, to generate energy by using wind. One of the main elements of this system is a copper coil pendulum with flapping wing attached to it which oscillates at a low frequency when wind strikes it. The wind blowing kinetic energy compels the pendulum to oscillate, so the kinetic energy of wind is converted into oscillating energy of pendulum. The oscillating energy is then converted into electrical energy by using a semicircle shaped permanent magnet, which is placed under the coil rod pendulum. Theoretical construction and electrical design of wing oscillating coil rod pendulum system is discussed in this paper.

1. Introduction.

The proposed system called “wing oscillating coil rod pendulum”. The purpose of this system is to generate energy from wind blowing power. Two types of this System are shown in fig. 1. Flapping wing is attached to a coil rod pendulum (C.R.P) which oscillates under the effect of magnetic field and generates electrical energy in type-1. North pole of semi circle magnet is placed under coil pendulum an south pole of magnet is placed above coil pendulum.

In type-2 of this system, a bar magnet is placed in a coil of copper wire wrapped on a glass core. Bar magnet is attached with a pendulum having bob of mass “m”. When wind thrust strikes at wing, pendulum oscillates and in result bar magnet moves up and down in coil. The phenomena of converting pendulum oscillating energy into electrical energy in both types uses the definition of faraday’s law, which states that, when a coil of conductor moves under the effect of magnetic field then magnetic lines cut the turns of coil or conversely, when a magnet moves in a coil of conductor an emf (electro motive force or voltage) will be induced in coil. Amount of emf generated is directly proportional to the rate at which coil cuts the magnetic lines and number of turns in the coil.

Wing oscillating coil pendulum generator has the following advantages.

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1) Construction of this system is simple and of low cost.
2) This system can be used to convert vibrational energy into electrical energy.

2. Related work and motivation.

Harvesting energy from kinetic energy of blowing wind and converting this kinetic energy into electrical energy is a cheap, green and renewable energy. In 2011 a paper, was published in 7th international symposium on advance topics in electrical engineering, the faculty of electrical engineering, U.P.B. Bucharest, May 12-14-2011 [1]. A new type of wind turbine called as “Wing Oscillating Wind system” was discussed. Good work done by authors by using the classical theory of pendulum in their wind system.

The equation used by author for time period “T” (frequency=1/T) of pendulum to calculate the filling factor of electrolytic capacitor is

\[ T = 2\pi \sqrt{\frac{l}{g}}. \]  

(1)

Where “l” is the length of pendulum and “g” is acceleration due to gravity which is 9.8 m/sec². By using equation (1), theorectical output voltage achieved by author and other calculations for capacitor can never match with the original values, because this equation is only valid if pendulum is displaced at very small angle such that \( \sin \theta \approx \theta \). Author used this equation (1) for maximum displacement of pendulum when angle \( \theta = 90 \), which is not appropriate and can never be used for large angle oscillation of pendulum.

New equations of time period for large angle oscillation of simple pendulum have been developed. One of the famous publication due to its simplicity in 2002 by Richard B. Kidd and Stuart L. Fogg [2]. In which time period formula for large angle oscillation of simple pendulum is derived. Kidd-Fogg formula based on experimental results and sophisticated reasoning. According to kidd-fogg the time period equation of simple pendulum for large angle oscillation is

\[ T = 2\pi \sqrt{\frac{l}{g\cos(\alpha/2)}}. \]  

(2)

Where \( \alpha \) is the maximum angular displacement of pendulum and \( \alpha \leq \pi/2 \). Kidd and Fogg derived their approximate expression for time period of pendulum at large angle oscillation by making an analogy between the equation of pendulum time period for small angle and period of simple harmonic motion. After it one of more known equation made
by L. Edward Millet [3], who justified Kidd-Fogg formula taking into account well known trigonometric relation for the half angle formula \( \sin(\theta) = 2\sin(\theta/2)\cos(\theta/2) \) and considering that \( \sin(\theta/2) \) would have to be replaced by \( \theta/2 \) to give linear function, and \( \cos(\theta/2) \) would have to be replaced by \( \cos(\theta_m/2) \). So that the standard approximation \( \sin(\theta) \approx \theta \) becomes \( \sin(\theta) \approx 2\cos(\theta_m/2) \). Where \( \theta_m \) is the maximum angular displacement of pendulum. Then the nonlinear differential equation for a simple pendulum becomes a linear differential equation analogous to the simple harmonic oscillator however frequency of oscillation dependent on angular displacement \( \theta \) or simply on amplitude of pendulum. Gerald E. Hite in 2005 made approximation for the period of simple pendulum [4]. In 2009 an approximation scheme to obtain the period for large amplitude oscillation of a simple pendulum is analysed and discussed [5]. The latest research on the approximation of period of simple pendulum for large angle oscillation by using Taylor series expansion [6]. According to him the expression for time period of pendulum by using taylor series have the following form

\[
\frac{T_{nl}}{T_0} = \frac{4}{(1 + \sqrt{\cos(\theta/2)})^2}.
\]

Where \( T_0 \) is the time period for small angle approximation which is equal to \( T = 2\pi \sqrt{l/g} \) and \( T_{nl} \) is the approximate time period by considering first four terms of taylor series expansion for pendulum oscillating at large angle. Secondly, there is friction of axel with support, lenz’s law effect, and attraction between iron core of coil (used for concentrating effect of magnetic flux) in the model proposed [1] by authors. These parameters will effect significantly the time period and frequency of pendulum. Time period of that model will be less than the time period of simple pendulum. So the equation of simple pendulum for time period is no longer valid for the model proposed by author. In the figure of author’s model [1]. Construction of coil is shown is a layered coil in which inner most loops of coil do not cut the same number of magnetic lines as outer most loops of coil because inner loops have less diameter. Consequently the length of inner most loops cut magnetic lines is very less than outer most loops, so inner most layer of loops will induce less amount of emf as compared to outer most layer of loops. Proposed model [1] out put voltage can only be taken if all number of turns (loops) having same length or same diameter and then these loops will cut the equal number of magnetic lines of flux. There is also a limitation that copper wire coil does not remain all the time under the influence of magnetic filed when the angular displacement of magnetic pendulum is maximum which is 90. To overcome this limitation a step forward to that model is taken, so that during the whole journey of pendulum the copper coil will remain under the influence of magnetic field as long as pendulum oscillates and output efficiency of the system increases.

3. instantaneous time calculations of a simple pendulum.

Consider a simple pendulum of length 1.5 meters, a concentrated spherical bob of mass “m” is hanging under the influence of gravity “g”. Assume that there is no air friction and weight of thread is negligible small. When pendulum is released from 90° it will move towards its mean position “O” and oscillates. Time period of this pendulum according to equation (3) is \( T = 2.89 \) s by putting \( \theta = 90° \), \( l = 1.5 \) m and “g” = 9.8 m/sec\(^2\). We are interested in finding instantaneous time between any two intervals in first quarter cycle of “T” or simply in first “T/4” time interval as shown in fig. 2.

3.1. Maximum Speed.

Speed of simple pendulum does not depend on weight of pendulum. With length of 150 cm = 1.5 m and maximum speed of pendulum is calculated by using following formula

\[
V_{max} = \sqrt{2gl(1 - \cos \theta)}.
\]

Where “\( \theta \)” is the angle of pendulum with y axis, “\( l \)” is length of pendulum and “\( g \)” is the gravitational acceleration. For maximum speed angle is 90 degree (\( \pi/2 \) rad). By putting the values of \( \theta \), \( g \) and \( l \) we get

\[
V_{max} = \sqrt{(1 - \cos(90))} \times \sqrt{2 \times 9.8 \times 1.5} = 5.42 \text{ m/sec}.
\]
3.2. Time Calculations At Different Points.

When pendulum is released, after an angular displacement of 30° from point “P”, bob is at point’A’. The velocity of pendulum at point “A” will be determined by considering the energy equations of pendulum. Kinetic energy of pendulum at point “A” is

\[ K.E = \frac{1}{2}(mV_a^2). \] (5)

Potential energy of pendulum at point “A” P.E = \( mgl - mg \cos \theta \), which can be written as

\[ P.E = mgl(1 - \cos \theta). \] (6)

At point “O” all the energy of pendulum is converted into its kinetic energy such as \( K.E = \frac{1}{2}(mV_a^2) \). So total energy at point “A” becomes \( \frac{1}{2}(mV_a^2) + mgh(1 - \cos \theta) = \frac{1}{2}(mV_{max}^2) \) and above equation can be written as

\[ V_a^2 = V_{max}^2 - 2gl(1 - \cos \theta). \] (7)

Linear velocities in above equation can be converted into angular velocities( angular frequencies) by using the following relation.

\[ V = rw. \] (8)

By putting value of “V” in equation(7) we get following equation.

\[ r^2w_a^2 = r^2w_{max}^2 - 2gl(1 - \cos \theta). \] (9)

Or instantaneous angular frequency at any point can be written as

\[ w_{inst} = w_{max}^2 - \frac{2gl(1 - \cos \theta)}{r^2}. \] (10)

\( w_{max} = 3.61 \text{ rad/s} \) \( l = 1.5 \text{ m} \) \( g = 9.8 \text{ m/sec}^2 \) \( \theta = 60^\circ \) or \( \pi/3 \) and calculating the value of \( w_a \), which is \( w_a = 2.55 \text{ rad/s} \)

When pendulum is released from \( \theta = 90^\circ \), as \( \theta \) approaches to 0° (which is the mean position of pendulum) velocity of pendulum approaches to \( V_{max}^2 \).

\( \theta \rightarrow 0^\circ \)

\( V_{max} \)

Or alternatively \( \theta \) may be taken as 30° so cosine function in equation (10) is replaced by sine function and we get the same value of \( w_a = 2.55 \text{ rad/s} \),as given below \( w_a^2 = w_{max}^2 - 2gl(1 - \sin 30)/r^2 \) Now calculating time taken by pendulum to reach at point “A” from point “P”. We have find out angular velocity at point “A” already, and must have to find out angular acceleration of coil pendulum at position “A” by using equations of circular motion.

\[ 2a(\theta_1 - \theta_f) = w_f^2 - w_i^2. \] (11)
Where, $\theta_i$ is the initial angle at which the pendulum is displaced which is $\theta_i = \pi/2$ rad at position “P”, and $\theta_f = \pi/3$ rad at position “A” with respect to mean position “O”. $w_i$ and $w_f$ are initial and final angular frequencies respectively.

Angular velocity at point “P” is 0 and equation (11) becomes

$$2a(\pi/2 - \pi/3) = w_a^2 - w_p^2$$

$$2a_s(\pi/6) = 2a_s(0.25) = 2.55^2 - 0$$

$$\alpha_a = 6.25 \text{ rad/s}^2.$$  

This is the acceleration of pendulum at point “A”. By using the values of $w_a$ and $\alpha_a$ we are able to calculate the time taken by coil pendulum to reach at point “A” from point “P”. Using the following equation of motion.

$$w_a = w_p + \alpha_a t_{pa}.$$  

$$2.55 = 0 + (6.25)t_{pa}.$$  

$t_{pa} = 0.408s.$

Where $t_{pa}$ is the time taken from “P” to “A”. Time taken by pendulum from point “A” to point “B” can be calculated in similar manner as above. By putting the the following values in equation (10) $w_{inst} = w_b$.

$$w_{max} = 3.61 \text{ rad/s}.$$  

$l = 1.5 \text{ m}.$  

$g = 9.8 \text{ m/sec}^2.$  

$\theta = 30^o$ or $\pi/6.$

The value obtained of angular frequency at point “B” is $w_b = 3.36 \text{ rad/s}$ and angular acceleration at “B” is calculated by using equation (11) $2a_b(\pi/3 - \pi/6) = w_b^2 - w_a^2$. we get

$$\alpha_b = 4.5 \text{ rad/sec}^2.$$  

Time taken from point “A” to point “B” is calculated by using equation (12) as $w_b = w_a + \alpha_b t_{ab}$

$t_{ab} = 0.18 \text{ s},$ where $t_{ab}$ is time taken by pendulum from point “A” to point “B”.

Similarly time taken from point “B” to point “O” is $t_{bo} = 0.148 \text{ s}.$

Adding all these times we get total time taken from point “P” to point “O”

$$t_{po} = t_{pa} + t_{ab} + t_{bo} = 0.736 \text{ sec}.$$  

Time period of this pendulum having length of 1.5 m by using equation (3) is

$$T = 2.89 \text{ sec} \text{ so } T/4 = 0.7225 \text{ s}.$$  

Clearly it can be seen that $t_{po} \approx T/4$, which shows the verification of equation 3 and we made correct instantaneous time calculations.


Mechanical design of oscillating coil pendulum as shown in fig. 1(a). It consist of C.R.P attached with a flapping wing hanging vertically with axle and supports. Flapping wing made from light weight glass-textolyte can rotate along axel by means of bushes. When air thrust strikes on wing with force“F”, it compels wing to rotate around axel as a result C.R.P oscillates. Coil of C.R.P is chosen as brick shape. Copper wire is wrapped on a brick shaped light weight plastic core.

There are number of layers of loops on one another. Brick shaped coil is chosen so that all conductors of loops will be perpendicular to the magnetic flux lines and maximum emf will induce in coil. C.R.P is hanging under the influence of gravity. A semi circle shaped electromagnet is placed beneath the C.R.P. Electromagnet is used for experimental point of view. A permanent magnet with same shape and same magnetic flux density will replace electromagnet. yoke is made of cast iron, holds the two poles of magnet and all flux passes through it. The reason to use the semi circle magnet is that the pendulum always oscillates in such a way to make a pattern of a semi circle if it is released from 90° angular displacement.


Mechanical design of type-2 “Wing oscillating coil pendulum system”, shown in fig. 1(b). It is simpler than type-1. A permanent bar magnet of high magnetic field intensity is placed in a copper coil which is wrapped on a glass tube.
This bar magnet is attached with the bob of pendulum via a nylon fiber or rope over a pulley. Nylon fiber is helpful to reduce the friction between pulley and itself. When air thrust strikes the wing, pendulum oscillates and it results bar magnet moves up and down in copper coil. As long as pendulum oscillates the coil induces emf in it.

6. Electrical design and output electrical energy.

6.1. Maximum Speed.

The weight of wood rod is very less than the weight of coil. If considering negligible weight of wood rod as compared with coil weight then center of mass of C.R.P should be the center of coil. And if weight of rod is taken into account then the center of mass C.R.P will be shift towards rod from center of coil as shown in fig. 3(b). Irregular shapes or non uniform distribution of mass effect the position of c.m (center of mass) so the effective length of physical pendulum changes and in response velocity of physical pendulum varies. Effective length of a physical pendulum is always measured from center of mass to pivot point. The shape of C.R.P is uniform, as rod’s and coil’s shape is uniform. C.m point of C.R.P is at bottom as shown in fig. 3. In the absence of pivot point friction, and air resistance, we can compare C.R.P with simple pendulum shown in fig. 2, both simple and C.R.P have their c.m point on bottom. If we take the length of rod from C.m to the pivot point which is equal to 1.5 m then we can apply instantaneous time calculations to C.R.P which are derived in section 3. One factor which comes from lenz’s law will definitely reduce the time period. According to lenz’s law “induced current has its own magnetic flux which opposes the the motion of source”. With length of rod 150 cm = 1.5 m the maximum velocity of C.R.P is calculated by using equation 4 which is $5.42 – k \text{ m/sec}$. Where k is the factor by which velocity will reduced due to lenz’s law and friction.

6.2. Flux Density and Total flux.

Flux density is denoted by “$B$” and its unit in Tesla. Copper coil is rectangular brick shaped consist of many layers of loops. The length of each turn is perpendicular to magnetic field lines. For inner most layer, the length of each turn which is perpendicular to magnetic lines is $l_1$. And for second layer, the length of each turn which is perpendicular to magnetic lines is $l_2$ and so on up to “$m$” number of layers. Total flux is total number of webers passing through a given area. Flux through coil is calculated as $\Phi_{max} = B.S_A$ Where $S_A$ is the surface area of whole coil.

6.3. output electrical energy.

According to Faraday’s law of electro magnetic induction, calculating the induced emf denoted by “$e$” at point “A. Time taken by C.R.P from point “P” to point “A” has been calculated in section III, so $t_{pa}$ for C.R.P = 0.408s – k. This $t_{pa}$ now called as differential time or short interval of time from point “P” to “A”.

In fig. 4(b) differential time
is shown as $dt$, and magnetic flux cut during this time interval as $d\phi$. Distance from point “P” to point “A” is $S_{PA} = L \theta$ (length of arc in a circle). Where “L” is length of pendulum (wood rod), and “$\theta$” is angle traversed by pendulum. The differential flux is the product of flux density and the total area traversed by coil. Area covered by coil from point “P” to point “A” will be $(Area)_{pa} = (L\theta).1/2(l_x)$ when specific layer “x”, in which each turn having same length is considered

“L” is the length of C.R.P, $l_x$ is the length of a single turn which is perpendicular to magnetic field lines of a specific layer “x”. Practically $l_x$ is total length of a single turn because some length of a turn is parallel to field at the edges of core of coil. $1/2l_x$ is below the core and $1/2l_x$ is above the core. Subscript “x” denotes specific layer number.

$$d\phi_x = B.(Area)_{pa} = B.(L\theta).1/2(l_x).$$ \hspace{1cm} (13)

Dividing both sides by $dt$ we get

$$d\phi_x/dt = e_{ind} = 1/2(B.l_x.L.\theta)/dt.$$ \hspace{1cm} (14)

Suppose inner most layer have $N_1$ turns in which each turn have length $l_1$ perpendicular to magnetic field lines. Induced emf at point “A” due to these turns is

$$e_1 = 1/2.N_1.(B.l_1. L.\theta)/dt.$$ 

Total induced emf at point “A” by considering all layers on coil is

$$e_A = \sum_{i=1}^{m} 1/2.N_i.B.l_i.L.\theta \quad dt.$$ \hspace{1cm} (15)

For simulation results of output voltage and current wave form, we will find induced emf at different positions with same angular displacements.

In fig. 5(b), light emitting diode is used to get output in the form of light. Light emitting diode is suitable because its energy consumption is suitable for proposed model.

7. Design of energy efficient door from type-2.

Type-2 of “Wing oscillating coil pendulum system” can be used to make our home’s door energy efficient by doing little changes in it. An energy efficient door is shown in fig. 5(a). The idea behind this energy efficient door is that, when a normal healthy man enters in a room he opens door up to distance “x” from point “A” to point “B” by applying force “F1” and energy “E1” as shown in fig. 9. Now if we consider that same man is entering in room with emotions such as in anger or in cheer, he will definitely open the door with a force “F2” and energy “E2” such as “F2” > “F1”, “E2” > “E1”. When “F2” is applied door will cover distance “y” which is greater than distance “x” and magnet in
coil will move in upward. When door is closed back magnet moves downward. Our system of magnet and coil will convert some amount of “E2” into electrical energy whenever “E2” is applied. A small battery or capacitors will be connected with terminal “C” and “D” which can glow LED’s in emergency.

Fig. 5. (a) Design of an energy efficient door system. (b) Electric scheme for output energy: L generator coil; V1 Schotky diode; C1 electrolytic capacitor; V2 light-emitting diode. [1].

8. Results and conclusions.

In this paper we proposed two models to convert wind thrust energy and vibrational energy into electrical energy. Time period formula for large angle oscillation of simple pendulum derived by Taylor series expansion is verified by using circular equations of motion. Expression for induced emf in layered coil is derived in equation 15. Model of an energy efficient door is presented. Hardware and simulation results of output electrical power of our presented systems will be described later.

References