Accuracy Evaluation of Generalized Prandtl-Ishlinskii Model in Characterizing Asymmetric Saturated Hysteresis Nonlinearity Behavior of Shape Memory Alloy Actuators

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Abstract – Prandtl-Ishlinskii (P-I) model is one of the powerful operator-based phenomenological models which is used in modeling complex hysteretic nonlinear behavior in piezoelectric, piezoceramic, magnetostrictive and shape memory alloy actuators. The most appealing and unique aspect of the Prandtl-Ishlinskii model comes back to this fact that this model is analytically invertible and therefore could be easily implemented as a feedforward controller for compensating the hysteretic nonlinearity behavior. However, the first form of this model, called the Classical Prandtl-Ishlinskii model, cannot describe systems with output saturation and also results to considerable error when there is an asymmetric in the input-output hysteresis loops. In order to eliminate these shortcomings, some modifications are applied to the Classical Prandtl-Ishlinskii model and these models are called the modified or generalized Prandtl-Ishlinskii models. There are some weaknesses in previous researches that it is not clear whether the developed generalized Prandtl-Ishlinskii is capable in predicting hysteresis minor loops of SMA actuators and how much its level of accuracy in the prediction task is. Some of these research shortcomings are perhaps due to the fact that the tested actuator was not available in the laboratory and therefore further experimental tests were not performable. In this paper the generalized Prandtl-Ishlinskii model is trained by some experimental measured data obtained from an experimental test set-up consisting of a flexible beam actuated by a shape memory alloy wire. The parameters of the generalized Prandtl-Ishlinskii model are identified in order to adapt the model response to the real hysteretic nonlinearity. Then the accuracy of the obtained generalized Prandtl-Ishlinskii model in predicting nonlinear hysteretic behavior of the system is validated with some different experimental data. Although the model has been trained with the data of first order descending reversal curves, it is shown that the generalized Prandtl-Ishlinskii model has good power in behavior prediction of first order ascending curves as well as higher order minor loops.

Keywords – Prandtl-Ishlinskii model, Shape Memory Alloy, Experimental Data

1. Introduction

Hysteresis is a mathematically attractive and challenging phenomenon that observed in different materials such as piezoelectrics, piezoceramics, magnetostrictives and shape memory alloys. In fact, hysteresis refers to a relation between two scalar quantities that cannot be expressed in terms of a single-valued function and takes the form of loops [6]. Also, in the mathematical literature the notation of the nonlinear-hysteresis usually means "rate-independent memory effect" [6]-[7] and [9]. This means that the present output signal value of a system with hysteresis depends not only on the present value of the input signal but also on the order of their amplitudes, especially their extremum values, but not on their rate in the past [3].

Since un-modeled hysteresis causes inaccuracy in trajectory tracking and decreases the performance of the control systems, an accurate modeling of hysteresis behavior for performance evaluation and identification as well as controller design is essentially required [10]. Therefore, it is necessary to develop hysteresis models that not only identification of the model parameters, in order to adapt the model to the real hysteretic nonlinearity, can easily and precisely be performed but also are suitable for real time control and compensation system design.

Two different methods of hysteresis modeling have been proposed to capture the observed hysteric characteristics [16]. The first group of models is derived from the underlying physics of hysteresis and combined with the empirical factors to describe the observed characteristics [17]-[19]. However, these models have limited applicability, as the physical basis of some of the hysteresis characteristics is not completely understood [20]. Furthermore, considerable effort is required in identifying and tuning the model parameters to accurately describe the hysteresis nonlinearity. Another major drawback of these physical models is that they are specific to a particular system, and this implies separate controller design techniques for each system [21].

The second group of models is based on the phenomenological nature and mathematically describes the observed phenomenon without necessarily providing physical insight into the problems. The most important phenomenological hysteresis models include operator based hysteresis models and differential equation-based hysteresis models [11]. Preisach model [7], Krasnosel’s’kii-Pokrovskii model [6], and Prandtl-Ishlinskii model [8] are some important operator-based hysteresis models while Duhem
model and Bouc-Wen model [9] are most widely used differential equation-based hysteresis models. Choosing a proper phenomenological model among the mentioned models is a crucial task since the mathematical complexity of the identification and inversion problem depends directly on the phenomenological modeling method and strongly influences the practical use of the design concept [3]. Furthermore, the accuracy of the modeling method in characterizing system hysteretic behavior consequently affects the whole compensator design task.

Prandtl-Ishlinskii (P-I) model is one of the powerful operator-based phenomenological models which is used in modeling complex hysteretic nonlinear behavior in piezoelectric, piezoceramic, magnetostrictive and shape memory alloy actuators. The most appealing and unique aspect of the Prandtl-Ishlinskii model is that, unlike the Preisach and Krasnosel’skii-Pokrovskii models which their inverses are obtained numerically, this model is analytically invertible and therefore can be easily implemented as a feedforward controller for compensating the hysteretic nonlinearity behavior. In other words, the exact inverse of this model is accessible, consequently making it more attractive for real-time control applications of smart actuators. Like preisach model, Prandtl-Ishlinskii model is defined in terms of play or stop operator with a density function determining the shape of hysteresis [6].

Although the Prandtl-Ishlinskii model (like Krasnosel’skii-Pokrovskii model) is a subclass and extension of the Preisach model [6], the first from of this model, called the Classical Prandtl-Ishlinskii model, cannot describe systems with output saturation because of unbounded nature of the classical play operator. In addition, owing to the symmetric nature of the play operator, applying the classical Prandtl-Ishlinskii model results to considerable error when there is an asymmetric in the input-output hysteresis loops, like what is observed in shape memory alloy and magnetostrictive actuators [12]. In order to eliminate these shortcomings, some modifications are applied to the Classical Prandtl-Ishlinskii model. Janocha and Kuhnen [1] proposed a modified Prandtl-Ishlinskii model by integrating a number of deadzone operators into the classical Prandtl-Ishlinskii to characterize hysteresis nonlinearities of some smart actuators. Kuhnen [3] introduced a modified Prandtl-Ishlinskii hysteresis operator by the serial combination of a classical Prandtl-Ishlinskii hysteresis operator and a memory-free nonlinearity with an asymmetrical function in order to describe the saturated asymmetric hysteresis nonlinearities available in many of smart actuators. In fact, in this method, the classical Prandtl-Ishlinskii is extended to a so-called modified Prandtl-Ishlinskii approach for the modeling, identification and compensation of complex hysteretic nonlinearities with convex branches and symmetrical hysteresis loops. In another research Sjostroma and Visone [4] suggested a modified Prandtl-Ishlinskii model by integrating a nonlinear function in the classical Prandtl-Ishlinskii formulation to describe saturated hysteresis properties. In another approach Bashash and Jalili [14] proposed a modified play hysteresis operator in the Prandtl-Ishlinskii model to characterize hysteresis in a piezoceramic actuator. Brokate and Sperkels [6] and Visintin [9] proposed a nonlinear play operator which is applied to the classical Prandtl–Ishlinskii model to describe asymmetric hysteresis loops in addition to output saturation.

In [12] Al Janaideh et al applied an asymmetric generalized play hysteresis operator to the classical Prandtl-Ishlinskii model in conjunction with density function to characterize asymmetric hysteresis behavior in smart actuators. In contrast to the deadzone operators of Kuhnen modified model [3], which are applied together with the classical operators, the proposed generalized operator can be directly applied in conjunction with the Prandtl–Ishlinskii hysteresis model for characterizing symmetric as well as asymmetric hysteretic properties of smart actuators with output saturation. The results also showed that even though the classical model can characterize the symmetric hysteretic properties accurately, the proposed generalized Prandtl-Ishlinskii model revealed a relatively smaller error, in comparison to the classical model results, when is used for predicting the hysteretic behavior of piezoceramic actuators. In another paper [13], they modified the generalized model to ensure its continuity, and its validity in characterizing different hysteresis properties is briefly demonstrated by comparing the model responses with the measured data of magnetostrictive, SMA (Shape Memory Alloys) and piezoelectric micropositioning actuators. Since the output of the developed generalized Prandtl-Ishlinskii model strongly depends upon the shape of the envelope function and the density function, these functions should be chosen with respect to hysteresis behavior of the material. It means that output saturation as well as asymmetric hysteresis behavior is the critical point in the envelope function and the density function selection. After this stage the parameters of the generalized play operator, the envelope function, and the density function are obtained with optimization method in order to have minimum error with respect to some experimental data.

Unfortunately in all of these papers only the ability of the generalized Prandtl-Ishlinskii model in characterizing the hysteretic behavior of SMA actuators is demonstrated with respect to some specified experimental data and the accuracy of the developed model with respect to other data is not validated. In other words only the parameters of a selected Prandtl-Ishlinskii model are identified in order to adapt the model response to the real hysteretic nonlinearity with some specific experimental data and finally the outputs of the obtained generalized Prandtl-Ishlinskii are compared with those same data. Therefore, it is not clear whether the developed generalized Prandtl-Ishlinskii is capable in predicting hysteresis minor loops of SMA materials and how much its accuracy in this prediction task is. Some of these research shortcomings are due to the fact that the tested actuator was not available in the laboratory and therefore further experimental tests were not performable.

In this paper, first the generalized Prandtl-Ishlinskii model is trained by some experimental measured data obtained from an experimental test set-up consisting of a flexible beam actuated by a shape memory alloy wire. The parameters of the generalized Prandtl-Ishlinskii model are identified in order to adapt the model response to the real hysteretic nonlinearity. Then the accuracy of the obtained generalized Prandtl-Ishlinskii model with known parameters in predicting nonlinear hysteretic behavior of first order ascending curves and higher order minor loops, is validated.
with some other experimental data. Although the model has been trained with data of first order descending reversal curves, it has good power in behavior prediction of first order ascending curves as well as higher order minor loops.

2. Generalized Prandtl-Ishlinskii Model

The classical Prandtl-Ishlinskii (P-I) model uses the classical play (or stop) operator with a density function to characterize the hysteretic behavior of materials. Figure 1(a) illustrates the Input–output relation of the classical play hysteresis operator. This operator, characterized by the input \( u \) and the threshold \( r \) determining the width of the hysteresis operator, is a continuous rate-dependent operator which further details about it can be found in [6]. Assume that \( C_m[0,T] \) is the space of the piecewise monotone continuous functions and the input \( u(t) \in C_m[0,T] \) is monotone on each of the sub-intervals \([t_i, t_{i+1}]\), where \( 0 = t_0 < t_1 < t_2 < \cdots < t_N = T \). Then the output of the classical P-I model, \( y_{\text{classical}} \), can be obtained as:

\[
y_{\text{classical}}(t) = q u(t) + \int_0^t p(r) F_r[u](t)dr
\]

where in this equation \( q \) is a positive constant, \( p(r) \) is an integrable positive density function, \( r \) is the positive threshold as \( 0 = r_0 < r_1 < \cdots < r_{t_{i+1}} < \cdots < r_N = R \), and \( F_r[u] \) is the classical play hysteresis operator that is analytically expressed for \( r_i < t \leq r_{i+1} \) (i = 0,1,...,N – 1) as:

\[
F_r[u](t) = f_r(u(t),0) = w(0)
\]

where \( f_r(u,w) = \max\{u-r,\min(u+r,w)\} \).

In practical applications a finite number of hysteresis play operators are used to model hysteretic behavior, the output of the classical P-I model can also be expressed as:

\[
y_{\text{classical}}(t) = q u(t) + \sum_{i=1}^{N} p(r_i) F_{r_i}[u](t)\Delta r
\]

where \( \Delta r = (r_i - r_{i-1}) \) is a positive constant.

It should be mentioned here that in practical applications the values of \( q \) and density function \( p(r) \) is derived from experimental data of a specific actuators by solving the minimization problem in order to adapt the classical P-I model response to the real hysteretic nonlinearity.

Since the classical play hysteresis operator has a symmetric unbounded nature, the classical P-I model cannot characterize the behavior of systems with output saturation or asymmetric input-output loops. In order to eliminate these shortcomings, Brokate and Sperkels [6], and Visitin [9] have suggested an alternative generalized play operator, as a nonlinear play operator, for which the increases and decrease in input \( u \) yields to increase and decrease of the play operator output along the curves \( \gamma_1 \) and \( \gamma_r \), respectively.

The \( \gamma_1 \) and \( \gamma_r \) function, with constraint \( \gamma_1 < \gamma_r \), are continuous envelope functions over the input domain. According to (2) the output of the generalized play hysteresis operator is analytically expressed for \( t_i < t \leq t_{i+1} \) (i = 0,1,...,N – 1) as:

\[
S_r[u](t) = g_r(u(t),0) = z(0)
\]

where \( g_r(u,z) = \max\{\gamma_1(u) - r, \min(\gamma_r(u) + r, z)\} \).

Consequently, the output of the generalized P-I model, \( y_{\text{generalized}} \), can be expressed as [12]:

\[
y_{\text{generalized}}(t) = H[u(t)] + \int_0^t p(r) S_r[u](t)dr
\]

where \( H \) is a non-decreasing Lipschitz continuous function. In the case of a finite number of generalized hysteresis play operators, (5) would be expressed as:

\[
y_{\text{generalized}}(t) = H[u(t)] + \sum_{i=1}^{N} p(r_i) S_{r_i}[u](t)\Delta r
\]
identical envelope functions are selected (i.e. \( \gamma_r(u) = \gamma_r(u) = u \)) and in addition \( H[u] = qu \).

Since the output of the generalized P-I model strongly depends upon the shape of the envelope function and the density function, these functions should be chosen with respect to the material hysteresis behavior. For example if under increasing and decreasing the input, the input-output loops are symmetric, the identical envelope functions (i.e. \( \gamma_r(u) = \gamma_r(u) \)) should be selected. Also, in special cases when there is output saturations by increasing and decreasing the input (like SMA actuators), the hyperbolic tangent functions may be the best choice due to their continuity and bounded properties [12].

Al Janaideh et al. [12] have demonstrated that the generalized hysteresis model is capable in characterizing symmetric as well as asymmetric hysteresis properties of smart actuators with (or without) output saturation, like magnetostrictive, SMA (Shape Memory Alloys) and piezo actuators.

In the training process they identified the model parameters (like the envelope function and the density function parameters) with solving optimization problem to adapt the model response to the real hysteretic nonlinearity with some specific experimental data (training data). Finally, the generalized hysteresis model responses of these actuators are compared with the training data and the results showed that the developed generalized hysteresis model (with the selected envelope function, the density function and corresponding obtained parameters) have very good accuracy with respect to training data. Unfortunately, the validation process, in which the model response should be compared with other experimental measured data, was only limited to a piezo micro-positioning actuator that was available in the laboratory [13]. Since the SMA materials and actuators have much more nonlinear hysteretic behavior than piezo actuators and in addition they have output saturation in input-output hysteresis loop, predicting their hysteretic behavior by the generalized P-I, especially when there are some high order minor loops, seems be a more difficult task.

3. **Formulation of envelope function, density function and threshold function for SMA actuators**

As it is stated before and according to (4) and (6), the generalized P-I model output depends on the selection of envelope, density and threshold functions. Usually the shapes of these functions are defined on the basis of hysteresis loop of a particular system and considering that whether such system has asymmetric hysteresis loops and (or) output saturation or not. Also, the output of the mentioned functions strongly depends on the parameters of these functions. Therefore, these parameters need to be obtained on the basis of some experimental data of the actuator or system in order to gain this ability to correctly predict the behavior of such actuator or system.

Owing to the continuity, bonded and invertible property of the hyperbolic tangent function, Al Janaideh et al. [12] suggests choosing this function as an envelope function for the shape memory alloy (SMA) actuators. In addition, such functions can facilitate describing the output saturation property available in SMA actuators. Furthermore, if the input-output hysteresis loops of the actuator are asymmetric, according to (4), different envelope functions (i.e. \( \gamma_r(u) \neq \gamma_r(u) \)) should be selected. Therefore, in this research the following functions are chosen for the envelope functions of the generalized play operator:

\[
\begin{align*}
\gamma_r(u) &= P_1 \tanh(P_2 u + P_3) + P_4 \\
\gamma_r(u) &= P_5 \tanh(P_6 u + P_7) + P_8
\end{align*}
\]

Also, the following forms are selected for the density and threshold functions [15].

\[
\begin{align*}
p(r) &= P_9 e^{-P_{10}r} \\
r_j &= P_{11}j
\end{align*}
\]

Finally, the Lipchitz continuous function \( H(t) \) of the generalized P-I hysteresis model should be chosen based on the shape and form of the hysteresis loop of the actuators. For SMA actuators that have output saturation property, this function should be selected as following to guarantee satisfying mentioned property as well as asymmetric input-output hysteresis loops [12]:

\[
H(u) = \begin{cases} P_{12} \tanh(P_{13}u + P_{14}) + P_{15} & \text{if } u \geq 0 \\ P_{16} \tanh(P_{17}u + P_{18}) + P_{19} & \text{else} \end{cases}
\]

In order to use the generalized P-I hysteresis model for behavior prediction of a particular SMA actuators, first the abovementioned 19 constants, including \( P_1, P_2, \ldots, P_{19} \) should be identified using the measured input-output experimental data. In the current research, this process, called training process (or preprocessing process), is performed with the MATLAB optimization Toolbox, in order to have minimum error with respect to some experimental data. Also, the experimental data are collected from an experimental test set-up, consisting a flexible beam actuated by a shape memory alloy wire, and the details about this set-up will be explained in the following section.

4. **Experimental test set-up**

Figure 2 and 3 present a PC-based experimental test set-up and its associated instruments which are used to investigate the capability of the generalized Prandtl-Ishlinskii model in prediction of a flexible beam behavior under a SMA wire actuation. The main properties of the SMA wire and the cantilever aluminum beam respectively, are presented in Table I and II. This SMA actuator is made of Nitinol (Ni-Ti) alloy which has excellent electrical and mechanical properties, long fatigue life, and high corrosion resistance and due to these properties this material is used in many SMA actuators today [22]. In this experimental setup a Flexinol TM actuator wire, manufactured by Dynalloy Inc is used. This Ni–Ti SMA actuator wire is a one way high temperature (90 °C) shape memory wire with 0.01 inch diameter which some of its thermomechanical properties are presented in Table II, respectively. The SMA wire is placed horizontally (parallel to the beam neutral axis) with one end fixed to the end of the beam and the other end to the base of the beam.
Table 1. Parameters of Flexible Aluminum Beam

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Length</th>
<th>Thickness</th>
<th>Width</th>
<th>Young Module</th>
<th>Yield Stress</th>
<th>SMA Wire Offset Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>400 mm</td>
<td>1.27 mm</td>
<td>25 mm</td>
<td>70 GPa</td>
<td>410 MPa</td>
<td>10 mm</td>
</tr>
</tbody>
</table>

Table 2. Thermomechanical Parameters of SMA Wire Actuator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Martensite Finish Temperature</th>
<th>Martensite Start Temperature</th>
<th>Austenite Start Temperature</th>
<th>Austenite Finish Temperature</th>
<th>Maximum Recoverable Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>43.9 °C</td>
<td>48.4 °C</td>
<td>68 °C</td>
<td>73.75 °C</td>
<td>4.10 %</td>
</tr>
</tbody>
</table>

Figure 2. Schematic of the cantilever flexible beam set-up actuated by a SMA wire.

Figure 3. Experimental test set-up used for verification of the current analysis results.

Figure 4. Top view of the deformed beam after the wire actuation.
Since the available SMA actuator for this set-up has a moderate maximum recoverable strain (about 4%) and the purpose of constructing this set-up is achieving large deflection of the beam, the length of SMA wire is enlarged at the back of the beam base (the added length is 55cm) in such a way that the connection point of wire with the base of the beam does not change during cooling and heating process.

Since the tip of the beam does not move on a straight line after the SMA wire actuation, the tip of the beam is connected to a precise frictionless rectilinear displacement transducer (PZ12-A-125, GEFRAN Inc.,) while the other side of the transducer is joined to a high resolution rotary encoder (E50S series, Autonics Corporation). By measuring the length of the transducer and its angle, with respect to their initial quantities, the tip deflection of the beam can easily be computed. In addition, the output voltage of these sensors are fed to a computer-based data acquisition (not shown in Figure 22) using a AD/DA PCI multifunction card (PCI 1711, Advantech Inc.,) and Matlab Real Time Data Acquisition Toolbox (Matlab R2008a, Mathworks Ltd.,).

The activation electric voltage over the SMA wire is set by the computer generated voltage controlling a current amplifier which is capable of delivering up to 3 A current. Figure 4 shows the top view of the deformed structure after the heating process.

### 5. Identification and Validation Processes

The input voltage applied to the current amplifier of the SMA actuator in the training process is a slow decaying ramp signal and is shown in fig. 5. The rate of change of the input voltage is selected so small in order to allow the SMA temperature to stabilize, as in the steady state temperature will be determined by applied electrical current. In the training process of the generalized P-I model, 439 data set, consist of the major loop and 10 first order descending reversal curves attached to the major loop, is used. The switching values of these descending reversal curves are selected as: [2.4, 2, 1.8, 1.75, 1.7, 1.65, 1.6, 1.55, 1.5, 1.45, and 1.4] (volt). For switching values less than 1.4 (volt), the change in the beam deflection is not considerable. The experimental input-output hysteresis loops of the flexible beam with SMA wire actuator, under the abovementioned input voltage of the current amplifier is shown in fig 6. The 19 generalized P-I model parameters, identified by using MATLAB optimization Toolbox in order to minimize the error between the model output and experimental data, are tabulated in table III. Since, unlike the Preisach model, the Prandtl-Ishlinskii model has not exact output even for the training data, the output of the generalized P-I model in time domain under the voltage profile of fig .5 as the input, is compared with the experimental data in figure 7. This figure clearly shows that the generalized P-I model, with selected envelope, density and threshold functions and their corresponding parameters in table IV, can effectively characterized the behavior of the flexible beam structure with SMA wire actuation and there are only small differences for some data.

For evaluation and comparing the prediction of the beam end deflection by the generalized P-I model with respect to

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>P1</td>
<td>0.45</td>
</tr>
<tr>
<td>P2</td>
<td>4.10</td>
</tr>
<tr>
<td>P3</td>
<td>-</td>
</tr>
<tr>
<td>P4</td>
<td>0.49</td>
</tr>
<tr>
<td>P5</td>
<td>9.24</td>
</tr>
<tr>
<td>P6</td>
<td>-</td>
</tr>
<tr>
<td>P7</td>
<td>2.70</td>
</tr>
<tr>
<td>P8</td>
<td>10.0</td>
</tr>
<tr>
<td>P9</td>
<td>4.97</td>
</tr>
<tr>
<td>P10</td>
<td>7.8568</td>
</tr>
<tr>
<td>P11</td>
<td>6.89</td>
</tr>
<tr>
<td>P12</td>
<td>3.4759</td>
</tr>
<tr>
<td>P13</td>
<td>-</td>
</tr>
<tr>
<td>P14</td>
<td>11.5116</td>
</tr>
<tr>
<td>P15</td>
<td>16.8</td>
</tr>
<tr>
<td>P16</td>
<td>027</td>
</tr>
<tr>
<td>P17</td>
<td>3.95</td>
</tr>
<tr>
<td>P18</td>
<td>2.9923</td>
</tr>
<tr>
<td>P19</td>
<td>-</td>
</tr>
<tr>
<td>P20</td>
<td>5.09</td>
</tr>
<tr>
<td>P21</td>
<td>4.07</td>
</tr>
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</table>

Figure 5. The decaying ramp input voltage applied in the training process.
the experimental measured data, in the first validation process the voltage profile shown in Fig. 8 is applied to the current amplifier of SMA actuator. It should be mentioned that although the form of the voltage applied in this validation process looks like what was applied in the training process, the extremums are different. The switching values of these descending reversal curves attached to the major loop are selected as: [2.325, 1.975, 1.725, 1.675, 1.625, 1.575, 1.525, 1.475, and 1.425] (volt). The experimental input-output hysteresis loops of the flexible beam with SMA wire actuator, under the abovementioned input voltage of the current amplifier is also shown in fig 9. The experimental measured data of the

Figure 6. Experimental data of hysteresis behavior between the beam tip deflection and the input voltage of the current amplifier in the training process.

Figure 7. Comparison between the displacement prediction of the generalized Prandtl-Ishlinskii model and the measured data of the flexible smart beam in the time domain.

Figure 8. The decaying ramp input voltage applied in the first validation process.

Figure 9. Experimental data of hysteresis behavior between the beam tip deflection and the input voltage of the current amplifier in the first validation process.

Figure 10. Comparison between the displacement response of the generalized Prandtl-Ishlinskii model and the measured data of the flexible smart beam in the first validation process.

Figure 11. Time history of absolute error between the generalized Prandtl-Ishlinskii model and experimental measured displacement responses in the first validation process.
beam tip deflection is compared with the prediction of the
generalized P-I model in fig. 10. The effectiveness of the
generalized P-I model can be seen from the absolute error plot,
in the time domain, presented in fig 11. As it is clear from these figures, the generalized P-I model has good
 capability in predicting the beam behavior under the voltage actuations which are same as ones implemented in the
training process. In order to show this property more clearly, the maximum, mean and mean squared values of the absolute error are also presented in Table IV. Since the maximum deflection of the beam under the SMA wire actuation is 134.5 mm, the peak prediction error in this case is about 10.5% of the maximum output.

For better evaluation of the generalized P-I model in predicting the beam hysteresis behavior, in the second validation process the voltage profile shown in fig. 12 is selected as the input of SMA wire current amplifier. Unlike the first validation process, this form of actuation voltage leads to some first order reversal ascending curves attached to the descending part of major loop.

The experimental input-output hysterisis loops of the flexible beam with SMA wire actuator, under the abovementioned input voltage is shown in fig 13. The switching values of these ascending reversal curves are selected as: [0.3, 0.9, 1.1, 1.15, 1.2, 1.25, 1.3, 1.35, 1.4, 1.45 and 1.5] (volt). The experimental measured displacement, as a result of this actuation voltage, is compared with the prediction of the generalized P-I model in fig. 12. The absolute error response of the generalized P-I model with respect to the experimental measured data, in the time domain, is also presented in fig 13. The maximum, mean and mean squared values of the absolute error are also presented in Table V. The result clearly shows that in this validation process, in which the form of the actuation voltage differs from what is applied in the training process, the prediction of the generalized P-I model is weaker than the first validation process but acceptable. In this case the peak prediction error is about 16.3% of the maximum beam deflection.

Most of the phenomenological hysteresis models have difficulty in predicting higher order hysteresis minor loops and this cause to poor results when their inverse is implemented as a feedforward controller. For appraising the ability of generalized P-I model in these situations, in the third validation process a damped voltage profile shown in Fig. 16 is applied to the current amplifier of SMA actuator.
Table V. Error of the generalized Prandtl-Ishlinskii model in second validation process

<table>
<thead>
<tr>
<th></th>
<th>Mean of Absolute Error (mm)</th>
<th>Max of Absolute Error (mm)</th>
<th>Mean of Squared Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.0818</td>
<td>21.9253</td>
<td>21.1693</td>
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</tbody>
</table>

The prediction of the higher order hysteresis minor loops by the generalized P-I model is compared with the experimental measured data in figure 17. The absolute error response with respect to measured data, in time domain, is also presented in figure 18. The maximum, mean and mean square values of the absolute error are also presented in Table VI. The peak prediction error in this case is about 9.1% of the maximum output.

As it was expected and it is obvious from figure 18 and Table VII, the generalized P-I model has moderate accuracy in predicting the higher order hysteresis minor loops especially when, like this case, it has been only trained with some first order hysteresis reversal curves attached to the major loop. Nevertheless these results demonstrate that the ability of the generalized P-I model in behavior prediction of SMA actuated smart structures is reliable. Also, as it was mentioned before, since the exact inverse of this model is accessible and with respect to the results of this research, using the inverse of the generalized P-I model for real-time control applications of SMA actuated smart structures seems well-suited attractive.

Table VI. Error of the generalized Prandtl-Ishlinskii model in third validation process

<table>
<thead>
<tr>
<th></th>
<th>Mean of Absolute Error (mm)</th>
<th>Max of Absolute Error (mm)</th>
<th>Mean of Squared Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.4253</td>
<td>12.1803</td>
<td>18.4006</td>
</tr>
</tbody>
</table>

6. Conclusion

The first form of the Prandtl-Ishlinskii model, called the Classical Prandtl-Ishlinskii model, cannot describe systems with output saturation and also applying it results to considerable error when there is an asymmetric in the input-output hysteresis loops. In order to eliminate these shortcomings, some modifications are applied to the Classical Prandtl-Ishlinskii model and these models are called the modified or generalized Prandtl-Ishlinskii models.

Unfortunately in all of the researches performed in this area only the ability of the generalized Prandtl-Ishlinskii model in characterizing the hysteretic behavior of SMA actuators is demonstrated with respect to some specified experimental data and the accuracy of the developed model with respect to different data is not validated. Therefore, it is not clear whether the developed generalized Prandtl-Ishlinskii is capable in predicting hysteresis minor loops of SMA materials and how much its accuracy in this prediction task is. Some of these research shortcomings are due to the fact that the tested actuator was not available in the laboratory and therefore further experimental tests were not performable.

To demonstrate the ability of the generalized Prandtl-Ishlinskii model in predicting the SMA actuator behavior, in this paper the experimental data of a flexible beam actuated by a SMA actuator is used. The model has been trained with the data of some first order descending reversal curves attached to the major loop. The result shows that the generalized Prandtl-Ishlinskii model can effectively predict the saturated asymmetric input-output hysteresis loops of the SMA actuators. The maximum prediction error of the developed model in characterizing the first order ascending curves and higher order minor loops are, respectively, 16.3% and 9.1% of the maximum beam deflection. In order to achieve better results more data should be applied. According to result of this paper and due to inevitability the Prandtl-Ishlinskii model, it is a good candidate for feedforward
controller to compensate the hysteretic nonlinearity behavior of SMA actuated structures.

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