Speed Sensorless Vector Control of Unbalanced Three-Phase Induction Motor with Adaptive Sliding Mode Control

Mohammad Jannati, Ali Monadi, Nik Rumzi Nik Idris, Mohd Junaidi Abdul Aziz
UTM-PROTON Future Drive Laboratory, Faculty of Electrical Engineering, Universiti Teknologi Malaysia, Johor Bahru, Malaysia

1. INTRODUCTION

AC motor drives are widely used in industry. In these drives, AC motors like Induction Motors (IMs) and permanent magnet synchronous motors are used. These drives are employed applications such as Heating, Ventilation and Air Conditioning (HVAC), fans, mixers, robots etc. Squirrel cage IM offers many advantages to DC motors. The main problem with a DC motor is its commutators and brush maintenance, which renders this type of motor inoperable in dirty environments. In recent years, DC motors have been replaced by AC motor drives. There are many types of controlling methods for 3-phase IMs [1]. One of the most common methods for controlling the speed and torque of 3-phase IMs is Field Oriented Control (FOC). In last four decades many researchers have investigated this method [2], [3].

Open circuit is one of most familiar failures in the IM stator windings. Blown fuses, the opening of coils, mechanical shaking of the machine and etc. causes this fault. Recently, different techniques have been developed to detect stator-winding faults in IMs [4]-[8]. In [4] an approach based on look up tables and neural networks, in [5] a method based on negative sequence current estimation and in [6], a technique based on unknown input observer and Extended Kalman Filter (EKF), have been presented for stator windings fault detection in IM. Fault detection in this paper is implemented using on-line calculations that require motor voltage and current as introduced in [7]. The method enables almost immediate detection depending on the sampling of the system by reconstructing the current space phasor based on the IM equation and comparing it with its actual measured value. Variables that are needed to generate the detection signal are available from the vector control algorithm, thus detection can be performed almost immediately [7]. Another method that is used to detect open stator winding is introduced in [8] whereby a balanced 3-phase high frequency signal
with small amplitude was used to detect faults. Both methods provide almost immediate open stator winding detection and are assumed in this paper.

Modeling of a faulty or unbalanced 3-phase IM is extremely important in some critical applications, such as traction drive in military and space exploration, to ensure fault-tolerant operation. In order for the same control algorithms as used in the balanced 3-phase IM to be directly applied to the faulty motor, a model of the faulty motor should have the same equation structure as the balanced 3-phase IM model. The model of the unbalanced 3-phase IMs is, in principle, similar to the single-phase IM model. Several controlling methods have been introduced to control single-phase IMs, which can also be applied to the unbalanced or faulty 3-phase IMs [9]-[16]. In [9], Rotor Field Oriented Control (RFOC) of single-phase IM with hysteresis current controller has been presented. In [10], Stator Field Oriented Control (SFOC) of single-phase IM with current double sequence controller was presented. The use of a current double sequence controller causes the controlling system to be very complex. Asymmetry in the stator main and auxiliary winding in single-phase IM causes torque and current oscillations, even in the vector control of single-phase IMs. To solve this problem, in [11] a new decoupling vector control of single-phase IM has been proposed. In [12], [13] Direct Torque Control (DTC) method for single-phase IM has been discussed. In [15], a technique for sensorless single-phase IM efficiency maximization control using variable speed drives has been proposed and implemented. In [16], SFOC for two-phase induction motor with rotor speed estimation using Model Reference Adaptive System (MRAS) has been shown. Using the MRAS technique does not yield good results in the low speed IM drive operation. Moreover, this method is sensitive to resistance variations. In [17]-[21], several methods for 3-phase IM in faulty mode or single-phase IM were shown. By introducing new rotational transformations for stator current and voltage variables, these papers offer some method to control faulty or single-phase motors. By using the transformation matrices, it is shown in [17]-[21] that the equations of the unbalanced 2-phase IM for RFOC control strategy can be transformed into a structure of equations, which are similar to the balanced RFOC IM. It was shown that the exact RFOC block diagram used for balanced IM could be directly used for the unbalanced IM provided that some adjustments are made to the machine parameters [17]-[21].

Accurate speed estimation is an essential requirement for robust and highly accurate IM control. Using optical and mechanical sensors (such as tachometer, encoders etc) increases the complexity, cost, and drive system size. In addition, using these sensors decrease the reliability and robustness of the variable frequency drives. These disadvantages can be removed by rotor speed estimation. In recent years, speed estimation of single-phase IMs (unbalanced 2-phase IMs) techniques has attracted the interest of researchers [14]-[16] and [22]. In the work of Jemli et al. [14], a new method to estimate rotor speed by measuring stator currents and the reference q-axis current for single-phase IM has been presented. In [15] a speed estimation technique for single-phase IMs based on a machine model in the stator flux reference frame is proposed. In [22], speed sensorless RFOC of single-phase IM by using Extended Kalman Filter (EKF) has been presented.

The performance of the previous FOC for single-phase IMs or faulty IMs strongly depends on the load disturbances, unknown parameters, parameter variations (such as rotor and stator resistance), tuning of PI controller coefficients etc. Many researchers have been done on the IM drives to protect the performance of the controlling system under parameter variations such as neural control [23], fuzzy control [24], adaptive control [25], [26] and genetic algorithm [27]. One of the methods to overcome the parameter variation and system uncertainties in IMs is the use of adaptive sliding mode [28]-[30]. This method can provide a fast dynamic response, can be used in the observers design and can be employed for speed control of AC motor drives. The suggested method in [28] showed that using adaptive techniques could satisfy aching conditions. In this paper, the zero tracking error is obtained in the case of unfamiliar upper bound on norm of uncertainties as well as demonstrating the stability of a closed loop system. In [29], a new sensorless second order sliding mode control is applied to current fed IM. In addition to speed, load torque, rotor flux and rotor time constant are also estimated in this work. The proposed Higher Order Sliding Mode (HOSM) decreased the chattering problem, which causes an increase mechanical stress. The method presented in [29] requires an accurate knowledge of motor parameters. Paper [30] proposed a robust vector control for 3-phase IM drives with an adaptive sliding mode control. In this paper, the proposed variable structure with adaptive algorithm to calculate the sliding gain value is used. The results of [30] showed the proposed controller provides dynamic, high performance characteristics and it is robust with respect to parameter variations and external load disturbances.

In the proposed speed sensorless RFOC strategy for faulty IM, based on equivalent circuit of single-phase IM, rotational transformations are obtained and applied to the faulty 3-phase IM equations. Using these rotational transformations, the structure of the unbalanced 3-phase IM begins to resemble the balanced mode. Therefore, with some modifications in the conventional vector control of balanced 3-phase IM, vector control of unbalanced IM is possible. In the proposed speed sensorless RFOC method for faulty motors, for the robust vector control of faulty 3-Phase IM, an adaptive sliding mode control based on [30] is employed. As
the upper bound is used in the calculation of switching gain in the sliding mode control method, this method requires previous information of the upper bound for the system uncertainties. Choosing an accurate upper bound simply cannot be found in practical control systems as the uncertainties are very complicated [28], [31]. This research employs an adaptive sliding mode control with on-line estimation of sliding gain with the purpose of compensating for the system uncertainties. This report is organized as follows:

In section 2, the mathematical model of faulty IM in the dq frame is presented. The RFOC equations for faulty 3-phase IM are then presented using conventional rotational transformation. In section 3, the main idea behind using new rotational transformations and equations of RFOC for faulty IM by using proposed rotational transformations is provided. A method for speed estimation of faulty IM is presented in section 4. Vector control of faulty IM with adaptive sliding mode control and checking the stability of presented controller is shown in section 5. In section 6, the performance of the proposed method is evaluated and checked using Matlab software and section 7 concludes the paper.

2. MATHEMATICAL MODEL OF FAULTY IM

Suppose that a phase cut off fault has occurred in phase “c” of a 3-phase IM. The dq equations of faulty IM can be described as following equations:

\[
\begin{align*}
\frac{d\lambda_s'}{dt} &= r_s i'_s + \frac{d\lambda_q'}{dt}, \\
\frac{d\lambda_q}{dt} &= L_{ds} i_d + M_{d} i_{dr}, \quad \lambda_{qs} = L_{qs} i_{qs} + M_{q} i_{qr} \\
0 &= r_s i'_q + \frac{d\lambda_s}{dt} + \omega \lambda_{qs}, \\
0 &= r_q i'_q + \frac{d\lambda_q}{dt} - \omega \lambda_{qs}, \quad \lambda_{qr} = M_{q} i_{qr} + L_{qr} i'_r.
\end{align*}
\]

(1)

Where:

\[
L_{ds} = L_{qs} + L_{rs} + L_{qs}, \\
L_{qs} = \frac{3}{2} L_{rs}, \\
L_{mr} = \frac{1}{2} L_{ms}, \\
L_{ms} = \frac{3}{2} L_{mr}, \\
M_s = \frac{\sqrt{3}}{2} L_{ms}
\]

(2)

Moreover, \(\lambda_{qs}, \lambda_{qr}, \lambda'_{qs}, \lambda'_{qr}, \lambda'_{qs}, \lambda'_{qr}, \lambda'_{qs}, \lambda'_{qr}\) and \(\lambda_{qs}, \lambda_{qr}\) are the dq axes voltages, currents, and fluxes of the stator and rotor in the stator reference frame (superscript “s”). \(r_s\) and \(r_q\) denote the stator and rotor resistances. \(L_{ds}, L_{qs}, L_{rs} \), \(M_s\) and \(M_q\) denote the stator, the rotor self and mutual inductances. \(\omega\) is the machine speed. Electromagnetic torque and moving equation are as following equations:

\[
\tau_e = \frac{P}{2} (M_{q} i_{qs} i_{rq} - M_{d} i_{dq} i_{rd})
\]

\[
\tau_e - \tau_i = \frac{2}{P} \left( J \frac{d\omega}{dt} + F\omega \right)
\]

(3)

As can be seen from (1)-(3), the equations of the unbalanced motor are similar to the equations of the balanced one. In fact, by substituting \(M_s = M_q = \frac{3}{2} L_{ms}\) and \(L_{ds} = L_{qs} = L_{rs} + L_{rs} + \frac{3}{2} L_{ms}\) in the unbalanced equations, we can obtain the familiar equation of balanced motor. It can be said that the two stator windings of ”a,” and ”b,” with an equal number of turns and a spatial displacement of 120°, are transformed to ”d,” and ”q,” windings with an unequal number of turns and a spatial displacement of 90°.

3. RFOC EQUATIONS OR FAULTY IM

In the RFOC method and in the balanced condition, conventional rotational transformation (balanced rotational transformation), which is applied to the machine equations is as follows (in RFOC method, the motor equations are transferred to the rotor reference frame) [1]:

\[
[T_r] = \begin{bmatrix} \cos \theta_r & \sin \theta_r \\ -\sin \theta_r & \cos \theta_r \end{bmatrix}
\]

(4)

In this equation, \(\theta_r\) is the angle between the stationary reference frame and the rotor flux oriented reference frame. By applying Equation (4) to the faulty IM equations (Equation (1)), faulty motor equations...
are divided into forward and backward components [19]. The backward components are created because of
different inductances in faulty IM. Controlling faulty IM is possible by split control of forward and backward
components but the control system will be complex. To solve this problem (generating backward terms in the
faulty IM equation), transformation matrixes can be applied to the faulty motor equations, and by applying
these matrixes, faulty motor equations are obtained as balanced motor equations. Using these rotational
transformation matrixes is obtained from the steady state equivalent circuit of single-phase IM. The single-
phase IM circuit is shown in Figure 1 [32].

![Steady state equivalent circuit of the single-phase IM](image)

In this figure, $V_m$, $V_a$, $I_m$ and $I_a$ are the main and auxiliary voltages and currents, "$a$" is the turn ratio
($a=N_a/N_m$) and "$j$" is the square root of "-1". $E_{mf}$ and $E_{af}$ are the forward magnetizing branch voltages of the
main and auxiliary winding. $E_{mb}$ and $E_{ab}$ are the backward magnetizing branch voltages of the main and
auxiliary winding. $R_f$ and $R_b$ are the forward and backward stator resistance in main winding. $X_f$ and $X_b$ are
the forward and backward stator inductance in main winding. $R_{lm}$, $R_{la}$, $X_{lm}$ and $X_{la}$ are the leakage resistant
and inductance of the main and auxiliary winding. From Figure 1, the following equations can be written:

$$
V_m = Z_{lm} I_m + E_{mf} - \frac{j}{a} E_{fa} + E_{bm} + \frac{j}{a} E_{ba}
$$

$$
V_a = Z_{la} I_a + E_{fa} + j\alpha E_{fa} + E_{ba} - j\alpha E_{bm}
$$

(5)

Where:

$$
E_{fa} = Z_f I_a \quad , \quad E_{bm} = Z_b I_m \quad , \quad E_{fa} = a^2 Z_f I_a \quad , \quad E_{ba} = a^2 Z_b I_a
$$

(6)

By applying the following change of variables,

$$
I_m = \frac{1}{2} (I_1 + I_2)
$$

$$
I_a = \frac{j}{2\alpha} (I_1 - I_2)
$$

(7)

Ratio of windings currents is obtained as follows:
\[
I_m = \frac{Z_{la} + \alpha^2 (Z_f + Z_h) + j \alpha (Z_f - Z_h)}{Z_{ln} + Z_f + Z_h - j \alpha (Z_f - Z_h)}
\]
\[
I_a = \frac{Z_{ln} + Z_f + Z_h - j \alpha (Z_f - Z_h)}{Z_{ln} + Z_f + Z_h - j \alpha (Z_f - Z_h)} - \alpha Z_{ln} + jZ_{la} + 2 \alpha Z_h (\alpha j + 1)
\]
\[
I_1 = \frac{\alpha Z_{ln} + jZ_{la} + 2 \alpha Z_h (\alpha j + 1)}{- \alpha Z_{ln} + jZ_{la} + 2 \alpha Z_f (\alpha j - 1)}
\]

By using the following change of variables,

\[
V_1 = Z_3 V_m + jZ_4 V_a
\]

Figure 1 can be simplified as Figure 2.

![Simplified equivalent circuit of single-phase IM](image)

Moreover, \(Z_1\) and \(Z_4\) are the function in terms of \(M_d\) and \(M_q\). As shown, the equivalent circuit of a single-phase IM is changed into a balanced circuit. Equation (7) and (9) can be rewritten as the following equations:

\[
\begin{bmatrix}
    jI_1 \\
    I_1 \\
    jV_1 \\
    V_1
\end{bmatrix} = 
\begin{bmatrix}
    N_x \\
    N_a \\
    -jN_x \\
    N_a
\end{bmatrix}
\begin{bmatrix}
    j \\
    1 \\
    -Z_3 \\
    jZ_3
\end{bmatrix}
\begin{bmatrix}
    I_x \\
    I_a \\
    V_x \\
    V_a
\end{bmatrix}
\]

With the following substitutions:

\[
j \rightarrow \sin \theta_e, \quad 1 \rightarrow \cos \theta_e, \quad \frac{N_a}{N_d} \rightarrow \frac{N_q}{M_d}, \quad jV_1 \rightarrow v_{d^e}, \quad V_1 \rightarrow v_{q^e}
\]

\[
V_a \rightarrow v_{d^e}, \quad V_m \rightarrow v_{q^e}, \quad jI_1 \rightarrow i_{d^e}, \quad I_1 \rightarrow i_{q^e}, \quad I_a \rightarrow i_{d^e}, \quad I_m \rightarrow i_{q^e}
\]

The rotational transformations for stator voltage and current variables are obtained as follows:

\[
\begin{bmatrix}
    i_{d^e} \\
    i_{q^e}
\end{bmatrix} = 
\begin{bmatrix}
    M_d \cos \theta_e \sin \theta_e \\
    -M_d \sin \theta_e \cos \theta_e
\end{bmatrix}
\begin{bmatrix}
    i_{d^e} \\
    i_{q^e}
\end{bmatrix}
\]
Equation (13) and (14) are transformation matrixes for variable transformation from unbalanced mode to the balanced mode. It is expected that by using these transformation matrixes, the unbalanced faulty IM equations become similar to the balanced motor equations. In RFOC method, the rotor flux vector is aligned with d-axis ($\lambda_{dr} = |\lambda|$, $\lambda_{qr} = 0$). With this supposition and by applying (13) and (14) to the faulty motor equations and after simplifying, RFOC equations are obtained as:

$$\tau_e = \frac{P M_e}{2 L_e} |\lambda|^2$$

In (15), $T_r$ is rotor time constant. As shown by using the proposed rotational transformations (Equation (13) and (14)), RFOC equations for faulty IM begin to resemble balanced equations. The only difference between these equations and balanced IM equations is that in the balanced mode, we have $M$ instead of $M_q$. Necessary modifications to the conventional vector control, to make it suitable for the unbalanced motor, are summarized in Table 1.

### Table 1. Comparison between Two Vector Control Methods

<table>
<thead>
<tr>
<th>Conventional Vector Control for the Balanced Motor</th>
<th>Modified Vector Control for Faulty IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 to 2 transformation of the stator currents:</td>
<td>2 to 2 transformation of the stator currents:</td>
</tr>
<tr>
<td>$\begin{bmatrix} i_{ds} \ i_{qs} \end{bmatrix} = \begin{bmatrix} T_1 \ T_2 \end{bmatrix} \begin{bmatrix} i_d \ i_q \end{bmatrix}$</td>
<td>$\begin{bmatrix} i_{ds} \ i_{qs} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} &amp; 1 \ -\frac{\sqrt{2}}{2} &amp; 1 \end{bmatrix} \begin{bmatrix} i_d \ i_q \end{bmatrix}$</td>
</tr>
<tr>
<td>Balanced rotational transformation of the stator currents:</td>
<td>Unbalanced rotational transformation of the stator currents:</td>
</tr>
<tr>
<td>$\begin{bmatrix} i_{ds} \ i_{qs} \end{bmatrix} = \begin{bmatrix} \cos \theta_e &amp; \sin \theta_e \ -\sin \theta_e &amp; \cos \theta_e \end{bmatrix} \begin{bmatrix} i_{ds} \ i_{qs} \end{bmatrix}$</td>
<td>$\begin{bmatrix} i_{ds} \ i_{qs} \end{bmatrix} = \begin{bmatrix} M_e \cos \theta_e &amp; \sin \theta_e \ -M_e \sin \theta_e &amp; \cos \theta_e \end{bmatrix} \begin{bmatrix} i_{ds} \ i_{qs} \end{bmatrix}$</td>
</tr>
<tr>
<td>Mutual inductance: $M = \frac{3}{2} L_{sw}$</td>
<td>Mutual inductance $M = M_e = \frac{\sqrt{3}}{2} L_{sw}$</td>
</tr>
</tbody>
</table>

In this paper, rotor flux oriented faulty IM control with hysteresis current controller has been presented. Based on Equation (15) and Table 1, a block diagram of the fault-tolerant RFOC can be shown as Figure 3.
4. Rotor Speed Calculation

In this section a method for an accurate estimation of rotor speed based on motor voltages and currents in the stationary reference frame is presented. Based on (1), the angle \( \theta_e \) (the angle between the stationary reference frame and the rotor flux oriented reference frame) and its derivation can be defined as follows:

\[
\theta_e = \arctan \left( \frac{\lambda_{qr}}{\lambda_{dr}} \right) \Rightarrow \frac{d\theta_e}{dt} = \frac{d\lambda'_{qr}}{\lambda'_{dr} \lambda_{qr}^2 + \lambda_{dr}^2} = \frac{d\lambda'_{dr}}{\lambda'_{dr} \lambda_{qr}^2 + \lambda_{dr}^2}
\]  

(16)

From Equation (1), it is obtained:

\[
\frac{d\lambda'_{qr}}{dt} = \frac{L_r}{M_d} v'_{dr} - \frac{L_r}{M_d} \left( r_q + \sigma_d L_s \frac{d}{dt} \right) i'_{dr} = \frac{M_d}{T_r} i'_{dr} - \frac{1}{T_r} \lambda'_{dr} - \omega_r \lambda'_{qr}
\]  

(17)

\[
\frac{d\lambda'_{dr}}{dt} = \frac{L_r}{M_q} v'_{qr} - \frac{L_r}{M_q} \left( r_q + \sigma_q L_q \frac{d}{dt} \right) i'_{qr} = \frac{M_q}{T_r} i'_{qr} - \frac{1}{T_r} \lambda'_{qr} + \omega_r \lambda'_{dr}
\]

(18)

Where,

\[
\sigma_q = 1 - \frac{M_q^2}{L_r L_{qr}}
\]  

\[
\sigma_d = 1 - \frac{M_d^2}{L_r L_{dr}}
\]  

(19)

Substituting Equation (17)-(19) in Equation (16) obtains:

\[
\omega_e = \omega_r + \frac{1}{T_r} \left( \frac{M_q \lambda'_{dr} i'_{qr} - M_d \lambda'_{dr} i'_{dr}}{\lambda'_{dr}^2 + \lambda'_{qr}^2} \right)
\]

(20)

Then, substituting Equation (16) in Equation (20), \( \omega_r \) can be written as (21).

\[
\omega_r = \frac{1}{\lambda'_{dr}^2 + \lambda'_{qr}^2} \left( \frac{d\lambda'_{qr}}{dt} \lambda'_{dr} - \lambda'_{qr} \frac{d\lambda'_{dr}}{dt} - \frac{M_q \lambda'_{dr} i'_{qr} - M_d \lambda'_{dr} i'_{dr}}{T_r} \right)
\]

(21)

Therefore, in terms of the stator measured currents, voltages, and the rotor flux, \( \omega_r \) is obtained as (22).

\[
\omega_r = \frac{1}{\left| \lambda_e \right|^2} \left( \frac{L_r}{M_q} v'_{qr} - \frac{L_r}{M_q} \left( r_q + \sigma_q L_q \frac{d}{dt} \right) i'_{qr} \lambda'_{dr} \right)
\]

\[
- \lambda'_{qr} \left( \frac{L_r}{M_d} v'_{dr} - \frac{L_r}{M_d} \left( r_q + \sigma_d L_s \frac{d}{dt} \right) i'_{dr} \lambda'_{dr} \right) \left( \frac{M_q \lambda'_{dr} i'_{qr} - M_d \lambda'_{dr} i'_{dr}}{T_r} \right)
\]

(22)

Where,

\[
\left| \lambda_e \right|^2 = \lambda'_{dr}^2 + \lambda'_{qr}^2
\]

(23)

The structure of Equation (22) is like the structure of balanced IM. The only difference between these equations and balanced IM equations is that, in the balanced mode we have: \( M \) instead of \( M_q \) and \( L_s \) instead of \( L_{qr} \).
5. VECTOR CONTROL OF FAULTY IM WITH ADAPTIVE SLIDING MODE

From Figure 3, the controlling of faulty IM is sensitive to the variation of speed PI controller coefficients. In other words, from balanced mode to unbalanced mode, the coefficients of speed PI controller should change. In this paper, an adaptive sliding mode observer is replaced instead of speed PI controller. The mechanical equations of an IM can be written as:

\[ \tau_r - \tau_i = J \frac{d\omega_m}{dt} + F \omega_m \]

Equation (24) can be shown as:

\[ \frac{d\omega_m}{dt} + a \omega_m + c = bi_{qs}^* \]

In the RFOC and for faulty IM the parameters of a, b and c are defined as:

\[ a = \frac{F}{J}, \quad b = \frac{P M_s}{2 L_r} |i_{qs}|, \quad c = \frac{\tau_j}{J} \]

Equation (25) is considered with uncertainties as follows:

\[ \frac{d\omega_m}{dt} = -(a + \Delta a) \omega_m + (b + \Delta b)i_{qs}^* - (c + \Delta c) \]

Where the terms \( \Delta a, \Delta b \) and \( \Delta c \) denote the uncertainties of the terms a, b and c which depend on system parameters. The speed error can be shown as:

\[ e(t) = \omega_m(t) - \omega_m^*(t) \]

Where. \( \omega_m^*(t) \) is the reference speed. Taking the derivative of Equation (28) yields:

\[ \dot{e}(t) = \frac{e(t)}{dt} = -ae(t) + u(t) + d(t) \]

Where:

\[ u(t) = bi_{qs}^*(t) - a\omega_m^*(t) - \dot{\omega}_m^*(t) - c(t) \]

\[ d(t) = \Delta b i_{qs}^*(t) - \Delta a \omega_m(t) - \Delta c(t) \]

The switching surface with integral component for sliding mode speed control is considered as follows:

\[ S(t) = e(t) + \int_0^t (a + k)e(\tau) d\tau = 0 \]

Where, \( k \) is constant gain. The speed controller, which is considered in this paper, is as:

\[ u(t) = -ke(t) - \tilde{\rho}(t) a \text{sgn}(S(t)) \]

In Equation (32):

\[ \tilde{\rho}(t) \leq a \left| S(t) \right| \]

\[ \text{sgn}(S(t)) = \begin{cases} +1 & S(t) > 0 \\ -1 & S(t) \leq 0 \end{cases} \]
Moreover, $\rho(t)$ and $\alpha$ are estimated switching gain and a positive constant respectively.

**Theorem 1.** The adaptive structure speed controller with the adaptation algorithm (33) makes the controlled system (32) convergent to the switching surface $S(t) = 0$ and the stability for the speed control can be guaranteed as well.

Proof: Choosing a Lyapunov function candidate:

$$V(t) = \frac{1}{2} \left( S^2(t) + \hat{\rho}^2(t) \right)$$

(34)

Where:

$$\hat{\rho}(t) = \tilde{\rho}(t) - \rho$$

(35)

Taking the derivative of the Lyapunov function:

$$\dot{V}(t) = S(t) \dot{S}(t) + \hat{\rho}(t) \dot{\hat{\rho}}(t)$$

$$= S(t)(\dot{e}(t) + (a + k)e(t)) + \hat{\rho}(t)\dot{\hat{\rho}}(t)$$

$$= S(t)(-ae(t) + u(t) + d(t) + (a + k)e(t)) + \alpha \tilde{\rho}(t) S(t)$$

$$= S(t)(u(t) + d(t) + ke(t)) + \alpha (\tilde{\rho}(t) - \rho) S(t)$$

(36)

Assumption 1: $\rho > d_{\text{max}}$

Therefore, from Equation (36):

$$S(t) d(t) - \alpha \rho S(t) \leq \|S(t)\| d(t) - \|d(t)\| d_{\text{max}} \alpha$$

(37)

Assumption 2: $\alpha > 1$

Therefore, from Equation (37):

$$\|S(t)\| d(t) - \|d(t)\| d_{\text{max}} \alpha \leq 0 \Rightarrow \dot{V}(t) \leq 0$$

(38)

Using Lyapunov theorem, the controlled system is stable. Since $S(t)$ is bounded, $e(t)$ is also bounded. From Equation (30):

$$\dot{S}(t) = \dot{e}(t) + (a + k)e(t) = ke(t) + u(t) + d(t)$$

(39)

Because $e(t)$, $u(t)$ and $d(t)$ are bounded, $\dot{S}(t)$ is also bounded. From equation (36) it is deduced that:

$$\dot{V}(t) = \dot{S}(t) d(t) + S(t) - \alpha \rho \frac{d S(t)}{dt}$$

(40)

Equation (40) is also bounded. Barbalat’s lemma lets us conclude that,

$$\dot{V} \to 0 \Rightarrow S(t) \to 0$$

(41)

When the sliding mode occurs on the sliding surface, then:

$$S(t) = \dot{S}(t) = 0 \Rightarrow \dot{e}(t) = -(a + k)e(t)$$

(42)
Assumption 3: \( a + k > 0 \)

From Equation (17) and Assumption 3, it is obvious the tracking error \( e(t) \) converges to zero. Therefore, we have:

\[
i_q^c(t) = \frac{1}{b} \left( -ke(t) - \bar{p}(t) \alpha \text{sgn}(S(t)) + a\omega^*_q(t) + \omega^*_q(t) + c \right)
\]

(43)

Therefore, the proposed structure of the speed control with adaptive sliding mode resolves the sensitivity of the proposed RFOC to the speed PI controller coefficients and system parameters variations. The block diagram for the speed control with adaptive sliding mode can be presented, as in Figure 4.

![Figure 4. Block diagram of the speed control with adaptive sliding mode](image)

In conclusion, based on Figure 3 and Figure 4, Figure 5 can be recommended as a speed sensorless vector control of the faulty 3-phase IM with adaptive sliding mode control.

![Figure 5. Block diagram of the proposed speed sensorless vector control of the faulty 3-phase IM with adaptive sliding mode control](image)

6. SIMULATION RESULTS

A 3-phase IM which is fed from a 3 hysteresis band current SPWM (Sine Pulse Width Modulation) Voltage Source Inverter (VSI) was simulated by Matlab software. Motor data are presented in Appendix A. The controller, which was used for the speed control of the balanced motor, is a conventional RFO vector controller as can be seen in Figure 5. To verify the effectiveness of the proposed speed sensorless control method for faulty 3-phase IM, vector control drive system based on Figure 5 is also simulated. To demonstrate the better performance of the proposed drive system, an uncertainty of around 10% in the system parameters after applying load torque moreover and an uncertainty of around 10% in the system parameters after fault occurrence is supposed.

Figure 6 shows the simulation results of the conventional vector controller. In starting and loading, the motor is healthy. At time \( t=0.5s \) the load torque steps from 0N.m to 1N.m (in 3-phase IM and under open phase fault, the maximum permissible torque is about 30% of the rated motor torque as mentioned in [33]). A phase cut out fault then occurs and the motor becomes unbalanced (a phase cut off occurs at \( t=1.5s \)).
Simulation shows that the conventional vector controller cannot properly control the unbalanced motor. A considerable oscillation is seen in the electromagnetic torque (about 1N.m around the average amount of 1N.m).

Figure 6. Simulation results of the conventional speed sensorless R.F.O vector controller; Speed reference = 500rpm, Load torque = 1N.m (a) Stator a-axis current, (b) Rotor a-axis current, (c) Electromagnetic torque, (d) Speed

Figure 7. Simulation results of the proposed speed sensorless R.F.O vector controller; Speed reference = 500rpm, Load torque = 1N.m (a) Stator a-axis current, (b) Rotor a-axis current, (c) Electromagnetic torque, (d) Speed
In Figure 7, the same process is repeated but this time after the fault occurrence the proposed modifications in the vector controller are applied (Figure 5). Simulation results show that the proposed vector controller reduces the torque oscillation considerably (this time the torque oscillation is about 0.4N.m around the average amount of 1N.m).

Using sliding mode control in the proposed faulty 3-phase IM drive causes the drive system to become insensitive to variations in load torque disturbances and motor parameters. Due to the variable structure control nature, this proposed drive system is robust under uncertainties caused by changes in the load torque and parameter errors.

7. CONCLUSION

In this paper, a new scheme for the robust vector control of 3-phase IM under open phase fault has been presented. In the proposed method for controlling faulty motor, two new transformation matrixes are invented which are different for the current and the voltage variables. With these new transformations the transformed equations of the unbalanced motor are obtained, such as the balanced motor equations. Using this similarity we present a new vector control, which is obtained from the conventional one by applying a few modifications. Moreover, in this paper, based on current, voltage and flux equations in the stationary reference frame, the rotor speed for both balanced and unbalanced IM is estimated. In the proposed method, we need to change the speed PI controller coefficients from balanced mode to unbalanced mode. To solve this problem, the sliding control scheme with on-line adaptive law is used in the presented adaptive sliding mode control in order to calculate the sliding gain. In the presented sliding control system, previous knowledge of the upper bound for the system uncertainties is not necessary. The stability of the adaptive sliding mode control in this work has been proven through Lyapunov stability theory. The advantage of the presented method is that this method can be used for the vector controlling of the single-phase IMs as well. In fact, a single-phase IM with dissimilar main and auxiliary windings is an unbalanced IM. The performance of the presented RFOC scheme is highly satisfactory for controlling faulty 3-phase IM especially in decreasing the torque ripples. In this way, this technique seems to be suitable method for critical industrial applications where there is a need a fault-tolerant control.

APPENDIX A

Ratings and parameters of 3-phase IM:

\[ v = 125V , \quad f = 50HZ , \quad f_{pole} = 4 , \quad r_s = 20.6\Omega , \quad r_r = 19.15\Omega \]

\[ L_i = 0.0814 , \quad L_d = 0.0814H , \quad L_m = 0.851H , \quad J = 0.0038kg.m^2 \]

REFERENCES


Speed Sensorless Vector Control of Unbalanced Three-Phase Induction Motor with... (Mohammad Jannati)


