Localization and Control of Tracked Mobile Robots under Slip Conditions

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Abstract—This paper deals with the localization and trajectory tracking control problems of tracked mobile robots under slip conditions. The proposed control law consists of the modification of a well-known control algorithm based on the feedback linearization technique, in which additional parameters have been included in order to compensate for the slip effects. Furthermore, an Indirect Kalman Filter has been used to improve the localization of the robot. Real tests show promising results.

I. INTRODUCTION

Tracked Mobile Robots (TMR) are increasingly being used on rough off-road terrains for applications such as forestry, mining, agriculture, army, and in general in different kinds of applications on unpaved terrains [1], [2]. These applications usually require robots to travel across unprepared terrain to operate or transport material.

On this kind of terrains, the mobility and the controllability of the robot is strongly influenced by the physical properties of the terrain. Track-soil interaction produces a reaction force to push the vehicle forward and it imposes a longitudinal slip component [1], [3]. This slip component is conceptually different from that of wheeled vehicles, in which it is motivated by the pneumatic tire deformation [4]. However, in practice both slip terms can be calculated in a similar way [3].

The slip phenomenon has been addressed by many researchers and is currently a key issue in the field of mobile robots working on off-road environments [5], [6]. In [7], the problem is addressed for an Ackermann-type agricultural vehicle in which adaptive control techniques were used to face the lateral slip effects. The work [8] proposes a kinematic approach for a TMR adding the different values of the instantaneous centers of rotation (ICRs) of the tracks. The position of these ICRs depends on the track-soil interactions.

This paper presents a linear feedback controller based on a modification of the well-known linear feedback controller described in [9]. Additional parameters have been included to compensate the longitudinal slip effect. Furthermore, an indirect Kalman filter (IKF) has been included to localize the TMR. Finally, real tests show a substantial improvement against the original controller under slip conditions.

The motivation of this work comes from a research project at the University of Almería (Spain), which is devoted to the development of a TMR to perform different tasks on slippery terrains (greenhouses) [2]. Greenhouses are composed of sandy loose soils, which have an important slip component (around 10-15%) [1].

The paper is organized as follows: the IKF used to localize the mobile robot is described in section 2. The next section is devoted to describe the modified control law. Validation of the IKF and real tests for the control algorithm are discussed in section 4. Finally, some conclusions are summarized in section 5.

II. LOCALIZATION OF TMR UNDER SLIP CONDITIONS

It is well-known that one of the main issues in mobile robotics is the robot localization (position and orientation), that is, the process in which a mobile robot determines its current position and orientation [10].

In our case, IKF has been used to fusing data from different sensors to improve the localization of the mobile robot [10], [11], [12]. Odometry has been used with calibrated track radius and effective distance between tracks centers in order to avoid systematic errors, and the radar-compass combination to avoid non-systematic errors due to slip effects [13].

A. Kinematic models

The approach presented in this paper is based on the kinematic model of a differential-drive mechanism [14] since a TMR can be approximated by this configuration.

For odometry, the discrete-time kinematic model is given by

\[
\begin{align*}
    x^{odo}(k+1) &= x^{odo}(k) + T_s \frac{t_r}{2} [\phi_{mr}(k) + \phi_{ml}(k)] \cos \theta^{odo}(k), \\
    y^{odo}(k+1) &= y^{odo}(k) + T_s \frac{t_r}{2} [\phi_{mr}(k) + \phi_{ml}(k)] \sin \theta^{odo}(k), \\
    \theta^{odo}(k+1) &= \theta^{odo}(k) + T_s \frac{t_r}{b} [\phi_{mr}(k) - \phi_{ml}(k)],
\end{align*}
\]

(1)

where \( [x^{odo}, y^{odo}, \theta^{odo}] \) represents the position of the robot using odometry, \( k \) is the current time instant, \( T_s \) is the sampling time, \( \phi_{mr} \) and \( \phi_{ml} \) are the angular velocity measured by the encoders of the right and left tracks respectively, \( b \) is the distance between the tracks centers, and \( t_r \) is the track radius.
For a radar-compass system, the previous equation can be rearranged as,
\[ x^{rc}(k+1) = x^{rc}(k) + T_s v_m(k) \cos \theta^{rc}(k), \]
\[ y^{rc}(k+1) = y^{rc}(k) + T_s v_m(k) \sin \theta^{rc}(k), \]
\[ \theta^{rc}(k+1) = \theta^{rc}(k) + T_s \omega_m(k), \]
where \([x^{rc} \ y^{rc} \ \theta^{rc}]\) represents the position of the robot using radar-compass [10], \(v_m\) is the linear velocity of the vehicle measured with a radar, and \(\omega_m\) is the angular velocity of the vehicle measured with a magnetic compass.

### B. Feedback indirect Kalman filter

The indirect formulation of the Kalman Filter is used to improve the position calculated by a primary localization method (i.e. odometry). This approach improves the position obtained from a primary source by means of differences with data from other auxiliary sources [11]. In this paper, we present an implementation of the feedback IKF where odometry and radar-compass sources are compared to improve the localization, and then the estimated errors are fed back into the odometry to correct it. 

As commented in [10], the predicted error is obtained as the error between reference and odometry (prediction stage),
\[ \hat{e}_{k+1|k} = A \hat{e}_{k|k}, \]
where the error between reference and odometry position is determined as
\[ \hat{e}_{k|k} = p^{ref}(k) - p^{odo}(k), \]
where \([x^{ref} \ y^{ref} \ \theta^{ref}]^T\) is related to the reference, and \(p^{odo} = [x^{odo}, \ y^{odo}, \ \theta^{odo}]^T\) is the position of the robot obtained from (1). Generally, in the IKF formulation the state matrix \(A\) is equal to the identity matrix [11]. In this case \(A = I_3\). Then, error covariance matrix \(P\) is calculated,
\[ P_{k+1|k} = AP_{k|k} + Q^{odo}, \]
where \(Q^{odo}\) is the noise covariance matrix for odometry,
\[ Q^{odo} = \begin{bmatrix} \sigma^2(x^{ref} - x^{odo}) & 0 & 0 \\ 0 & \sigma^2(y^{ref} - y^{odo}) & 0 \\ 0 & 0 & \sigma^2(\theta^{ref} - \theta^{odo}) \end{bmatrix}. \]

In the next step, the Kalman gain \(K\) is calculated as follows
\[ K_k = P_{k+1|k}[P_{k+1|k} + Q^{rc}]^{-1}, \]
where \(Q^{rc}\) is the noise covariance matrix for radar-compass,
\[ Q^{rc} = \begin{bmatrix} \sigma^2(x^{rc} - x^{ref}) & 0 & 0 \\ 0 & \sigma^2(y^{rc} - y^{ref}) & 0 \\ 0 & 0 & \sigma^2(\theta^{rc} - \theta^{ref}) \end{bmatrix}. \]

Matrices \(Q^{odo}\) and \(Q^{rc}\) are experimentally determined measuring the raw data from sensors and statistically calculating the variance of the error for each component of the position (between these data and the reference) over \(n\) sampling data [11]. In this paper, this process was repeated for several experiments taking as result the average of them. Then, the error state is estimated using the Kalman gain, \(K\), and the error between odometry and radar-compass (correction stage) producing
\[ \hat{e}_{k+1|k+1} = \hat{e}_{k+1|k} + K_k(z_k - \hat{e}_{k+1|k}), \]
where \(z_k = p^{odo} - p^{rc}\). Now, the error covariance matrix is updated,
\[ P_{k+1|k+1} = (I_3 - K_k)P_{k+1|k}. \]

Finally, the position of the robot is calculated using the odometry and the estimated error as
\[ \hat{p}_{k+1|k+1} = \hat{p}_{k|k} + \hat{e}_{k+1|k+1}. \]

### III. CONTROL OF TMR UNDER SLIPPERY CONDITIONS

It is well known that a nonholonomic system, such as a TMR, cannot be stabilized by smooth static state feedback laws [9]. This systems fails in the Brockett’s Condition for the existence of a continuously differentiable control law (the dimension of the state space is three and the number of control signals are only two) [15].

In [9], a linear feedback control law is presented based on the dynamic feedback linearization technique, which is used to linearize the kinematic model of an unicycle-type robot. This section describes a modification of this well-known control law where additional terms have included in order to compensate for slip effects. The same notation that in [9] is used, for that reason we present a continuous-time formulation of the control law. However, for the combination with IKF and for real-time implementation purposes, the algorithm was discretized.

The control architecture uses a four-layer structure (Fig. 1). The upper layer is devoted to path planning. This layer generates the reference or desired trajectories and velocities [2]. The second layer includes the motion control and uses the trajectory tracking control law given in this paper. As it can be observed, this controller uses the online slip estimation. In the third layer, two PI controllers have been implemented in order to reach the desired trajectories given by the upper layer. Finally, IKF is used to estimate the localization of the mobile robot, such as described in section II-B.
A. Control law with slip compensation

The kinematic model of the robot including slip effects can be expressed as [6],

\[ p(t) = \begin{bmatrix} \cos \theta(t) & 0 & \frac{v(t)}{\omega(t)} \\ \sin \theta(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sigma(t)}{\lambda(t)} \end{bmatrix}, \]  \hspace{1cm} (13)

where \( p = [x \ y \ \theta]^T \) is the position of the robot, \( v \) and \( \omega \) are the control signals (linear velocity and angular velocity of the vehicle), and \( \sigma \) and \( \lambda \) are the slip terms defined as

\[ \sigma(t) = \frac{t_r \phi_{mr}(t) i_r + t_r \phi_{ml}(t) i_l}{2}, \]  \hspace{1cm} (14)

\[ \lambda(t) = \frac{t_r \phi_{mr}(t) i_r - t_r \phi_{ml}(t) i_l}{b}, \]  \hspace{1cm} (15)

where \( i_r \) and \( i_l \) are the longitudinal slips of the right and left tracks respectively.

From equations (13)-(15), it can be observed how the variables \( \sigma \) and \( \lambda \) can be interpreted as pseudo-disturbances in the motion of the vehicle.

The trajectory tracking problem can be seen as the problem in which a mobile robot must follow a virtual mobile robot representing the desired positions and velocities. For that reason, the objective is that the difference (error) between the virtual mobile robot and the real robot is kept as small as possible.

Such as described in [9], [6], and considering a coordinate change for the global frame, the dynamic equation of the error can be expressed as

\[ e(t) = \begin{bmatrix} \cos \theta(t) & \sin \theta(t) \\ -\sin \theta(t) & \cos \theta(t) \end{bmatrix} \begin{bmatrix} x(t) - x_r(t) \\ y(t) - y_r(t) \end{bmatrix}, \] \hspace{1cm} (16)

where \( e = [e_x \ e_y \ e_{\theta}]^T \) is the state composed of the longitudinal deviation or longitudinal error, the lateral deviation or lateral error, and the orientation deviation or orientation error respectively.

Differentiating (16) with respect to time and using (13), it is obtained that [6],

\[ \dot{e}(t) = \begin{bmatrix} \frac{\sigma(t)}{\lambda(t)} \end{bmatrix} + \begin{bmatrix} \cos \theta(t) & \sin \theta(t) \\ -\sin \theta(t) & \cos \theta(t) \end{bmatrix} \begin{bmatrix} \omega(t) \ \omega_r(t) \end{bmatrix} + \begin{bmatrix} \sigma(t) \ \lambda(t) \end{bmatrix}, \] \hspace{1cm} (17)

where \( \alpha(t) = \omega(t) - \lambda(t) \).

As it can be observed from previous equation, the linearization is required due to the presence of nonlinear terms. Hence, the dynamic feedback linearization technique is used, following the idea proposed in [9]. Thus, it produces

\[ u_1(t) = v_r(t) \cos \epsilon_\theta(t) - v(t) + \sigma(t) - \lambda(t) \epsilon_y(t), \] \hspace{1cm} (18)

\[ u_2(t) = \omega_r(t) - \omega(t) + \lambda(t), \] \hspace{1cm} (19)

where \( u_1 \) and \( u_2 \) are called virtual control signals.

Notice that the virtual control signals include the terms depending on the linear velocity in the case of \( u_1 \) and angular velocity in the case of \( u_2 \). Now, the slip effects by means of \( \sigma \) and \( \lambda \) are also included in the control strategy.

As commented above, the control signals, which must be sent to the low-level controllers, are obtained transforming previous equations such as

\[ v(t) = v_r(t) \cos \epsilon_\theta(t) - u_1(t) + \sigma(t) - \lambda(t) \epsilon_y(t), \] \hspace{1cm} (20)

\[ \omega(t) = \omega_r(t) - u_2(t) + \lambda(t). \] \hspace{1cm} (21)

Since the control signals that can be manipulated in the TMR are the velocities of each track, previous signals \( (v, \omega) \) must be translated as

\[ v_r(t) = v(t) + \frac{\omega(t)b}{2}, \] \hspace{1cm} (22)

\[ v_l(t) = v(t) - \frac{\omega(t)b}{2}, \] \hspace{1cm} (22)

where \( v_r \) and \( v_l \) constitute the desired trajectories to the low-level PI controllers.

Now, if (17) is linearized around the equilibrium point, we obtain the following time-variant linear system

\[ \dot{e}(t) = \begin{bmatrix} 0 & \frac{\sigma(t)}{\lambda(t)} - \omega_r(t) \\ 0 & 0 \end{bmatrix} e(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} u(t), \]
where \( u = [u_1 \ u_2]^T \).

As mentioned above, this paper presents a modification of a feedback control law [9] in order to take into account slip effects. This feedback control law is described in the following equations [9]

\[
\begin{align*}
    u_1(t) &= -k_1(t)\omega_1(t), \\
    u_2(t) &= -k_2(t)\omega_1(t) + k_3(t)\omega_2(t),
\end{align*}
\]

where \( k_i \) are the time-dependent gains of the controller using the virtual control signals \( u_1 \) and \( u_2 \).

Finally, in order to obtain these gains, we apply the same procedure described previously in [6] resulting

\[
\begin{align*}
    k_1(t) &= 2\delta(\omega_1^2(t) + \beta\omega_2^2(t))^{\frac{1}{2}}, \\
    k_2(t) &= \beta(\omega_2(t) + \omega_2(t)\lambda(t)), \\
    k_3(t) &= 2\delta(\omega_1^2(t) + \beta\omega_2^2(t))^{\frac{1}{2}},
\end{align*}
\]

where \( \beta > 0 \) and \( \delta > 0 \) allow the determination of the desired closed-loop behavior of the system.

Notice that, equations (25)-(27) differ with respect to those obtained in [9] because of the presence of \( \lambda \) parameter in (26). In this sense, \( k_2 \) is online updated based on the slip disturbances.

B. Estimation of the slip

There are different ways to determine the slip [16], [3]. In this work, we have used the solution proposed by [3], where the theoretical velocity of the tracks is measured using encoders, and the real forward velocity of the vehicle is calculated using the radar. The main advantage of this approach is that the slip is easily determined and vehicle immobilization is successfully detected

\[
i_j(t) = 1 - \frac{v_{n_j}(t)}{v_{m_j}(t)},
\]

where \( i_j \) is the slip of the track \( j \).

IV. RESULTS AND DISCUSSION

The localization and control strategies described in this paper have been tested using a TMR available at the University of Almería (Spain) called Fitorobot (see Fig. 2). More details about this TMR can be found in [2].

A. Validating the localization techniques

The first tests were devoted to evaluate the IKF. For that, different well-known localization techniques were also implemented for comparison, namely, odometry simulating a calibrated track radius and a non-calibrated track radius, and radar-compass combination.

The real track radius of the testbed is 0.15 [m] and the calibrated track radius is 0.10 [m]\(^1\). The distance between the tracks centers is 0.5 [m], and the noise covariance matrices \( Q^{odo} \) and \( Q^{rc} \) are

\[
\begin{align*}
    Q^{odo} &= \begin{bmatrix} 12.1 & 0 & 0 \\ 0 & 24.8 & 0 \\ 0 & 0 & 9.8 \end{bmatrix}, \\
    Q^{rc} &= \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 1.9 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}
\end{align*}
\]

In order to test the performance of the localization techniques, the mobile robot was teleoperated over some trajectories. In this case, we show a U-shaped reference trajectory on a sandy soil. The size of the trajectory was approximately 14 meters long and 10 meters width. Since it was an open-loop test, the real trajectory followed by the mobile robot differs slightly from the reference trajectory.

Fig. 3 shows the reference trajectory and the trajectories obtained using the localization methods (trajectory with calibrated radius is denoted as \( cr \)). As observed, the trajectory using the IKF fits correctly the reference trajectory (taking into account the errors due to open-loop pointed above). Techniques based on odometry show an unacceptable behavior, mainly, at the turns.

The errors are better observed from Fig. 4. To calculate these errors the Euclidean norm (between the error in the forward and lateral directions) has been used. As advised, the error using odometry grows considerably. The lowest error is achieved using the feedback IKF.

B. Real tests

Some closed-loop tests were carried out by means of several trajectories in order to compare the performance of the proposed modified control law with the original formulation. In this case, we present a circular trajectory on a sandy loose soil. The diameter of the trajectory was 6 [m]. Furthermore, due to the test was realized on a sandy loose soil, an appreciable slip was expected. The initial condition for the trajectory was

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\(^1\)Since a track is different to a wheel, some experiments have been carried out to find out the relationship between the linear velocity of the vehicle and the angular velocities of the tracks. That is the reason by which appears two different radii.
The control signals (linear and angular velocities), are smaller than those obtained with the original control law. The modified controller presents less aggressive control signals than the original formulation.

V. CONCLUSIONS AND FUTURE WORKS

This paper presents the modification of a well-known linear feedback controller with the aim of compensating slip effects. Furthermore, an IKF has been used to improve the localization of the mobile robot. This localization technique has been validated through real experiments. Afterwards, the proposed control approach was tested through real tests under slip conditions showing substantial improvements. As future works, we will attempt the inclusion of a dynamic model instead of kinematic ones, and other advanced control policies.

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REFERENCES

Fig. 5. (a) Trajectories in the real experiment (b) Values of $\sigma$ and $\lambda$ parameters

Fig. 6. Longitudinal, lateral and orientation errors

Fig. 7. (a) Slip of each track (b) Gains of both controllers

Fig. 8. (a) Control Signals (linear and angular velocities) (b) Virtual Control Signals


